Contents


```
% ce 710 hmk2
clear all
clc
close all
```

Variables

layer 1 (top) ... nl (to bottom)

```
theta = fliplr([0 45 -45 90 0].* pi/180);
thk = zeros(1,length(theta)) + 0.0025;
nl = length(thk);
a =   20;  % plate width;
b =   10;  % plate height
q0_ = 5.7; % plate load;
% Transversly isotropic material properties
Ell = 150e9;
Ett = 12.1e9;
vlt = 0.248;
Glt = 4.4e9;
vtt = 0.458;
Gtt = Ett / (2*(1+vtt));
% Failure Strengths
SLLt =  1500e6;
SLLc = -1250e6;
STTt =  50e6;
STTc = -200e6;
SLTs =  100e6;
Sxzs =  100e6;
Strength = [SLLt SLLc;
            STTt STTc;
            SLTs Sxzs];
```

Stiffness Matrix

```
syms th
% tranformation
Tij6 = [cos(th)^2 sin(th)^2 0 0 0 -sin(2*th);
        sin(th)^2 cos(th)^2 0 0 0  sin(2*th);
        0 0 1 0 0 0;
        0 0 0 cos(th) sin(th) 0;
        0 0 0 -sin(th) cos(th) 0;
```

```
       cos(th)*sin(th) -cos(th)*sin(th) 0 0 0 (cos(th)^2-sin(th)^2)];

Tij = [cos(th)^2 sin(th)^2 2*sin(th)*cos(th);
       sin(th)^2 cos(th)^2 -2*sin(th)*cos(th);
       -cos(th)*sin(th) sin(th)*cos(th) (cos(th)^2-sin(th)^2)];

% compliance matrix
Sij6 = [1/Ell -vlt/Ell -vlt/Ell 0 0 0;
       -vlt/Ell 1/Ett -vtt/Ett 0 0 0;
       -vlt/Ell -vtt/Ett 1/Ett 0 0 0;
       0 0 0 1/Gtt 0 0;
       0 0 0 0 1/Glt 0;
       0 0 0 0 0 1/Glt];

% Stiffnes matrix in material coordinates
Cijm6 = inv(Sij6);

% Stiffness matrix in Structural coordinates
Cij6 = Tij6*Cijm6*Tij6.';

% reduced stiffness in structural
Cij = [Cij6(1,1) Cij6(1,2) 0; Cij6(1,2) Cij6(2,2) 0; 0 0 Cij6(6,6)];
hlam = sum(thk);

% Create z dimensions of laminate
z_(1) = -hlam/2;
for i = 1:nl
   z_(i+1) =  z_(1) + sum(thk(1:i));
end
% extensional stiffness
Aij = zeros(6,6);
for i = 1:nl
    Aij = Aij + subs(Cij6,th,theta(i)) * (z_(i+1)-z_(i));
end
% coupling  stiffness
Bij = zeros(6,6);
for i = 1:nl
    Bij = Bij + 0.5* subs(Cij6,th,theta(i)) * (z_(i+1)^2-z_(i)^2);
end
% bending or flexural laminate stiffness relating moments to curvatures
Dij = zeros(6,6);
for i = 1:nl
    Dij = Dij + (1/3)* subs(Cij6,th,theta(i)) * (z_(i+1)^3-z_(i)^3);
end
```

Cylindrical Bending of a laminated plate

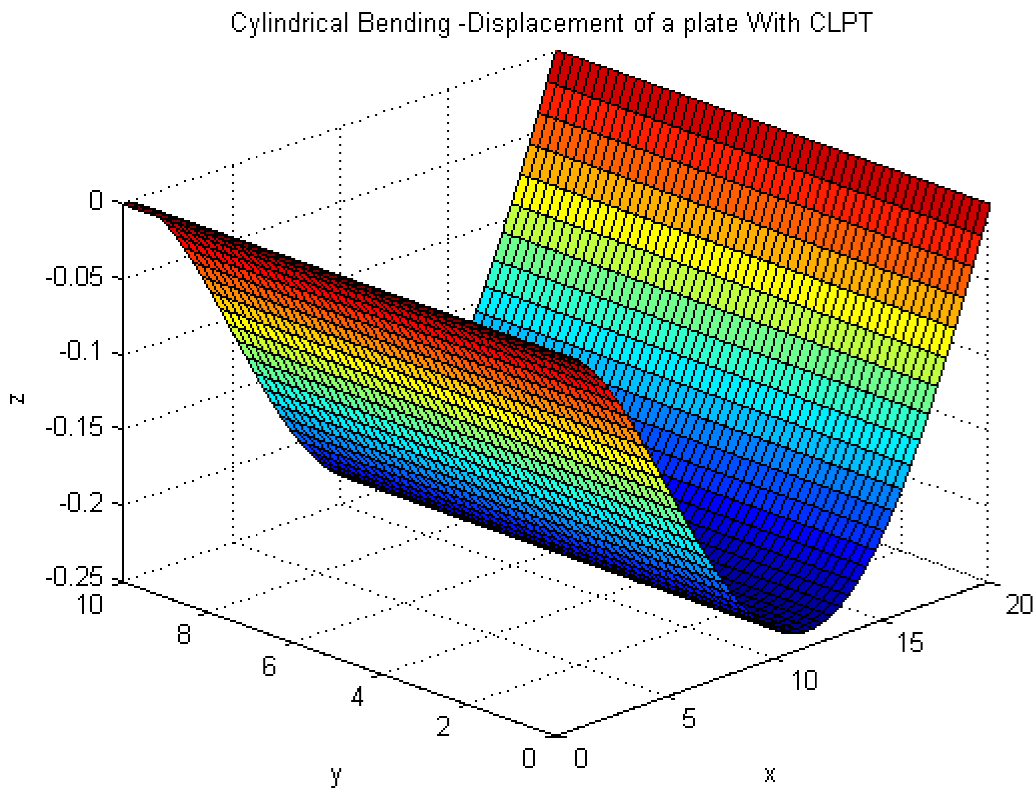
```
% displacement in w (z direction)
syms x y z q0 C1 C2 C3 C4 C5 C6 C7 A11 B11 D11 A16 B16


syms wfun ufun
% EQ 4.4.1a
eq1   = A11*diff(ufun,x,2) - B11*diff(wfun,x,3); % C5 C1
% EQ 4.4.1b
eq2   = A16*diff(ufun,x,2) - B16*diff(wfun,x,3); % C5 C1
% EQ 4.4.1c
eq3 = B11*diff(ufun,x,3) - D11*diff(wfun,x,4) + q0;
% solve eq1 eq2 and eq3 to get the w and u functions
```

```
% displacement in w (z direction) from eq1,eq2,eq3
wfun = A11*q0*x^4 / (4*(6*B11^2-6*A11*D11)) + C1 + C2*x + C3*x^2 + C4*x^3; %  C1 C2 C3 C4
% displacement in u (x direction) from eq1,eq2,eq3
ufun = B11*q0*x^3 / (6*(B11^2-A11*D11)) + C7 + x*C6 + 3*B11*x^2*C5/A11 ;% C5 C6 C7

% cond1 -> w(0)=0 at x(0), roller
C1sol = solve(subs(wfun, x, 0)==0,C1); % = 0
% cond2 -> angle at dw/dx at x(0) is 0, cantilever
C2sol = solve(subs(diff(wfun,x),x ,0),C2);  % =  0
% cond3 -> w(z) = 0 at x(a), roller
C4sol1 =  solve(subs(wfun,[x C1 C2],[a C1sol C2sol ]),C4); % C3
% cond4 u = 0 at x = 0
C7sol = solve(subs(ufun,x,0),C7); %=0
% u=0 at x = a
C5sol1 = solve(subs(ufun,[x C7],[a C7sol]),C5); %C6
% cond 5 EQ 4.4.14a Myy = 0 @ x(a) (Mxx , B11 D11) (Myy, B12 D12) roller no moment
C6sol1 = solve(subs(  [B11*(diff(ufun,x)+0.5*diff(wfun,x)^2 ) - D11*diff(wfun,x,2)]  ,...
               [x C1    C2    C4     C5     C7],...
               [a C1sol C2sol C4sol1 C5sol1 C7sol]),C6); % C6 C3
% EQ 4.4.13a, Nxx = 0 @ x(0) roller has no Nxx
C6sol2 = solve(subs([A11* (diff(ufun,x) +0.5*diff(wfun,x)^2)-B11*diff(wfun,x,2)],...
    [x C1 C2 C4 C5 C7],[a C1sol C2sol C4sol1 C5sol1 C7sol]),C6);% C6 C3
C3sol = solve(C6sol1 == C6sol2,C3);
C4sol = subs(C4sol1,C3,C3sol);
C6sol = simplify(subs(C6sol2,C3,C3sol));
C5sol = simplify(subs(C5sol1,C6,C6sol));
% substitute integration constants with actual values( _ is actual number)
C1_ = C1sol;
C2_ = C2sol;
C7_ = C7sol;
C3_ = subs(C3sol,[q0 A11 B11 D11],[q0_ Aij(1,1) Bij(1,1) Dij(1,1)]);
C4_ = subs(C4sol,[q0 A11 B11 D11],[q0_ Aij(1,1) Bij(1,1) Dij(1,1)]);
C5_ = subs(C5sol,[q0 A11 B11 D11],[q0_ Aij(1,1) Bij(1,1) Dij(1,1)]);
C6_ = subs(C6sol,[q0 A11 B11 D11],[q0_ Aij(1,1) Bij(1,1) Dij(1,1)]);

% function w(x) vertical displacement w along z with actual vaules
wsol = subs(wfun,[q0  C1  C2  C3  C4  A11      B11      D11],...
                 [q0_ C1_ C2_ C3_ C4_ Aij(1,1) Bij(1,1) Dij(1,1)]);
% function u(x) horizontal displacement u along x with actual vaules
usol = subs(ufun,[q0  C5  C6  C7  A11      B11      D11],...
                 [q0_ C5_ C6_ C7_ Aij(1,1) Bij(1,1) Dij(1,1)]);
ezsurf(x,y,wsol,[0,a,0,b])
view(-45,30)
xlabel('x')
ylabel('y')
zlabel('z')
title('Cylindrical Bending -Displacement of a plate With CLPT')
wsol_opt = matlabFunction(wsol);
[xmax,wmax] = fminsearch(wsol_opt,0);
```



Strain calculation

eq 3.3.8 (pg 116 reddy (pdf = 138))

```
epstotal = [diff(usol,x) + 0.5* diff(wsol,x)^2 - z*diff(wsol,x,2),0,0].';
epsx = epstotal(1);
```

Calculating and plotting Stress in each layer

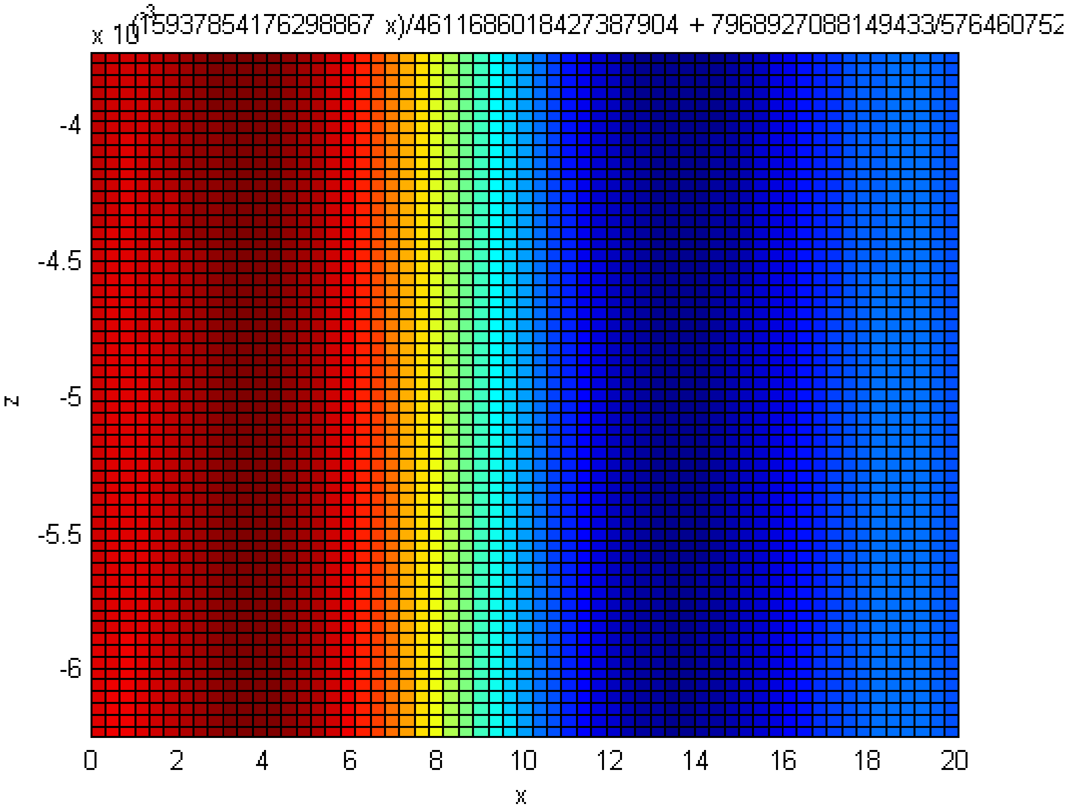
```
res = 8; % accuracy of finding max and min stress
xplot = linspace(0,a,res);
yplot = linspace(0,b,res);
for kstress = 1:3 % stress state s_x, s_y, s_xz
    figure(kstress+1)
    hold on
    for klay = 1:nl % loop through all layers
```

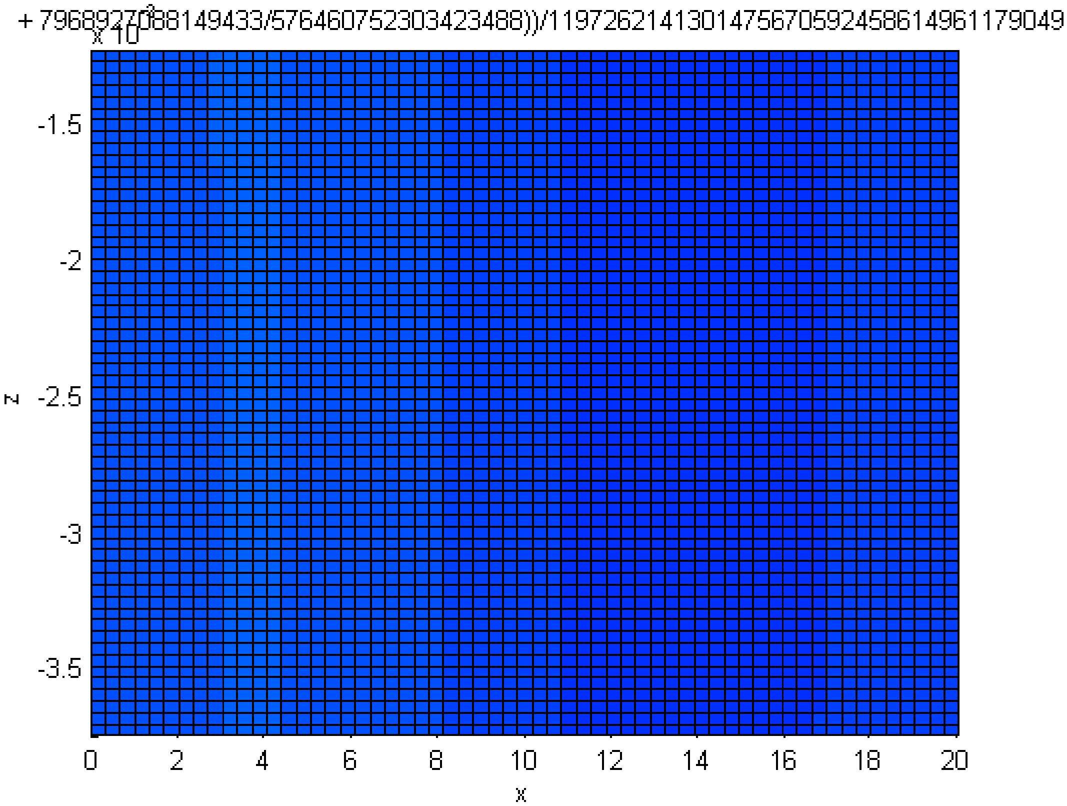
```
        thplot = theta(klay);
        zplot = linspace(z_(klay),z_(klay+1),res);
```

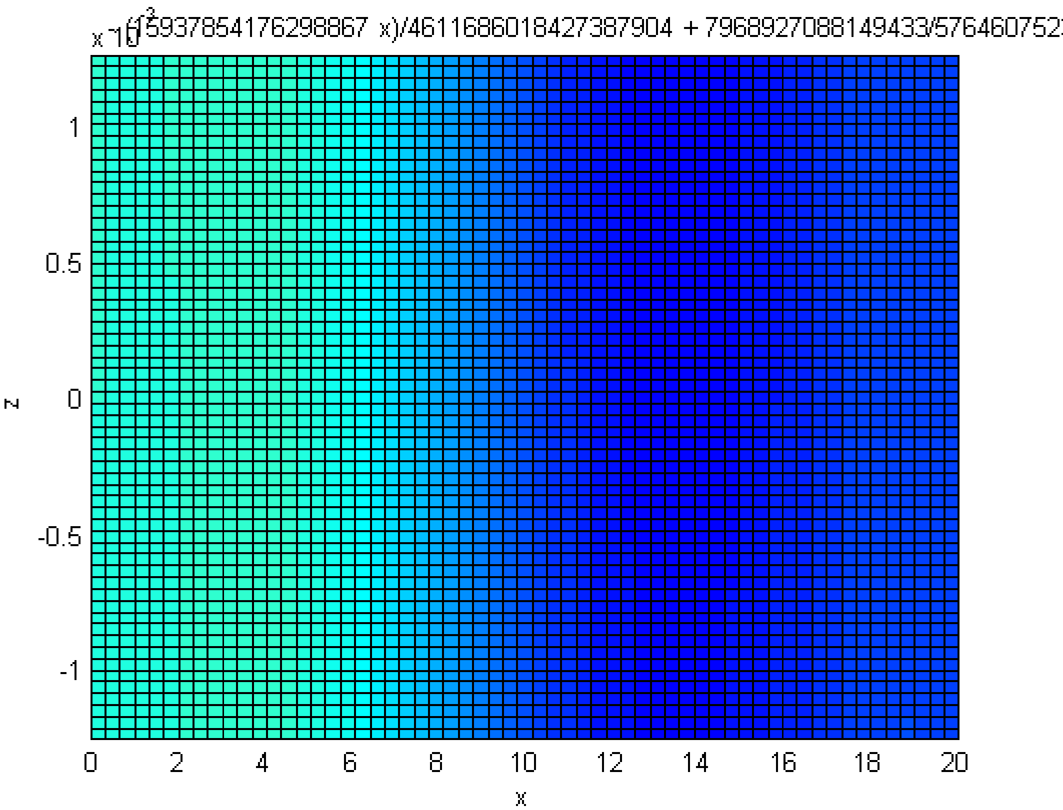
Calc Stresses

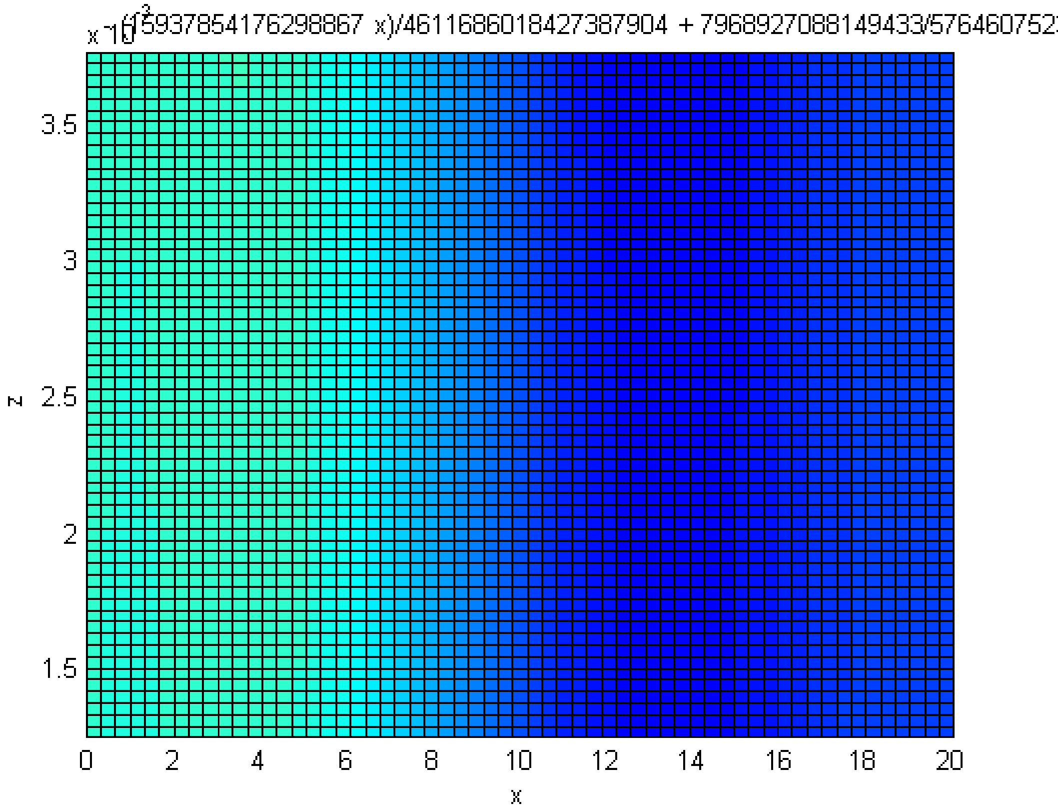
```
        if kstress == 3
            % Shear stresses
            syms G0
            G0_ = -int(diff(s_stress(1),x),z)+G0.';
            % solve for shear stresses from s_1
            s_xz = solve(G0_,G0);
```

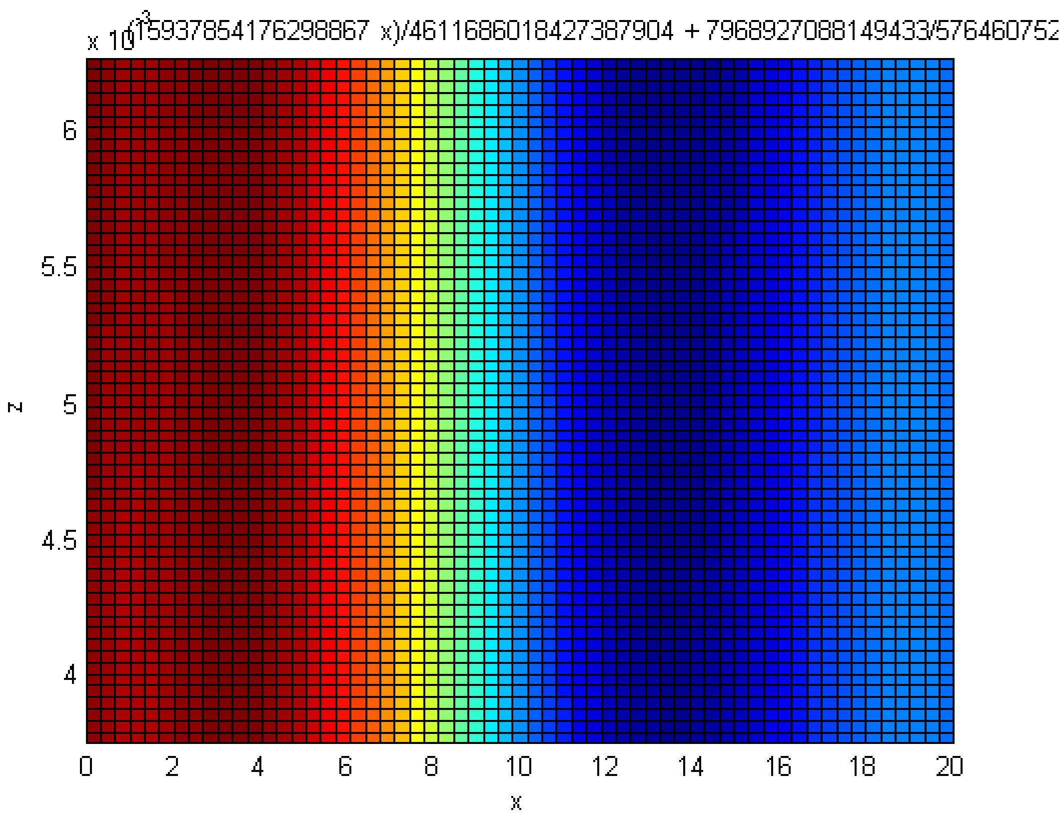
```
            % out of plane shear S_xz does not need to be transformed ??
            ezsurf(s_xz, [0, a, z_(klay), z_(klay+1)])
        else
            % normal stresses
            % Cij = reduced structural stiffness in strictural coordinates 3x3
            % stress in structural coordinates
            s_stress = subs(Cij,th,thplot)*epstotal;
            % stressin material coordinates
            m_stress = subs(Tij,th,thplot)*s_stress  ;
            ezsurf(m_stress(kstress),[0,a,z_(klay),z_(klay+1)])
        end
```




+ 7968927088149433/576460752303423488))/1197262141301475670592458614961179049
x 10^-3
-1.5
-2
-2.5
-3
-3.5
z
0
2
4
6
8
10
12
14
16
18
20
x


((15937854176298867 x)/4611686018427387904 + 7968927088149433/576460752
x 10^-3
1
0.5
0
-0.5
-1
z
0
2
4
6
8
10
12
14
16
18
20
x


x 10^-3
(15937854176298867 x)/4611686018427387904 + 7968927088149433/57646075Z
z
x
3.5
3
2.5
2
1.5
0 2 4 6 8 10 12 14 16 18 20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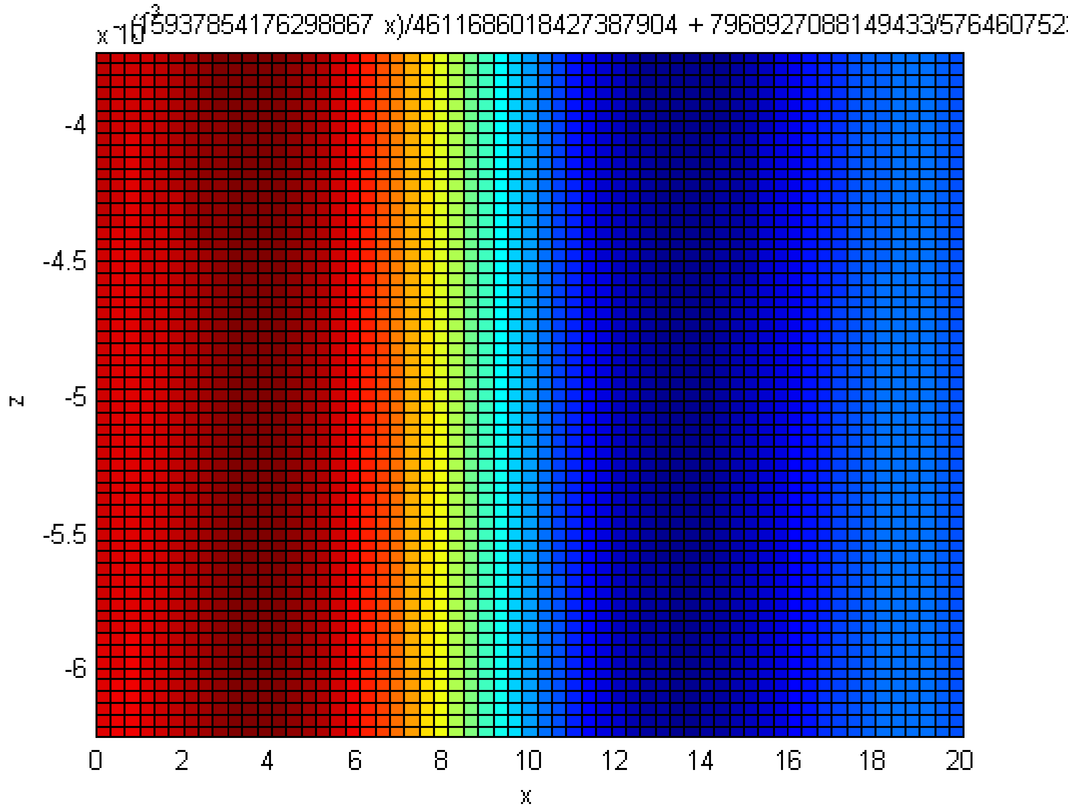
z
x
6
5.5
5
4.5
4
0 2 4 6 8 10 12 14 16 18 20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x 10^-3
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-4.5
-5
-5.5
-6
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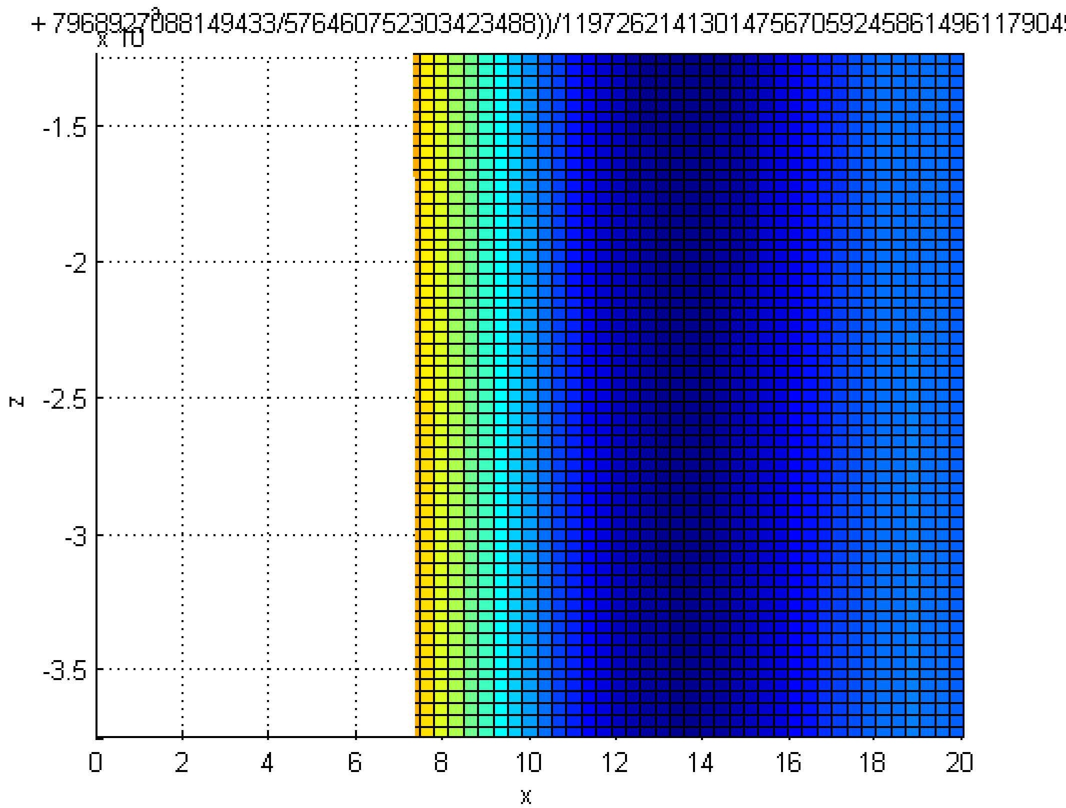
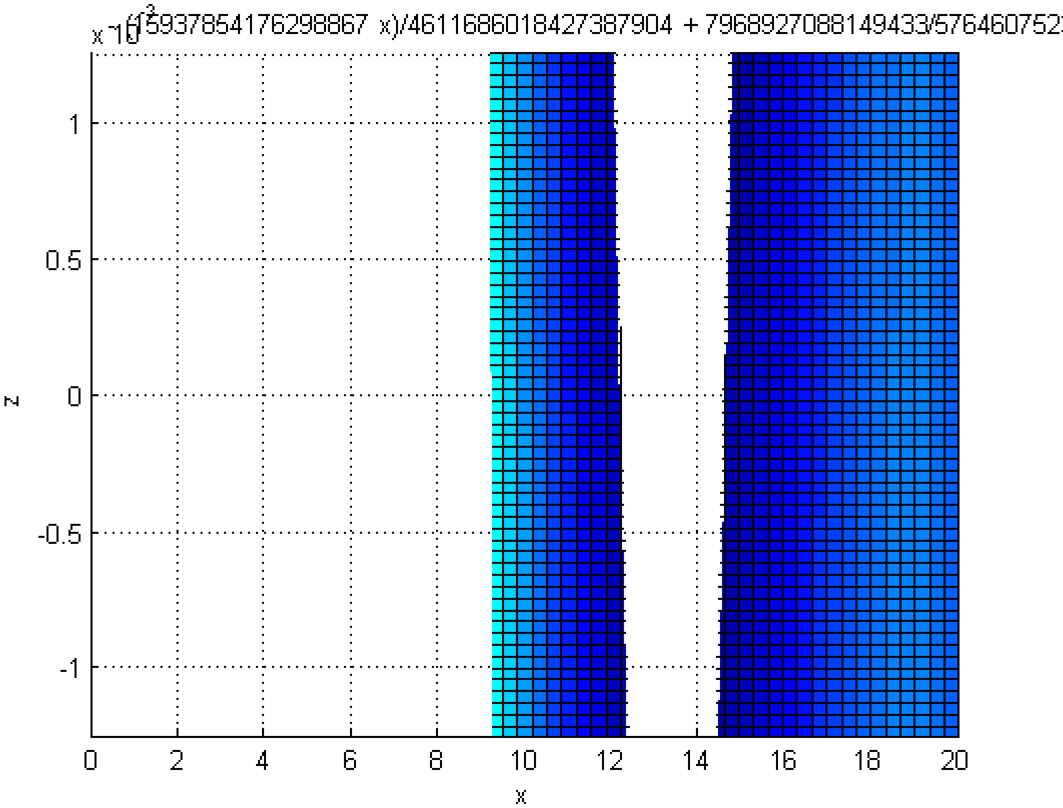
0 2 4 6 8 10 12 14 16 18 20
x


+ 7968927088149433/576460752303423488))/119726214130147567059245861496117904
x 10^-3
-1.5
-2
-2.5
-3
-3.5
z
0 2 4 6 8 10 12 14 16 18 20
x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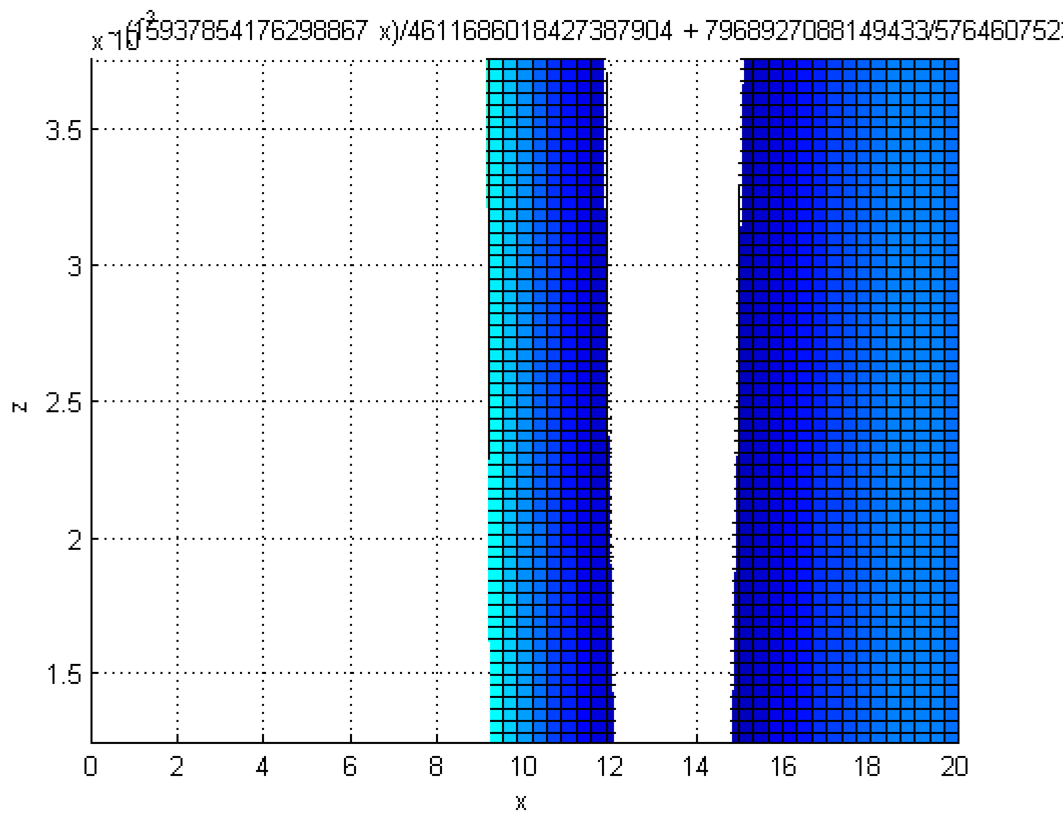
x 10^-3
1
0.5
0
-0.5
-1
z
0
2
4
6
8
10
12
14
16
18
20
x


(15937854176298867 x)/4611686018427387904 + 7968927088149433/576460752
x 10^-3
3.5
3
2.5
2
1.5
z
0
2
4
6
8
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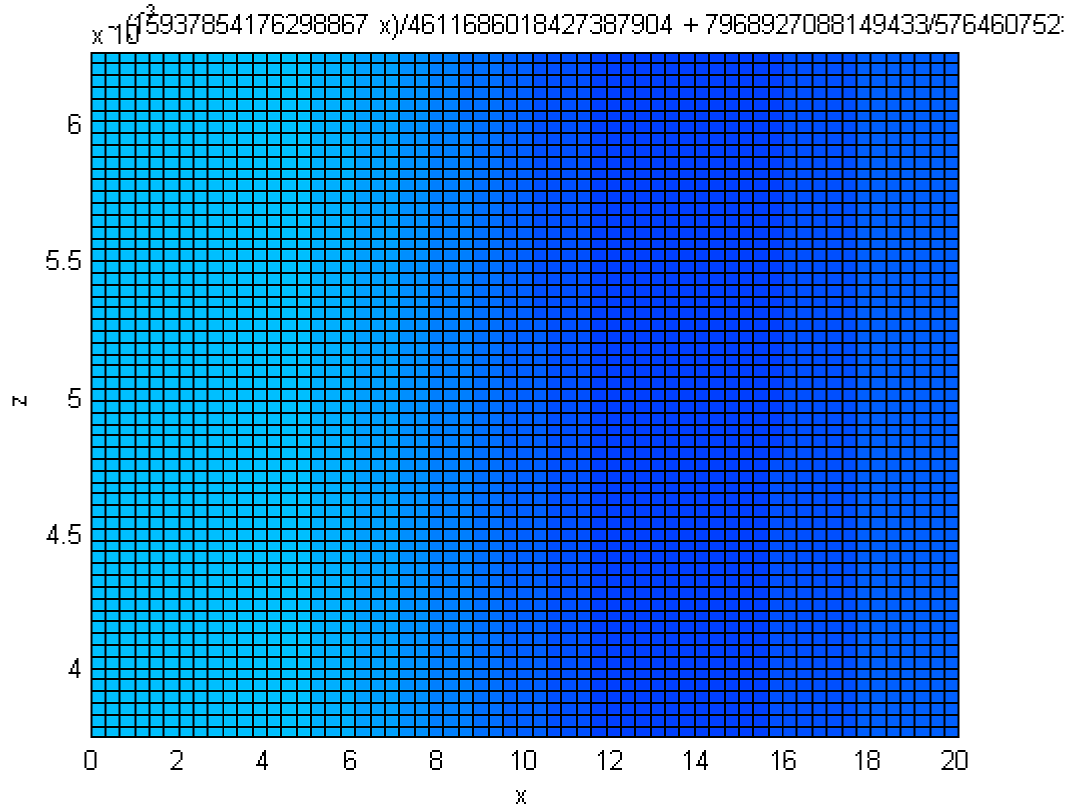
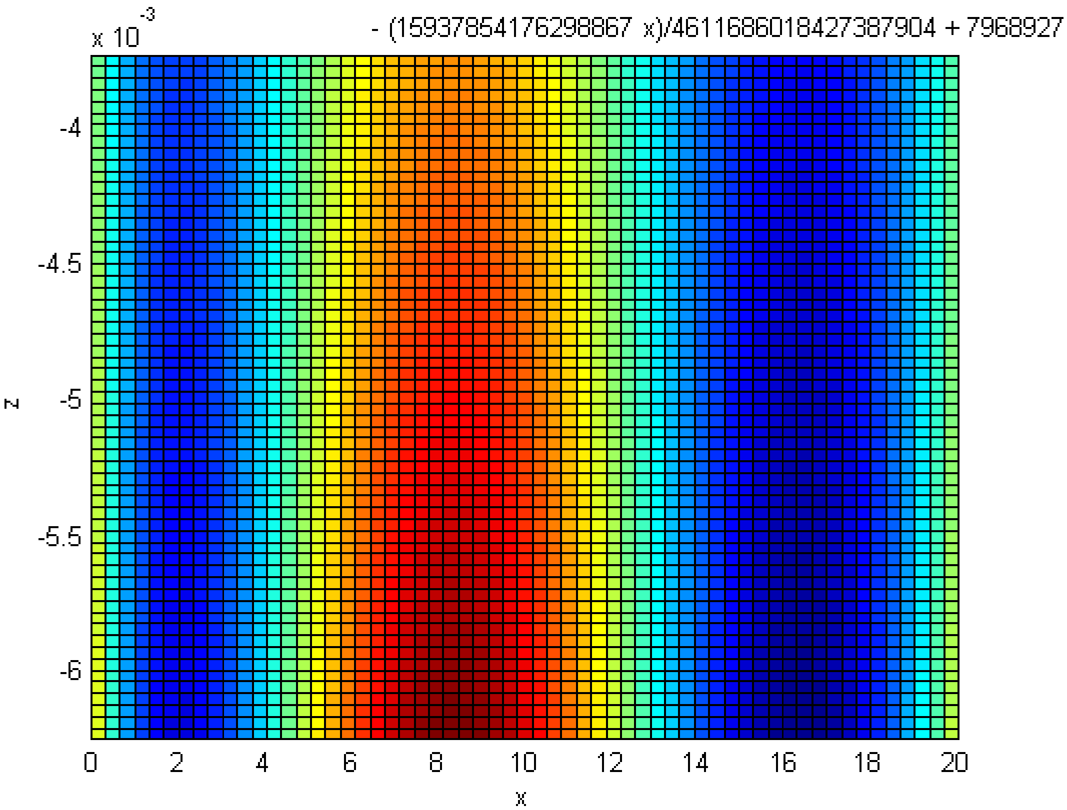
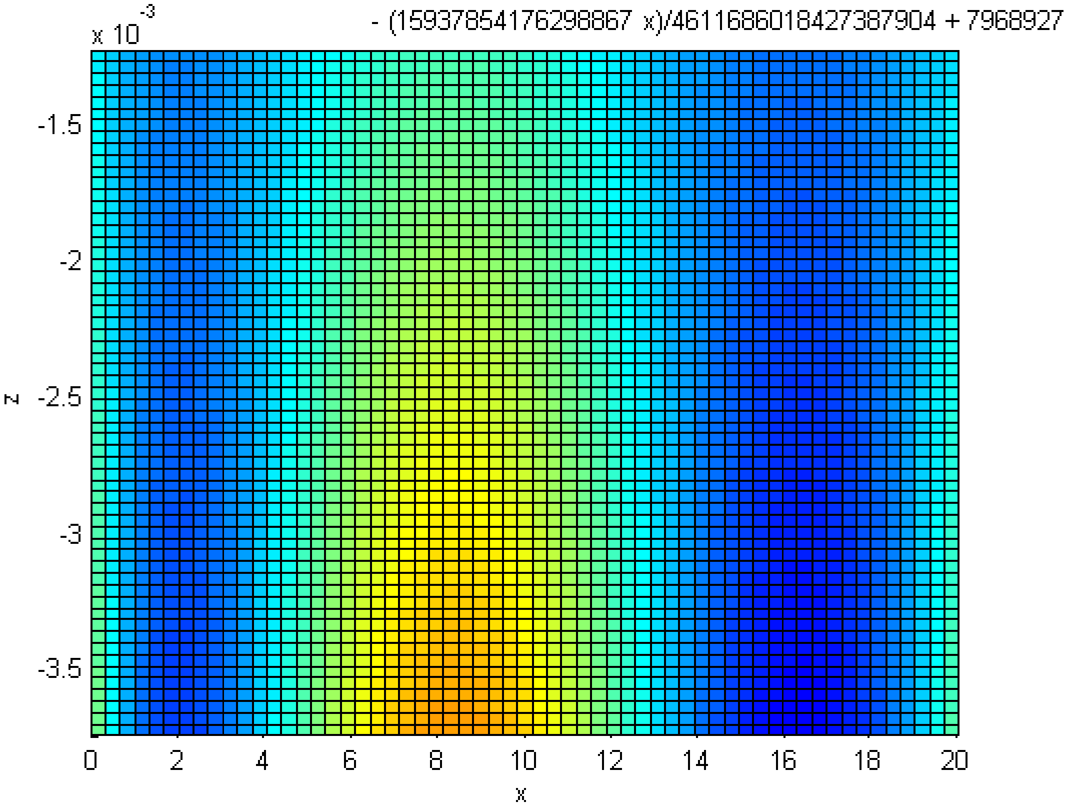
12
14
16
18
20
x


x 10^-3
(15937854176298867 x)/4611686018427387904 + 7968927088149433/576460752
6
5.5
5
4.5
4
z
0
2
4
6
8
10
12
14
16
18
20
x


x 10^-3
- (15937854176298867 x)/4611686018427387904 + 7968927
-4
-4.5
-5
-5.5
-6
z
0
2
4
6
8
10
12
14
16
18
20
x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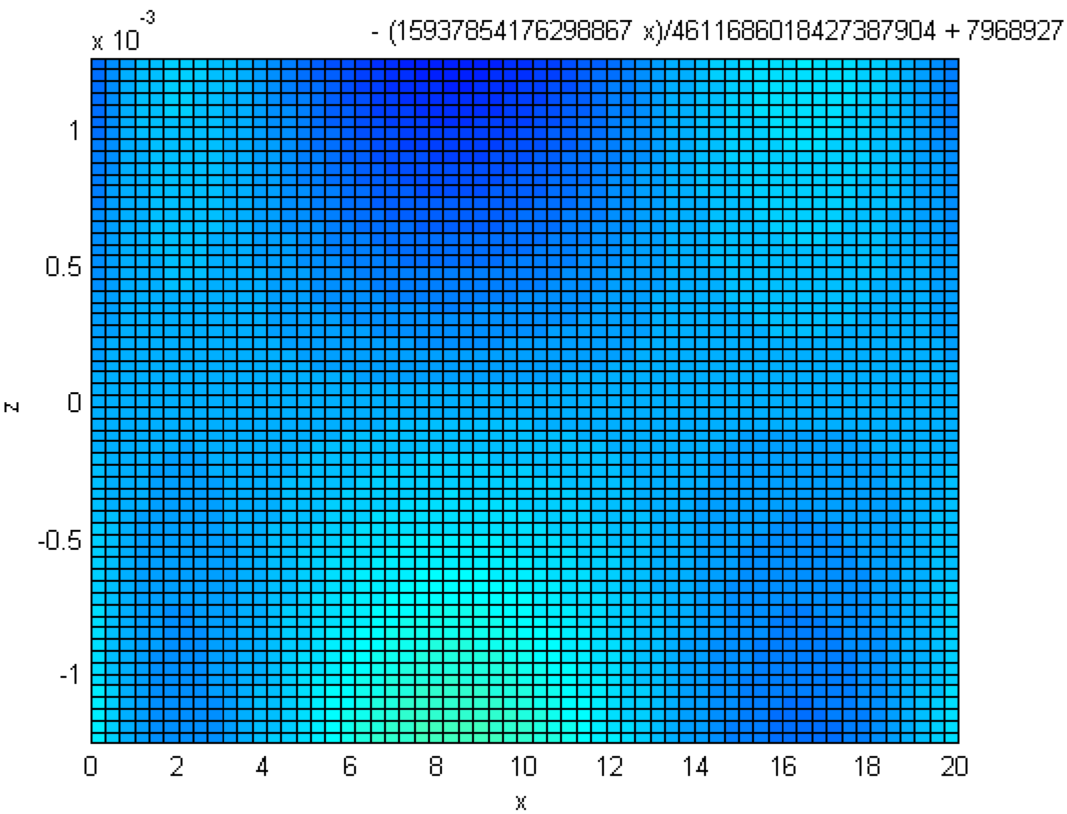
x 10^-3
-1.5
-2
-2.5
-3
-3.5
z
0
2
4
6
8
10
12
14
16
18
20
x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x 10^-3
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0
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-1
z
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2
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6
8
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14
16
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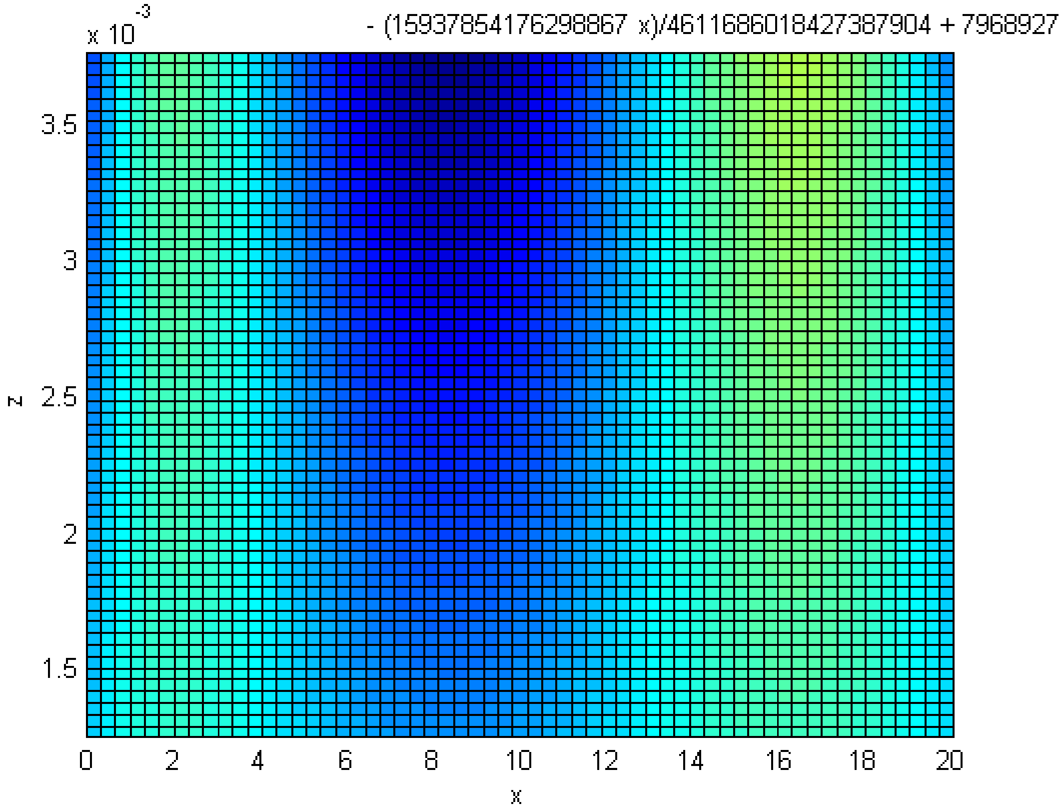
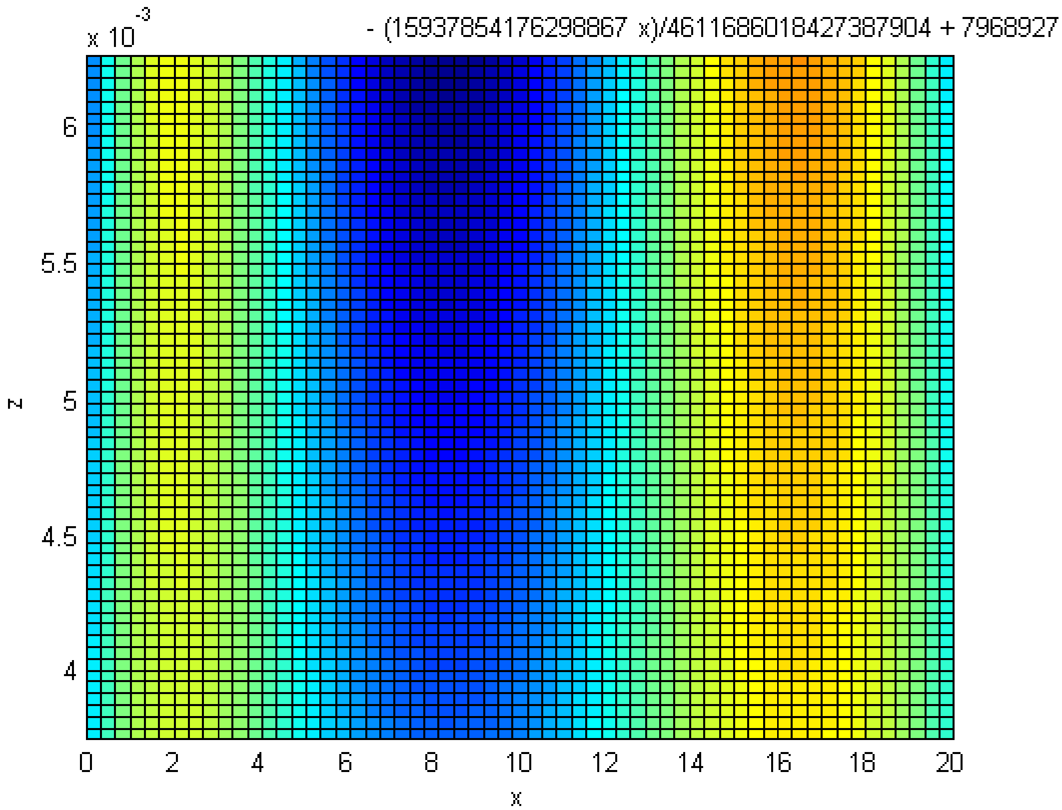
20
x



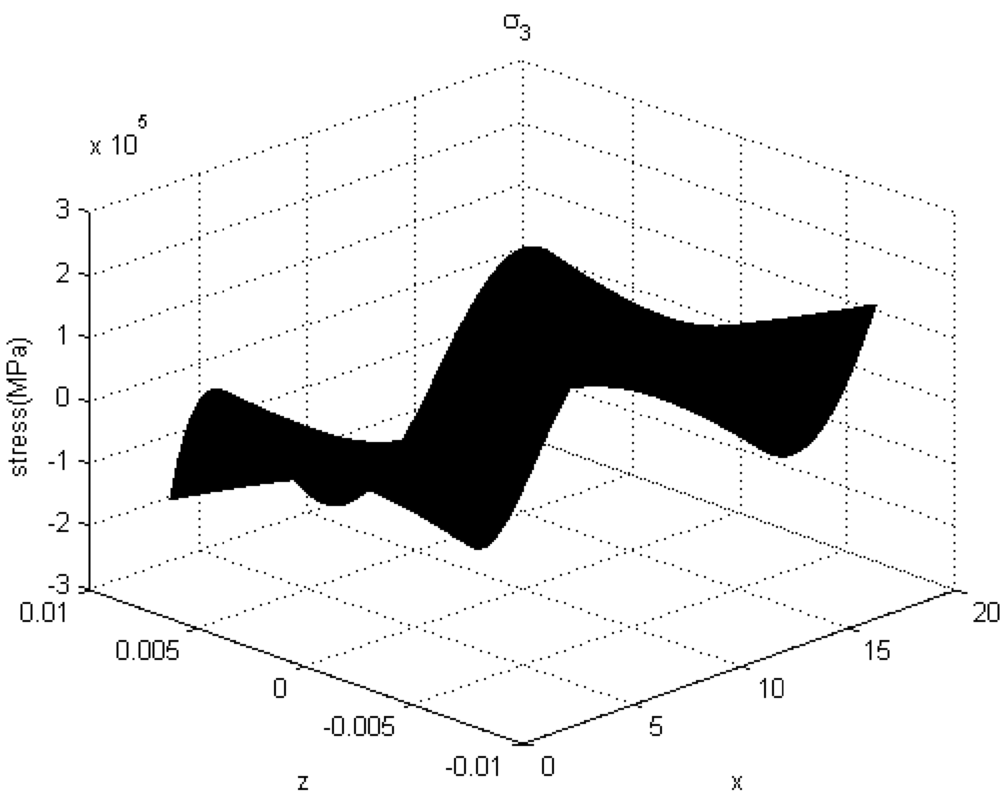


find max stress in each layer

```
ii=1;
for i = xplot
    jj=1;
```

```
            for j = zplot
                if kstress == 3
                    stressplot(ii,jj) = subs(s_xz,[x z],[i j]);
                else
                    stressplot(ii,jj) = subs(m_stress(kstress),[x z],[i j]);
                end
                jj=jj+1;
            end
            ii=ii+1;
        end
        Globalminstress(kstress,klay) = min(min(stressplot));
        Globalmaxstress(kstress,klay) = max(max(stressplot));
        %
```

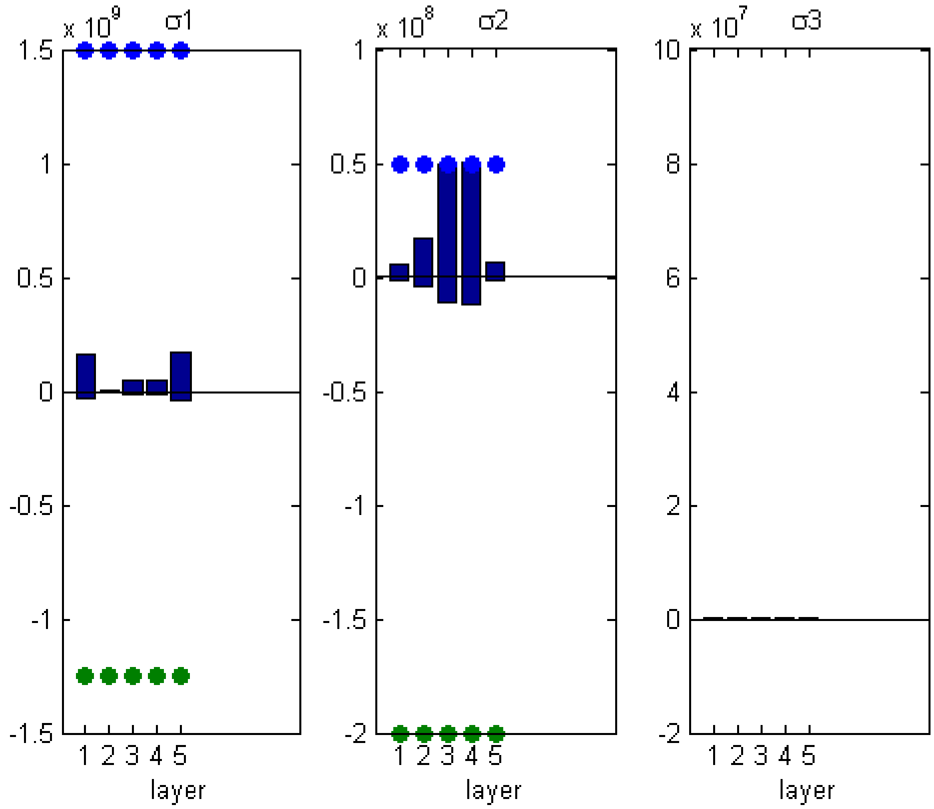
```
    end
    hold off
    axis auto
    title(strcat('\sigma_',num2str(kstress)))
    zlabel('stress(MPa)')
    view(-45,30)
end
```



Plot max stress and failure strength

```
figure
for i = 1:3
    subplot(1,3,i)
    bar(Globalmaxstress(i,:))
    hold on
    bar(Globalminstress(i,:))
```

```
    scatter(1:nl,ones(nl,1).*Strength(i,1),'filled')
    scatter(1:nl,ones(nl,1).*Strength(i,2),'filled')
    hold off
    xlabel('layer')
    title(strcat('\sigma',num2str(i)))
end
```



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