CE 710
Advanced Composites

Neal Gordon

Homework #2

Anti-symmetric Composite modeled with CLPT

- [0 45 -45 90 0] composite
- Plate a = 20”, b=10”
- Ply thickness = 0.0025”
- MATLAB was used
- A FSDT solution was not found due to errors in the programming

CLPT

Equations used from Reddy

$$
\begin{aligned}
\varepsilon_{xx} &= \frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 - z\frac{\partial^2 w_0}{\partial x^2} \\
\varepsilon_{xy} &= \frac{1}{2}\left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}\right) - z\frac{\partial^2 w_0}{\partial x \partial y} \\
\varepsilon_{yy} &= \frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 - z\frac{\partial^2 w_0}{\partial y^2} \\
\varepsilon_{xz} &= \frac{1}{2}\left(-\frac{\partial w_0}{\partial x} + \frac{\partial w_0}{\partial x}\right) = 0 \\
\varepsilon_{yz} &= \frac{1}{2}\left(-\frac{\partial w_0}{\partial y} + \frac{\partial w_0}{\partial y}\right) = 0 \\
\varepsilon_{zz} &= 0
\end{aligned}
\tag{3.3.8}
$$

$$
\left\{\begin{array}{c}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{array}\right\} = \left\{\begin{array}{c}\varepsilon_{xx}^{(0)}\\ \varepsilon_{yy}^{(0)}\\ \gamma_{xy}^{(0)}\end{array}\right\} + z\left\{\begin{array}{c}\varepsilon_{xx}^{(1)}\\ \varepsilon_{yy}^{(1)}\\ \gamma_{xy}^{(1)}\end{array}\right\}
\tag{3.3.9}
$$

$$
\{\varepsilon^0\} = \left\{\begin{array}{c}\varepsilon_{xx}^{(0)}\\ \varepsilon_{yy}^{(0)}\\ \gamma_{xy}^{(0)}\end{array}\right\} = \left\{\begin{array}{c}\frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\\ \frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2\\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}\end{array}\right\}, \quad \{\varepsilon^1\} = \left\{\begin{array}{c}\varepsilon_{xx}^{(1)}\\ \varepsilon_{yy}^{(1)}\\ \gamma_{xy}^{(1)}\end{array}\right\} = \left\{\begin{array}{c}-\frac{\partial^2 w_0}{\partial x^2}\\ -\frac{\partial^2 w_0}{\partial y^2}\\ -2\frac{\partial^2 w_0}{\partial x \partial y}\end{array}\right\}
\tag{3.3.10}
$$

CLPT

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \varepsilon_{yy}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} \quad (3.3.36)$$

$$\begin{aligned} \begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} &= \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} z\, dz \\ &= \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon_{xx}^{(0)} + z\varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(0)} + z\varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{(0)} + z\gamma_{xy}^{(1)} \end{Bmatrix} z\, dz \end{aligned}$$

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \varepsilon_{yy}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} \quad (3.3.37)$$

$$\begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix} = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} \begin{Bmatrix} \{\varepsilon^0\} \\ \{\varepsilon^1\} \end{Bmatrix} - \begin{Bmatrix} \{N^T\} \\ \{M^T\} \end{Bmatrix} - \begin{Bmatrix} \{N^P\} \\ \{M^P\} \end{Bmatrix} \quad (3.3.40)$$

$$\begin{aligned} \begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} &= \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \frac{\partial u_0}{\partial x} + \frac{1}{2}(\frac{\partial w_0}{\partial x})^2 \\ \frac{\partial v_0}{\partial y} + \frac{1}{2}(\frac{\partial w_0}{\partial y})^2 \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y} \end{Bmatrix} \\ &- \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \frac{\partial^2 w_0}{\partial x^2} \\ \frac{\partial^2 w_0}{\partial y^2} \\ 2\frac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix} \end{aligned} \quad (3.3.43)$$

$$\begin{aligned} \begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} &= \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \frac{\partial u_0}{\partial x} + \frac{1}{2}(\frac{\partial w_0}{\partial x})^2 \\ \frac{\partial v_0}{\partial y} + \frac{1}{2}(\frac{\partial w_0}{\partial y})^2 \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y} \end{Bmatrix} \\ &- \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \frac{\partial^2 w_0}{\partial x^2} \\ \frac{\partial^2 w_0}{\partial y^2} \\ 2\frac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix} \end{aligned} \quad (3.3.44)$$

CLPT

$$0 = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z}$$

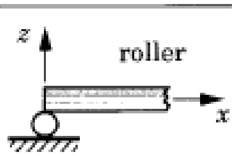
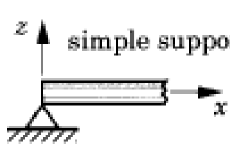
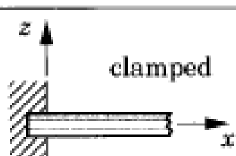
$$0 = \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z}$$

$$0 = \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} \tag{4.2.13}$$

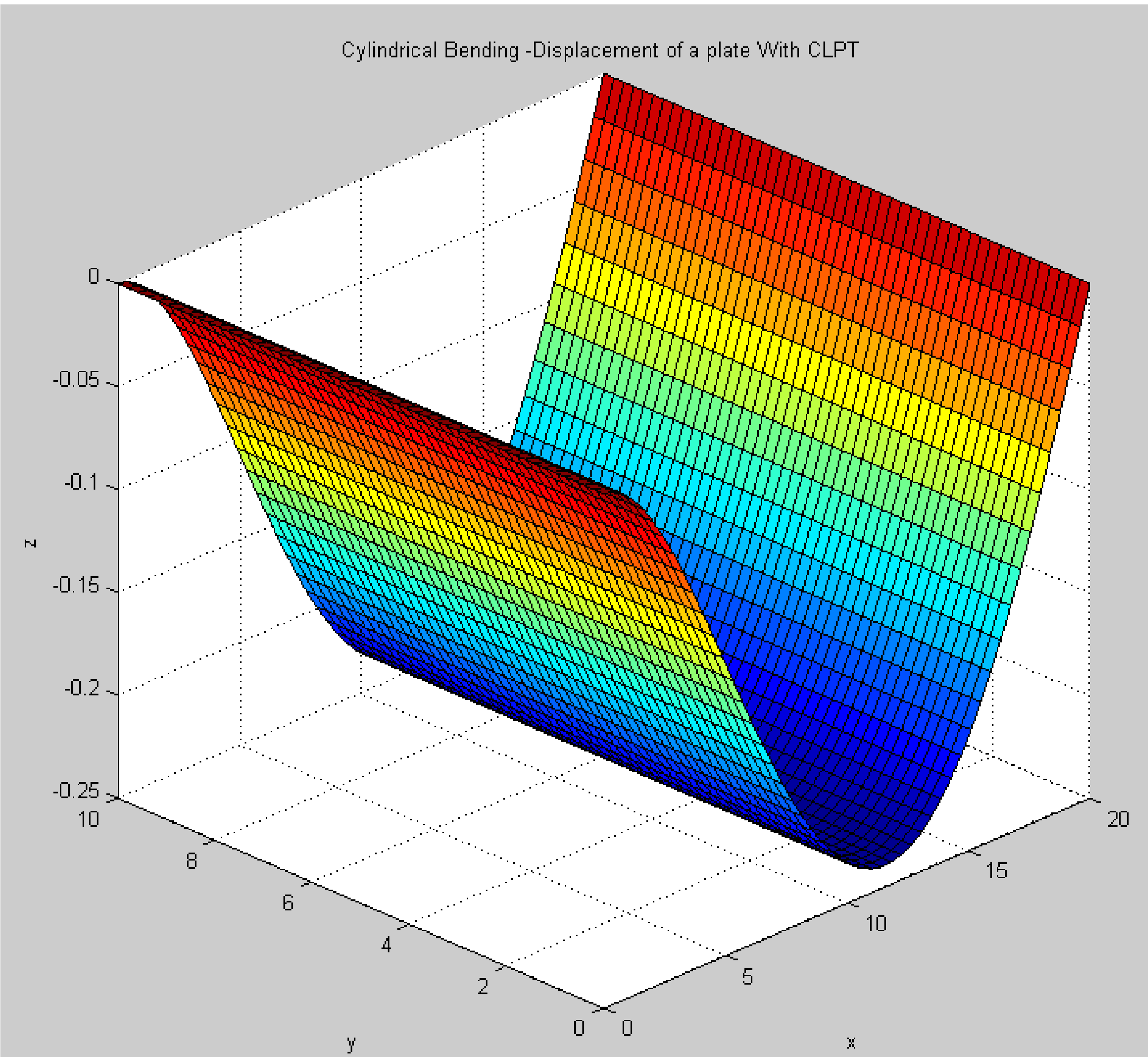
$$A_{11}\frac{\partial^2 u_0}{\partial x^2} + A_{16}\frac{\partial^2 v_0}{\partial x^2} - B_{11}\frac{\partial^3 w_0}{\partial x^3} - \frac{\partial N_{xx}^T}{\partial x} = I_0\frac{\partial^2 u_0}{\partial t^2} - I_1\frac{\partial^3 w_0}{\partial x \partial t^2} \tag{4.4.1a}$$

$$A_{16}\frac{\partial^2 u_0}{\partial x^2} + A_{66}\frac{\partial^2 v_0}{\partial x^2} - B_{16}\frac{\partial^3 w_0}{\partial x^3} - \frac{\partial N_{xy}^T}{\partial x} = I_0\frac{\partial^2 v_0}{\partial t^2} \tag{4.4.1b}$$

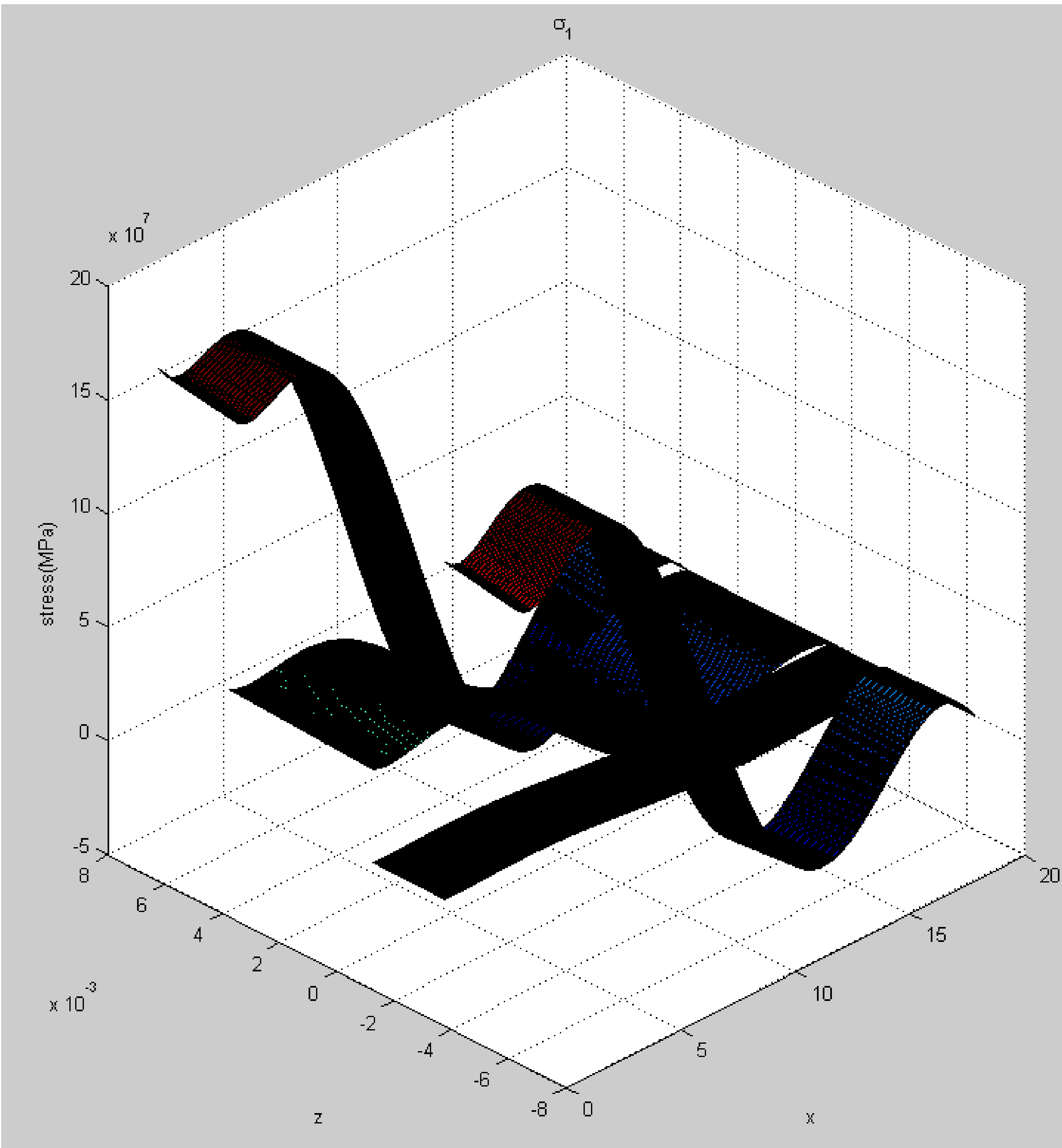
$$\begin{aligned} B_{11}\frac{\partial^3 u_0}{\partial x^3} + B_{16}\frac{\partial^3 v_0}{\partial x^3} - D_{11}\frac{\partial^4 w_0}{\partial x^4} + \frac{\partial}{\partial x}\left(\hat{N}_{xx}\frac{\partial w_0}{\partial x}\right) - \frac{\partial^2 M_{xx}^T}{\partial x^2} + q \\ = I_0\frac{\partial^2 w_0}{\partial t^2} - I_2\frac{\partial^4 w_0}{\partial x^2 \partial t^2} + I_1\frac{\partial^3 u_0}{\partial x \partial t^2} \end{aligned} \tag{4.4.1c}$$

Edge Condition	CLPT		FSDT	
free 	$N_{xx}=0$	$N_{xy}=0$	$N_{xx}=0$	$N_{xy}=0$
	$M_{xx}=0$	$\frac{dM_{xx}}{dx}=0$	$M_{xx}=0$	$Q_x=0$
roller 	$w_0=0$	$\frac{dv_0}{dx}=0$	$w_0=0$	$\frac{dv_0}{dx}=0$
	$N_{xx}=0$	$M_{xx}=0$	$N_{xx}=0$	$M_{xx}=0$
simple support 	$u_0=0$	$w_0=0$	$u_0=0$	$w_0=0$
	$\frac{dv_0}{dx}=0$	$M_{xx}=0$	$\frac{dv_0}{dx}=0$	$M_{xx}=0$
clamped 	$u_0=0$	$v_0=0$	$u_0=0$	$v_0=0$
	$w_0=0$	$\frac{dw_0}{dx}=0$	$w_0=0$	$\phi_x=0$

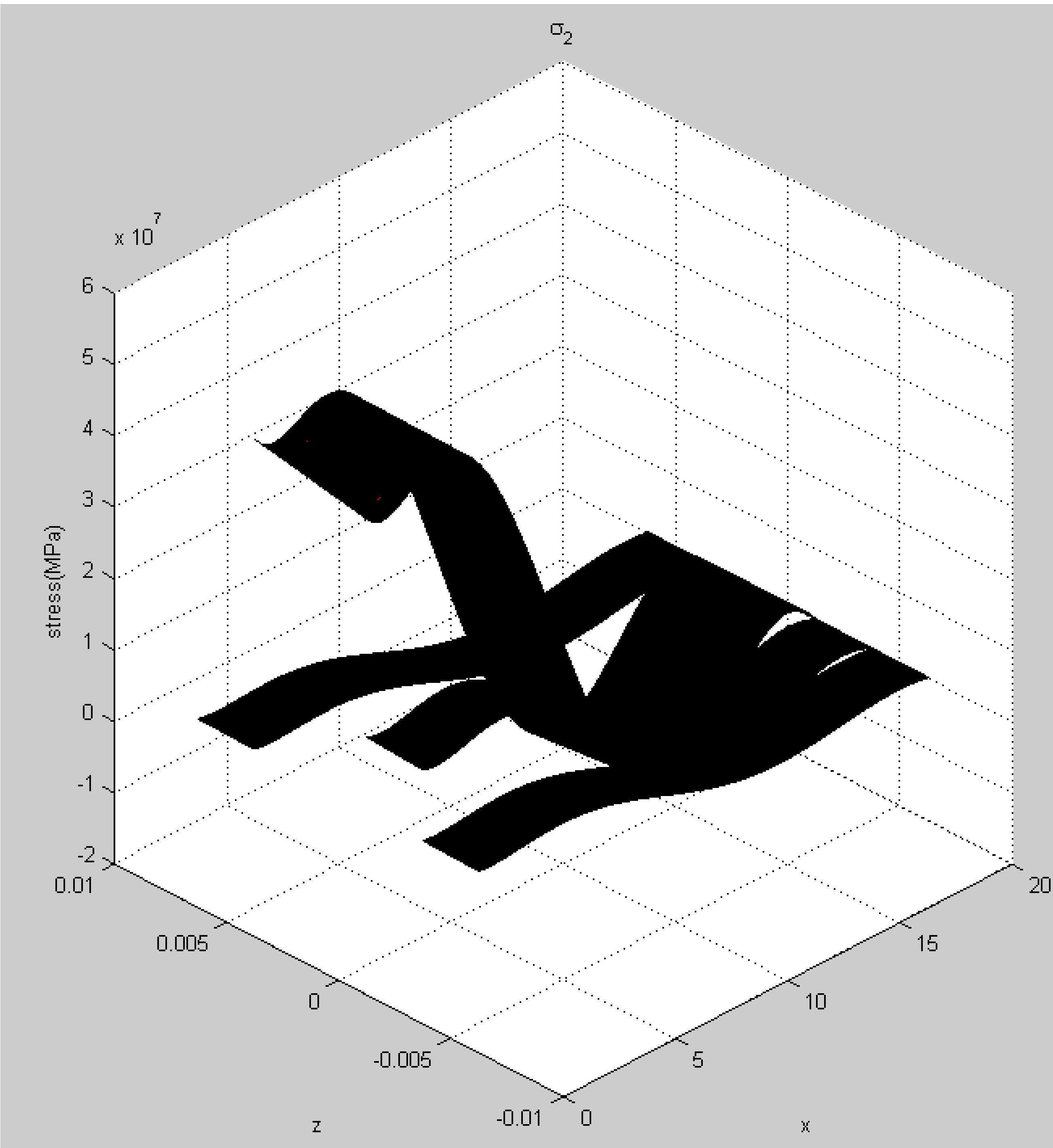
Displacement of the plate



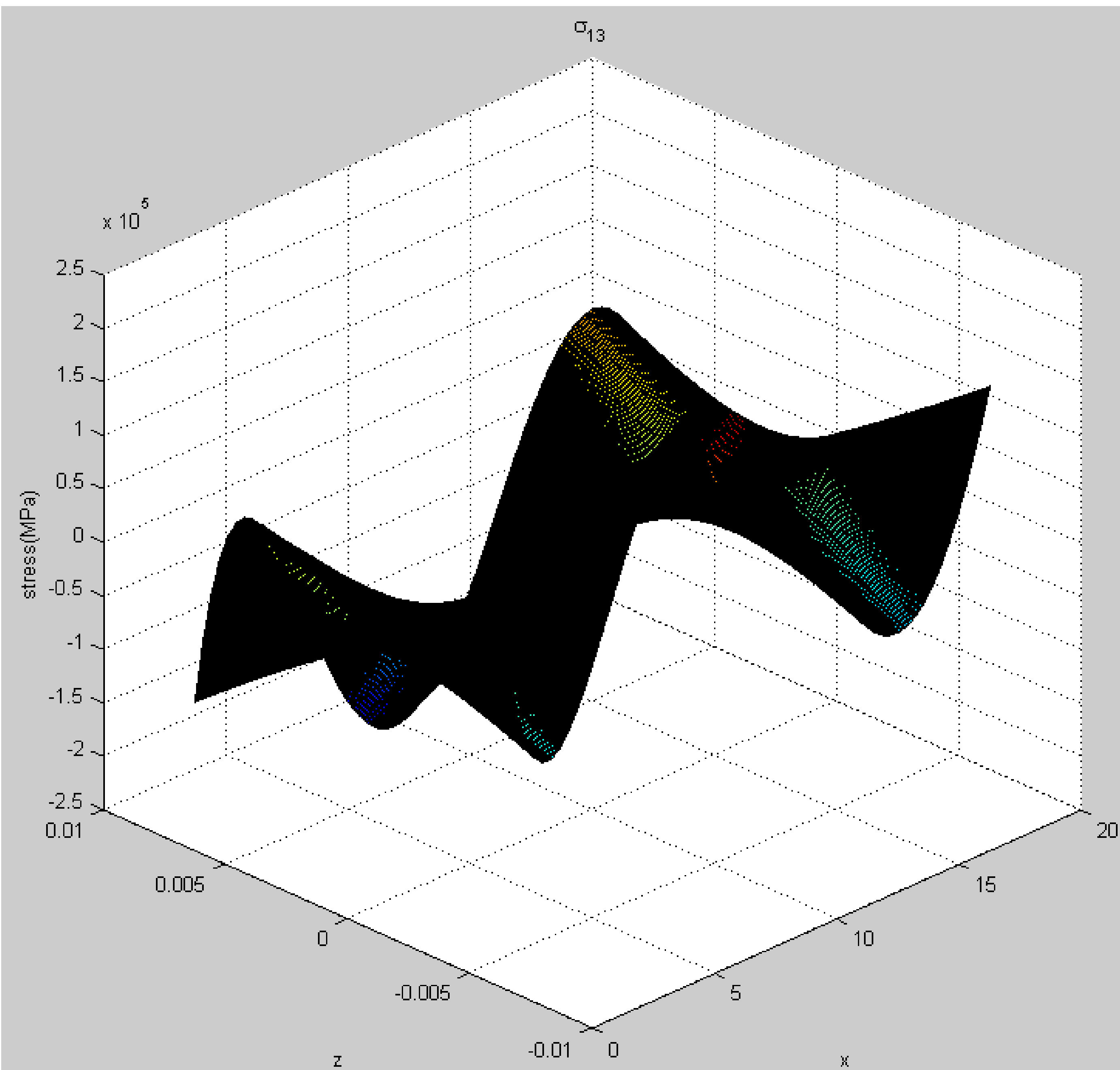
Stress σ_1 in each ply



Stress σ_2 in each ply



Stress σ_{13} in each ply



```
% ce 710 hmk2
clear all
clc
close all
%% Variables
% layer         1 (top)  ... nl (to bottom)
theta = fliplr([0 45 -45 90 0].* pi/180);
thk = zeros(1,length(theta)) + 0.0025;
nl = length(thk);
a =   20;  % plate width;
b =   10;  % plate height
q0_ = 5.7; % plate load;
% Transversly isotropic material properties
Ell = 150e9;
Ett = 12.1e9;
vlt = 0.248;
Glt = 4.4e9;
vtt = 0.458;
Gtt = Ett / (2*(1+vtt));
% Failure Strengths
SLLt =  1500e6;
SLLc = -1250e6;
STTt =  50e6;
STTc = -200e6;
SLTs =  100e6;
Sxzs =  100e6;
Strength = [SLLt SLLc;
            STTt STTc;
            SLTs Sxzs];
%% Stiffness Matrix
syms th
% tranformation
Tij6 = [cos(th)^2 sin(th)^2 0 0 0 -sin(2*th);
        sin(th)^2 cos(th)^2 0 0 0  sin(2*th);
        0 0 1 0 0 0;
        0 0 0 cos(th) sin(th) 0;
        0 0 0 -sin(th) cos(th) 0;
        cos(th)*sin(th) -cos(th)*sin(th) 0 0 0 (cos(th)^2-sin(th)^2)];

Tij = [cos(th)^2 sin(th)^2 2*sin(th)*cos(th);
        sin(th)^2 cos(th)^2 -2*sin(th)*cos(th);
        -cos(th)*sin(th) sin(th)*cos(th) (cos(th)^2-sin(th)^2)];

% compliance matrix
Sij6 = [1/Ell -vlt/Ell -vlt/Ell 0 0 0;
       -vlt/Ell 1/Ett -vtt/Ett 0 0 0;
       -vlt/Ell -vtt/Ett 1/Ett 0 0 0;
       0 0 0 1/Gtt 0 0;
       0 0 0 0 1/Glt 0;
       0 0 0 0 0 1/Glt];
```

```
% Stiffnes matrix in material coordinates
Cijm6 = inv(Sij6);

% Stiffness matrix in Structural coordinates
Cij6 = Tij6*Cijm6*Tij6.';

% reduced stiffness in structural
Cij = [Cij6(1,1) Cij6(1,2) 0; Cij6(1,2) Cij6(2,2) 0; 0 0 Cij6(6,6)];
hlam = sum(thk);

% Create z dimensions of laminate
z_(1) = -hlam/2;
for i = 1:nl
   z_(i+1) =  z_(1) + sum(thk(1:i));
end
% extensional stiffness
Aij = zeros(6,6);
for i = 1:nl
    Aij = Aij + subs(Cij6,th,theta(i)) * (z_(i+1)-z_(i));
end
% coupling  stiffness
Bij = zeros(6,6);
for i = 1:nl
    Bij = Bij + 0.5* subs(Cij6,th,theta(i)) * (z_(i+1)^2-z_(i)^2);
end
% bending or flexural laminate stiffness relating moments to curvatures
Dij = zeros(6,6);
for i = 1:nl
    Dij = Dij + (1/3)* subs(Cij6,th,theta(i)) * (z_(i+1)^3-z_(i)^3);
end

%% Cylindrical Bending of a laminated plate

% displacement in w (z direction)
syms x y z q0 C1 C2 C3 C4 C5 C6 C7 A11 B11 D11 A16 B16


syms wfun ufun
% EQ 4.4.1a
eq1   = A11*diff(ufun,x,2) - B11*diff(wfun,x,3); % C5 C1
% EQ 4.4.1b
eq2   = A16*diff(ufun,x,2) - B16*diff(wfun,x,3); % C5 C1
% EQ 4.4.1c
eq3 = B11*diff(ufun,x,3) - D11*diff(wfun,x,4) + q0;
% solve eq1 eq2 and eq3 to get the w and u functions

% displacement in w (z direction) from eq1,eq2,eq3
wfun = A11*q0*x^4 / (4*(6*B11^2-6*A11*D11)) + C1 + C2*x + C3*x^2 + C4*x^3; %  C1 C2 C3 C4
% displacement in u (x direction) from eq1,eq2,eq3
ufun = B11*q0*x^3 / (6*(B11^2-A11*D11)) + C7 + x*C6 + 3*B11*x^2*C5/A11 ;% C5 C6 C7
```

```
% cond1 -> w(0)=0 at x(0), roller
C1sol = solve(subs(wfun, x, 0)==0,C1); % = 0
% cond2 -> angle at dw/dx at x(0) is 0, cantilever
C2sol = solve(subs(diff(wfun,x),x ,0),C2);  % =  0
% cond3 -> w(z) = 0 at x(a), roller
C4sol1 =  solve(subs(wfun,[x C1 C2],[a C1sol C2sol ]),C4); % C3
% cond4 u = 0 at x = 0
C7sol = solve(subs(ufun,x,0),C7); %=0
% u=0 at x = a
C5sol1 = solve(subs(ufun,[x C7],[a C7sol]),C5); %C6
% cond 5 EQ 4.4.14a Myy = 0 @ x(a) (Mxx , B11 D11) (Myy, B12 D12) roller no moment
C6sol1 = solve(subs(  [B11*(diff(ufun,x)+0.5*diff(wfun,x)^2 ) - D11*diff(wfun,x,2)]  ,...
               [x C1    C2    C4     C5     C7],...
               [a C1sol C2sol C4sol1 C5sol1 C7sol]),C6); % C6 C3
% EQ 4.4.13a, Nxx = 0 @ x(0) roller has no Nxx
C6sol2 = solve(subs([A11* (diff(ufun,x) +0.5*diff(wfun,x)^2)-B11*diff(wfun,x,2)],...
    [x C1 C2 C4 C5 C7],[a C1sol C2sol C4sol1 C5sol1 C7sol]),C6);% C6 C3
C3sol = solve(C6sol1 == C6sol2,C3);
C4sol = subs(C4sol1,C3,C3sol);
C6sol = simplify(subs(C6sol2,C3,C3sol));
C5sol = simplify(subs(C5sol1,C6,C6sol));
% substitute integration constants with actual values( _ is actual number)
C1_ = C1sol;
C2_ = C2sol;
C7_ = C7sol;
C3_ = subs(C3sol,[q0 A11 B11 D11],[q0_ Aij(1,1) Bij(1,1) Dij(1,1)]);
C4_ = subs(C4sol,[q0 A11 B11 D11],[q0_ Aij(1,1) Bij(1,1) Dij(1,1)]);
C5_ = subs(C5sol,[q0 A11 B11 D11],[q0_ Aij(1,1) Bij(1,1) Dij(1,1)]);
C6_ = subs(C6sol,[q0 A11 B11 D11],[q0_ Aij(1,1) Bij(1,1) Dij(1,1)]);

% function w(x) vertical displacement w along z with actual vaules
wsol = subs(wfun,[q0  C1  C2  C3  C4  A11      B11      D11],...
                 [q0_ C1_ C2_ C3_ C4_ Aij(1,1) Bij(1,1) Dij(1,1)]);
% function u(x) horizontal displacement u along x with actual vaules
usol = subs(ufun,[q0  C5  C6  C7  A11      B11      D11],...
                 [q0_ C5_ C6_ C7_ Aij(1,1) Bij(1,1) Dij(1,1)]);
ezsurf(x,y,wsol,[0,a,0,b])
view(-45,30)
xlabel('x')
ylabel('y')
zlabel('z')
title('Cylindrical Bending -Displacement of a plate With CLPT')
wsol_opt = matlabFunction(wsol);
[xmax,wmax] = fminsearch(wsol_opt,0);
%% Strain calculation
% eq 3.3.8 (pg 116 reddy (pdf = 138))
epstotal = [diff(usol,x) + 0.5* diff(wsol,x)^2 - z*diff(wsol,x,2),0,0].';
epsx = epstotal(1);
%% Calculating and plotting Stress in each layer
res = 8; % accuracy of finding max and min stress
xplot = linspace(0,a,res);
```

```
yplot = linspace(0,b,res);
for kstress = 1:3 % stress state s_x, s_y, s_xz
    figure(kstress+1)
    hold on
    for klay = 1:nl % loop through all layers
        thplot = theta(klay);
        zplot = linspace(z_(klay),z_(klay+1),res);
        %% Calc Stresses
        if kstress == 3
            % Shear stresses
            syms G0
            G0_ = -int(diff(s_stress(1),x),z)+G0.';
            % solve for shear stresses from s_1
            s_xz = solve(G0_,G0);
            % out of plane shear S_xz does not need to be transformed ??
            ezsurf(s_xz, [0, a, z_(klay), z_(klay+1)])
        else
            % normal stresses
            % Cij = reduced structural stiffness in strictural coordinates 3x3
            % stress in structural coordinates
            s_stress = subs(Cij,th,thplot)*epstotal;
            % stressin material coordinates
            m_stress = subs(Tij,th,thplot)*s_stress  ;
            ezsurf(m_stress(kstress),[0,a,z_(klay),z_(klay+1)])
        end
        %% find max stress in each layer
        ii=1;
        for i = xplot
            jj=1;
            for j = zplot
                if kstress == 3
                    stressplot(ii,jj) = subs(s_xz,[x z],[i j]);
                else
                    stressplot(ii,jj) = subs(m_stress(kstress),[x z],[i j]);
                end
                jj=jj+1;
            end
            ii=ii+1;
        end
        Globalminstress(kstress,klay) = min(min(stressplot));
        Globalmaxstress(kstress,klay) = max(max(stressplot));
        %
    end
    hold off
    axis auto
    title(strcat('\sigma_',num2str(kstress)))
    zlabel('stress(MPa)')
    view(-45,30)
end
%% Plot max stress and failure strength
figure
```

```
for i = 1:3
    subplot(1,3,i)
    bar(Globalmaxstress(i,:))
    hold on
    bar(Globalminstress(i,:))
    scatter(1:nl,ones(nl,1).*Strength(i,1),'filled')
    scatter(1:nl,ones(nl,1).*Strength(i,2),'filled')
    hold off
    xlabel('layer')
    title(strcat('\sigma',num2str(i)))
end
```