# CMPS 130
# Homework 7

## Myhill-Nerode Theorem Problems

Use the Myhill-Nerode Theorem to prove that the languages in Exercise 1.29 (a and c) and Problem 1.46 (b and c) are not regular.

1.29 a. $A_1 = \{0^n 1^n 2^n \mid n \geq 0\}$

Let $S = \{0^n \mid n \geq 0\}$

For $i \neq j$,

$0^i 1^i 2^i \in A_1$

$0^j 1^i 2^i \notin A_1$

Since S is an infinite set and each string in S belongs to a different equivalence class in $R_{A_1}$, $R_{A_1}$ has an infinite index. Therefore, $A_1$ is not regular.

c. $A_3 = \{a^{2^n} \mid n \geq 0\}$

Let $S = \{a^{2^n} \mid n \geq 0\}$

For $i \neq j$,

$a^{2^i} a^{2^i} = a^{2^{i+1}} \in A_3$

$a^{2^j} a^{2^i} = a^{2^j + 2^i} \notin A_3$

Since S is an infinite set and each string in S belongs to a different equivalence class in $R_{A_3}$, $R_{A_3}$ has an infinite index. Therefore, $A_3$ is not regular.

Note: $2^j + 2^i \neq 2^k$

Proof: Assume toward contraction that $2^j + 2^i = 2^k$ $(i \neq j)$. Also assume without loss of generality that $i < j$. Dividing both sides by $2^i$ gives us $2^{j-i} + 1 = 2^{k-i}$.

This is a contradiction (left side is clearly odd, and right side is clearly even). Therefore, $2^j + 2^i \neq 2^k$.

1.46 b. $L = \{0^m 1^n \mid m \neq n\}$

Let $S = \{0^m \mid m \geq 0\}$

For $i \neq j$,

$0^i 1^i \notin L$

$0^j 1^i \in L$

Since S is an infinite set and each string in S belongs to a different equivalence class in $R_L$, $R_L$ has an infinite index. Therefore, L is not regular.

c. $L = \{w \mid w \in \{0,1\}^* \text{ is not a palindrome}\}$

Let $S = \{0^n 1 \mid n \geq 0\}$

For $i \neq j$,

$0^i 1 0^i \notin L$

$0^j 1 0^i \in L$

Since S is an infinite set and each string in S belongs to a different equivalence class in $R_L$, $R_L$ has an infinite index. Therefore, L is not regular.

Prove any language of your choice is regular with the Myhill-Nerode theorem. Could you have done the proof with the Pumping Lemma?

Let $L = \{w \in \{a, b\}^* \mid w \text{ has an even number of } a's\}$

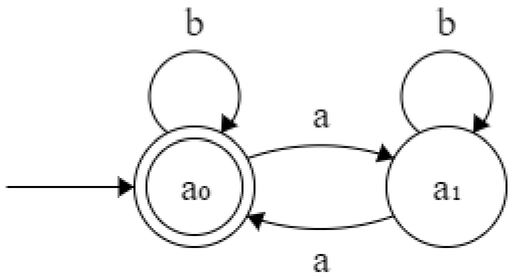
$R_L$ has two equivalence classes:

$[odd]_{R_L} = \{w \in \{a, b\}^* \mid w \text{ has an odd number of } a's\}$

$[even]_{R_L} = \{w \in \{a, b\}^* \mid w \text{ has an even number of } a's\}$

Since $R_L$ has a finite index (2), L is a regular language.

Note: Below is the minimized DFA for L (2 states)




I could not have done this proof with the Pumping Lemma, because the Pumping Lemma cannot prove a language is regular (it can only prove that some languages are non-regular).

## Textbook Exercises

2.3 a. R, S, T, X
b. a, b
c. R
d. ab, ba, aab
e. a, b, aa
f. False
g. True
h. False
i. True
j. True
k. False
l. True
m. True
n. False
o. L(G) = {w | w is not a palindrome}

2.4 a. {w | w contains at least three 1s}
$S \rightarrow T1T1T1T$
$T \rightarrow 0T \mid 1T \mid \varepsilon$
b. {w | w starts and ends with the same symbol}
$S \rightarrow 0T0 \mid 1T1 \mid 0 \mid 1$
$T \rightarrow 0T \mid 1T \mid \varepsilon$
c. {w | the length of w is odd}
$S \rightarrow 0S0 \mid 0S1 \mid 1S0 \mid 1S1 \mid 0 \mid 1$
d. {w | the length of w is odd and its middle symbol is a 0}
$S \rightarrow 0S0 \mid 0S1 \mid 1S0 \mid 1S1 \mid 0$
e. $\{w \mid w = w^R$ (w is a palindrome)$\}$
$S \rightarrow 0S0 \mid 1S1 \mid 0 \mid 1 \mid \varepsilon$
f. The empty set
$S \rightarrow S$

## Extra CFG Problem

Give a CFG for the language:
$\{x \in \{a, b\}^* \mid x \neq ww \text{ for some } w \in \{a, b\}^*\}$

$S \rightarrow A \mid B \mid AB \mid BA$
$A \rightarrow LaL \mid a$
$B \rightarrow LbL \mid b$
$L \rightarrow a \mid b$