Calculus II

MAT187 Student Slides

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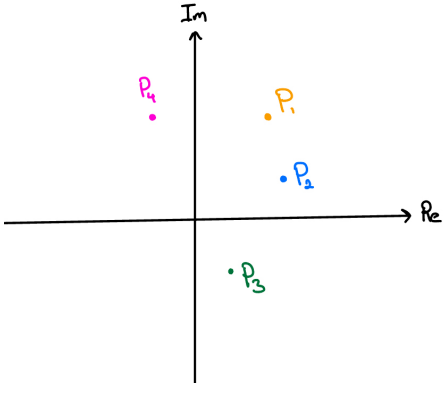
Exercise 1

Consider the plot of the complex numbers p_1, p_2, p_3, p_4 in the complex plane.



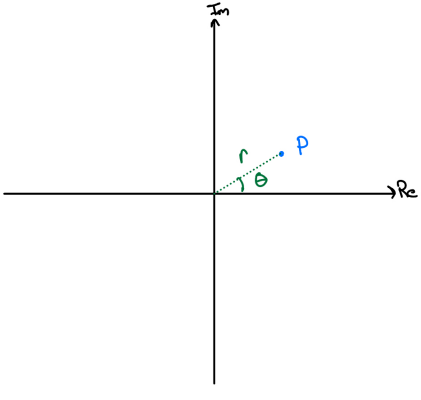
1.1 For which complex numbers is the real part grater than the imaginary part?

1.2 Which complex number has the smallest *modulus/absolute value*?

1.3 Which complex number has the largest *argument*? Is your answer at all ambiguous?

Exercise 2

Consider the plot of the complex number p in the complex plane.



2.1 Sketch the complex number $2p$.

2.2 Sketch the complex number p^2.

2.3 Sketch the complex numbers p^n for $n = 3, 4, \ldots$. Will your answer depend on r?

2.4 Use the geometry of the complex plane to find $\sqrt{i}$. Express your answer in both polar and rectangular form.

Exercise 3

Consider the equation

$$z^3 = -1 \tag{1}$$

3.1 Find a solution to Equation (1).

3.2 If $z = re^{i\theta}$ is a solution to Equation (1), what conditions must r and θ satisfy? Justify your conclusions.

3.3 Find all solutions to Equation (1).

Exercise 4

Consider the equation

$$z^n = 1, \tag{2}$$

where n is a positive integer.

4.1 Solutions to Equation (2) are called *roots of unity*. How many roots of unity are there (for a fixed value of n)?

4.2 Find the roots of unity for $n = 4$.

4.3 Let $n = 4$. Geometrically, what should the *sum* of the roots of unity be? Verify your answer algebraically.

4.4 Let $n = 5$. What should the sum of the roots of unity be?