Calculus II

MAT187 Student Slides

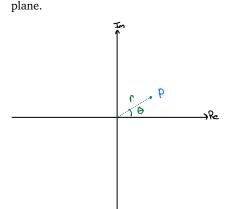
Geoff McGregor Arman Pannu Jason Siefken

Consider the plot of the complex numbers p_1 , p_2 , p_3 , p_4 in the complex plane.



- 1.1 For which complex numbers is the real part grater than the imaginary part?
- 1.2 Which complex number has the smallest modulus/absolute value?
- 1.3 Which complex number has the largest argument? Is your answer at all ambiguous?

Consider the plot of the complex number p in the complex 2.1 Sketch the complex number 2p.



- 2.2 Sketch the complex number p^2 .
- 2.3 Sketch the complex numbers p^n for n = 3, 4, ... Will your answer depend on r?
- 2.4 Use the geometry of the complex plane to find \sqrt{i} . Express your answer in both polar and rectangular form.

Consider the equation

3.1 Find a solution to Equation (1).

3.3 Find all solutions to Equation (1).

- 3.2 If $z = re^{i\theta}$ is a solution to Equation (1), what conditions must r and θ satisfy? Justify your conclusions.

 $z^3 = -1$

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(1)

For each situation, decide whether *least squares* curve fitting or *polynomial interpolation* would be more appropriate.

- 4.1 You are modelling the arch used in the construction of a particular Roman aqueduct. You have collected several hundred data points of height of the arch vs. distance from the base of the aqueduct.
- 4.2 You are creating a function to govern the brightness of a light which will be used for signalling a computer. There are three different brightnesses that must be achieved exactly and the transition between those brightnesses must be smooth.
- 4.3 You are given data from a lab and told that the data was created with a 4th degree polynomial. You are asked to find the coefficients of the polynomial.

A baseball is thrown on the moon. You are trying to find the function

• h(t), the height (in meters) of the baseball above the moon's surface at time t (in seconds).

You collected the following data

h(t)
4
3.8
2

- 5.1 What degree polynomial would best model *h*?
- 5.2 Use polynomial interpolation to find h.
- 5.3 Find the maximum height of the baseball above the moon's surface.

While developing a robotics control system, you find the 6.2 Complete the following table need for a function f which satisfies the following properties:

(i)
$$f(0) = -1$$
 and $f(1) = 2$

(ii)
$$f'(0) = -1$$
 and $f'(1) = 3$

Your friend suggests that you could use the following polynomial to come up with f:

$$L_1(x) = -(x-1)$$
 $L_2(x) = x$
 $S_1(x) = (x-1)^2 x$ $S_2(x) = (x-1)x^2$

Can Lagrange interpolation be used to directly find f? Explain.

g	g(0)	g(1)	g'(0)	g'(1)
L_1				
L_2				
S_1				
S_2				

- 6.3 Use L_1 , L_2 , S_1 , and S_2 to find a polynomial satisfying the properties of f.
- 6.4 Explain how Lagrange interpolation can be generalized to allow finding a polynomial that passes through particular points and takes on particular derivatives at those points.

