

# Calculus II

MAT187 Student Slides

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## Exercise 1

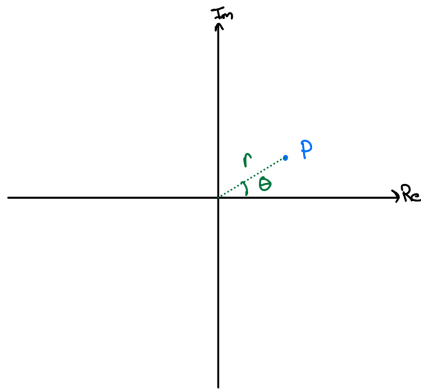
Consider the plot of the complex numbers  $p_1, p_2, p_3, p_4$  in the complex plane.



- 1.1 For which complex numbers is the real part greater than the imaginary part?
- 1.2 Which complex number has the smallest *modulus/absolute value*?
- 1.3 Which complex number has the largest *argument*? Is your answer at all ambiguous?

## Exercise 2

Consider the plot of the complex number  $p$  in the complex plane.



- 2.1 Sketch the complex number  $2p$ .
- 2.2 Sketch the complex number  $p^2$ .
- 2.3 Sketch the complex numbers  $p^n$  for  $n = 3, 4, \dots$ . Will your answer depend on  $r$ ?
- 2.4 Use the geometry of the complex plane to find  $\sqrt{i}$ . Express your answer in both polar and rectangular form.

### Exercise 3

Consider the equation

$$z^3 = -1 \tag{1}$$

3.1 Find a solution to Equation (1).

3.2 If  $z = re^{i\theta}$  is a solution to Equation (1), what conditions must  $r$  and  $\theta$  satisfy? Justify your conclusions.

3.3 Find all solutions to Equation (1).

## Exercise 4

Consider the equation

$$z^n = 1 \tag{2}$$

- 4.1 Solutions to Equation (2) are called *roots of unity*. How many roots of unity are there (for a fixed value of  $n$ )?
- 4.2 Find the roots of unity for  $n = 4$ .
- 4.3 Let  $n = 4$ . Geometrically, what should the *sum* of the roots of unity be? Verify your answer algebraically.
- 4.4 Let  $n = 5$ . What should the sum of the roots of unity be?