CMPS 130
Homework 6

Textbook Problems

1.51. Proof that $\equiv_L$ is an equivalence relation:

Let a, b, and c be strings and L be any language.

- Reflexive: For any string z, it is trivial that $az \in L$ implies $az \in L$ ($a \equiv_L a$). Therefore, $\equiv_L$ is reflexive.
- Symmetric: Let $a \equiv_L b$. If a is indistinguishable from b, then a is not distinguishable from b. Therefore, it is NOT the case that exactly one of the strings az or bz is in L. Therefore, either they are both in L, or both not in L. It follows that $bz \in L$ whenever $az \in L$ ($b \equiv_L a$). We have just shown that $a \equiv_L b$ implies $b \equiv_L a$, so $\equiv_L$ is a symmetric relation.
- Transitive: Let $a \equiv_L b$ and $b \equiv_L c$. For any string z, if az is in L, then bz must be in L, and if bz is in L, then cz must be in L. Alternatively, if az is not in L, then bz must not be in L, and if bz is not in L, then cz must not be in L. Therefore, $a \equiv_L b$ and $b \equiv_L c$ implies $a \equiv_L c$ and $\equiv_L$ is a transitive relation.

$\equiv_L$ is reflexive, symmetric, and transitive, and therefore an equivalence relation.

State Minimization Problems

A. Original DFA   Equivalent States   Minimized DFA

Original DFA:

	a	b
>1	6	3
2	5	6
3F	4	5
4F	3	2
5	2	1
6	1	4

Equivalent States:

1					
X''	2				
X'	X'	3			
X'	X'		4		
X''		X'	X'	5	
	X''	X'	X'	X''	6

[1]=[6]
[2]=[5]
[3]=[4]

Minimized DFA:

	a	b
>[1]	[1]	[3]
[2]	[2]	[1]
[3]F	[3]	[2]

B. Original DFA   Equivalent States   Minimized DFA

Original DFA:

	a	b
>1	2	3
2	5	6
3F	1	4
4F	6	3
5	2	1
6	5	4

Equivalent States:

1					
X''	2				
X'	X'	3			
X'	X'		4		
X''		X'	X'	5	
	X''	X'	X'	X''	6

[1]=[6]
[2]=[5]
[3]=[4]

Minimized DFA:

	a	b
>[1]	[2]	[3]
[2]	[2]	[1]
[3]F	[1]	[3]

C. Original DFA   Equivalent States   Minimized DFA

Original DFA:

	a	b
>0F	3	2
1F	3	5
2	2	6
3	2	1
4	5	4
5	5	3
6	5	0

Equivalent States:

0						
	1					
X'	X'	2				
X'	X'	X''	3			
X'	X'	X'''	X''	4		
X'	X'		X''	X''	5	
X'	X'	X''		X''	X''	6

[0]=[1]
[2]=[5]
[3]=[6]

Minimized DFA:

	a	b
>[0]F	[3]	[2]
[2]	[2]	[3]
[3]	[2]	[1]
[4]	[2]	[4]

D. Original DFA   Equivalent States   Minimized DFA

Original DFA

	a	b
> 0	3	5
1	2	4
2	6	3
3	6	6
4F	0	2
5F	1	6
6	2	6

Equivalent States

0						
	1					
X"	X"	2				
X"	X"		3			
X'	X'	X'	X'	4		
X'	X'	X'	X'		5	
X"	X"			X'	X'	6

[0]=[1]
[2]=[3]=[6]
[4]=[5]

Minimized DFA

	a	b
> [0]	[2]	[4]
[2]	[2]	[2]
[4]F	[0]	[2]