Calculus II

MAT187 Student Slides

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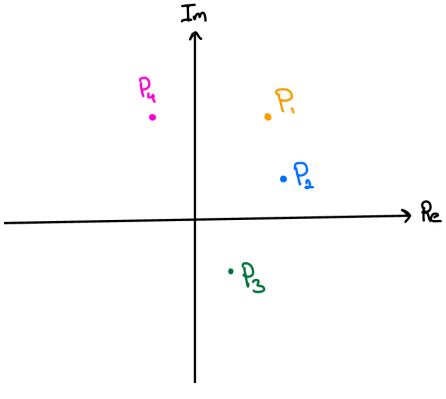
Exercise 1

Consider the plot of the complex numbers p_1, p_2, p_3, p_4 in the complex plane.



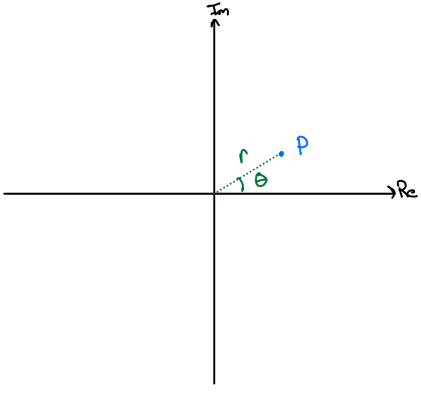
1.1 For which complex numbers is the real part grater than the imaginary part?

1.2 Which complex number has the smallest *modulus/absolute value*?

1.3 Which complex number has the largest *argument*? Is your answer at all ambiguous?

Exercise 2

Consider the plot of the complex number p in the complex plane.



2.1 Sketch the complex number $2p$.

2.2 Sketch the complex number p^2.

2.3 Sketch the complex numbers p^n for $n = 3, 4, \ldots$. Will your answer depend on r?

2.4 Use the geometry of the complex plane to find $\sqrt{i}$. Express your answer in both polar and rectangular form.

Exercise 3

Consider the equation

$$z^3 = -1 \tag{1}$$

3.1 Find a solution to Equation (1).

3.2 If $z = re^{i\theta}$ is a solution to Equation (1), what conditions must r and θ satisfy? Justify your conclusions.

3.3 Find all solutions to Equation (1).

Exercise 4

For each situation, decide whether *least squares* curve fitting or *polynomial interpolation* would be more appropriate.

4.1 You are modelling the arch used in the construction of a particular Roman aqueduct. You have collected several hundred data points of height of the arch vs. distance from the base of the aqueduct.

4.2 You are creating a function to govern the brightness of a light which will be used for signalling a computer. There are three different brightnesses that must be achieved exactly and the transition between those brightnesses must be smooth.

4.3 You are given data from a lab and told that the data was created with a 4th degree polynomial. You are asked to find the coefficients of the polynomial.

Exercise 5

A baseball is thrown on the moon. You are trying to find the function

- $h(t)$, the height (in meters)of the baseball above the moon's surface at time t (in seconds).

You collected the following data

t	$h(t)$
1	4
2	3.8
3	2

5.1 What degree polynomial would best model h?

5.2 Use polynomial interpolation to find h.

5.3 Find the maximum height of the baseball above the moon's surface.

Exercise 6

While developing a robotics control system, you find the need for a function f which satisfies the following properties:

(i) $f(0) = -1$ and $f(1) = 2$

(ii) $f'(0) = -2$ and $f'(1) = 3$

Your friend suggests that you could use the following polynomial to come up with f:

$$L_1(x) = -(x-1) \qquad L_2(x) = x$$

$$S_1(x) = (x-1)^2 x \qquad S_2(x) = (x-1)x^2$$

6.1 Can Lagrange interpolation be used to directly find f? Explain.

6.2 Complete the following table

g	$g(0)$	$g(1)$	$g'(0)$	$g'(1)$
L_1				
L_2				
S_1				
S_2				

6.3 Use L_1, L_2, S_1, and S_2 to find a polynomial satisfying the properties of f.

6.4 Explain how Lagrange interpolation can be generalized to allow finding a polynomial that passes through particular points and takes on particular derivatives at those points.