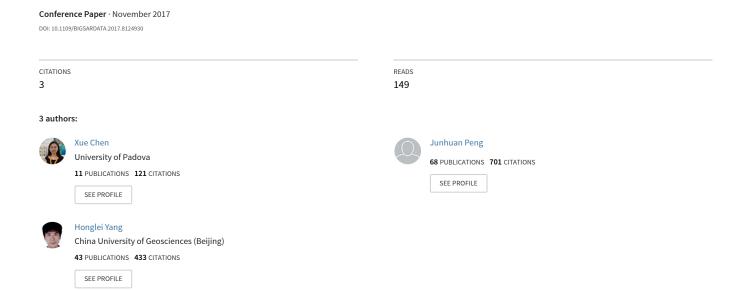
Orbital error modeling and analysis of spaceborne InSAR



ORBITAL ERROR MODELING AND ANALYSIS OF SPACEBORNE INSAR

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ABSTRACT

Spaceborne synthetic aperture radar interferometry is a key earth observation technique for large-area deformation monitoring. Measurements can be biased by error originating from different sources. Satellite orbital error affects interferograms in the form of an almost linear signal. The commonly used methods to mitigate the orbital error are to calculate the residual baseline error by the Fast Fourier Transform on the residual phase or to fit it with the quadratic surface. These two methods are applicable in most cases, but do not apply in the presence of a large-scale deformation signal that has similar spatial characteristics, such as tectonic movement or tides. Based on the current research on modeling and analysis of baseline error, this paper realized the orbital error network adjustment of Envisat ASAR data in Los Angeles area according to the principle of network adjustment. For quality control on the network level, iterative data snooping has been used, and the result proved its capability to detect and reject outliers.

Index Terms—Synthetic Aperture Radar Interferometry, Baseline Estimation, Orbital Error, Network Adjustment

1. INTRODUCTION

Synthetic Aperture Radar Interferometry (InSAR) is an earth observation technique to obtain the elevation data or surface deformation information base on two or more than two times imaging of a radar satellite in the same area. InSAR technique has played an important role in the study of surface deformation, ground subsidence, landslides, glaciers, volcanoes [1] and so on. Differential Synthetic Aperture Radar Interferometry (D-InSAR) can obtain the phase difference with high accuracy, which can't reflect deformation phase directly as it may be influenced by imprecise orbit state vector, atmospheric delay during electromagnetic wave propagation and Digital Elevation Model(DEM) error. Orbital error affects the accuracy of InSAR technique significantly. Firstly, small orbital error may propagate to interferograms causing obvious residual phase. Secondly, influence factors like atmospheric propagation delay phase, deformation phase, DEM error and

random error are inevitable. The time series InSAR technique such as Persisrent Scatterers InSAR [2][3][4] and Small Baseline Subsets InSAR [5] can detect linear and nonlinear deformation, and eliminate the effect of orbital error in theory, but in data processing, the influence of orbital error on phase unwrapping and elevation phase can't be ignored. Therefore, it is necessary to model and analyze the orbital error.

Since Graham [6] proposed the concept of InSAR 3D imaging in 1974, many countries and institutions had begun to study InSAR technique. Orbital error is considered to be the main limiting factor for long wavelength displacement measurement by InSAR technique [7].

Li and Goldstein [8] have proved that InSAR can be used to measure topography. Many methods have been taken to mitigate the orbital error. Some simple methods have been proposed such as fitting inteferometric phase with linear or two quadric surface [9][10], reestimating baseline, and calculating the fringes caused by orbital error. More accurate methods are making consistent estimation of phase fitting surface [9], or making compensation to orbits[11][12]. Zhang et al. [13] proposed a model for joint estimation of deformation rates and orbital error from multitemporal interferograms without unwrapping. Liu et al. [14] established a nonlinear error model for the InSAR baseline. Bähr and Hanssen [12] proposed reliable and rigorous geometric modeling of the orbital error signal, and systematic biases can be detected and iteratively eliminated.

2. METHODOLOGY

2.1. Baseline Error Estimation

Various method of estimating baseline error has been concluded by Bähr [15]. He has described the effect of an baseline error components yielding a distinct inteferometric signal (see Figure 1). Here, we choose the least square estimation to obtain baseline error [15][16].

Reference phase is caused by the difference between sensors and ground object [17]. The bias of range between master satellite and ground object and range between slave satellite and ground object is regarded as orbital error. Thus, orbital error can be expressed by the equation:

error can be expressed by the equation :
$$E\{\varphi\} = \delta\phi_{\rm orb} = -\delta\phi_{\rm ref} = \frac{4\pi}{\lambda} \left(\delta R_{\rm M,ref} - \delta R_{\rm S,ref}\right) + \varphi_0(1)$$

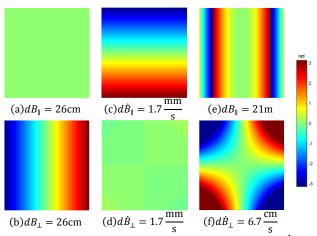


Figure 1 Baseline Error Components dB_{\perp} and \dot{B}_{\parallel}

 φ_0 is the relative bias constant. After a series of complex mathematical operations, considering N pixels, baseline error parameter $\mathbf{b}^T = (\delta B_h \ \delta B_h \ \delta B_v \ \delta B_v)$ can be estimated by the function model:

$$E\{\varphi\} = \begin{pmatrix} \vdots & \vdots & \vdots & \vdots \\ a_{h,i} & a_{h,i}t_i & a_{v,i} & a_{v,i}t_i \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix} \begin{pmatrix} \delta B_h \\ \delta \dot{B}_h \\ \delta B_v \\ \delta \dot{B}_v \end{pmatrix} + \begin{pmatrix} \varphi_0 \\ \varphi_0 \\ \varphi_0 \\ \varphi_0 \end{pmatrix}$$

$$=: \mathbf{A}_h \mathbf{b} + \mathbf{1} \varphi_0 \tag{2}$$

Stochastic model:

$$D\{\varphi\} = \sigma_0^2 \boldsymbol{Q}_{\varphi} \tag{3}$$

The equation is solved by least square estimation [15].

$$\widehat{\boldsymbol{b}}_{\theta} = \left(\boldsymbol{T} \overline{\boldsymbol{A}}_{b}^{T} \boldsymbol{Q}_{\omega}^{-1} \overline{\boldsymbol{A}}_{b} \boldsymbol{T}^{T}\right)^{-1} \boldsymbol{T} \overline{\boldsymbol{A}}_{b}^{T} \boldsymbol{Q}_{\omega}^{-1} \boldsymbol{\varphi} \tag{4}$$

$$D\{\widehat{\boldsymbol{b}}_{\theta}\} = \sigma_0^2 \boldsymbol{Q}_{\theta} = \sigma_0^2 (\boldsymbol{T} \overline{\boldsymbol{A}}_b^{\mathrm{T}} \boldsymbol{Q}_{\varphi}^{-1} \overline{\boldsymbol{A}}_b \boldsymbol{T}^{\mathrm{T}})^{-1}$$
 (5)

With

$$\boldsymbol{b}_{\theta}^{T} = \left(\delta \dot{B}_{\parallel} \ \delta B_{\perp}\right) \tag{6}.$$

$$\overline{A}_b = \left(I - \mathbf{1} (\mathbf{1}^T \mathbf{Q}_{\omega}^{-1} \mathbf{1})^{-1} \mathbf{1}^T \mathbf{Q}_{\omega}^{-1}\right) A_b \tag{7}.$$

$$\overline{A}_{b} = (I - \mathbf{1}(\mathbf{1}^{T} \boldsymbol{Q}_{\varphi}^{-1} \mathbf{1})^{-1} \mathbf{1}^{T} \boldsymbol{Q}_{\varphi}^{-1}) \boldsymbol{A}_{b} \qquad (7)$$

$$\boldsymbol{T} = \begin{pmatrix} 0 & \sin(\theta_{0}) & 0 & -\cos(\theta_{0}) \\ \cos(\theta_{0}) & 0 & \sin(\theta_{0}) & 0 \end{pmatrix} \qquad (8)$$

$$\theta_{0} := \frac{\overline{\theta}_{0} + \overline{\theta}_{1}}{2} \qquad (9)$$

$$\hat{\sigma}_{0}^{2} = \frac{\mathbf{v}_{\varphi}^{T} \boldsymbol{Q}_{\varphi}^{-1} \mathbf{v}_{\varphi}}{n_{\varphi} - u} \qquad (10)$$

$$\theta_0 := \frac{\theta_0 + \theta_1}{2} \tag{9}$$

$$\hat{\sigma}_0^2 = \frac{\mathbf{v}_{\varphi}^{\mathsf{T}} \mathbf{Q}_{\varphi}^{-1} \mathbf{v}_{\varphi}}{n_{\varphi} - u} \tag{10}$$

2.2. Network Adjustment

The sequential approach of network adjustment combines individual estimation of baseline error with a subsequent adjustment of misclosures between baseline Functional model [15]:

$$E\left\{ \begin{pmatrix} \vdots \\ \hat{\mathbf{b}}_{\theta,k} \end{pmatrix} \right\} = \begin{pmatrix} \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{0} & -\mathbf{I_2} & \mathbf{0} & \mathbf{I_2} & \mathbf{0} \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix} \begin{pmatrix} \vdots \\ \mathbf{x}_{s,i} \\ \vdots \\ \mathbf{x}_{s,j} \\ \vdots \end{pmatrix}$$
(11)

 I_2 is a 2×2 identity matrix.

Stochastic model:

$$D\{I_s\} = \varsigma_s^2 \boldsymbol{P}_s^{-1} = \varsigma_s^2 I_n \otimes \begin{pmatrix} \left(\delta \dot{B}_{\parallel,2\pi}\right)^2 & 0\\ 0 & \left(\delta \dot{B}_{\parallel,2\pi}\right)^2 \end{pmatrix}$$
 (12)

Where \otimes is the Kronecker product.

With a datum definition, two zero-mean conditions [15]:

$$\sum_{k=1}^{m} \hat{\mathbf{x}}_{s,k} = \sum_{k=1}^{m} \begin{pmatrix} \delta \hat{\hat{\mathbf{x}}}_{\parallel,s,k} \\ \delta \hat{\mathbf{x}}_{\perp,s,k} \end{pmatrix} = \mathbf{0}$$
 (13)

The predicted corrections:

$$\boldsymbol{v}_{s} = \boldsymbol{A}_{s} \widehat{\boldsymbol{x}}_{s} - \boldsymbol{l}_{s} \tag{14}$$

$oldsymbol{v}_{\scriptscriptstyle S} = oldsymbol{A}_{\scriptscriptstyle S} \widehat{oldsymbol{x}}_{\scriptscriptstyle S} - oldsymbol{l}_{\scriptscriptstyle S}$ 2.3. Quality Control

Baarda [18] data snooping has been widely used in adjustment. Assuming observations are normal distribution. $T_{\phi.i}$ follows a t-distribution [18][19].

$$T_{\phi,i} = \frac{v_{\phi,i}}{\overline{\sigma}_0^2 \sqrt{e_i^T \boldsymbol{Q}_{v_{\phi}} e_i}} \sim t_{n_{\phi} - u - 1}$$
 (15)

 $Q_{v_{\phi}}$ is the cofactor matrix of the corrections $v_{\phi} = (v_{\phi,i})$:

$$\boldsymbol{Q}_{v_{\phi}} = \boldsymbol{Q}_{\phi} - \overline{\boldsymbol{A}}_{b} \boldsymbol{T}^{\mathrm{T}} \boldsymbol{Q}_{\hat{x}\hat{x}} \boldsymbol{T} \overline{\boldsymbol{A}}_{b}^{\mathrm{T}}$$
 (16)

When $T_{\phi,i}$ exceeds a dedicated threshold, the *i*th observation is rejected. The procedure is repeated until all test statistics fall below the threshold.

3. EXPERIMENT AND RESULT

3.1. Research Area and Data

As Los Angeles area (see Figure 2) has dense GPS observations from Southern California Integrated GPS Network (see Figure 3), it has been used frequently to test InSAR techniques. Besides, this area has moderate moving and low image decorrelation [9][20][21][13][22]. Envisat satellite is equipped with ASAR and MERIS sensors to get SAR data and atmospheric water vapor data. We choose a set of m=24 Envisat acquisitions from a scence in Los Angeles (tack 170, frame 2925) between October 6, 2007 and September 25, 2010. Topographic height variations have been accounted for with a 3" DEM product from the Shuttle Radar Topography Mission (SRTM). All interferograms have been multilooked by a factor 2 in range and 10 in azimuth, yielding pixels of approximately 40m×40m.

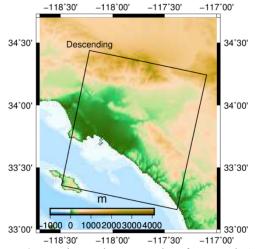


Figure 2 Location and Topography of Research Area



Figure 3 SCIGN

3.2. Data Processing

A network of n=99 interferograms has been set up with a maximum perpendicular baseline of 300 m and a maximum temporal baseline of 500 days. Selection of interferograms and network graph is showed in Figure4 and Figure5 respectively.

Images are sorted and numbered in chronological order. Then interferograms are combined with the expression of master and slave number (see Table 1 and Table 2).

Initial baselines are calculated through precise orbit state vectors. To prevent leverage effect, Ground Control Points (GCP) with high coherence (0.8~1) and uniform distribution has been selected (see Figure 6). Unwrapped differential phase of these GCPs has been used to reestimate baseline, we regard the difference of two baseline value as baseline error.

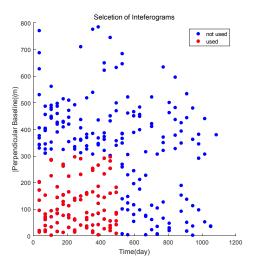


Figure 4 Selection of Interferograms

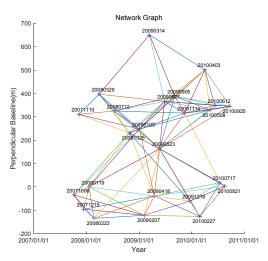


Figure 5 Network Graph

Table 1 Number of Image Time of Image Number of Image 20071006 2 20071110 3 20071215 20080119 4 5 20080223 20080329 6 7 20080712

24

20100925

Table 2 Inteferograms Combination	1
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Master Number	Slave Number						
1	3						
1	4						
1	5						
1	8						
1	9						
1	10						
2	6						
22	23						

3.3. Network Adjustment

Based on the methodology in section 2.2, the functional model of this experiment is:

Solving this equation, we get the estimated value of baseline components and orbital error:

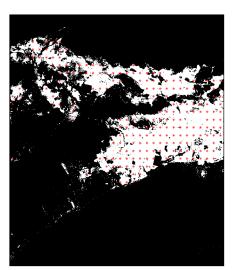


Figure 6 Coherence and GCP(red *)

$$\boldsymbol{b}_{\theta,k} = \begin{cases} 0.0106 \\ -0.0729 \\ 0.0042 \\ -0.1381 \\ 0.0303 \\ -0.1080 \\ 0.0102 \\ -0.2573 \\ \vdots \\ -0.0168 \\ 0.0490 \end{cases} \qquad \boldsymbol{x}_{s,i} = \begin{cases} 0.0183 \\ -0.1501 \\ -0.0073 \\ -0.1002 \\ 0.0012 \\ -0.0650 \\ 0.0085 \\ -0.0167 \\ \vdots \\ 0.0027 \\ 0.1181 \end{cases}$$

With data snooping, two values are detected and rejected (see Figure 7). Orbit errors are complementarily quantified in terms of the total number of fringes that they induce into an interferogram ($\delta \dot{B}_{\parallel,2\pi}=1.7$ mm/s, $\delta B_{\perp,2\pi}=26$ cm):

$$\delta \hat{B}_{fr} := \left| \frac{\delta \hat{B}_{\parallel}}{\delta \hat{B}_{\parallel,2\pi}} \right| + \left| \frac{\delta \hat{B}_{\perp}}{\delta B_{\perp,2\pi}} \right|$$

$$\delta \hat{x}_{fr} := \left| \frac{\delta \hat{x}_{\parallel}}{\delta \hat{B}_{\parallel,2\pi}} \right| + \left| \frac{\delta \hat{x}_{\perp}}{\delta B_{\perp,2\pi}} \right|$$

$$vB_{fr} := \left| \frac{v \hat{B}_{\parallel}}{\delta \hat{B}_{\parallel,2\pi}} \right| + \left| \frac{v B_{\perp}}{\delta B_{\perp,2\pi}} \right|$$

$$vB_{fr} := \left| \frac{v \hat{B}_{\parallel}}{\delta \hat{B}_{\parallel,2\pi}} \right| + \left| \frac{v B_{\perp}}{\delta B_{\perp,2\pi}} \right|$$

$$vB_{fr} := \left| \frac{v \hat{B}_{\parallel}}{\delta \hat{B}_{\parallel,2\pi}} \right| + \left| \frac{v B_{\perp}}{\delta B_{\perp,2\pi}} \right|$$

$$vB_{fr} := \left| \frac{v \hat{B}_{\parallel}}{\delta \hat{B}_{\parallel,2\pi}} \right| + \left| \frac{v B_{\perp}}{\delta B_{\perp,2\pi}} \right|$$

$$vB_{fr} := \left| \frac{v \hat{B}_{\parallel}}{\delta \hat{B}_{\parallel,2\pi}} \right| + \left| \frac{v B_{\perp}}{\delta B_{\perp,2\pi}} \right|$$

$$vB_{fr} := \left| \frac{v \hat{B}_{\parallel}}{\delta \hat{B}_{\parallel,2\pi}} \right| + \left| \frac{v B_{\perp}}{\delta B_{\perp,2\pi}} \right|$$

$$vB_{fr} := \left| \frac{v \hat{B}_{\parallel}}{\delta \hat{B}_{\parallel,2\pi}} \right| + \left| \frac{v B_{\perp}}{\delta B_{\perp,2\pi}} \right|$$

The result of network adjustment is showed in Table3.

Table 3 Result of Network Adjustment

	$d\hat{B}_{\parallel}$	$d\widehat{B}_{\perp}$	$\delta \widehat{B}_{\mathrm{fr}}$	$\delta\hat{\hat{x}}_{\parallel}$	$\delta \hat{x}_{\perp}$	$\delta \hat{x}_{ m fr}$	$v_{\dot{B}_\parallel}$	$v_{B_{\perp}}$	vB_{fr}
	mm/s	cm	fr.	mm/s	cm	fr.	mm/s	cm	fr.
Mel	0.15	-1.08	0.12	0	0	0	-0.22	0.12	0.13
Ma1	40.47	35.22	25.18	18.20	15.82	11.31	17.11	16.43	10.69
Me2	0.42	-0.95	0.28	0	0	0	-0.38	-0.14	0.23
Ma2	40.47	35.22	25.18	18.32	13.94	11.31	13.47	7.74	8.23

Here, Me1 and Ma1 represent the mean and max value before data snooping, and Me2 and Ma2 represent the mean and max value after data snooping. The max value is improved through data snooping. Orbital error before and after data snooping is illustrated in Figure 8, with rejected two interferograms (green line in Figure 8).

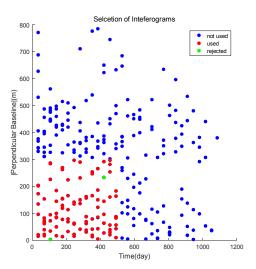


Figure 7 Rejected points

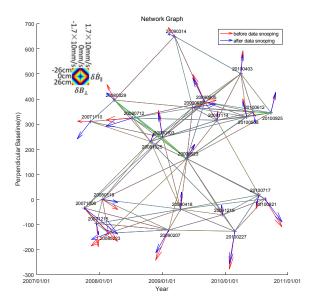


Figure 8 Orbital Error Before and After Data Snooping

4. CONCLUSIONS

This paper mainly focuses on the modeling and analysis of orbital error in spaceborne InSAR, least square and network adjustment are applied in calculating baseline error and orbital error. For quality control, iterative data snooping has proven its capability and reliability to detect and reject outliers. Envisat ASAR data in Los Angeles has the advantages in validating the experiment results. Some more works need doing to indicate the validity of modeling and analysis of orbital error in spaceborne InSAR.

5. ACKNOWLEDGEMENTS

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