Math 1540 Spring 2023 - Homework 2

Instructions: This assignment is worth twenty points. Please complete the following problems assigned below. Submissions with insufficient explanation may lose points due to a lack of reasoning or clarity. If you are handwriting your work, please ensure it is readable and well-formatted for the grader.

Be sure when uploading your work to **assign problems to pages**. Problems with pages not assigned to them **may not be graded**.

Textbook Problems: 55, 56, 61, 62

Additional Problems:

1. Let $f(x)$ be in $F[x]$ where F is a field. Show that f has no repeated roots if and only if $(f, f') = 1$.

2. Let E/F be a field extension of F. Prove that $(f, g)_F = (f, g)_E$ for any two polynomials $f(x), g(x) \in F[x]$.

3. Recall Eisenstein's Criterion says that if you have a polynomial $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$, and a prime p dividing each a_i for $0 \leq i < n$, but p doesn't divide a_n and p^2 doesn't divide a_0, then $f(x)$ is irreducible in $\mathbb{Q}[x]$. Using Eisenstein's criterion, show that $x^n - m$ is irreducible in $\mathbb{Q}[x]$ provided m is square-free, and $n \geq 2$.

4. Prove that $x^4 + 6x^3 + 12x^2 + 6x + 1$ is irreducible over $\mathbb{Q}[x]$.