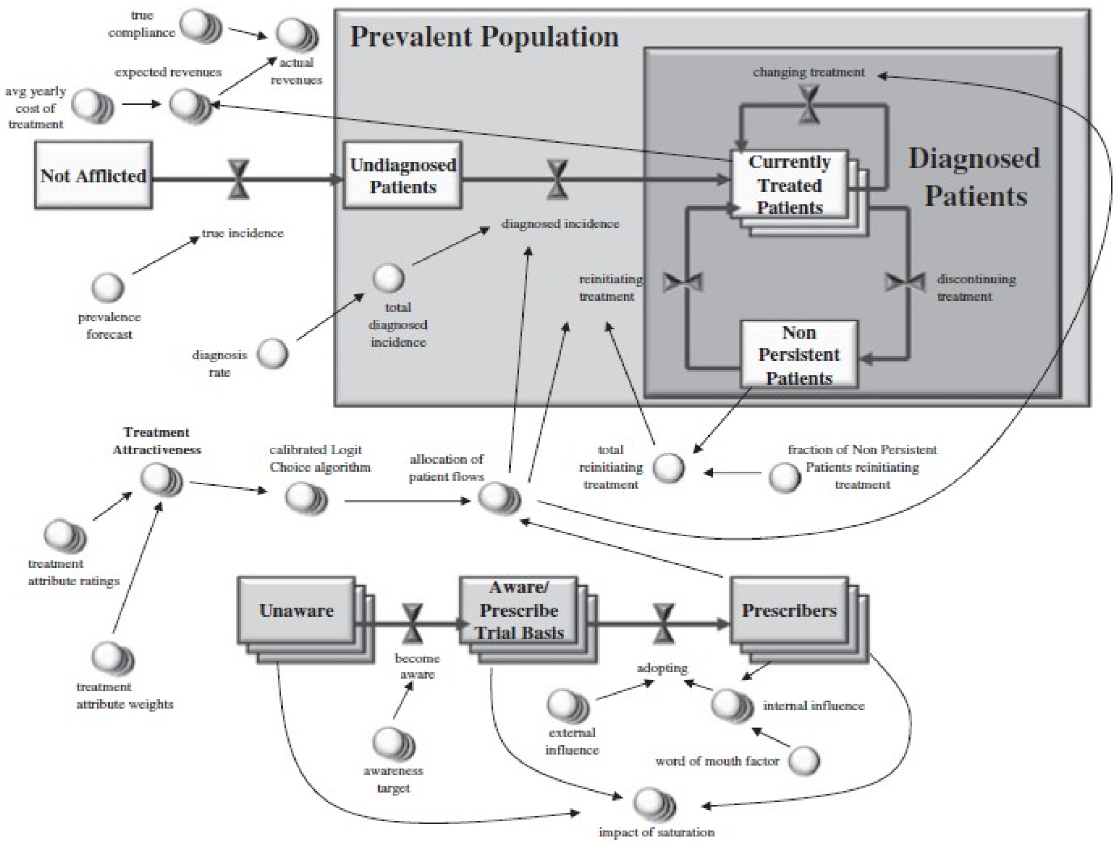
Lecture 9: LOTKE-VOLTERRA PREDATOR PREY MODELS AND SYSTEMS DYNAMICS



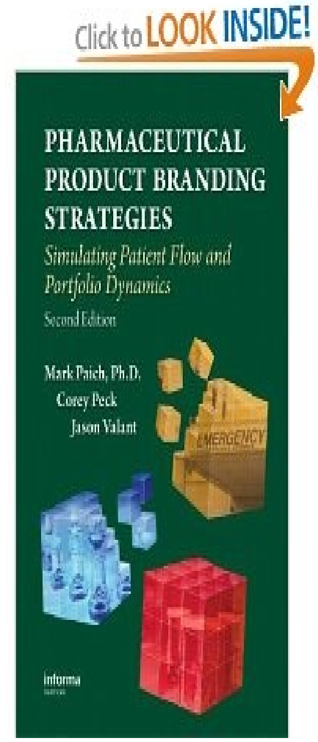
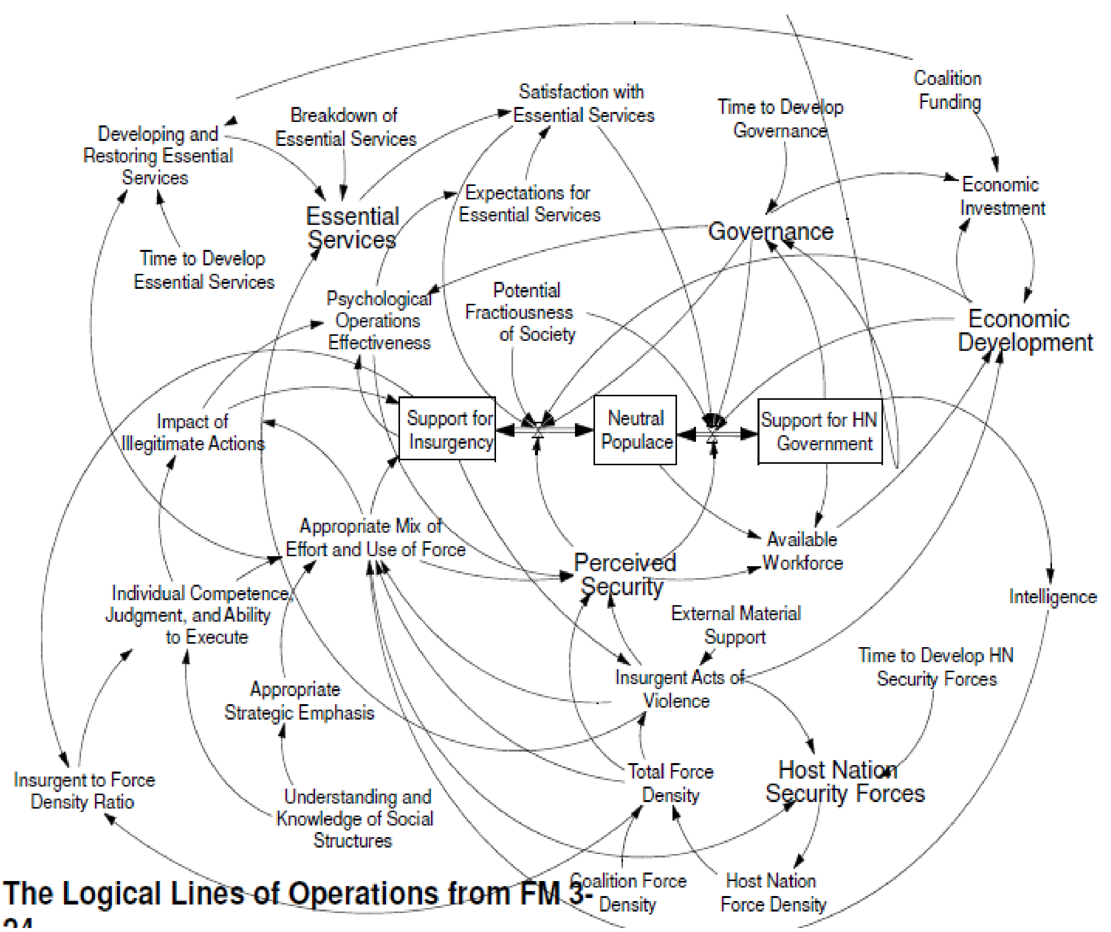
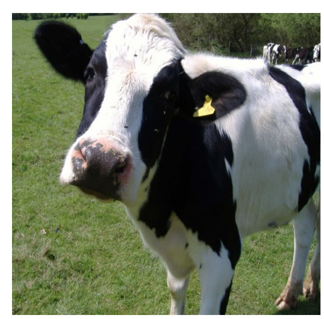


Fig. 6. Integrated model of three main components found in pharmaceutical markets (Reproduced from Paich





Systems dynamics is a tool for simulating links between agents by **differential/difference equations.** Systems dynamics models exploit non-linearities and feedback loops to capture the dynamic relations in the "actual process". There are a bunch of programs. Stella, http://www.iseesystems.com/ and Vensimhttp://www.vensim.com/; netlogo; Powersim (www.powersim.com/). For more see (http://en.wikipedia.org/wiki/List_of_system_dynamics_software)

Systems Dynamics was developed by MIT engineer Jay Forrester, inventor of flight simulators, who decided that social science was nonsense because it did not have a systems perspective and sought to cure the problem. He wrote Urban Dynamics, and World Dynamics. In 1960s the Club of Rome, businessfolk/intellectuals interested in the future of the world publicized Dennis Meadows' et al Limits of Growth book NYT columnist Antony Lewis called work "one of the most important documents of our age". http://en.wikipedia.org/wiki/Jay_Wright_Forrester .

How did the systems dynamicist determine their model of the world? Forrester: "... from intensive discussion with a group of people who know the system first hand". This lead to parameters and relations based on no verified information. In 1973 Bill Nordhaus critiqued Forrester's book " World Dynamics: Measurement without Data", noting that not a single relationship or variable was drawn from actual data/empirical studies. This might work if you know certain relations exist and you have plausible a priori bounds on parameters. But ...

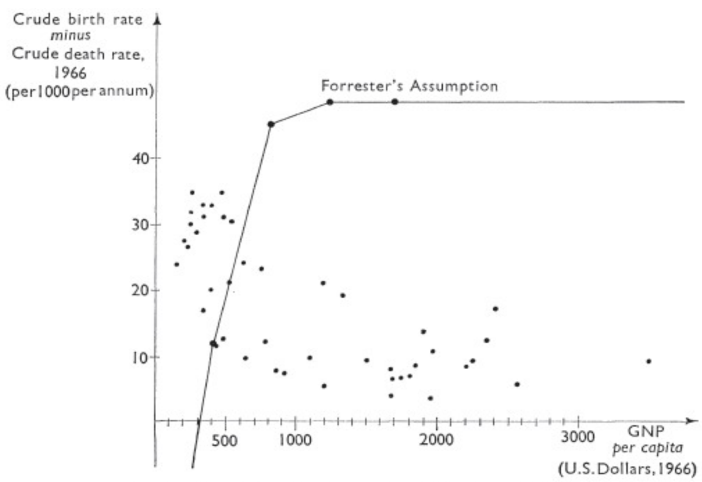


Fig. 2. Assumed and Cross Section Population Growth, 1966.
The line marked " Forrester's assumption " assumes that food and consumption rise in proportion with *per capita* G.N.P.; and that crowding and pollution are at 1970 levels.
Each of the unconnected dots represents one country: the sources of the data are the U.N. *Statistical Yearbook* and U.N. *National Accounts Statistics*, Vol. II.

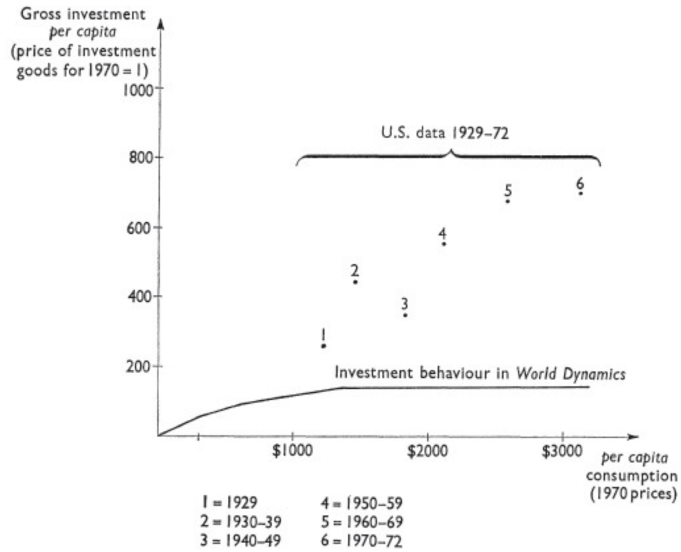


Fig. 5.
Solid line shows implicit investment behaviour in *World Dynamics*. Dots show actual investment behaviour for the U.S. for the period 1929–1972.

The prediction of the systems dynamicists was a Malthusian overpopulation disaster. In 1992 Meadows published a revision of his work entitled Limits of Growth Revisited. Nordhaus critiques this book in Lethal Model 2. Climate change models have similar structure but based on scientific evidence on relations and size of parameters.

There are two virtues to Systems Dynamics models: 1) they set up differential/difference equation models that forces you to think about the PROCESS underlying a problem; 2) they show that x → y flow diagrams are differential equation that turns hand waving "theories" into testable propositions depending on estimable parameter.

On the process side, systems dynamics mock regression modeling as LAUNDRY LISTS that lead to kitchen sink regressions that ultimately end up a descriptive static model that lives on correlations rather than THE PROCESS. The STELLA program has the following:

WHERE ARE THE COWS? "A prestigious economics journal contained an article which presented a model that was designed to forecast milk production in the US. By all statistical measures of validity the model was quite sound … Milk Production = a GNA + b Interest rates + … The equation. … assumes that ... Milk Production is a function of a set of macroeconomic variables … does not purport to represent how milk is *actually* produced … **for nowhere in the equation do we see any cows!**

Systems dynamics holds that the **structure** – shown in the nodes/arrows in flow diagram –is more informative than correlations or regressions. If you know the structure and can get reasonable estimate of parameters you can predict better and identify places to intervene. Causal model not data-mongering. Data-mining should drive a systems dynamicist mad. But in many problems the structure is unknown, opening the door to ideological choice of structures.

When does the structural approach work?

1. When the way the organization/operation is **put together** determines its behavior.

Example: The structure of AFL-CIO (leadership selected by union presidents) arguably determines how the organization behaves, so that changes in leadership – from Kirkland to Sweeney to Trumpke – produce only modest changes in AFL-CIO actions. Why? Because union presidents concerned about their own fiefdoms can block almost anything. US Senate filibuster creates a similar structure that makes it hard to deal with problems.

2. When the behavior fits a **generic process:** oscillations, exponential growth or decline for which we have a well-established model or pattern of feedback loops to describe data.

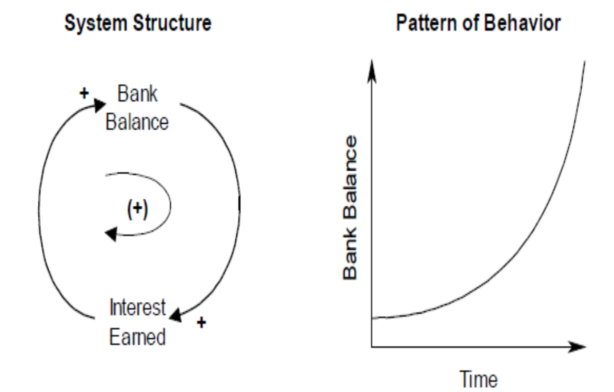


Figure 1.5 *Positive (reinforcing) feedback loop: Growth of bank balance*

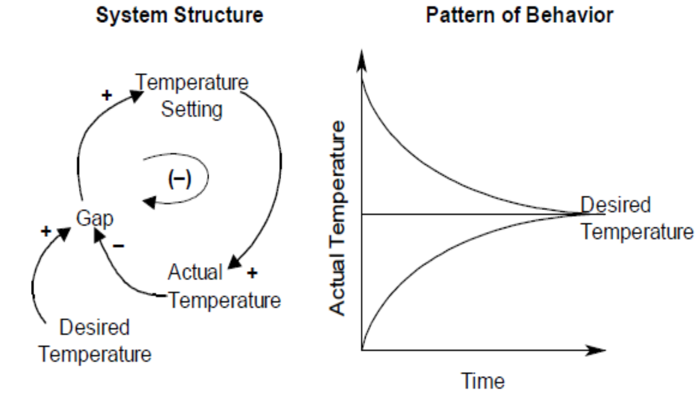


Figure 1.6 *Negative (balancing) feedback loop: Regulating an elective blanket*

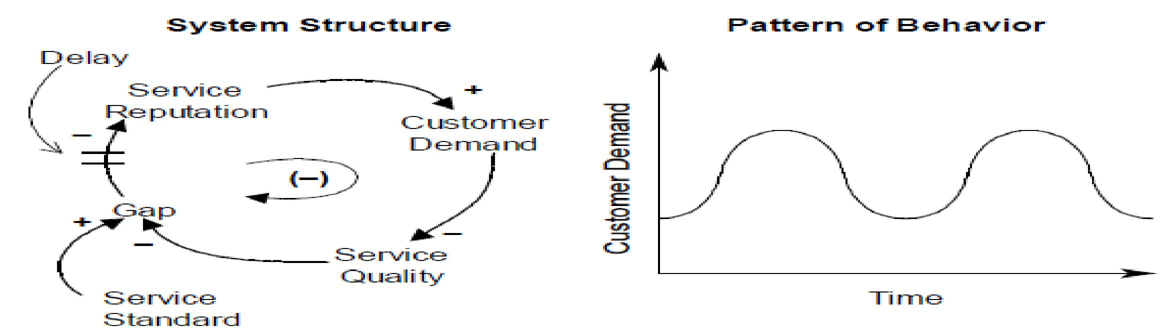
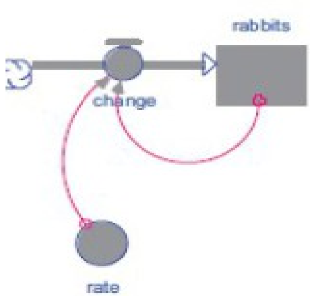


Figure 1.7 *Negative feedback loop with delay: Service quality*

KEY MODEL: BIRTH-DEATH MODEL: POP = POP(-1) + Births – Deaths,

Births = bPOP(-1), a positive feedback -- INFLOW = q STOCK where feedback is variable to itself.

Births → compound growth. POP = (1+b) POP(-1) or dPOP/POP(-1) = b, percentage growth of population due to births is an exponential growth relation.



"*Stella*" has several levels. By clicking on a down arrow you can move to the equation level to see the mathematics and structure of the model. For the population growth model above the structure is:

rabbits(t) = rabbits(t-dt) + (change)*dt
INT rabbits = 100 (a specification I choose for this example)
INFLOWS:
change = rate*rabbits

Deaths = -dPOP(-1). This is a draining process. OUTFLOW = d STOCK.

The birth-death process is a difference/differential equation: POP - POP(-1) = (b-d) POP(-1) so that Δln POP = (b-d) with an equilibrium when b=d. If b and d are unequal, the model delivers exponential growth or decline.

Since unlikely to get b=d from random draw, next step in model construction is to make these parameters vary with other things in the model. For example, assume that birth rate declines as the population grows: b = c - c'POP(-1). This gives a NON-LINEAR/second order difference equation: d POP = (c-d) POP(-1) - c'POP(-1)^2

Apply birth-death model to a different problem – Union growth/decline in US. Union membership U in period depends on past membership U(-1); organization of a new work site, which occurs at rate g; and lose of membership due to closure of union plants, which occurs at rate r. So we have **U-U(-1) = (g-r) U(-1).** But people usually focus on union density (UD), which is U divided by employment (E).

Then write the equation as U=U(-1) + (g-r) U(-1)). Divide both sides by E and we have UD = (1+g-r) U(-1)/E Now multiply the right hand side by E(-1)/E(-1) to get UD = (1+g-r) UD (-1) (E(-1)/E), where (E(-1)/E) ~ 1- e, the growth employment, so UD(-1) ~ g-r– e. (NB, this ignores employment changes at union plants that keep functioning.
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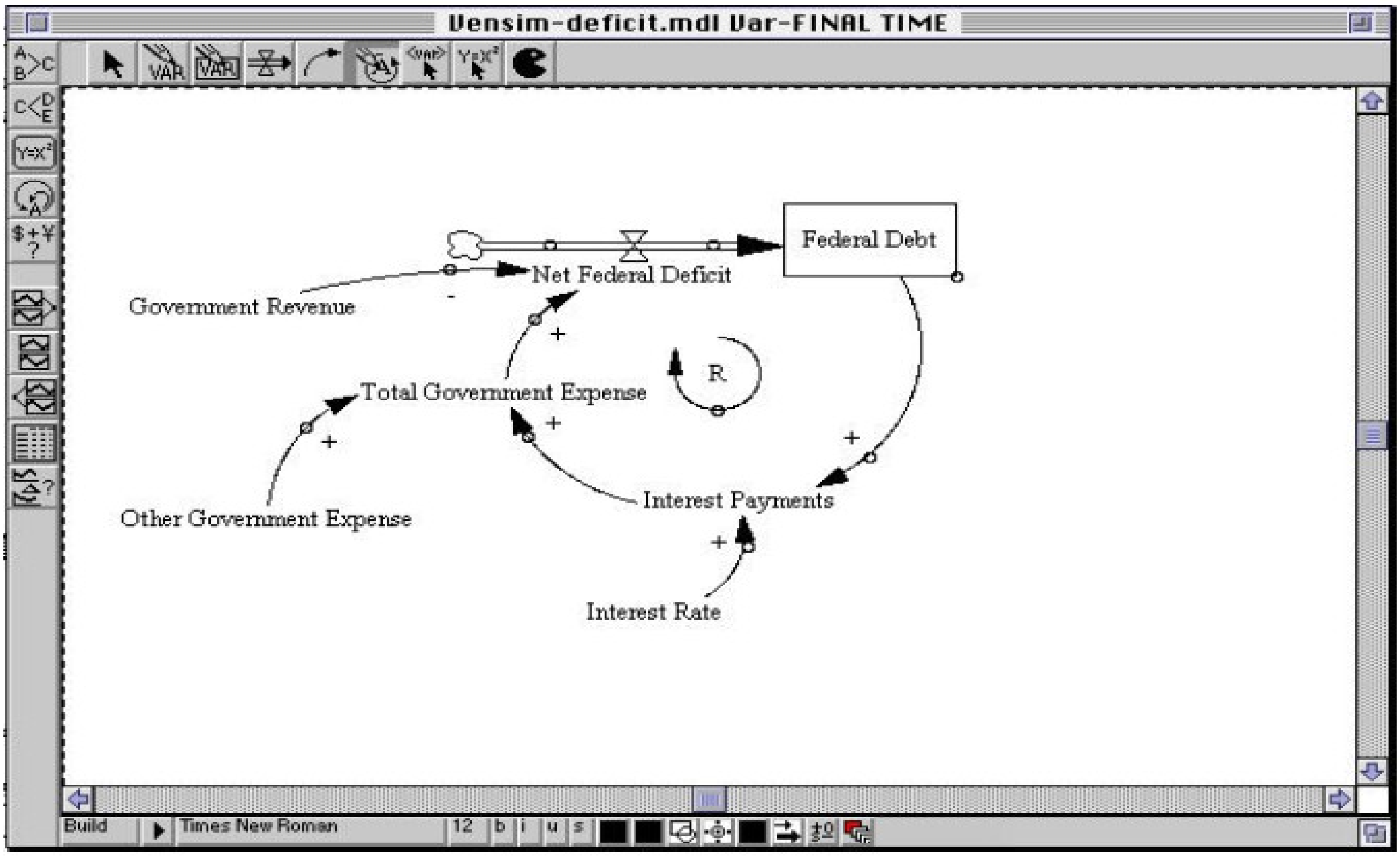
Government deficit and debt is similar. Debt is stock. Deficit is flow. But more meaningful ratio is Debt/GDP:

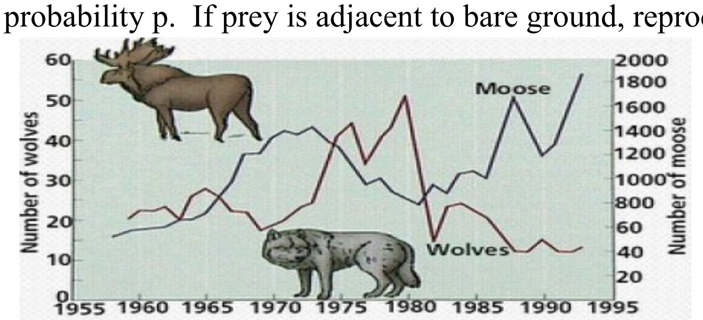


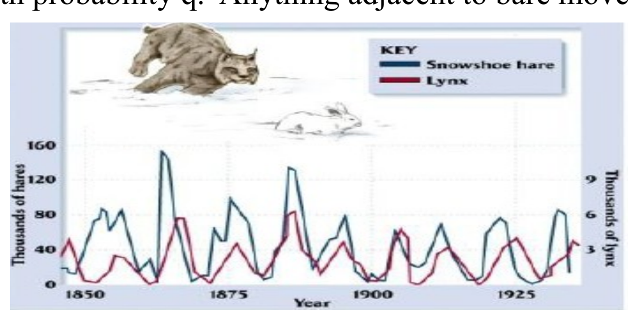
Model II: PREDATOR (y) PREY (x) model with two interacting stocks:

Prey: dX /dt = (a-bY) X = a X -b XY. Interpretation: a is birth rate and b as death rate from running into predator
or in growth rate form: dX/X = a-bY birth rate and then die from meeting predator

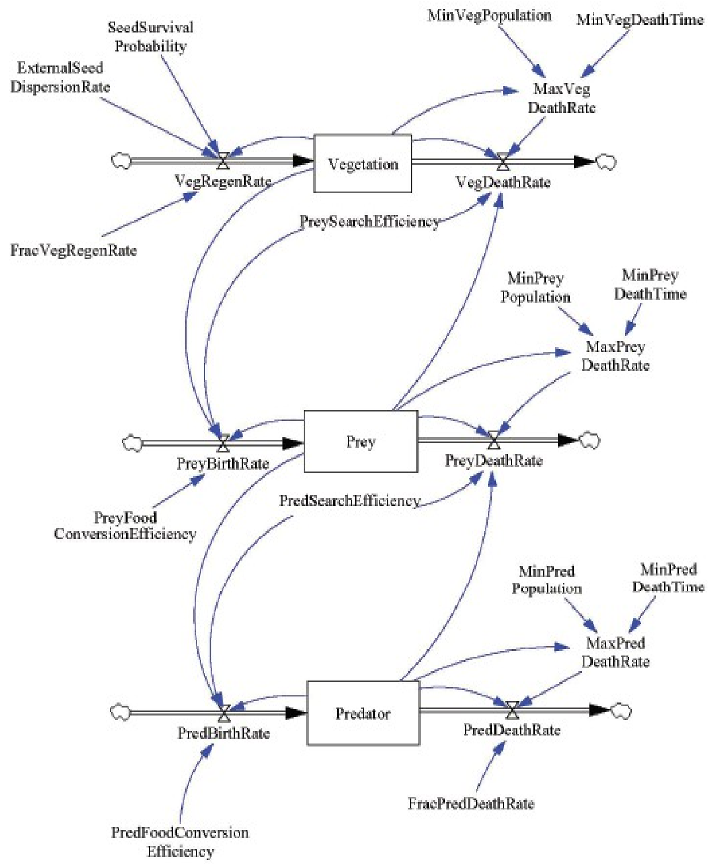
Predator: dY/dt = -(λ-uX)Y= - λY + uXY. Interpretation: λ is death rate and birth/growth comes from eating up prey
or in growth rate form: dY/Y = - λ + uX. Die unless you meet prey

Model can be represented by a cellular automata. Sites can be predator, prey, or empty. If a predator is adjacent to prey, the prey becomes predator with probability r or bare with 1-r. If no prey adjacent to predator, predator dies with probability p. If prey is adjacent to bare ground, reproduces with probability q. Anything adjacent to bare moves there.





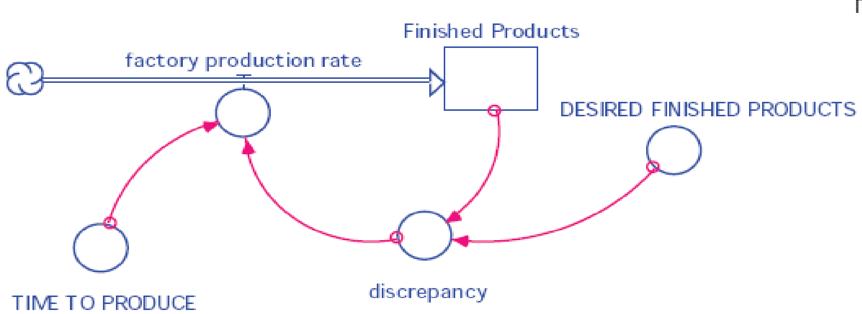
A Three stock version from Vensim (http://www.cleanmetrics.com/pages/WildlifeManagement-2.pdf)

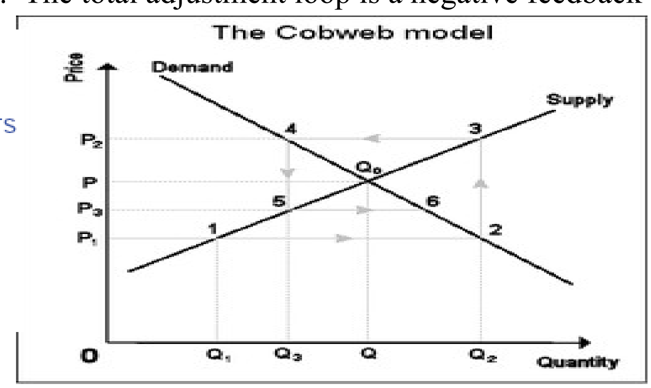


Vensim Model Equations
(15) Predator= INTEG (PredBirthRate-PredDeathRate, 10) Units: pred
(16) PredBirthRate= PredFoodConversionEfficiency * PredSearchEfficiency * Prey * Predator Units: pred/Month
(17) PredDeathRate= MIN(FracPredDeathRate * Predator, MaxPredDeathRate) Units: pred/Month
(18) PredFoodConversionEfficiency= 0.1 Units: pred/prey
(19) PredSearchEfficiency= 0.02 Units: 1/pred/Month
(20) Prey= INTEG (PreyBirthRate-PreyDeathRate, 500) Units: prey
(21) PreyBirthRate= PreyFoodConversionEfficiency * PreySearchEfficiency * Vegetation * Prey Units: prey/Month
(22) PreyDeathRate= MIN(PredSearchEfficiency * Predator * Prey, MaxPreyDeathRate) Units: prey/Month
(23) PreyFoodConversionEfficiency= 0.005 Units: prey/veg
(24) PreySearchEfficiency= 0.2 Units: 1/prey/Month
(25) SAVEPER = TIME STEP Units: Month [0,?]
(26) SeedSurvivalProbability= 0.25 Units: Dmnl
(27) TIME STEP = 1 Units: Month [0,?]
(28)VegDeathRate= MIN(PreySearchEfficiency * Prey * Vegetation, MaxVegDeathRate) Units: veg/Month

Model III: link between DESIRED and ACTUAL gives logistic approach to equilibrium:

Assume you are initially short of desired stock so discrepancy is positive. Then higher production reduces discrepancy –> Minus sign to that loop. If discrepancy falls, lower production rate –> positive sign to that loop. The process is one of adjusting to make discrepancy zero or close to zero. The total adjustment loop is a negative feedback





Model IV: Cobweb. Consider the supply of college graduates, CG, which depends on the salary of graduates relative to less educated person in the preceding year S(-1): **CG = A + a S(-1)**, where A is a shift term. Demand for graduates is **CGd = D -b S** , where D is shift term. This market equilibrates by supply = demand. So **D -b S** = **A + a S(-1).** Thus, S = D-A -a/b S(-1), which oscillates depending on the values of a/b. The assumption that CG depends solely on S(-1) can be modified in many ways to allow for more sophisticated forecasts of the future.

The time delay between deciding what field to study and graduating into the job market lies at the heart of the dynamics, producing oscillations. Minority Game expectations highlights the problem and shows that fluctuations tend to be modest around the equilibrium value. In global labor market, with huge differences in earnings between graduates in advanced and low income countries immigration becomes potential stabilizer.

Beyond Predator/Prey model.

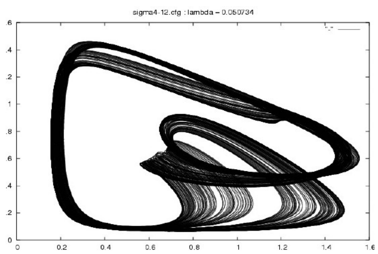
The solution to the general Lotke-Volera system produces oscillations in which large number of prey → large number of predators → small number of prey and so on (ecological oscillations that resemble the cobweb structure). But data show more complicated patterns that lead to modified models.

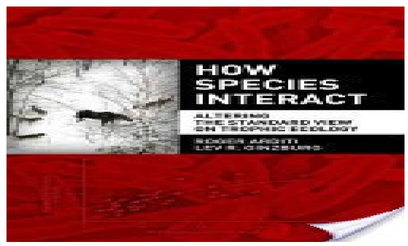
1) Ratios rather than absolute values as predator/prey size matters for own survival (Arditi & Ginsburg):

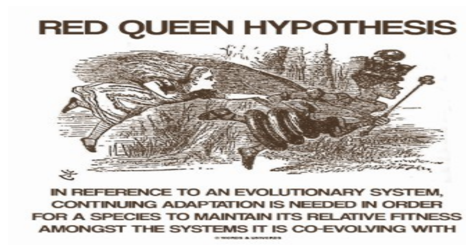
In L-V model prey death rate and predator rate of increase are proportional to the rate of feeding -- dY/Y = -(λ +uX) which as linear relation of number of prey. But predators they will be competing for prey. So the Lotka-Volterra is not "clearly" the correct form. Instead of uX how about uX/Y? This says that should do analysis in relative measures rather than absolute. Below g(X,Y) is interaction on the notion that more X and Y around more get eaten.

CREATURE	GENERAL EQUATION	LOTKE-VOLTERRA	ARDITI-GINZBURG
PREY dX/dt =	f(X)X – g(X, Y) Y	aX - b XY	AX - g(X/Y)Y
PREDATOR dY/dt =	u g(X, Y)Y – λY	u XY – λY	u g(X/Y)Y - λY

Arditi &Ginsburg argue it should depend on X/Y This is ratio-dependent model. Why? Chance of individual prey being caught should be lower when prey/predator is large and higher when prey/predator is small so constant b is not realistic. Issue of functional form and measures of b and u. Relatively easy to handle in systems dynamics where you can use lookup tables to add more non-linearity to equations.





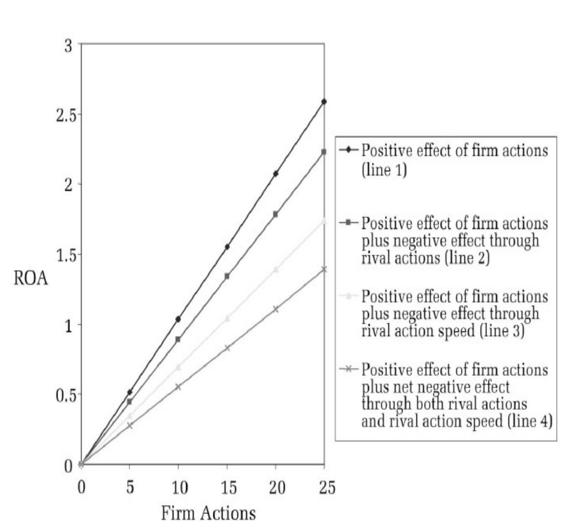


"Red Queen Dynamics" in Business/Economics

Missing element in Predator/Prey dynamics are changes in the policies/attributes of the predators/preys to improve their potential for survival – ie evolutionary pressures and outcomes. Red Queen adds to the predator-prey analysis Parasite-Host issues where evolution produces hosts more able to resist parasites, per the impending/current crisis with anti-biotics. Applied to economics Baumol (2004) writes: A "Red-Queen game" in which every player's success requires her to match or exceed the current efforts or expenditures of rival .. describes one of the most powerful economic mechanisms in economic development and in history … it underlies the exponential behavior of investment in innovation" If so how can we measure it?

"The Red Queen Effect: Competitive Actions and Firm Performance (Derfus et al Academy of Management Journal 2008, Vol. 51, No. 1, 61–80). The only way rival firms in competitive races can maintain performance relative to others is by taking actions of their own so they put together a data set of actions based on coding observable competitive actions by structured content analysis of newspaper and trade magazine articles on the Lexus-Nexus article index, by electronic searching of full-text articles. For industries chosen, they searched at least one industry trade magazine, NYT and WSJ. We identified 76,963 article citations. Coders then content-analyzed the full texts of articles yielding a database containing 4,474 actions. Results below:

Variables	Model 1: ROA		Model 2: ROA	
Lagged ROA	35.74**	(4.81)	35.73**	(4.82)
Lagged ROS				
Lagged rival ROS				
Firm sales	0.29†	(0.21)	0.30†	(0.24)
Quick ratio	1.84*	(0.75)	1.89*	(0.75)
Rival sales				
Industry quick ratio				
Market share rank	0.93	(1.27)	0.84	(1.27)
Herfindahl index	−5.52	(5.57)	−5.78	(5.59)
Industry growth	6.12*	(3.11)	5.06*	(3.06)
Firm total actions	0.08**	(0.03)	0.12**	(0.04)
Rival total actions	−0.05**	(0.01)		
Rival speed of actions			−17.02**	(5.21)

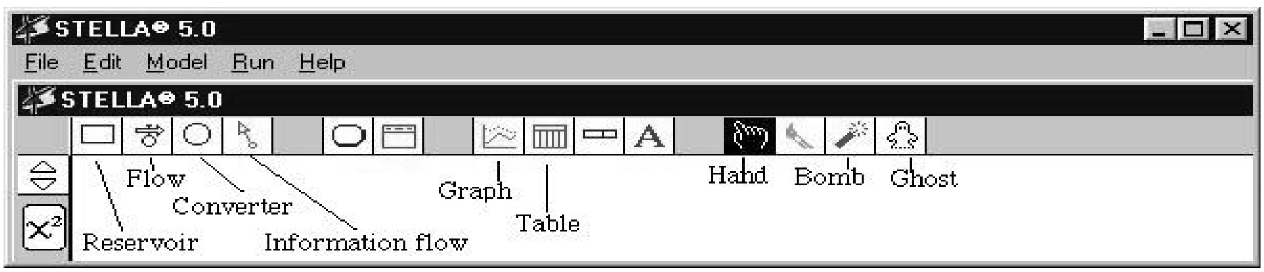


Building a Systems Dynamics Model

Systems dynamics uses arrows to represent difference equations. Because of feedbacks, an outcome will depend not simply on the immediate equation but on the entire system. The systems dynamics program imposes some internal consistency and a set of GENERIC PROCESSES that you can call on to yield a set of potential outcomes:

exponential growth (positive or negative feedbacks) ; stable equilibrium (sufficient negative feedbacks)
multiple equilibrium (some stable, some not); chaos (all you need is simple logistic)
oscillations – cobwebs or overshoot and collapse

STOCKS and FLOW are the essential building blocks.
A **Stock** is a BOX that is an accumulation. It is inert unless you have flows coming into it and out of it. It lasts from period to period. It can be qualitative -- esteem; thinking capacity; mental model of the world. It can be a price, if we think of prices as having some inertia. Stocks are slow-moving.

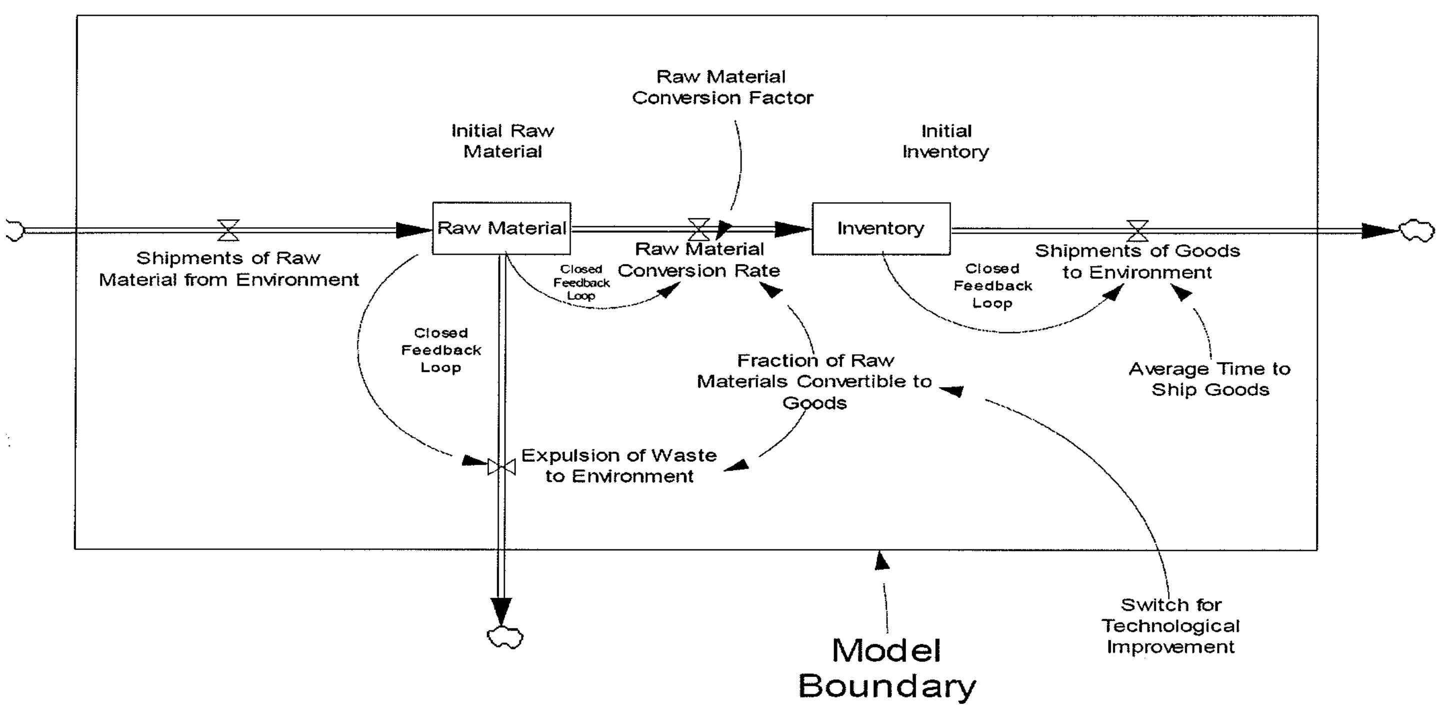


Some stocks decline with use -- inventory; consumables -- or with time due to a depreciation rate. Other stocks do not change or maybe even grow -- stock of useful knowledge – possibly acting as a catalyst that causes other things to happen but does not itself enter the process.

A **flow** (arrow in a stock- flow diagram) is a circle/valve that is attached to an arrow that leads to box that represents a stock It reflects a differential/difference equation that changes a stock. The flow can be in both directions -- biflow. The flow is influenced by stocks -- the change in population depends on the population and the birth rate.

Flows can allocate a fixed stock among various categories. In a model of a union/firm, there will be some resources/cash that will be allocated among different departments/functions, including for the firm dividends.

A flow begins with a cloud or can end in a cloud, meaning where it comes from or where it goes is outside the model. Here is a production function represented by a flow (compare to Y=F(K,L):



Link stocks through a process using flows. This directs attention at the units of the parts of the process, the key parameters, and opens the door for more subtle relations, so that, for instance, the flow may depend on multiple factors. Flows turn diagrams that might have a static flavor into dynamics.

The final element are **converters/auxiliary variables.** These are arrows that convert things, such as units, and make it easier to specify and later modify parameters. These will often take the form of ratios.

The models are based on "causal loops"

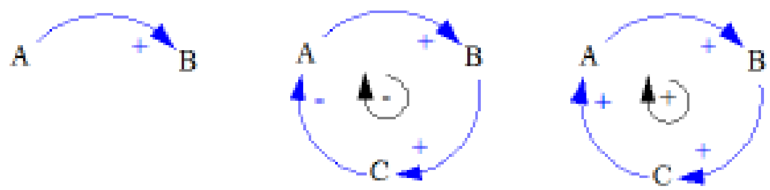


Figure 1.1: Causal links and feedback loops: a positive causal link (left hand side), a negative feedback loop (middle), and a positive feedback loop (right hand side)

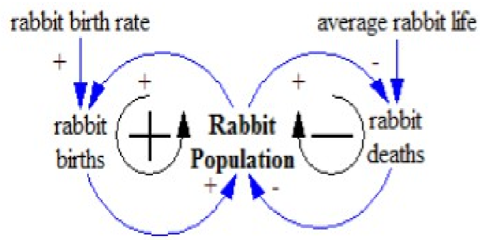


Figure 1.2: Example of a Causal Loop Diagram

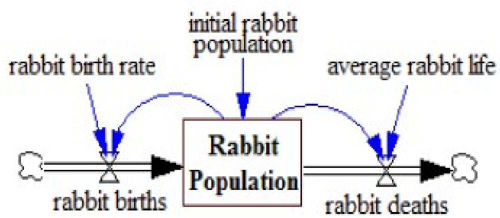


Figure 1.3: Example of a Stock-Flow Diagram

Systems Dynamics Research into whether people understand the causal links in dynamic predator/prey system.

The Rheindeer Herd Game: (B. Kopainsky and A. Sawicka: Simulator-Supported Descriptions of Complex Dynamic Problems Syst. Dyn. Rev. (2010)

You own a reindeer herd. Your task is to produce as much reindeer meat as possible each year. Note, however, you should make sure that your operation is sustainable. This means that you should aim for the highest possible sustainable slaughtering rate. You should also try to reach this desired state as quickly as possible. For your information, sustainable meat production will be maximized when the sustain able herd size is maximized. Thus your focus should be on the maximum sustainable herd size. Each year your only decision is to set the desired number of reindeer for the next year. You get only 15 years to reach the desired state, and no new trial. Do the best you can. The participant who gets the best results will receive a symbolic prize. … The limiting resource is lichen to support the reindeer throughout the winter. Lichen is a small plant growing on the ground. Biologically it is a combination of fungus and algae. The lichen plant grows in the summer time, growth stops in the winter, and then the plant continues to grow "on top of itself" the next summer, and so on. When there is very little lichen present, there is only little growth. When there is a lot of lichen, the net growth of lichen tends towards zero, what grows up is just compensating for what rots at the bottom of the plant. In between these extremes, net lichen growth reaches a maximum. When the reindeer graze, they eat the top of the plant, and the plant continues to grow on top of what is left.

GO TO IT. Restore the maximum sustainable herd size as quickly as possible.

The evidence from experiments is that people have great difficulty in understanding dynamics and managing dynamic systems. "Poor performance also seems robust in the case of subjects with relevant backgrounds. Most recent studies (Cronin et al., 2008; Booth Sweeney and Sterman, 2000) indicate that simple stock–flow problems are unintuitive and difficult even for people with a background in mathematics (including calculus). Similar misperceptions regarding dynamics have also been found among physics students, many of whom have difficulty with understanding the basic concepts in kinematics (Trowbridge and McDermott, 1980; McDermott et al., 1987), thermodynamics (Johnstone et al., 1977; Thomaz et al., 1995), or electricity (Chen and Kwen, 1998; Grotzer and Sudbury, 2000).