The Moderation of Higher School Certificate Assessments using a Quadratic Polynomial Transformation: a Technical Paper

Dr Robert MacCann
Head, Measurement & Research

1. Introduction

This booklet gives a technical explanation of the moderation procedures especially developed by the author for the moderation of assessments at the Higher School Certificate (HSC). These procedures have been in use from the 1992 HSC. Previously, the moderation procedures used a linear adjustment to convert a school's Raw Assessments to have the same mean and standard deviation as the school's Examination Marks. A linear adjustment is of the form

$$y = ax + b$$

where x is the Raw Assessment, y the Moderated assessment and a, b are constants to be determined for a particular school group. This linear adjustment, involving the mean and standard deviation, has a long history. It was originally devised by Francis Galton in the late nineteenth century, put into its current form by Karl Pearson around the turn of the century, and used in an educational context by Cyril Burt before the first world war (Howard, 1958). It has been used for many years in Australia and in most countries throughout the world that require the adjustment of school assessments.

A linear adjustment is most suitable where a school's Raw Assessments and Examination Marks have similar distribution shapes. However, when these distributions have significantly different shapes (for example, substantially different skews) the linear moderation process is less successful. In such cases a transformation which equates percentiles, giving a *curved* line of adjustment, would be the appropriate procedure if the school groups were sufficiently large (see for example, Angoff, 1971; Braun and Holland, 1982). However, most school groups are not large enough to employ equipercentile procedures.

A simple algebraic way of generating a curved line of adjustment is to move from a polynomial of degree 1 (the above linear model) to a polynomial of degree 2 (the quadratic model). This adjustment is of the form

$$y = ax^2 + bx + c$$

where there are now three constants a, b, and c to be determined for a particular school group.

Both the linear and curved models produce Moderated Assessments having the same rank order as the Raw Assessments. The linear method also has the property of retaining the ratio of differences between pairs of Raw Assessments. The curved method does not do this. Within a localised region the curved method may approximate the linear method: to this extent the ratio of differences may approximately hold for adjacent pairs, but not for pairs widely separated on the Raw Assessments.

Whereas the linear model has two constants to be determined, the quadratic model has three. To determine the two constants under the linear model, the two conditions imposed were that both the mean and standard deviation of the Moderated Assessments be set equal to their corresponding values in the Examination Marks.

The quadratic model requires three conditions to determine the three constants. The condition stated above, involving the equality of means, is retained but the standard deviation condition is not used. Instead, more control over the level of the maximum and minimum Moderated Assessments is gained by explicitly defining the values that these will take. These three conditions are given in the next section.

When the school group Raw Assessments and Examination Marks have the same distribution shape (which still allows the rank orders and the respective means and standard deviations to be different), the particular curved method to be employed here reduces to the usual linear, mean and standard deviation method. Thus the previous linear method may be viewed as a special case of the more general curved method. A mathematical proof of this is given in Appendix A.

2. The Three Conditions for Determining the Curve

In requesting that a new moderation moderation procedure be developed for the HSC, the Board specified that, in normal circumstances, the top Moderated Assessment awarded to a school group should be equal to the top Examination Mark obtained within the group. This condition was necessary in a very competitive situation to protect the interests of outstanding candidates. Under the previous linear model, an outstanding student may have received the top raw school assessment and obtained the top Examination Mark, but received a Moderated Assessment which was less than the obtained Examination Mark.

This situation is exacerbated when, within a school group, the assessments are negatively skewed (which could occur if the assessment tasks are easy) and the Examination Marks are positively skewed (which could occur if the examination is difficult for the group). To a large degree, the type and extent of this skewing is not easily controllable by the teacher giving the assessments. Certainly, the teacher has little control over the Examination Mark distribution. It is quite possible for the teacher to pitch class tests and assignments at a level to encourage the weaker students (and thus obtain negatively skewed assessments) but for the students to find the statewide examination much harder (and thus obtain positively skewed Examination Marks).

Under these circumstances, using the previous linear model, the top Moderated Assessment tended to be less than the top Examination Mark. If a student has topped both the assessment and the examination, it is difficult to explain why the Moderated Assessment should be lower than the Examination Mark, particularly if the Raw Assessment is at the top of the mark scale (e.g. 100 out of 100).

The opposite effect tended to occur when the assessments were positively skewed (which could occur with difficult assessment tasks) and the Examination Marks were negatively skewed (which could occur if the group found the examination to be easy). In these circumstances, with the previous linear model, the top Moderated Assessment tended to exceed the top Examination Mark.

In an intensely competitive situation such as the HSC, it is undesirable that the relationship between the two distribution shapes, as described for the two cases above, should effect the top Moderated Assessment. For this reason, the Board elected to set the top Moderated Assessment equal to the top Examination Mark scored by the school group. This provided one constraint for fixing the equation of the quadratic polynomial.

A second constraint required by the Board was that, under normal circumstances, the mean of the Moderated Assessments be set equal to that of the Examination Marks. This condition also held under the long-standing linear model. Schools are used to this condition and often think about it in the following way: students in a school group sit for the examination and earn a certain total of marks. This total of marks is then returned to the school for distributing as the school sees fit, apportioned to students according to performance on the school’s assessment program.

These two constraints leave only one further condition to be devised which will determine the equation of the quadratic polynomial. This condition was taken to be that the bottom Moderated Assessment be set equal to the bottom Examination Mark obtained by the school group. This condition could not always be applied as it sometimes created moderation curves that were not monotonic increasing (see Section 4 below). It also required that a check be made to ensure that the bottom Examination Mark is not such an atypically poor performance that an outrider mark is obtained.

To summarise: the following three conditions are used to determine the constants of the quadratic polynomial.

A. The maximum Moderated Assessment in the school group is equal to the maximum Examination Mark in the group.

B. The mean of the school group Moderated Assessments is equal to the mean of the Examination Marks.

C. If possible, the minimum Moderated Assessment in the school group is equal to the minimum Examination Mark in the group.

Condition C is modified in the following three cases.

C1 Atypically Low Minimum Mark

A test is made as to whether the minimum school group Examination Mark is much lower than expected in relation to the pattern of Raw Assessments. When this occurs both the minimum Examination Mark and minimum Raw Assessment (not necessarily belonging to the same candidate) are excluded from the analysis and conditions A, B and C re-applied to the reduced group.

The minimum Raw Assessment is then moderated by extrapolation. (For extrapolation procedures see Section 5.2.)

The test *estimates* a minimum Examination Mark using the principle that the difference between the second lowest and lowest Examination Marks (in standard deviation units) should not be grossly dissimilar to the difference between the second lowest and lowest Raw Assessments (in standard deviation units).

In symbols the *estimated* lowest Examination Mark is given by

$$E_1(est) = E_2 + \left(\frac{s_E}{s_x}\right)(x_1 - x_2)$$

where x_1 and x_2 are the lowest and second lowest Raw Assessments, E_2 is the second lowest Examination Mark, s_E and s_x are the standard deviations of the Examination Marks and Raw Assessments.

If the difference between $E_1(est)$ and E_1, the lowest Examination Mark, exceeds a certain value then the result is considered atypically low. Currently this value is set at 7.5 marks per unit.

C2 Inadequate Discrimination due to Shallow Slope

In some cases, the slope of the curve at an endpoint is too shallow to give adequate discrimination between the Moderated Assessments *in that region.* At the HSC, the Moderated Assessments are held on file to the nearest 0.1 of a mark. It requires a slope at the endpoint of at least 0.1 of a mark to ensure that Raw Assessments in this region, which differ by one mark, are separated by at least 0.1 of a mark in the Moderated Assessments.

If the slope at the minimum Raw Assessment (curve concave up) or at the maximum Raw Assessment (curve concave down) is *between 0 and 0.1*, then a *new* minimum Moderated Assessment is calculated which sets the slope at 0.1 exactly. The procedures for calculating this new minimum Moderated Assessment are given in Section 4.

It is not always possible to avoid clumping in the Moderated Assessments. For example, a set of Raw Assessments with a large spread may have a corresponding set of Examination Marks with a very small spread. In this case, even a linear conversion may have clumping.

Sometimes in such cases, the setting of an endpoint slope at 0.1 causes the curve to change concavity; i.e. the curve flips from concave up to concave down (or vice versa). This occurs because the curve is nearly linear: a slight change in gradient is sufficient for it to 'flip over'. In these cases, the gradient is set to the appropriate value (less than 0.1) such that a straight line is generated. The calculated value of the minimum Moderated Assessment that generates this straight line is given in Equation 26 (Section 4.3).

C3 Curve not Monotonic Increasing

Quadratic moderation can produce a non monotonic curve if the skew of the school group Raw Assessments is markedly different from that of the group's Examination Marks. In this case the minimum Moderated Assessment is determined by calculation rather than by set equal to the minimum Examination Mark. This calculation determines a value of the minimum Moderated Assessment such that the slope at the appropriate endpoint (minimum Raw Assessment for concave up, maximum Raw Assessment for concave down) is set at 0.1. The procedures for calculating this new minimum Moderated Assessment are given in Section 4.

3. Derivation of the Quadratic Polynomial Constants

Notation

Raw Assessments:		x	
	Maximum:	h	
	Minimum:	l	
	Mean:	m	
	SD:	s	
Moderated Assessments:		y	
	Maximum:	H	(the maximum Examination Mark)
	Minimum:	L	(the minimum Examination Mark)
	Mean:	M	(the mean of the Examination Marks)

The general equation of the polynomial is given by

$$y = ax^2 + bx + c .$$

Using Condition A,

$$H = ah^2 + bh + c . \tag{1}$$

Using Condition C,

$$L = al^2 + bl + c . \tag{2}$$

Now the mean of the Moderated Assessments is given by

$$\frac{\sum_{i=1}^{N} y_i}{N} = \frac{\sum_{i=1}^{N}\left(ax_i^2 + bx_i + c\right)}{N} .$$

Using Condition B to substitute with M on the left hand side and extracting the constants from the summations on the right hand side gives

$$M = a\frac{\sum_{i=1}^{N} x_i^2}{N} + b\frac{\sum_{i=1}^{N} x_i}{N} + c .$$

Using the basic formulae for the standard deviation and mean, this simplifies to

$$M = a(s^2 + m^2) + bm + c . \tag{3}$$

From Equations 1 and 3 we eliminate c

$$H - M = a(h^2 - s^2 - m^2) + b(h - m) .$$

Therefore, making the coefficient of b equal to unity we obtain

$$\frac{H-M}{h-m} = \frac{a(h^2 - s^2 - m^2)}{(h-m)} + b\,. \tag{4}$$

Similarly, from Equations 2 and 3

$$\frac{L-M}{l-m} = \frac{a(l^2 - s^2 - m^2)}{(l-m)} + b\,. \tag{5}$$

Solving Equations 4 and 5 simultaneously gives

$$a = \frac{H(l-m) - L(h-m) + M(h-l)}{(h-l)\left[s^2 + (h-m)(l-m)\right]}\,. \tag{6}$$

From Equation 5, b is written as a function of a

$$b = \frac{L - M - a(l^2 - m^2 - s^2)}{l-m}\,. \tag{7}$$

From Equation 2, c is written as a function of a and b

$$c = L - l(al + b)\,. \tag{8}$$

4. Calculation of the Minimum Moderated Assessment

This is required if Conditions C2 or C3 apply, as discussed in Section 2.

4.1 Curve is concave up ($a > 0$)

For the concave up case we require the slope at the lowest point of the assessment range, at $x = l$, to be a small positive number, k, so that Raw Assessments in this region, that are only one mark apart, produce Moderated Assessments that are distinct marks. Currently, k is set at a value of 0.1.

Differentiating the equation of the quadratic with respect to x we obtain a general expression for the slope

$$\frac{\partial y}{\partial x} = 2ax + b\,.$$

At $x = l$, the slope is equal to k which yields

$$2al + b = k\,. \tag{9}$$

Substituting from Equation 7 this gives

$$2al + \frac{L - M - a(l^2 - m^2 - s^2)}{l-m} = k\,. \tag{10}$$

After some algebra we obtain

$$L = M - a\left[(l-m)^2 + s^2\right] + k(l-m). \tag{11}$$

Let F_1 be defined as follows

$$F_1 = (l-m)^2 + s^2. \tag{12}$$

Therefore

$$L = M - aF_1 + k(l-m). \tag{13}$$

As a is a function of L (see Equation 6), it is necessary to substitute for a. Before doing this, however, we abbreviate the formulae by introducing the following definitions:

$$d_1 = h - m, \tag{14}$$

$$d_2 = l - m, \tag{15}$$

$$d_3 = h - l, \tag{16}$$

$$d_4 = l + m. \tag{17}$$

Now substituting Equations 14-17 into Equation 6 and expressing a as a linear function of L we may write

$$a = pL + q \tag{18}$$

where

$$p = \frac{-d_1}{d_3\left[s^2 + d_1 d_2\right]} \tag{19}$$

$$q = \frac{d_2 H + d_3 M}{d_3\left[s^2 + d_1 d_2\right]}. \tag{20}$$

Substituting Equation 18 into Equation 13 we obtain

$$L = M - (pL + q)F_1 + kd_2.$$

Solving for L we obtain

$$L = \frac{M - qF_1 + kd_2}{1 + pF_1}. \tag{21}$$

This calculated value of L replaces the observed value of the minimum Examination Mark in determining the equation of the moderation curve.

4.2 Curve is concave down ($a < 0$)

For the concave down case we require the slope at the highest point of the assessment range, at $x = h$, to be a small positive number, k. For discrimination purposes in the Moderated Assessments, k is again set equal to 0.1.

This gives the following expression

$$2ah + b = k\ .$$

Therefore, substituting for b from Equation 7 gives

$$2ah + \frac{L - M - a(l^2 - m^2 - s^2)}{l - m} = k\ .$$

After some algebra, we make L the subject of the expression

$$L = M - a\left[(m-h)^2 - (h-l)^2 + s^2\right] + k(l-m)\ . \qquad (22)$$

To simplify the equations we then define F_2 as follows

$$F_2 = (m-h)^2 - (h-l)^2 + s^2. \qquad (23)$$

Substituting in Equation 22 from Equations 23 and 15 gives

$$L = M - aF_2 + kd_2. \qquad (24)$$

We now eliminate a, which is a function of L, from Equation 24 by substituting from Equation 18, to obtain an expression for our calculated value of L

$$L = \frac{M - qF_2 + kd_2}{1 + pF_2}. \qquad (25)$$

4.3 Change of Concavity with New Minimum

As discussed in relation to Condition C2, an attempt to change an endpoint gradient from a value between 0 and 0.1 to 0.1 itself may on rare occasions cause a change in concavity. This occurs when the curve is close to horizontal and nearly a straight line.

Under these circumstances, it is best to calculate L such that a straight line is generated. This requires setting the constant a to zero exactly (it is already near zero).

From Equation 18 this gives

$$L = \frac{-q}{p}. \qquad (26)$$

5. Other Features

5.1 Ties on the Maximum or Minimum Assessments

If the top n Raw Assessments for a school group are tied on the one mark, then the top Moderated Assessment is established by averaging the top n Examination Marks. For example, if the top two Raw Assessments are tied, and the top two Examination Marks are 84 and 80, then the top Moderated Assessment is set at 82. A similar scheme holds for the case where the bottom Raw Assessments are tied.

5.2 Interpolation/Extrapolation of Exclusions

Certain students, such as Illness/Misadventure applicants and other special cases, may be excluded from the school group when the equation of the moderation curve is determined. After the curve has been determined, they receive their Moderated Assessments by interpolation or extrapolation. Interpolation (within the range of the curve) is performed by use of the quadratic polynomial. Extrapolation (outside the range of the curve) is done in a linear fashion to avoid possible problems of a non monotonic adjustment beyond the current range of the curve.

If the Raw Assessment to be extrapolated is greater than the maximum Raw Assessment used, then the following are used as the anchor points (for assessments out of 50):

$$\begin{array}{lll} 50 & \rightarrow & 50 \\ h & \rightarrow & H. \end{array}$$

where h is the maximum Raw Assessment used and H is the current maximum Moderated Assessment.

The corresponding equation is

$$y = H + \frac{(50 - H)(x - h)}{(50 - h)}. \tag{27}$$

If the Raw Assessment to be extrapolated is less than the minimum Raw Assessment used, then the following are used as the anchor points:

$$\begin{array}{lll} l & \rightarrow & L \\ 0 & \rightarrow & 0. \end{array}$$

where l is the minimum Raw Assessment used and L is the current minimum Moderated Assessment.

The corresponding equation is

$$y = \left(\frac{L}{l}\right) x. \tag{28}$$

6. Special Cases

The following cases cannot be handled by the quadratic polynomial formulae as they would involve a division by zero. An algebraic analysis of these cases is given in Appendix B. Note that the procedures in 6.1, 6.2 and 6.3 have been used for many years at the HSC when a linear moderation was in operation.

6.1 n = 1 Case

For the case of single candidate in the school group, the Moderated Assessment is set equal to the student's Examination Mark.

6.2 n = 2 Case

For the case of two students in the school group, the following applies:

* The maximum Moderated Assessment is set equal to the maximum Examination Mark.

* The Minimum Moderated Assessment is set equal to either the minimum Examination Mark or W, *whichever is the higher,* where

$$W = \left(\frac{l}{h}\right) H . \tag{29}$$

6.3 All Assessments Tied

For the case where all assessments are tied, each Moderated Assessment is equal to the average of all the Examination Marks.

6.4 Only Two Distinct Assessment Values

In this case there may be more than two students in the group but there are only two distinct Raw Assessment values. For example, the series of Raw Assessments (70, 70, 70, 63, 63) has only the two *distinct* values of 70 and 63, despite the fact that there are five assessments. Here Equation 6, which gives the constant a, has a denominator which is undefined. For a mathematical analysis of this case, see Appendix B. When this occurs, an alternative set of formulae is used to calculate the three constants of the quadratic polynomial as follows:

$$a = 0 \tag{30}$$

$$b = \frac{(H - L)}{(h - l)} \tag{31}$$

$$c = L - bl . \tag{32}$$

where h, l are the highest and lowest Raw Assessments, and H, L are the highest and lowest Moderated Assessments.

Summary of Quadratic Formulae

Raw Assessment: (x)		**Moderated Assessment**: (y)		
Maximum:	h	Maximum:	H	(Exam Maximum)
Minimum:	l	Minimum:	L	(Exam Minimum)
Mean:	m	Mean:	M	(Exam Mean)
SD:	s			

General equation: $y = ax^2 + bx + c$

where

$$d_1 = h - m$$
$$d_2 = l - m$$
$$d_3 = h - l$$
$$d_4 = l + m$$

$$a = \frac{d_2 H - d_1 L + d_3 M}{d_3 \left[s^2 + d_1 d_2 \right]}$$

$$b = \frac{L - M + a\left[s^2 - d_2 d_4 \right]}{d_2}$$

$$c = L - l(al + b)$$

Modification of L for Conditions C2 and C3

k is the slope at $x = l$ (concave up) or at $x = h$ (concave down). Currently, k =0.1.

$$L = \frac{M - qF + kd_2}{1 + pF}$$

where

$$p = \frac{-d_1}{d_3\left[s^2 + d_1 d_2 \right]}, \qquad q = \frac{d_2 H + d_3 M}{d_3 \left[s^2 + d_1 d_2 \right]}$$

and F has the following values:

Case A (concave up): $F = d_2{}^2 + s^2$

Case B (concave down): $F = d_1{}^2 - d_3{}^2 + s^2$

If the concavity changes when L is calculated as above, then set

$$L = \frac{-q}{p}$$

References

Angoff, W.H. (1971). Scales, norms and equivalent scores. In R.L. Thorndike (Ed.), *Educational Measurement* (2nd ed.). Washington, D.C.: American Council on Education, 508-600

Braun, H.I. and Holland, P.W. (1982). Observed-score test equating: a mathematical analysis of some ETS equating procedures. In P.W. Holland and D.B. Rubin (Eds.), *Test Equating*. New York: Academic Press, 9-49.

Howard, M. (1958). The conversion of scores to a uniform scale. *The British Journal of Statistical Psychology*, Vol. XI, Part II, 199-206.

APPENDIX A

Reduction of the Curvilinear Method to the
Linear Method when the Raw Assessments and the Examination Marks have the Same Distribution Shapes

Symbols Used

Raw Assessment:	x
Curvilinear moderated assessment:	y
Linear moderated assessment:	Y
Examination mark:	z
Re-ordered Examination mark:	z'

Overview of the Proof

The basic assumption for this demonstration is that the distribution shape of the Raw Assessments is equal to that of the Examination Marks. The equivalence of these shapes still allows these two variables to differ in their rank order, their mean score, and their standard deviation.

Let z' be the measure obtained by re-ordering the Examination Mark to the same order as the Raw Assessment. That is, by assigning the top Examination Mark to the student with the top Raw Assessment, the second top Examination Mark to the student with the second top Raw Assessment, and so on.

Given the basic assumption stated above, it will first be established that the Linear (mean and standard deviation method) Moderated Assessment will be equal to z', the re-ordered Examination Mark. Secondly, it will be shown that the Curvilinear Moderated Assessment will be also equal to z'. Since both Linear and Curvilinear Moderated Assessments are equal to z', then it follows that they are equal to each other.

Equivalence of the Linear Moderation and Re-ordered Examination Mark

If the distribution shapes of x and z are identical, it follows that the shapes of x and z' are identical, as the re-ordering does not change the shape of z'. As x and z' have the same rank order and the same distribution shape, then this implies that they have a linear relationship. That is

$$x = uz' + v \qquad \text{A1}$$

where u, v are constants and $u > 0$.

The Linear Moderated Assessment, Y, is a linear function of x. That is

$$Y = fx + g \qquad \text{A2}$$

where f, g are constants and $f > 0$.

From Equations A1 and A2 we obtain

$$Y = fuz' + v + g. \qquad \text{A3}$$

From Equation A3, the standard deviation of Y is given by

$$s_Y = fus_{z'}.$$

But the standard deviation of Y is equal to that of z (and hence that of z') by the properties of the moderation method. Therefore it must follow that

$$fu = 1. \qquad \text{A4}$$

Taking the mean of each side of Equation A3 and substituting from Equation A4 we obtain

$$\overline{Y} = \overline{z'} + v + g.$$

But the mean of Y is equal to that of z (and hence that of z') by the properties of the moderation method. Therefore it must follow that

$$v + g = 0. \qquad \text{A5}$$

Therefore from Equations A3, A4 and A5

$$Y = z'. \qquad \text{A6}$$

Thus, the Linear Moderated Assessment is equal to the re-ordered Examination Mark.

Equivalence of the Curvilinear Moderation and Re-ordered Examination Mark

From Equation A1 the following three equations are derived by (respectively) taking the mean of both sides, substituting the maximum scores, and substituting the minimum scores:

$$m = uM + v \qquad \text{A7}$$

$$h = uH + v \qquad \text{A8}$$

$$l = uL + v. \qquad \text{A9}$$

Now by definition from Equation 14

$$d_1 = h - m.$$

Substituting from Equations A7 and A8 gives

$$d_1 = u(H - M). \qquad \text{A10}$$

Also by definition from Equation 15

$$d_2 = l - m.$$

Substituting from Equations A7 and A9

$$d_2 = u(L - M). \qquad \text{A11}$$

Also by definition from Equation 16

$$d_3 = h - l.$$

Substituting from Equations A8 and A9

$$d_3 = u(H - L)\,. \qquad \text{A12}$$

Substituting Equations A10, A11 and A12 into the formulae for the constants a, b and c (see Section 7) gives

$$a = 0 \qquad \text{A13}$$

$$b = \frac{1}{u} \qquad \text{A14}$$

$$c = L - \frac{1}{u}\,. \qquad \text{A15}$$

The curvilinear moderation polynomial is given by

$$y = ax^2 + bx + c\,.$$

Substituting from Equations A13, A14 and A15 gives

$$uy = x + uL - l\,. \qquad \text{A16}$$

Substituting in Equation A16 from Equation A9 gives

$$uy = x + l - v - l\,.$$

Therefore

$$x = uy + v\,. \qquad \text{A17}$$

But from Equation A1

$$x = uz' + v\,.$$

It therefore follows that

$$y = z'\,. \qquad \text{A18}$$

That is, the Curvilinear Moderated Assessment is equal to the re-ordered Examination Mark. It therefore follows from Equation A6 that the Curvilinear Moderated Assessment is equal to the Linear Moderated Assessment under the assumptions made.

APPENDIX B

Analysis of the Special Cases of Section 6

Analysis of the Special Cases of Section 6

The formulae derived for the quadratic polynomial constants are not appropriate for the cases of Section 6. This can be seen from Equation 6.

For both the $n = 1$ case (Section 6.1) and the case where all assessments are tied (Section 6.3), $h = l$, $s = 0$, $h = m$, $l = m$, so that the constant a is undefined.

The $n = 2$ case (Section 6.2) is actually a special case of the category where there are only two distinct assessment values (Section 6.4). Therefore it is sufficient to treat the latter case.

Suppose that there are n_1 Raw Assessments having the value h and n_2 Raw Assessments having the value l. That is, the Raw Assessments have only two distinct values. We will now consider the components of the denominator of Equation 6 and demonstrate that the denominator is zero under the above condition.

The variance of the assessments appears in the denominator and is given by

$$s^2 = \frac{n_1 h^2 + n_2 l^2}{n_1 + n_2} - m^2.$$

The other term in the denominator is the product $(h - m)(l - m)$. Upon expansion we obtain

$$(h - m)(l - m) = m^2 + hl - m(h + l).$$

Therefore the denominator of Equation 6 is given by

$$\begin{aligned}
s^2 + (h - m)(l - m) \quad &= \quad \frac{n_1 h^2 + n_2 l^2}{n_1 + n_2} + hl - m(h + l) \\
&= \quad \frac{(n_1 h + n_2 l)(h + l)}{n_1 + n_2} - m(h + l) \\
&= \quad (h + l)\left[\frac{(n_1 h + n_2 l)}{n_1 + n_2} - m\right] \\
&= \quad (h + l)(m - m) \\
&= \quad 0.
\end{aligned}$$

As the denominator is zero, the constant a is again undefined. The values of the constants a, b and c given in Section 6.4 are obtained by a linear conversion between the points (l, L) and (h, H).

This linear conversion gives

$$y = L + \left(\frac{H - L}{h - l}\right)(x - l).$$

Expressing this in quadratic form so that the constants are displayed, we obtain

$$y = 0x^2 + \left(\frac{H-L}{h-l}\right)x + L - \left(\frac{H-L}{h-l}\right)l .$$

By inspection, the three constants are given by

$$a = 0 ,$$

$$b = \frac{(H-L)}{(h-l)} ,$$

$$c = L - bl .$$