CHAPTER 11
Analytic Geometry in Three Dimensions

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CHAPTER 11
Analytic Geometry in Three Dimensions

Section 11.1 The Three-Dimensional Coordinate System

- You should be able to plot points in the three-dimensional coordinate system.
- The distance between the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is
  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.
- The midpoint of the line segment joining the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is
  $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}, \dfrac{z_1 + z_2}{2}\right)$.
- The equation of the sphere with center (h, k, j) and radius r is
  $(x - h)^2 + (y - k)^2 + (z - j)^2 = r^2$.
- You should be able to find the trace of a surface in space.

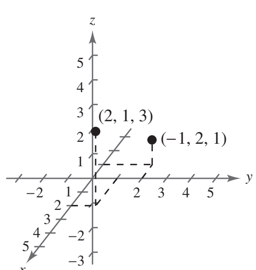
Vocabulary Check

1. three-dimensional	**2.** xy-plane, xz-plane, yz-plane	**3.** octants
4. Distance Formula	**5.** $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}, \dfrac{z_1 + z_2}{2}\right)$	**6.** sphere
7. surface, space	**8.** trace	

1. $A(-1, 4, 3)$, $B(1, 3, -2)$, $C(-3, 0, -2)$

2. $A(6, 2, -3)$, $B(2, -1, 2)$ $C(-2, 3, 0)$

3.



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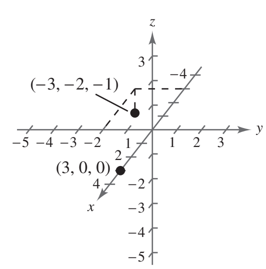

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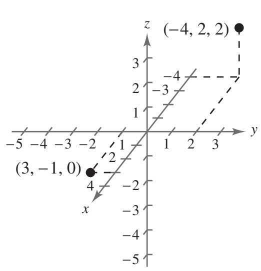

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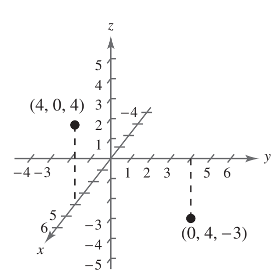


7. $x = -3, y = 3, z = 4$: $(-3, 3, 4)$

8. $x = 6, y = -1, z = -1 \implies (6, -1, -1)$

9. $y = z = 0, x = 10$: $(10, 0, 0)$

10. $x = 0, y = 2, z = 8 \implies (0, 2, 8)$

11. Octant IV

12. Octant VI

13. Octants I, II, III, IV (above the xy-plane)

14. Octants III, IV, VII, or VIII

15. Octants II, IV, VI, VIII

16. Octants I, II, VII, or VIII

17. $$\begin{aligned} d &= \sqrt{(5-0)^2 + (2-0)^2 + (6-0)^2} \\ &= \sqrt{25 + 4 + 36} \\ &= \sqrt{65} \text{ units} \end{aligned}$$

18. $$\begin{aligned} d &= \sqrt{(7-1)^2 + (0-0)^2 + (4-0)^2} \\ &= \sqrt{36 + 16} \\ &= \sqrt{52} \\ &= 2\sqrt{13} \end{aligned}$$

19. $$\begin{aligned} d &= \sqrt{(7-3)^2 + (4-2)^2 + (8-5)^2} \\ &= \sqrt{4^2 + 2^2 + 3^2} \\ &= \sqrt{16 + 4 + 9} \\ &= \sqrt{29} \\ &\approx 5.385 \end{aligned}$$

20. $$\begin{aligned} d &= \sqrt{(4-2)^2 + (1-1)^2 + (9-6)^2} \\ &= \sqrt{4 + 9} \\ &= \sqrt{13} \end{aligned}$$

21. $$\begin{aligned} d &= \sqrt{[6-(-1)]^2 + [0-4]^2 + [-9-(-2)]^2} \\ &= \sqrt{7^2 + 4^2 + 7^2} \\ &= \sqrt{49 + 16 + 49} \\ &= \sqrt{114} \\ &\approx 10.677 \end{aligned}$$

22. $$\begin{aligned} d &= \sqrt{(1-(-2))^2 + (1-(-3))^2 + (-7-(-7))^2} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

23. $$\begin{aligned} d &= \sqrt{(1-0)^2 + [0-(-3)]^2 + (-10-0)^2} \\ &= \sqrt{1 + 9 + 100} \\ &= \sqrt{110} \approx 10.488 \end{aligned}$$

24. $$\begin{aligned} d &= \sqrt{(2-0)^2 + (-4-6)^2 + (0-(-3))^2} \\ &= \sqrt{4 + 100 + 9} \\ &= \sqrt{113} \end{aligned}$$

25. $$\begin{aligned} d_1 &= \sqrt{(-2-0)^2 + (5-0)^2 + (2-2)^2} = \sqrt{4 + 25} = \sqrt{29} \\ d_2 &= \sqrt{(0-0)^2 + (4-0)^2 + (0-2)^2} = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5} \\ d_3 &= \sqrt{(0+2)^2 + (4-5)^2 + (0-2)^2} = \sqrt{4 + 1 + 4} = \sqrt{9} = 3 \\ d_1{}^2 &= d_2{}^2 + d_3{}^2 = 29 \end{aligned}$$

26. $$\begin{aligned} d_1 &= \sqrt{(-4-2)^2 + (4+1)^2 + (1-2)^2} = \sqrt{36 + 25 + 1} = \sqrt{62} \\ d_2 &= \sqrt{(-4+2)^2 + (4-5)^2 + (1-0)^2} = \sqrt{4 + 1 + 1} = \sqrt{6} \\ d_3 &= \sqrt{(2+2)^2 + (-1-5)^2 + (2-0)^2} = \sqrt{16 + 36 + 4} = \sqrt{56} = 2\sqrt{14} \\ d_1^2 &= d_2^2 + d_3^2 = 62 \end{aligned}$$

27. $d_1 = \sqrt{(2-0)^2 + (2-0)^2 + (1-0)^2} = \sqrt{4+4+1} = \sqrt{9} = 3$

$d_2 = \sqrt{(2-0)^2 + (-4-0)^2 + (4-0)^2} = \sqrt{4+16+16} = \sqrt{36} = 6$

$d_3 = \sqrt{(2-2)^2 + (-4-2)^2 + (4-1)^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$

$d_1{}^2 + d_2{}^2 = 9 + 36 = 45 = d_3{}^2$

28. $d_1 = \sqrt{(1-1)^2 + (3-0)^2 + (1-1)^2} = 3$

$d_2 = \sqrt{(1-1)^2 + (0-0)^2 + (3-1)^2} = 2$

$d_3 = \sqrt{(1-1)^2 + (0-3)^2 + (3-1)^2} = \sqrt{13}$

$d_3^2 = 13 = d_1^2 + d_2^2$

29. $d_1 = \sqrt{(5-1)^2 + (-1+3)^2 + (2+2)^2} = \sqrt{16+4+16} = \sqrt{36} = 6$

$d_2 = \sqrt{(5+1)^2 + (-1-1)^2 + (2-2)^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$

$d_3 = \sqrt{(-1-1)^2 + (1+3)^2 + (2+2)^2} = \sqrt{4+16+16} = \sqrt{36} = 6$

$d_1 = d_3$ Isosceles triangle

30. $d_1 = \sqrt{(7-5)^2 + (1-3)^2 + (3-4)^2} = \sqrt{4+4+1} = \sqrt{9} = 3$

$d_2 = \sqrt{(3-7)^2 + (5-1)^2 + (3-3)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

$d_3 = \sqrt{(3-5)^2 + (5-3)^2 + (3-4)^2} = \sqrt{4+4+1} = \sqrt{9} = 3$

$d_1 = d_3 = 3$. Isosceles triangle

31. $\left(\frac{3+0}{2}, \frac{-2+0}{2}, \frac{4+0}{2}\right) = \left(\frac{3}{2}, -1, 2\right)$

32. Midpoint: $\left(\frac{1+2}{2}, \frac{5+2}{2}, \frac{-1+2}{2}\right) = \left(\frac{3}{2}, \frac{7}{2}, \frac{1}{2}\right)$

33. Midpoint: $\left(\frac{3-3}{2}, \frac{-6+4}{2}, \frac{10+4}{2}\right) = (0, -1, 7)$

34. Midpoint: $\left(\frac{-1+3}{2}, \frac{5+7}{2}, \frac{-3-1}{2}\right) = (1, 6, -2)$

35. Midpoint: $\left(\frac{6-4}{2}, \frac{-2+2}{2}, \frac{5+6}{2}\right) = \left(1, 0, \frac{11}{2}\right)$

36. Midpoint: $\left(\frac{-3-6}{2}, \frac{5+4}{2}, \frac{5+8}{2}\right) = \left(-\frac{9}{2}, \frac{9}{2}, \frac{13}{2}\right)$

37. Midpoint: $\left(\frac{-2+7}{2}, \frac{8-4}{2}, \frac{10+2}{2}\right) = \left(\frac{5}{2}, 2, 6\right)$

38. Midpoint: $\left(\frac{9+9}{2}, \frac{-5-2}{2}, \frac{1-4}{2}\right) = \left(9, -\frac{7}{2}, -\frac{3}{2}\right)$

39. $(x-3)^2 + (y-2)^2 + (z-4)^2 = 16$

40. $(x+3)^2 + (y-4)^2 + (z-3)^2 = 4$

41. $(x-0)^2 + (y-4)^2 + (z-3)^2 = 3^2$

$x^2 + (y-4)^2 + (z-3)^2 = 9$

42. $(x-2)^2 + (y+1)^2 + (z-8)^2 = 36$

43. Radius $= \frac{\text{Diameter}}{2} = 5$

$(x+3)^2 + (y-7)^2 + (z-5)^2 = 5^2 = 25$

44. Radius $= \frac{\text{Diameter}}{2} = 4$: $(x-0)^2 + (y-5)^2 + (z+9)^2 = 4^2 = 16$

45. Center: $\left(\frac{3+0}{2}, \frac{0+0}{2}, \frac{0+6}{2}\right) = \left(\frac{3}{2}, 0, 3\right)$

Radius: $\sqrt{\left(3 - \frac{3}{2}\right)^2 + (0-0)^2 + (0-3)^2} = \sqrt{\frac{9}{4} + 9} = \sqrt{\frac{45}{4}}$

Sphere: $\left(x - \frac{3}{2}\right)^2 + (y-0)^2 + (z-3)^2 = \frac{45}{4}$

46. Center: $\left(\frac{2-1}{2}, \frac{-2+4}{2}, \frac{2+6}{2}\right) = \left(\frac{1}{2}, 1, 4\right)$

Radius: $\sqrt{\left(2 - \frac{1}{2}\right)^2 + (-2-1)^2 + (2-4)^2} = \sqrt{\frac{9}{4} + 9 + 4} = \sqrt{\frac{61}{4}}$

Sphere: $\left(x - \frac{1}{2}\right)^2 + (y-1)^2 + (z-4)^2 = \frac{61}{4}$

47. $\left(x^2 - 5x + \frac{25}{4}\right) + y^2 + z^2 = \frac{25}{4}$

$\left(x - \frac{5}{2}\right)^2 + y^2 + z^2 = \frac{25}{4}$

Center: $\left(\frac{5}{2}, 0, 0\right)$

Radius: $\frac{5}{2}$

48. $x^2 + y^2 - 8y + 16 + z^2 = 16$

$x^2 + (y-4)^2 + z^2 = 16$

Center: $(0, 4, 0)$

Radius: 4

49. $(x^2 - 4x + 4) + (y^2 + 2y + 1) + (z^2 - 6z + 9) = -10 + 4 + 1 + 9$

$(x-2)^2 + (y+1)^2 + (z-3)^2 = 4$

Center: $(2, -1, 3)$

Radius: 2

50. $(x^2 - 6x + 9) + (y^2 + 4y + 4) + z^2 = -9 + 9 + 4$

$(x-3)^2 + (y+2)^2 + z^2 = 4$

Center: $(3, -2, 0)$

Radius: 2

51. $(x^2 + 4x + 4) + y^2 + (z^2 - 8z + 16) = -19 + 4 + 16$

$(x+2)^2 + y^2 + (z-4)^2 = 1$

Center: $(-2, 0, 4)$

Radius: 1

52. $x^2 + (y^2 - 8y + 16) + (z^2 - 6z + 9) = -13 + 16 + 9$

$x^2 + (y-4)^2 + (z-3)^2 = 12$

Center: $(0, 4, 3)$

Radius: $\sqrt{12} = 2\sqrt{3}$

53. $$x^2 + y^2 + z^2 - 2x - \tfrac{2}{3}y - 8z = -\tfrac{73}{9}$$
$$(x^2 - 2x + 1) + \left(y^2 - \tfrac{2}{3}y + \tfrac{1}{9}\right) + (z^2 - 8z + 16) = -\tfrac{73}{9} + 1 + \tfrac{1}{9} + 16$$
$$(x - 1)^2 + \left(y - \tfrac{1}{3}\right)^2 + (z - 4)^2 = 9$$

Center: $\left(1, \tfrac{1}{3}, 4\right)$

Radius: 3

54. $x^2 + y^2 + z^2 - x - 3y - 2z = -\tfrac{5}{2}$
$$\left(x^2 - x + \tfrac{1}{4}\right) + \left(y^2 - 3y + \tfrac{9}{4}\right) + (z^2 - 2z + 1) = -\tfrac{5}{2} + \tfrac{1}{4} + \tfrac{9}{4} + 1$$
$$\left(x - \tfrac{1}{2}\right)^2 + \left(y - \tfrac{3}{2}\right)^2 + (z - 1)^2 = 1$$

Center: $\left(\tfrac{1}{2}, \tfrac{3}{2}, 1\right)$

Radius: 1

55. $$9x^2 - 6x + 9y^2 + 18y + 9z^2 = -1$$
$$x^2 - \tfrac{2}{3}x + \tfrac{1}{9} + y^2 + 2y + 1 + z^2 = -\tfrac{1}{9} + \tfrac{1}{9} + 1$$
$$\left(x - \tfrac{1}{3}\right)^2 + (y + 1)^2 + z^2 = 1$$

Center: $\left(\tfrac{1}{3}, -1, 0\right)$

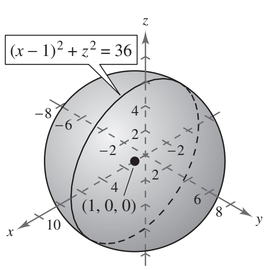
Radius: 1

56. $$x^2 - x + \frac{1}{4} + y^2 - 8y + 16 + z^2 + 2z + 1 = \frac{-33}{4} + \frac{1}{4} + 16 + 1$$
$$\left(x - \frac{1}{2}\right)^2 + (y - 4)^2 + (z + 1)^2 = 9$$

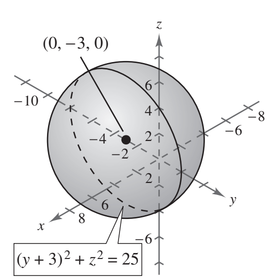
Center: $\left(\dfrac{1}{2}, 4, -1\right)$

Radius: 3

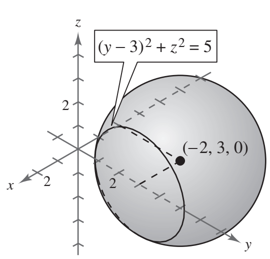
57.



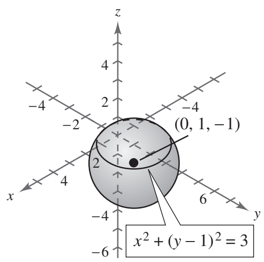
58. yz − trace $(x = 0)$: $(y + 3)^2 + z^2 = 25$ Circle



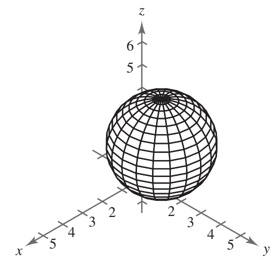
59.



60. xy − trace $(z = 0)$: $x^2 + (y - 1)^2 = 3$ Circle



61.

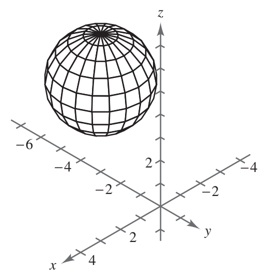


62. $x^2 + y^2 + 6y + (z^2 - 8z + 16) = -21 + 16$

$x^2 + y^2 + 6y + (z - 4)^2 = -5$

$z_1 = 4 + \sqrt{-5 - x^2 - y^2 - 6y}$

$z_2 = 4 - \sqrt{-5 - x^2 - y^2 - 6y}$



63. The length of each side is 3. Thus, $(x, y, z) = (3, 3, 3)$.

64. $x = 4, y = 4, z = 8 \quad (4, 4, 8)$

65. $d = 165 \Rightarrow r = \frac{165}{2} = 82.5$

$x^2 + y^2 + z^2 = \left(\frac{165}{2}\right)^2$

66. (a) $x^2 + y^2 + z^2 = 3963^2$.

(b) Assume the north and south poles are on the z-axis. Lines of longitude that run north-south are traces of planes containing the z-axis. These shapes are circles of radius 3963 miles.

(c) Latitudes are traces of planes perpendicular to the z-axis. These shapes are circles.

(d) The prime meridian is a trace of a plane containing the z-axis. It is a semi-circular arc running from pole to pole.

(e) The equator is the trace of the plane containing the x- and y-axes.

67. False. x is the directed distance from the yz-plane to P.

68. False. The trace could be a single point, or empty.

69. In the xy-plane, the z-coordinate is 0.
In the xz-plane, the y-coordinate is 0.
In the yz-plane, the x-coordinate is 0.

70. It is a plane.

71. The trace is a circle, or a single point.

72. The trace will be a line in the xy-plane (unless the plane is the xy-plane).

73. $x_m = \dfrac{x_2 + x_1}{2} \Rightarrow x_2 = 2x_m - x_1$

Similarly for y_2 and z_2,

$(x_2, y_2, z_2) = (2x_m - x_1, 2y_m - y_1, 2z_m - z_1)$.

74. $x_2 = 2x_m - x_1 = 2(5) - 3 = 7$

$y_2 = 2y_m - y_1 = 2(8) - 0 = 16$

$z_2 = 2z_m - z_1 = 2(7) - 2 = 12$

$(7, 16, 12)$

75. $v^2 + 3v + \frac{9}{4} = 2 + \frac{9}{4}$

$$\left(v + \frac{3}{2}\right)^2 = \frac{17}{4}$$

$$v + \frac{3}{2} = \pm\frac{\sqrt{17}}{2}$$

$$v = -\frac{3}{2} \pm \frac{\sqrt{17}}{2}$$

76. $z^2 - 7z + \frac{49}{4} = 19 + \frac{49}{4}$

$$\left(z - \frac{7}{2}\right)^2 = \frac{125}{4}$$

$$z - \frac{7}{2} = \pm\frac{5\sqrt{5}}{2}$$

$$z = \frac{7}{2} \pm \frac{5}{2}\sqrt{5}$$

77. $x^2 - 5x + \frac{25}{4} = -5 + \frac{25}{4}$

$$\left(x - \frac{5}{2}\right)^2 = \frac{5}{4}$$

$$x - \frac{5}{2} = \pm\frac{\sqrt{5}}{2}$$

$$x = \frac{5}{2} \pm \frac{\sqrt{5}}{2}$$

78. $x^2 + 3x + \frac{9}{4} = 1 + \frac{9}{4}$

$$\left(x + \frac{3}{2}\right)^2 = \frac{13}{4}$$

$$x + \frac{3}{2} = \pm\frac{\sqrt{13}}{2}$$

$$x = \frac{-3}{2} \pm \frac{\sqrt{13}}{2}$$

79. $4y^2 + 4y = 9$

$$y^2 + y + \frac{1}{4} = \frac{9}{4} + \frac{1}{4}$$

$$\left(y + \frac{1}{2}\right)^2 = \frac{10}{4}$$

$$y + \frac{1}{2} = \pm\frac{\sqrt{10}}{2}$$

$$y = -\frac{1}{2} \pm \frac{\sqrt{10}}{2}$$

80. $x^2 + \frac{5}{2}x + \frac{25}{16} = 4 + \frac{25}{16}$

$$\left(x + \frac{5}{4}\right)^2 = \frac{89}{16}$$

$$x + \frac{5}{4} = \pm\frac{\sqrt{89}}{4}$$

$$x = \frac{-5}{4} \pm \frac{\sqrt{89}}{4}$$

81. $\mathbf{v} = 3\mathbf{i} - 3\mathbf{j}$, Quadrant IV

$$\|\mathbf{v}\| = \sqrt{3^2 + (-3)^2} = \sqrt{18} = 3\sqrt{2}$$

$$\tan\theta = -\frac{3}{3} = -1 \implies$$

$$\theta = -45° \text{ or } 315°$$

82. $\mathbf{v} = \langle -1, 2\rangle$ Quadrant II

$$\|\mathbf{v}\| = \sqrt{1^2 + 2^2} = \sqrt{5}$$

$$\tan\theta = \frac{2}{-1} \implies \theta \approx 116.6°$$

83. $\mathbf{v} = 4\mathbf{i} + 5\mathbf{j}$, Quadrant I

$$\|\mathbf{v}\| = \sqrt{16 + 25} = \sqrt{41}$$

$$\tan\theta = \frac{5}{4} \implies \theta \approx 51.34°$$

84. $\mathbf{v} = \langle 10, -7\rangle$ Quadrant IV

$$\|\mathbf{v}\| = \sqrt{100 + 49} = \sqrt{149}$$

$$\tan\theta = \frac{-7}{10} \implies \theta \approx 325.0°$$

85. $\mathbf{u} \cdot \mathbf{v} = \langle -4, 1\rangle \cdot \langle 3, 5\rangle$

$$= -4(3) + 1(5)$$

$$= -7$$

86. $\mathbf{u} \cdot \mathbf{v} = \langle -1, 0\rangle \cdot \langle -2, -6\rangle$

$$= 2 + 0$$

$$= 2$$

87. $a_0 = 1, a_n = a_{n-1} + n^2$

$a_1 = 1 + 1^2 = 2$

$a_2 = 2 + 2^2 = 6$

$a_3 = 6 + 3^2 = 15$

$a_4 = 15 + 4^2 = 31$

	1		2		6		15		31
First differences:		1		4		9		16	
Second differences:			3		5		7		

Neither model

88. $a_0 = 0, a_n = a_{n-1} - 1$

$a_1 = 0 - 1 = -1$

$a_2 = -1 - 1 = -2$

$a_3 = -3$

$a_4 = -4$

	0		-1		-2		-3		-4
First difference		-1		-1		-1		-1	
Second difference			0		0		0		

Linear model

89. $a_1 = -1, a_n = a_{n-1} + 3$

$a_2 = -1 + 3 = 2$

$a_3 = 2 + 3 = 5$

$a_4 = 5 + 3 = 8$

$a_5 = 8 + 3 = 11$

	-1		2		5		8		11
First differences:		3		3		3		3	
Second differences:			0		0		0		

Linear model

90. $a_1 = 4, a_n = a_{n-1} - 2n$

$a_2 = 4 - 2(2) = 0$

$a_3 = 0 - 2(3) = -6$

$a_4 = -6 - 2(4) = -14$

$a_5 = -14 - 2(5) = -24$

	4		0		-6		-14		-24
First difference		-4		-6		-8		-10	
Second difference			-2		-2		-2		

Quadratic model

91. $(x + 5)^2 + (y - 1)^2 = 49$

92. $(x - 3)^2 + (y + 6)^2 = 81$

93. $(y - 1)^2 = 4p(x - 4),\ p = -3$

$(y - 1)^2 = 4(-3)(x - 4)$

$(y - 1)^2 = -12(x - 4)$

94. $(x - h)^2 = 4p(y - k)\ \ p = -5, (h, k) = (-2, 5)$

$(x + 2)^2 = 4(-5)(y - 5)$

$(x + 2)^2 = -20(y - 5)$

95. $a = 3, b = 2$, center: $(3, 3)$, horizontal major axis

$$\frac{(x - 3)^2}{9} + \frac{(y - 3)^2}{4} = 1$$

96. Center: $(0, 3)$ Vertical major axis length 9 $\implies a = \dfrac{9}{2}$

$c = 3 \implies b^2 = a^2 - c^2 = \dfrac{81}{4} - 9 = \dfrac{45}{4}$

$$\frac{(x-0)^2}{(45/4)} + \frac{(y-3)^2}{(81/4)} = 1$$

97. Center: $(6, 0)$, horizontal transverse axis

$a = 2, c = 6, b^2 = c^2 - a^2 = 36 - 4 = 32$

$$\frac{(x-6)^2}{4} - \frac{y^2}{32} = 1$$

98. Center: $(3, 5)$ Vertical transverse axis

$a = 4, c = 5, b^2 = c^2 - a^2 = 25 - 16 = 9$

$$\frac{(y-5)^2}{16} - \frac{(x-3)^2}{9} = 1$$

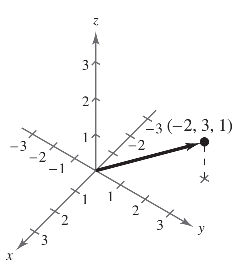
Section 11.2 Vectors in Space

- Vectors in space $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ have many of the same properties as vectors in the plane.
- The dot product of two vectors $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ in space is $\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3$.
- Two nonzero vectors $\mathbf{u}$ and $\mathbf{v}$ are said to be parallel if there is some scalar c such that $\mathbf{u} = c\mathbf{v}$.
- You should be able to use vectors to solve real life problems.

Vocabulary Check

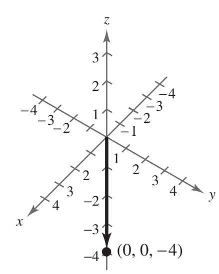
1. zero
2. $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$
3. component form
4. orthogonal
5. parallel

1. $\mathbf{v} = \langle 0 - 2, 3 - 0, 2 - 1 \rangle = \langle -2, 3, 1 \rangle$



2. (a) $\mathbf{v} = \langle 1 - 1, 4 - 4, 0 - 4 \rangle = \langle 0, 0, -4 \rangle$

(b)



3. (a) $\mathbf{v} = \langle 1 - (-6), -1 - 4, 3 - (-2) \rangle$

$= \langle 7, -5, 5 \rangle$

(b) $\|\mathbf{v}\| = \sqrt{7^2 + (-5)^2 + 5^2}$

$= \sqrt{49 + 25 + 25}$

$= \sqrt{99}$

$= 3\sqrt{11}$

(c) $\dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{1}{3\sqrt{11}}\langle 7, -5, 5 \rangle = \dfrac{\sqrt{11}}{33}\langle 7, -5, 5 \rangle$

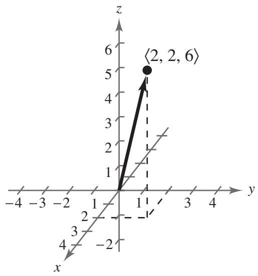
4. (a) $\mathbf{v} = \langle 0 + 7, 0 - 3, 2 - 5 \rangle = \langle 7, -3, -3 \rangle$

(b) $\|\mathbf{v}\| = \sqrt{49 + 9 + 9} = \sqrt{67}$

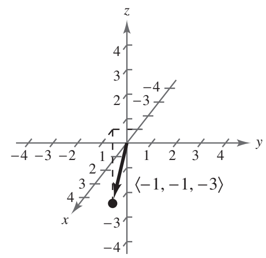
(c) Unit vector:

$\dfrac{1}{\sqrt{67}}\langle 7, -3, -3 \rangle = \dfrac{\sqrt{67}}{67}\langle 7, -3, -3 \rangle$

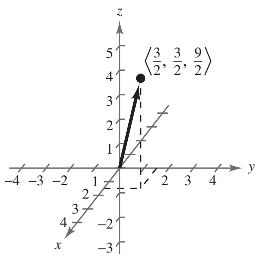
5. (a)



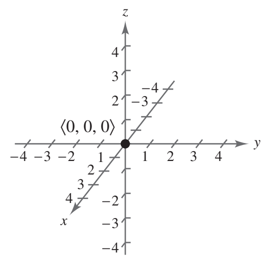
(b)



(c)

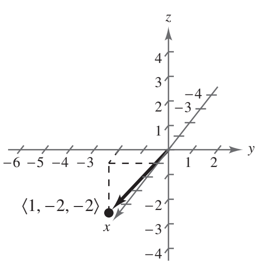


(d)

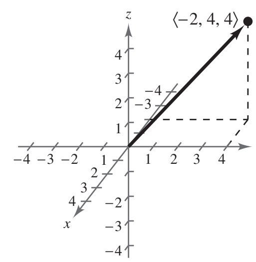


6. $\mathbf{v} = \langle -1, 2, 2 \rangle$

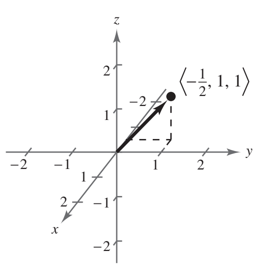
(a) $-\mathbf{v} = \langle 1, -2, -2 \rangle$



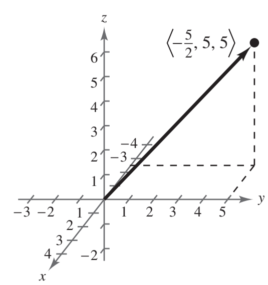
(b) $2\mathbf{v} = \langle -2, 4, 4 \rangle$



(c) $\frac{1}{2}\mathbf{v} = \left\langle -\frac{1}{2}, 1, 1 \right\rangle$



(d) $\frac{5}{2}\mathbf{v} = \left\langle \frac{-5}{2}, 5, 5 \right\rangle$



7. $\mathbf{z} = \mathbf{u} - 2\mathbf{v} = \langle -1, 3, 2 \rangle - 2\langle 1, -2, -2 \rangle = \langle -3, 7, 6 \rangle$

8. $\mathbf{z} = 7\langle -1, 3, 2 \rangle + \langle 1, -2, -2 \rangle - \frac{1}{5}\langle 5, 0, -5 \rangle = \langle -7, 19, 13 \rangle$

9. $2\mathbf{z} - 4\mathbf{u} = \mathbf{w} \implies \mathbf{z} = \frac{1}{2}(4\mathbf{u} + \mathbf{w}) = \frac{1}{2}(4\langle -1, 3, 2 \rangle + \langle 5, 0, -5 \rangle) = \left\langle \frac{1}{2}, 6, \frac{3}{2} \right\rangle$

10. $\mathbf{z} = -\mathbf{u} - \mathbf{v} = -\langle -1, 3, 2 \rangle - \langle 1, -2, -2 \rangle = \langle 0, -1, 0 \rangle$

11. $\|\mathbf{v}\| = \|\langle 7, 8, 7\rangle\|$
$= \sqrt{49 + 64 + 49} = \sqrt{162} = 9\sqrt{2}$

12. $\|\mathbf{v}\| = \sqrt{(-2)^2 + 0^2 + (-5)^2} = \sqrt{4 + 25} = \sqrt{29}$

13. $\|\mathbf{v}\| = \sqrt{4^2 + (-3)^2 + (-7)^2}$
$= \sqrt{16 + 9 + 49} = \sqrt{74}$

14. $\|\mathbf{v}\| = \sqrt{2^2 + (-1)^2 + 6^2} = \sqrt{41}$

15. $\mathbf{v} = \langle 1 - 1, 0 - (-3), -1 - 4\rangle = \langle 0, 3, -5\rangle$
$\|\mathbf{v}\| = \sqrt{0 + 3^2 + (-5)^2} = \sqrt{34}$

16. $\mathbf{v} = \langle 1 - 0, 2 - (-1), -2 - 0\rangle = \langle 1, 3, -2\rangle$
$\|\mathbf{v}\| = \sqrt{1 + 9 + 4} = \sqrt{14}$

17. (a) $\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \dfrac{\langle 8, 3, -1\rangle}{\sqrt{74}}$
$= \dfrac{1}{\sqrt{74}}(8\mathbf{i} + 3\mathbf{j} - \mathbf{k}) = \dfrac{\sqrt{74}}{74}\langle 8, 3, -1\rangle$
(b) $-\dfrac{1}{\sqrt{74}}(8\mathbf{i} + 3\mathbf{j} - \mathbf{k}) = -\dfrac{\sqrt{74}}{74}\langle 8, 3, -1\rangle$

18. (a) $\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \dfrac{\langle -3, 5, 10\rangle}{\sqrt{134}} = \dfrac{1}{\sqrt{134}}(-3\mathbf{i} + 5\mathbf{j} + 10\mathbf{k})$
(b) $\dfrac{-1}{\sqrt{134}}(-3\mathbf{i} + 5\mathbf{j} + 10\mathbf{k})$

19. $\mathbf{u} \cdot \mathbf{v} = \langle 4, 4, -1\rangle \cdot \langle 2, -5, -8\rangle$
$= 8 - 20 + 8 = -4$

20. $\mathbf{u} \cdot \mathbf{v} = 3(4) + (-1)(-10) + 6(1) = 28$

21. $\mathbf{u} \cdot \mathbf{v} = \langle 2, -5, 3\rangle \cdot \langle 9, 3, -1\rangle$
$= 18 - 15 - 3 = 0$

22. $\mathbf{u} \cdot \mathbf{v} = 0(6) + 3(-4) + (-6)(-2) = 0$

23. $\cos\theta = \dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \dfrac{-8}{\sqrt{8}\sqrt{25}} \implies \theta \approx 124.45°$

24. $\cos\theta = \dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \dfrac{5}{\sqrt{10}\sqrt{6}} \implies \theta \approx 49.80°$

25. $\cos\theta = \dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \dfrac{-120}{\sqrt{1700}\sqrt{73}} \implies \theta \approx 109.92°$

26. $\cos\theta = \dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \dfrac{100}{\sqrt{464}\sqrt{125}} \implies \theta \approx 65.47°$

27. $-\frac{3}{2}\langle 8, -4, -10\rangle = \langle -12, 6, 15\rangle \implies$ parallel

28. $\mathbf{u} \cdot \mathbf{v} = -2 - 3 - 5 = -10 \neq 0$ and
$\mathbf{u} \neq c\mathbf{v} \implies$ neither

29. $\mathbf{u} \cdot \mathbf{v} = 3 - 5 + 2 = 0 \implies$ orthogonal

30. $-8\mathbf{u} = -8\left\langle -1, \frac{1}{2}, -1\right\rangle = \langle 8, -4, 8\rangle = \mathbf{v} \implies$ parallel

31. $\mathbf{v} = \langle 7 - 5, 3 - 4, -1 - 1\rangle = \langle 2, -1, -2\rangle$
$\mathbf{u} = \langle 4 - 7, 5 - 3, 3 - (-1)\rangle = \langle -3, 2, 4\rangle$
Since $\mathbf{u}$ and $\mathbf{v}$ are not parallel, the points are not collinear.

32. $\mathbf{v} = \langle -4 - (-2), 8 - 7, 1 - 4\rangle = \langle -2, 1, -3\rangle$
$\mathbf{u} = \langle 0 - (-4), 6 - 8, 7 - 1\rangle = \langle 4, -2, 6\rangle$
Since $\mathbf{u} = -2\mathbf{v}$, the points are collinear.

33. $\mathbf{v} = \langle -1 - 1, 2 - 3, 5 - 2\rangle = \langle -2, -1, 3\rangle$
$\mathbf{u} = \langle 3 - (-1), 4 - 2, -1 - 5\rangle = \langle 4, 2, -6\rangle$
Since $\mathbf{u} = -2\mathbf{v}$, the points are collinear.

34. $\mathbf{v} = \langle -1 - 0, 5 - 4, 6 - 4\rangle = \langle -1, 1, 2\rangle$
$\mathbf{u} = \langle -2 - (-1), 6 - 5, 7 - 6\rangle = \langle -1, 1, 1\rangle$
Since $\mathbf{u}$ and $\mathbf{v}$ are not parallel, the points are not collinear.

35. $\mathbf{v} = \langle 2, -4, 7 \rangle = \langle q_1 - 1, q_2 - 5, q_3 - 0 \rangle \implies$

$$\left.\begin{aligned} 2 &= q_1 - 1 \\ -4 &= q_2 - 5 \\ 7 &= q_3 \end{aligned}\right\} \implies \left.\begin{aligned} q_1 &= 3 \\ q_2 &= 1 \\ q_3 &= 7 \end{aligned}\right\} \implies$$

Terminal point is (3, 1, 7).

36. $\langle 4, -1, -1 \rangle = \langle x - 6, y + 4, z - 3 \rangle \implies (x, y, z) = (10, -5, 2)$

37. $\mathbf{v} = \left\langle 4, \frac{3}{2}, -\frac{1}{4} \right\rangle = \left\langle q_1 - 2, q_2 - 1, q_3 + \frac{3}{2} \right\rangle$

$$4 = q_1 - 2 \implies q_1 = 6$$

$$\tfrac{3}{2} = q_2 - 1 \implies q_2 = \tfrac{5}{2}$$

$$-\tfrac{1}{4} = q_3 + \tfrac{3}{2} \implies q_3 = -\tfrac{7}{4}$$

Terminal point: $\left(6, \frac{5}{2}, -\frac{7}{4}\right)$

38. $\left\langle \frac{5}{2}, -\frac{1}{2}, 4 \right\rangle = \left\langle x - 3, y - 2, z + \frac{1}{2} \right\rangle \implies (x, y, z) = \left(\frac{11}{2}, \frac{3}{2}, \frac{7}{2}\right)$

39. $c\mathbf{u} = c\mathbf{i} + 2c\mathbf{j} + 3c\mathbf{k}$

$\|c\mathbf{u}\| = \sqrt{c^2 + 4c^2 + 9c^2} = |c|\sqrt{14} = 3 \implies$

$$c = \pm\frac{3}{\sqrt{14}} = \pm\frac{3\sqrt{14}}{14}$$

40. $\|c\,\mathbf{u}\| = |c|\,\|\mathbf{u}\| = |c|\sqrt{4 + 4 + 16} = |c|\sqrt{24} = 12$

$$\implies |c| = \frac{12}{\sqrt{24}} = \frac{6}{\sqrt{6}} = \sqrt{6} \implies c = \pm\sqrt{6}$$

41. $\mathbf{v} = \langle q_1, q_2, q_3 \rangle$

Since $\mathbf{v}$ lies in the yz-plane, $q_1 = 0$. Since $\mathbf{v}$ makes an angle of 45°, $q_2 = q_3$. Finally, $\|\mathbf{v}\| = 4$ implies that $q_2{}^2 + q_3{}^2 = 16$. Thus, $q_2 = q_3 = 2\sqrt{2}$ and $\mathbf{v} = \left\langle 0, 2\sqrt{2}, 2\sqrt{2} \right\rangle$, or $q_2 = 2\sqrt{2}$ and $q_3 = -2\sqrt{2}$ and $\mathbf{v} = \left\langle 0, 2\sqrt{2}, -2\sqrt{2} \right\rangle$.

42. $\mathbf{v}$ lies in xz-plane $\implies y = 0$.

$\mathbf{v} = 10\langle \sin 60°, 0, \cos 60° \rangle = \left\langle 5\sqrt{3}, 0, 5 \right\rangle$, or

$\mathbf{v} = 10\langle -\sin 60°, 0, \cos 60° \rangle = \left\langle -5\sqrt{3}, 0, 5 \right\rangle$

43. $\overrightarrow{AB} = \langle 0, 70, 115 \rangle$. $F_1 = C_1\langle 0, 70, 115 \rangle$

$\overrightarrow{AC} = \langle -60, 0, 115 \rangle$. $F_2 = C_2\langle -60, 0, 115 \rangle$

$\overrightarrow{AD} = \langle 45, -65, 115 \rangle$. $F_3 = C_3\langle 45, -65, 115 \rangle$

$F_1 + F_2 + F_3 = \langle 0, 0, -500 \rangle$. Thus

$$\begin{aligned} -60C_2 + 45C_3 &= 0 \\ 70C_1 - 65C_3 &= 0 \\ 115C_1 + 115C_2 + 115C_3 &= -500 \end{aligned}$$

Solving this system yields $C_1 = \dfrac{-104}{69}, C_2 = \dfrac{-28}{23}, C_3 = \dfrac{-112}{69}$.

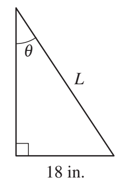
Thus,

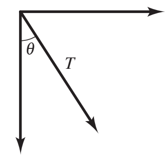
$\|\mathbf{F}_1\| \approx 202.919$ N

$\|\mathbf{F}_2\| \approx 157.909$ N

$\|\mathbf{F}_3\| \approx 226.521$ N

44. (a) $\sin\theta = \dfrac{18}{L}$





$$\theta = \sin^{-1}\left(\frac{18}{L}\right)$$

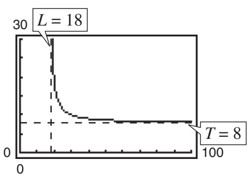
$$T = \frac{8}{\cos\theta} = \frac{8}{\cos\left(\sin^{-1}\left(\frac{18}{L}\right)\right)} = \frac{8}{\frac{\sqrt{L^2 - 18^2}}{L}} = \frac{8L}{\sqrt{L^2 - 18^2}}$$

Domain: $L > 18$

(b)

L	20	25	30	35	40	45	50
T	18.4	11.5	10	9.3	9.0	8.7	8.6

(c)



Vertical asymptote: $L = 18$

Horizontal asymptote: $T = 8$

The minimum tension in each cable is 8 pounds and the minimum cable length is 18 inches.

(d) $10 = \dfrac{8}{\cos\left(\sin^{-1}\left(\frac{18}{L}\right)\right)} \implies \cos\left(\sin^{-1}\left(\frac{18}{L}\right)\right) = \dfrac{8}{10} = \dfrac{4}{5}$

$$\sin^{-1}\left(\frac{18}{L}\right) = \cos^{-1}\left(\frac{4}{5}\right)$$

$$\frac{18}{L} = \sin\left(\cos^{-1}\left(\frac{4}{5}\right)\right) = \frac{3}{5}$$

$$L = \frac{90}{3} = 30 \text{ inches}$$

45. True. $\cos\theta = 0 \implies \theta = 90^\circ$

46. True

47. If $\mathbf{u}\cdot\mathbf{v} < 0$, then $\cos\theta < 0$ and the angle between $\mathbf{u}$ and $\mathbf{v}$ is obtuse, $180^\circ > \theta > 90^\circ$.

48. Let $\mathbf{v} = \langle v_1, v_2, v_3\rangle$ and $\mathbf{u} = \langle u_1, u_2, u_3\rangle$.

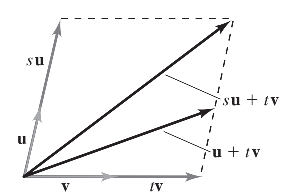
Then $t\mathbf{v} = \langle tv_1, tv_2, tv_3\rangle$

$$\mathbf{u} + t\mathbf{v} = \langle u_1 + tv_1, u_2 + tv_2, u_3 + tv_3\rangle$$

and $s\mathbf{u} + t\mathbf{v} = \langle su_1 + tv_1, su_2 + tv_2, su_3 + tv_3\rangle$

The endpoints of these three vectors are collinear, as indicated in the figure.

So, the figure is a line.



49. (a) $x = t, y = 3t + 2$

(b) $x = t - 1, y = 3(t - 1) + 2 = 3t - 1$

50. (a) $x = t, y = \dfrac{2}{t}$

(b) $x = t - 1, y = \dfrac{2}{t - 1}$

51. (a) $x = t, y = t^2 - 8$

(b) $x = t - 1, y = (t - 1)^2 - 8 = t^2 - 2t - 7$

52. (a) $x = t, y = 4t^3$

(b) $x = t - 1, y = 4(t - 1)^3$

Section 11.3 The Cross Product of Two Vectors

- The cross product of two vectors $\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j} + u_3\mathbf{k}$ and $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$ is given by

$$\mathbf{u} \times \mathbf{v} = (u_2v_3 - u_3v_2)\mathbf{i} - (u_1v_3 - u_3v_1)\mathbf{j} + (u_1v_2 - u_2v_1)\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}.$$

- The cross product satisfies the following algebraic properties.
  (a) $\mathbf{u} \times \mathbf{v} = -(\mathbf{v} \times \mathbf{u})$
  (b) $\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) + (\mathbf{u} \times \mathbf{w})$
  (c) $c(\mathbf{u} \times \mathbf{v}) = (c\mathbf{u}) \times \mathbf{v} = \mathbf{u} \times (c\mathbf{v})$
  (d) $\mathbf{u} \times \mathbf{0} = \mathbf{0} \times \mathbf{u} = \mathbf{0}$
  (e) $\mathbf{u} \times \mathbf{u} = \mathbf{0}$
  (f) $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$
- The following geometric properties of the cross product are valid, where θ is the angle between the vectors $\mathbf{u}$ and $\mathbf{v}$:
  (a) $\mathbf{u} \times \mathbf{v}$ is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.
  (b) $\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\|\,\|\mathbf{v}\| \sin\theta$
  (c) $\mathbf{u} \times \mathbf{v} = \mathbf{0}$ if and only if $\mathbf{u}$ and $\mathbf{v}$ are scalar multiples.
  (d) $\|\mathbf{u} \times \mathbf{v}\|$ is the area of the parallelogram having $\mathbf{u}$ and $\mathbf{v}$ as sides.
- The absolute value of the triple scalar product is the volume of the parallelepiped having $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ as sides.

$$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

Vocabulary Check

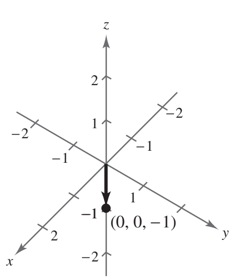
1. cross product

2. 0

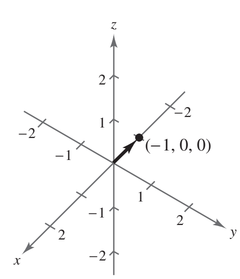
3. $\|\mathbf{u}\|\,\|\mathbf{v}\| \sin\theta$

4. triple scalar product

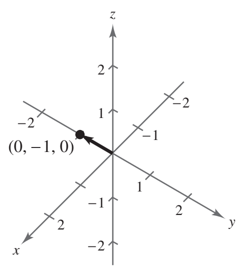
1. $\mathbf{j} \times \mathbf{i} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} = -\mathbf{k}$



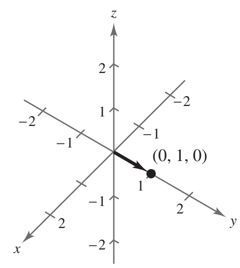
2. $\mathbf{k} \times \mathbf{j} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = -\mathbf{i}$



3. $\mathbf{i} \times \mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} = -\mathbf{j}$



4. $\mathbf{k} \times \mathbf{i} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} = \mathbf{j}$



5. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -2 & 5 \\ 0 & -1 & 1 \end{vmatrix} = \langle 3, -3, -3 \rangle$

$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u} = \langle 3, -3, -3 \rangle \cdot \langle 3, -2, 5 \rangle = 0$

$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v} = \langle 3, -3, -3 \rangle \cdot \langle 0, -1, 1 \rangle = 0$

6. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 8 & 3 \\ 4 & -1 & -4 \end{vmatrix} = \langle -29, 36, -38 \rangle$

7. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -10 & 0 & 6 \\ 7 & 0 & 0 \end{vmatrix} = \langle 0, 42, 0 \rangle$

$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u} = \langle 0, 42, 0 \rangle \cdot \langle -10, 0, 6 \rangle = 0$

$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v} = \langle 0, 42, 0 \rangle \cdot \langle 7, 0, 0 \rangle = 0$

8. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -5 & 5 & 11 \\ 2 & 2 & 3 \end{vmatrix} = \langle -7, 37, -20 \rangle$

9. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 2 & 1 \\ 1 & 3 & -2 \end{vmatrix} = \langle -7, 13, 16 \rangle$

$= -7\mathbf{i} + 13\mathbf{j} + 16\mathbf{k}$

10. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & \frac{3}{2} & -\frac{5}{2} \\ \frac{1}{2} & -\frac{3}{4} & \frac{1}{4} \end{vmatrix} = \left\langle -\frac{3}{2}, -\frac{3}{2}, -\frac{3}{2} \right\rangle = -\frac{3}{2}\mathbf{i} - \frac{3}{2}\mathbf{j} - \frac{3}{2}\mathbf{k}$

11. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 6 \\ -1 & 3 & 1 \end{vmatrix} = \langle -18, -6, 0 \rangle$

$= -18\mathbf{i} - 6\mathbf{j}$

12. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{2}{3} & 0 & 0 \\ 0 & \frac{1}{3} & -3 \end{vmatrix} = 2\mathbf{j} + \frac{2}{9}\mathbf{k}$

13. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 0 & 1 \\ 0 & 1 & -2 \end{vmatrix} = \langle -1, -2, -1 \rangle$

$= -\mathbf{i} - 2\mathbf{j} - \mathbf{k}$

14. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & -2 \\ 0 & -1 & 1 \end{vmatrix} = (0 - 2)\mathbf{i} - (1 - 0)\mathbf{j} + (-1 - 0)\mathbf{k} = -2\mathbf{i} - \mathbf{j} - \mathbf{k}$

15. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$

$\|\mathbf{u} \times \mathbf{v}\| = \sqrt{19}$

$$\text{Unit vector} = \frac{\mathbf{u} \times \mathbf{v}}{\|\mathbf{u} \times \mathbf{v}\|} = \frac{1}{\sqrt{19}}(\mathbf{i} - 3\mathbf{j} + 3\mathbf{k})$$
$$= \frac{\sqrt{19}}{19}\langle 1, -3, 3\rangle$$

16. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 0 \\ 1 & 0 & -3 \end{vmatrix} = -6\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}.$

$\|\mathbf{u} \times \mathbf{v}\| = \sqrt{36 + 9 + 4} = 7$

$$\text{Unit vector} = \frac{\mathbf{u} \times \mathbf{v}}{\|\mathbf{u} \times \mathbf{v}\|} = -\frac{6}{7}\mathbf{i} + \frac{3}{7}\mathbf{j} - \frac{2}{7}\mathbf{k}$$

17. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 2 & -5 \\ \frac{1}{2} & -\frac{3}{4} & \frac{1}{10} \end{vmatrix} = \left\langle -\frac{71}{20}, -\frac{11}{5}, \frac{5}{4} \right\rangle$

Consider the parallel vector $\langle -71, -44, 25\rangle = \mathbf{w}$.

$\|\mathbf{w}\| = \sqrt{71^2 + 44^2 + 25^2} = \sqrt{7602}$

$$\text{Unit vector} = \frac{1}{\sqrt{7602}}\langle -71, -44, 25\rangle$$
$$= \frac{\sqrt{7602}}{7602}\langle -71, -44, 25\rangle$$

18. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & -14 & 5 \\ 14 & 28 & -15 \end{vmatrix} = 70\mathbf{i} + 175\mathbf{j} + 392\mathbf{k}$

$\|\mathbf{u} \times \mathbf{v}\| = \sqrt{70^2 + 175^2 + 392^2}$

$= \sqrt{189{,}189} = 21\sqrt{429}$

$$\text{Unit vector} = \frac{\mathbf{u} \times \mathbf{v}}{\|\mathbf{u} \times \mathbf{v}\|} = \frac{1}{21\sqrt{429}}\langle 70, 175, 392\rangle$$
$$= \frac{1}{3\sqrt{429}}\langle 10, 25, 56\rangle$$
$$= \frac{\sqrt{429}}{1287}\langle 10, 25, 56\rangle$$

19. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix} = 2\mathbf{i} - 2\mathbf{j}$

$\|\mathbf{u} \times \mathbf{v}\| = 2\sqrt{2}$

$$\text{Unit vector} = \frac{\mathbf{u} \times \mathbf{v}}{\|\mathbf{u} \times \mathbf{v}\|} = \frac{1}{2\sqrt{2}}(2\mathbf{i} - 2\mathbf{j})$$
$$= \frac{1}{\sqrt{2}}\mathbf{i} - \frac{1}{\sqrt{2}}\mathbf{j}$$
$$= \frac{\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j}$$

20. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 2 \\ 2 & -1 & -2 \end{vmatrix} = 6\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$

$\|\mathbf{u} \times \mathbf{v}\| = \sqrt{36 + 36 + 9} = 9$

$$\text{Unit vector} = \frac{\mathbf{u} \times \mathbf{v}}{\|\mathbf{u} \times \mathbf{v}\|} = \frac{1}{9}(6\mathbf{i} + 6\mathbf{j} + 3\mathbf{k})$$
$$= \frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}$$

21. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{vmatrix} = \mathbf{j}$

Area $= \|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{j}\| = 1$ square unit

22. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 2 \\ 1 & 0 & 1 \end{vmatrix} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$

Area $= \|\mathbf{u} \times \mathbf{v}\| = \|2\mathbf{i} + \mathbf{j} - 2\mathbf{k}\|$

$= \sqrt{4 + 1 + 4} = 3$ square units

23. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 4 & 6 \\ 2 & -1 & 5 \end{vmatrix} = 26\mathbf{i} - 3\mathbf{j} - 11\mathbf{k}$

Area $= \|\mathbf{u} \times \mathbf{v}\| = \sqrt{26^2 + (-3)^2 + (-11)^2}$

$= \sqrt{806}$ square units

24. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 3 & 2 \\ 1 & 2 & 4 \end{vmatrix} = \langle 8, 10, -7\rangle$

Area $= \|\mathbf{u} \times \mathbf{v}\| = \sqrt{8^2 + 10^2 + (-7)^2}$

$= \sqrt{213}$ square units

25. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 2 & -3 \\ 0 & 2 & 3 \end{vmatrix} = \langle 12, -6, 4 \rangle$

Area $= \|\mathbf{u} \times \mathbf{v}\| = \sqrt{12^2 + (-6)^2 + 4^2}$

$= 14$ square units

26. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -3 & 2 \\ 5 & 0 & 1 \end{vmatrix} = \langle -3, 6, 15 \rangle$

Area $= \|\mathbf{u} \times \mathbf{v}\| = \sqrt{(-3)^2 + 6^2 + 15^2}$

$= \sqrt{270} = 3\sqrt{30}$ square units

27. (a) $\overrightarrow{AB} = \langle 3 - 2, 1 - (-1), 2 - 4 \rangle = \langle 1, 2, -2 \rangle$ is parallel to

$\overrightarrow{DC} = \langle 0 - (-1), 5 - 3, 6 - 8 \rangle = \langle 1, 2, -2 \rangle$.

$\overrightarrow{AD} = \langle -3, 4, 4 \rangle$ is parallel to $\overrightarrow{BC} = \langle -3, 4, 4 \rangle$.

(b) $\overrightarrow{AB} \times \overrightarrow{AD} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & -2 \\ -3 & 4 & 4 \end{vmatrix} = \langle 16, 2, 10 \rangle$

Area $= \|\overrightarrow{AB} \times \overrightarrow{AD}\| = \sqrt{16^2 + 2^2 + 10^2} = \sqrt{360} = 6\sqrt{10}$ square units

(c) $\overrightarrow{AB} \cdot \overrightarrow{AD} = \langle 1, 2, -2 \rangle \cdot \langle -3, 4, 4 \rangle \neq 0 \implies$ not a rectangle

28. (a) $\overrightarrow{AB} = \langle 1, 2, 3 \rangle$

$\overrightarrow{CD} = \langle 1, 2, 3 \rangle$

Opposites are parallel and same length. Thus ABCD form a parallelogram.

(b) $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ 5 & 4 & 1 \end{vmatrix} = \langle -10, 14, -6 \rangle$

Area $= \|\overrightarrow{AB} \times \overrightarrow{AC}\| = \sqrt{(-10)^2 + 14^2 + (-6)^2} = 2\sqrt{83}$ square units

(c) $\overrightarrow{AB} \cdot \overrightarrow{AC} = 5 + 8 + 3 = 16 \neq 0 \implies$ not a rectangle.

29. $\mathbf{u} = \langle 1, 2, 3 \rangle, \mathbf{v} = \langle -3, 0, 0 \rangle$

$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ -3 & 0 & 0 \end{vmatrix} = \langle 0, -9, 6 \rangle$

Area $= \frac{1}{2}\|\mathbf{u} \times \mathbf{v}\| = \frac{1}{2}\sqrt{81 + 36} = \frac{3}{2}\sqrt{13}$ square units

30. $\mathbf{u} = \langle 2 - 1, 0 - (-4), 2 - 3 \rangle = \langle 1, 4, -1 \rangle$

$\mathbf{v} = \langle -2 - 1, 2 - (-4), 0 - 3 \rangle = \langle -3, 6, -3 \rangle$

$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 4 & -1 \\ -3 & 6 & -3 \end{vmatrix} = \langle -6, 6, 18 \rangle$

Area $= \frac{1}{2}\|\mathbf{u} \times \mathbf{v}\| = \frac{1}{2}\sqrt{(-6)^2 + 6^2 + 18^2}$

$= \frac{1}{2}\sqrt{396} = 3\sqrt{11}$ square units

31. $\mathbf{u} = \langle -2 - 2, -2 - 3, 0 - (-5) \rangle = \langle -4, -5, 5 \rangle$

$\mathbf{v} = \langle 3 - 2, 0 - 3, 6 - (-5) \rangle = \langle 1, -3, 11 \rangle$

$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & -5 & 5 \\ 1 & -3 & 11 \end{vmatrix} = \langle -40, 49, 17 \rangle$

Area $= \frac{1}{2}\|\mathbf{u} \times \mathbf{v}\| = \frac{1}{2}\sqrt{(-40)^2 + 49^2 + 17^2}$

$= \frac{1}{2}\sqrt{4290}$ square units

32. $\mathbf{u} = \langle -2 - 2, -4 - 4, 0 - 0 \rangle = \langle -4, -8, 0 \rangle$

$\mathbf{v} = \langle 0 - 2, 0 - 4, 4 - 0 \rangle = \langle -2, -4, 4 \rangle$

$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & -8 & 0 \\ -2 & -4 & 4 \end{vmatrix} = \langle -32, 16, 0 \rangle$

Area $= \frac{1}{2}\|\mathbf{u} \times \mathbf{v}\| = \frac{1}{2}\sqrt{(-32)^2 + 16^2}$

$= \frac{1}{2}\sqrt{1280} = 8\sqrt{5}$ square units

33. $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 2 & 3 & 3 \\ 4 & 4 & 0 \\ 0 & 0 & 4 \end{vmatrix}$

$= 2(16) - 3(16) + 3(0) = -16$

34. $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 6$

35. $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 2 & 3 & 1 \\ 1 & -1 & 0 \\ 4 & 3 & 1 \end{vmatrix}$

$= 2(-1) - 3(1) + 1(7) = 2$

36. $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 1 & 4 & -7 \\ 2 & 0 & 4 \\ 0 & -3 & 6 \end{vmatrix}$

$= 1(0 + 12) - 4(12 - 0) - 7(-6) = 6$

37. $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 1 + 1 = 2$

Volume $= |\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})| = 2$ cubic units

38. $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 1 & 1 & 3 \\ 0 & 3 & 3 \\ 3 & 0 & 3 \end{vmatrix}$

$= 1(9) - 1(-9) + 3(-9) = -9$

Volume $= |\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})| = |-9| = 9$ cubic units

39. $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 0 & 2 & 2 \\ 0 & 0 & -2 \\ 3 & 0 & 2 \end{vmatrix}$

$= 0 - 2(6) + 2(0) = -12$

Volume $= |\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})| = 12$ cubic units

40. $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 1 & 2 & -1 \\ -1 & 2 & 2 \\ 2 & 0 & 1 \end{vmatrix}$

$= 1(2) - 2(-1 - 4) - 1(0 - 4) = 16$

Volume $= |\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})| = 16$ cubic units

41. $\mathbf{u} = \langle 4, 0, 0 \rangle$, $\mathbf{v} = \langle 0, -2, 3 \rangle$, $\mathbf{w} = \langle 0, 5, 3 \rangle$

$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 4 & 0 & 0 \\ 0 & -2 & 3 \\ 0 & 5 & 3 \end{vmatrix} = 4(-21) = -84$

Volume $= |-84| = 84$ cubic units

42. $\overrightarrow{AB} = \langle 1, 1, 0 \rangle$, $\overrightarrow{AC} = \langle 1, 0, 2 \rangle$, $\overrightarrow{AD} = \langle 0, 1, 1 \rangle$

$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & 1 \end{vmatrix} = 1(-2) - 1(1) = -3$

Volume $= 3$ cubic units

43. $\mathbf{V} = \frac{1}{2}(-\cos 40° \, \mathbf{j} - \sin 40° \, \mathbf{k})$

$\mathbf{F} = -p\mathbf{k}$

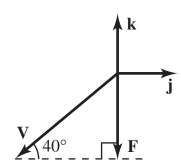
(a) $\mathbf{V} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -\frac{1}{2}\cos 40° & -\frac{1}{2}\sin 40° \\ 0 & 0 & -p \end{vmatrix}$

$= \frac{1}{2}p \cos 40° \, \mathbf{i}$

$T = \|\mathbf{V} \times \mathbf{F}\| = \frac{p}{2} \cos 40°$

(b)

p	T
15	5.75
20	7.66
25	9.58
30	11.49
35	13.41
40	15.32
45	17.24



44. $\mathbf{V} = 0.16(-\cos 30^\circ\, \mathbf{j} - \sin 30^\circ\, \mathbf{k})$

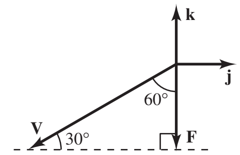
$\mathbf{F} = -2000\mathbf{k}$

$$\mathbf{V} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.16 \cos 30^\circ & -0.16 \sin 30^\circ \\ 0 & 0 & -2000 \end{vmatrix}$$

$$= (-2000)(-0.16) \cos 30^\circ\, \mathbf{i}$$

$$= 160\sqrt{3}\, \mathbf{i}$$

$$T = \|\mathbf{V} \times \mathbf{F}\| = 160\sqrt{3} \text{ ft-lb}$$



45. True. The cross product is not defined for two-dimensional vectors.

46. False. $\mathbf{u} \times \mathbf{v} = -(\mathbf{v} \times \mathbf{u})$

47. If the magnitudes of two vectors are doubled, the magnitude of the cross product will be four times as large.

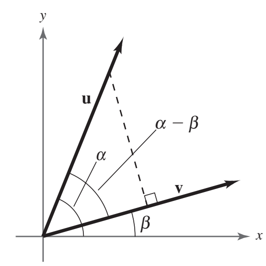
48. $$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos\alpha & \sin\alpha & 0 \\ \cos\beta & \sin\beta & 0 \end{vmatrix} = (\cos\alpha \ \sin\beta - \sin\alpha \ \cos\beta)\mathbf{k}$$

Area of triangle formed by the unit vectors $\mathbf{u}$ and $\mathbf{v}$ is
$\frac{1}{2}$(base)(height) $= \frac{1}{2}(1)\sin(\alpha - \beta)$.

The area is also given by $\frac{1}{2}\|\mathbf{u} \times \mathbf{v}\| = \frac{1}{2}|\cos\alpha \sin\beta - \sin\alpha\cos\beta|$

Notice that $\cos\alpha \sin\beta - \sin\alpha \ \cos\beta$ is negative.

Thus, $\sin(\alpha - \beta) = \sin\alpha \ \cos\beta - \cos\alpha \sin\beta$.



49. $\cos 480^\circ = \cos 120^\circ = -\frac{1}{2}$

50. $\tan 300^\circ = -\sqrt{3}$

51. $\sin 690^\circ = \sin 330^\circ = -\frac{1}{2}$

52. $\cos 930^\circ = \cos 210^\circ = -\dfrac{\sqrt{3}}{2}$

53. $\sin \dfrac{19\pi}{6} = \sin\left(\dfrac{7\pi}{6}\right) = -\dfrac{1}{2}$

54. $\cos \dfrac{17\pi}{6} = \cos \dfrac{5\pi}{6} = -\dfrac{\sqrt{3}}{2}$

55. $\tan \dfrac{15\pi}{4} = \tan \dfrac{7\pi}{4} = -1$

56. $\tan \dfrac{10\pi}{3} = \tan \dfrac{4\pi}{3} = \sqrt{3}$

57. $z = 6x + 4y$

At $(0, 5)$: $z = 6(0) + 4(5) = 20$

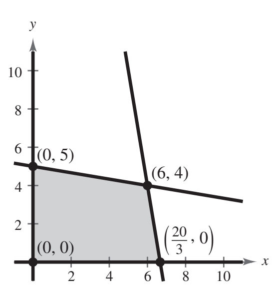
At $(0, 0)$: $z = 6(0) + 4(0) = 0$

At $\left(\frac{20}{3}, 0\right)$: $z = 6\left(\frac{20}{3}\right) + 4(0) = 40$

At $(6, 4)$: $z = 6(6) + 4(4) = 52$

The maximum value of z, $z = 52$, is found at $(6, 4)$.

The minimum value of z, $z = 0$, is found at $(0, 0)$.



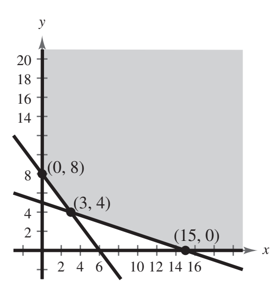
58. $(0, 8)$: $z = 6 \cdot 0 + 7 \cdot 8 = 56$

$(3, 4)$: $z = 6 \cdot 3 + 7 \cdot 4 = 46$

$(15, 0)$: $z = 6 \cdot 15 + 7 \cdot 0 = 90$

Minimum: 46 at $(3, 4)$.

Maximum: Unbounded



Section 11.4 Lines and Planes in Space

- The parametric equations of the line in space parallel to the vector $\langle a, b, c\rangle$ and passing through the point (x_1, y_1, z_1) are

  $x = x_1 + at, \quad y = y_1 + bt, \quad z = z_1 + ct.$
- The standard equation of the plane in space containing the point (x_1, y_1, z_1) and having normal vector (a, b, c) is

  $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0.$
- You should be able to find the angle between two planes by calculating the angle between their normal vectors.
- You should be able to sketch a plane in space.
- The distance between a point Q and a plane having normal $\mathbf{n}$ is

  $$D = \|\text{proj}_{\mathbf{n}}\overrightarrow{PQ}\| = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|}$$

  where P is a point in the plane.

Vocabulary Check

1. direction, $\dfrac{\overrightarrow{PQ}}{t}$
2. parametric equations
3. symmetric equations
4. normal
5. $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

1. $x = x_1 + at = 0 + t$

$y = y_1 + bt = 0 + 2t$

$z = z_1 + ct = 0 + 3t$

(a) Parametric equations: $x = t, y = 2t, z = 3t$

(b) Symmetric equations: $\dfrac{x}{1} = \dfrac{y}{2} = \dfrac{z}{3}$

2. (a) $x = x_1 + at = 3 + 3t$

$y = y_1 + bt = -5 - 7t$

$z = z_1 + ct = 1 - 10t$

Parametric equations:
$x = 3 + 3t, y = -5 - 7t, z = 1 - 10t$

(b) Symmetric equations: $\dfrac{x - 3}{3} = \dfrac{y + 5}{-7} = \dfrac{z - 1}{-10}$

3. $x = x_1 + at = -4 + \frac{1}{2}t,\ y = y_1 + bt = 1 + \frac{4}{3}t,\ z = z_1 + ct = 0 - t$

(a) Parametric equations: $x = -4 + \frac{1}{2}t, y = 1 + \frac{4}{3}t, z = -t$

Equivalently: $x = -4 + 3t, y = 1 + 8t, z = -6t$

(b) Symmetric equations: $\dfrac{x + 4}{3} = \dfrac{y - 1}{8} = \dfrac{z}{-6}$

4. $x = x_1 + at = 5 + 4t, y = y_1 + bt = 0 + 0t, z = z_1 + ct = 10 + 3t$

(a) Parametric equations: $x = 5 + 4t$, $y = 0$, $z = 10 + 3t$

(b) Symmetric equations: $\dfrac{x-5}{4} = \dfrac{z-10}{3}$, $y = 0$

5. $x = x_1 + at = 2 + 2t,\ y = y_1 + bt = -3 - 3t,\ z = z_1 + ct = 5 + t$

(a) Parametric equations: $x = 2 + 2t, y = -3 - 3t, z = 5 + t$

(b) Symmetric equations: $\dfrac{x-2}{2} = \dfrac{y+3}{-3} = z - 5$

6. (a) $\mathbf{v} = \langle 3, -2, 1\rangle$

$x = 1 + 3t, y = -2t, z = 1 + t$

(b) Symmetric equations: $\dfrac{x-1}{3} = \dfrac{y}{-2} = \dfrac{z-1}{1}$

7. (a) $\mathbf{v} = \langle 1 - 2, 4 - 0, -3 - 2\rangle = \langle -1, 4, -5\rangle$

Point: $(2, 0, 2)$

$x = 2 - t, y = 4t, z = 2 - 5t$

(b) $\dfrac{x-2}{-1} = \dfrac{y}{4} = \dfrac{z-2}{-5}$

8. (a) $\mathbf{v} = \langle 8, 5, 12\rangle$

Point: $(2, 3, 0)$

Parametric equations: $x = 2 + 8t, y = 3 + 5t, z = 12t$

(b) Symmetric equations: $\dfrac{x-2}{8} = \dfrac{y-3}{5} = \dfrac{z}{12}$

9. (a) $\mathbf{v} = \langle 1 - (-3), -2 - 8, 16 - 15\rangle = \langle 4, -10, 1\rangle$

Point: $(-3, 8, 15)$

$x = -3 + 4t, y = 8 - 10t, z = 15 + t$

(b) $\dfrac{x+3}{4} = \dfrac{y-8}{-10} = \dfrac{z-15}{1}$

10. $(2, 3, -1), (1, -5, 3)$

(a) Let $P = (2, 3, -1), Q = (1, -5, 3)$

$\vec{V} = \overrightarrow{PQ} = \langle 1 - 2, -5 - 3, 3 - (-1)\rangle = \langle -1, -8, 4\rangle$

Direction numbers: $a = -1, b = -8, c = 4$

Choose P as the initial point:

$x = 2 - t, y = 3 - 8t, z = -1 + 4t$

(b) $\dfrac{x-2}{-1} = \dfrac{y-3}{-8} = \dfrac{z+1}{4}$

11. $(3, 1, 2), (-1, 1, 5)$

(a) $\mathbf{v} = \langle -1 - 3, 1 - 1, 5 - 2 \rangle = \langle -4, 0, 3 \rangle$

Parametric: $x = 3 - 4t, y = 1, z = 2 + 3t$

(b) Since $b = 0$, there are no symmetric equations.

12. $(2, -1, 5), (2, 1, -3)$

(a) Let $P = (2, -1, 5), Q = (2, 1, -3)$

$\vec{V} = \overrightarrow{PQ} = \langle 2 - 2, 1 - (-1), -3 - 5 \rangle = \langle 0, 2, -8 \rangle$

Direction numbers: $a = 0, b = 2, c = -8$

Choose P as the initial point:

$x = 2, y = -1 + 2t, z = 5 - 8t$

(b) Since the direction number $a = 0$, no set of symmetric equations are possible.

13. $\left(-\frac{1}{2}, 2, \frac{1}{2}\right), \left(1, -\frac{1}{2}, 0\right)$

(a) $\mathbf{v} = \left\langle 1 - \left(-\frac{1}{2}\right), -\frac{1}{2} - 2, 0 - \frac{1}{2} \right\rangle = \left\langle \frac{3}{2}, -\frac{5}{2}, -\frac{1}{2} \right\rangle$

Direction numbers: $3, -5, -1$

Parametric: $x = -\frac{1}{2} + 3t, y = 2 - 5t, z = \frac{1}{2} - t$

(b) Symmetric: $\dfrac{2x + 1}{6} = \dfrac{y - 2}{-5} = \dfrac{2z - 1}{-2}$

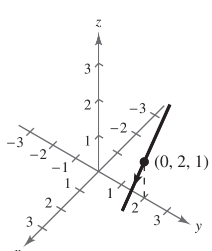
14. (a) $\mathbf{v} = \left\langle 3 - \left(-\frac{3}{2}\right), -5 - \frac{3}{2}, -4 - 2 \right\rangle = \left\langle \frac{9}{2}, -\frac{13}{2}, -6 \right\rangle$, or $\langle 9, -13, -12 \rangle$

Point: $(3, -5, -4)$

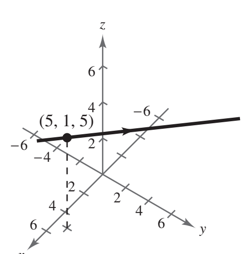
Parametric equations: $x = 3 + 9t, y = -5 - 13t, z = -4 - 12t$

(b) Symmetric equations: $\dfrac{x - 3}{9} = \dfrac{y + 5}{-13} = \dfrac{z + 4}{-12}$

15.



16.



17. $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

$1(x - 2) + 0(y - 1) + 0(z - 2) = 0$

$x - 2 = 0$

18. $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

$0(x - 1) + 0(y - 0) + 1(z + 3) = 0$

$z + 3 = 0$

19. $-2(x - 5) + 1(y - 6) - 2(z - 3) = 0$

$-2x + y - 2z + 10 = 0$

20. $0(x - 0) - 3(y - 0) + 5(z - 0) = 0$

$-3y + 5z = 0$

21. $\mathbf{n} = \langle -1, -2, 1 \rangle \implies -1(x-2) - 2(y-0) + 1(z-0) = 0$

$$-x - 2y + z + 2 = 0$$

22. $\mathbf{n} = \langle -1, 1, -2 \rangle$

$-1(x-0) + 1(y-0) - 2(z-6) = 0$

$-x + y - 2z + 12 = 0$

23. $\mathbf{u} = \langle 1-0, 2-0, 3-0 \rangle = \langle 1, 2, 3 \rangle$

$\mathbf{v} = \langle -2-0, 3-0, 3-0 \rangle = \langle -2, 3, 3 \rangle$

$$\mathbf{n} = \mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ -2 & 3 & 3 \end{vmatrix} = \langle -3, -9, 7 \rangle$$

$-3(x-0) - 9(y-0) + 7(z-0) = 0$

$-3x - 9y + 7z = 0$

$3x + 9y - 7z = 0$

24. $\mathbf{u} = \langle 2, -6, 2 \rangle$, $\mathbf{v} = \langle -3, -3, 0 \rangle$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -6 & 2 \\ -3 & -3 & 0 \end{vmatrix} = \langle 6, -6, -24 \rangle$$

$\mathbf{n} = \langle -1, 1, 4 \rangle$

Plane: $-1(x-4) + 1(y+1) + 4(z-3) = 0$

$-x + y + 4z - 7 = 0$

25. $\mathbf{u} = \langle 3-2, 4-3, 2+2 \rangle = \langle 1, 1, 4 \rangle$

$\mathbf{v} = \langle 1-2, -1-3, 0+2 \rangle = \langle -1, -4, 2 \rangle$

$$\mathbf{n} = \mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 4 \\ -1 & -4 & 2 \end{vmatrix} = \langle 18, -6, -3 \rangle$$

$18(x-2) - 6(y-3) - 3(z+2) = 0$

$18x - 6y - 3z - 24 = 0$

$6x - 2y - z - 8 = 0$

26. $\mathbf{u} = \langle 4, 0, 2 \rangle$, $\mathbf{v} = \langle 1, 2, -5 \rangle$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 0 & 2 \\ 1 & 2 & -5 \end{vmatrix} = \langle -4, 22, 8 \rangle$$

$\mathbf{n} = \langle -2, 11, 4 \rangle$

Plane: $-2(x-1) + 11(y+1) + 4(z-2) = 0$

$-2x + 11y + 4z + 5 = 0$

27. $\mathbf{n} = \mathbf{j}$: $0(x-2) + 1(y-5) + 0(z-3) = 0$

$y - 5 = 0$

28. $\langle -1-2, 1-2, -1-1 \rangle = \langle -3, -1, -2 \rangle$ and $\langle 2, -3, 1 \rangle$ are parallel to plane.

$$\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -1 & -2 \\ 2 & -3 & 1 \end{vmatrix} = \langle -7, -1, 11 \rangle$$

$-7(x-2) - 1(y-2) + 11(z-1) = 0$

$-7x - y + 11z + 5 = 0$

29. $\mathbf{n}_1 = \langle 5, -3, 1 \rangle$, $\mathbf{n}_2 = \langle 1, 4, 7 \rangle$

$\mathbf{n}_1 \cdot \mathbf{n}_2 = 5 - 12 + 7 = 0$; orthogonal

30. $\mathbf{n}_1 = \langle 3, 1, -4 \rangle$, $\mathbf{n}_2 = \langle -9, -3, 12 \rangle$

$3\mathbf{n}_1 = \langle 9, 3, -12 \rangle = -\mathbf{n}_2 \implies$ parallel planes

31. $\mathbf{n}_1 = \langle 2, 0, -1 \rangle$, $\mathbf{n}_2 = \langle 4, 1, 8 \rangle$

$\mathbf{n}_1 \cdot \mathbf{n}_2 = 8 - 8 = 0$; orthogonal

32. $\mathbf{n}_1 = \langle 1, -5, -1 \rangle$

$\mathbf{n}_2 = \langle 5, -25, -5 \rangle = 5\mathbf{n}_1 \implies$ parallel

33. (a) $\mathbf{n}_1 = \langle 3, -4, 5\rangle$, $\mathbf{n}_2 = \langle 1, 1, -1\rangle$; normal vectors to planes

$$\cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\,\|\mathbf{n}_2\|} = \frac{|-6|}{\sqrt{50}\sqrt{3}} = \frac{6}{\sqrt{150}} \implies \theta \approx 60.67^\circ$$

(b) $3x - 4y + 5z = 6$ Equation 1

$x + y - z = 2$ Equation 2

(-3) times Equation 2 added to Equation 1 gives

$-7y + 8z = 0$

$$y = \frac{8}{7}z.$$

Substituting back into Equation 2, $x = 2 - y + z = 2 - \frac{8}{7}z + z = 2 - \frac{1}{7}z.$

Letting $t = z/7$, we obtain $x = 2 - t$, $y = 8t$, $z = 7t$.

34. (a) $\mathbf{n}_1 = \langle 1, -3, 1\rangle$, $\mathbf{n}_2 = \langle 2, 0, 5\rangle$

$$\cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\,\|\mathbf{n}_2\|} = \frac{|7|}{\sqrt{11}\sqrt{29}} = \frac{7}{\sqrt{319}} \implies \theta \approx 66.93^\circ$$

(b) $2x + 5z + 3 = 0 \implies x = \frac{1}{2}(-5z - 3)$

Then $3y = x + z + 2 = \frac{1}{2}(-5z - 3) + z + 2 = -\frac{3}{2}z + \frac{1}{2} \implies y = -\frac{1}{2}z + \frac{1}{6}$

Let $z = t$. Parametric equations: $x = -\frac{5}{2}t - \frac{3}{2}$, $y = -\frac{1}{2}t + \frac{1}{6}$, $z = t$

or equivalently, let $z = 2t$ and you obtain $x = -5t - \frac{3}{2}$, $y = -t + \frac{1}{6}$, $z = 2t$.

35. (a) $\mathbf{n}_1 = \langle 1, 1, -1\rangle$, $\mathbf{n}_2 = \langle 2, -5, -1\rangle$; normal vectors to planes

$$\cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\,\|\mathbf{n}_2\|} = \frac{|-2|}{\sqrt{3}\sqrt{30}} = \frac{2}{\sqrt{90}} \implies \theta \approx 77.83^\circ$$

(b) $x + y - z = 0$ Equation 1

$2x - 5y - z = 1$ Equation 2

(-2) times Equation 1 added to Equation 2 gives

$-7y + z = 1$

$$y = \frac{z-1}{7}.$$

Substituting back into Equation 1, $x = z - y = z - \frac{z-1}{7} = \frac{6z}{7} + \frac{1}{7} = \frac{1}{7}(6z + 1).$

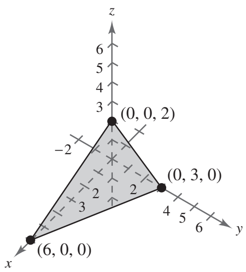
Letting $z = t$, $x = \frac{6t+1}{7}$, $y = \frac{t-1}{7}$.

Equivalently, let $y = t$, $z = 7t + 1$ and $x = 6t + 1$.

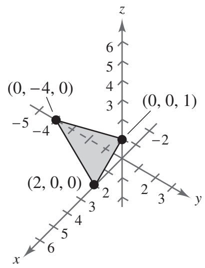
36. The planes are parallel because $\mathbf{n}_1 = \langle 2, 4, -2\rangle$ is a multiple of $\mathbf{n}_2 = \langle -3, -6, 3\rangle$.

The planes do not intersect.

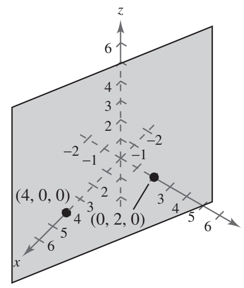
37. $x + 2y + 3z = 6$



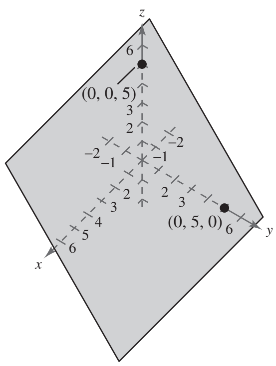
38. $2x - y + 4z = 4$



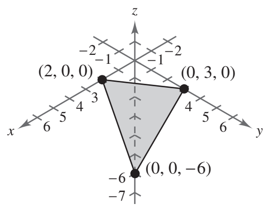
39. $x + 2y = 4$



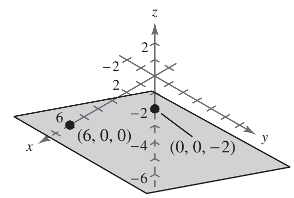
40. $y + z = 5$



41. $3x + 2y - z = 6$



42. $x - 3z = 6$



43. $D = \dfrac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|}$

$P = (1, 0, 0)$ on plane, $Q = (0, 0, 0)$,

$\mathbf{n} = \langle 8, -4, 1\rangle$, $\overrightarrow{PQ} = \langle -1, 0, 0\rangle$

$$D = \frac{|\langle -1, 0, 0\rangle \cdot \langle 8, -4, 1\rangle|}{\sqrt{64 + 16 + 1}} = \frac{|-8|}{\sqrt{81}} = \frac{8}{9}$$

44. $P = (4, 0, 0)$ on plane, $Q = (3, 2, 1)$, $\mathbf{n} = \langle 1, -1, 2\rangle$

$\overrightarrow{PQ} = \langle -1, 2, 1\rangle$

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{|-1|}{\sqrt{6}} = \frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{6}$$

45. $D = \dfrac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|}$

$P = (2, 0, 0)$ on plane, $Q = (4, -2, -2)$,

$\mathbf{n} = \langle 2, -1, 1\rangle$, $\overrightarrow{PQ} = \langle 2, -2, -2\rangle$

$$D = \frac{|\langle 2, -2, -2\rangle \cdot \langle 2, -1, 1\rangle|}{\sqrt{6}} = \frac{4}{\sqrt{6}} = \frac{2\sqrt{6}}{3}$$

46. $P = (6, 0, 0)$ on plane, $Q = (-1, 2, 5)$,

$\overrightarrow{PQ} = \langle -7, 2, 5\rangle$, $\mathbf{n} = \langle 2, 3, 1\rangle$

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{|-3|}{\sqrt{14}} = \frac{3}{\sqrt{14}} = \frac{3\sqrt{14}}{14}$$

47. (a) $z = 0.81x + 0.36y + 0.2$

Year	x	y	z (Actual)	z (Model)
1999	6.2	7.3	7.8	7.85
2000	6.1	7.1	7.7	7.70
2001	5.9	7.0	7.4	7.50
2002	5.7	7.0	7.3	7.34
2003	5.6	6.9	7.2	7.22

(b) The approximations are very similar to the actual values of z.

(c) If the consumption of the two types of milk increases (or decreases), so does the consumption of the third type of milk.

48. The plane containing $P(6, 0, 0)$, $S(0, 0, 0)$, $T(-1, -1, 8)$ has normal vector

$$\langle 6, 0, 0\rangle \times \langle -1, -1, 8\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 0 & 0 \\ -1 & -1 & 8 \end{vmatrix} = \langle 0, -48, -6\rangle$$

or $\mathbf{n}_1 = \langle 0, 8, 1\rangle$.

The plane containing $P(6, 0, 0)$, $Q(6, 6, 0)$, and $R(7, 7, 8)$ has normal vector

$$\langle 0, -6, 0\rangle \times \langle 1, 1, 8\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -6 & 0 \\ 1 & 1 & 8 \end{vmatrix} = \langle -48, 0, 6\rangle,$$

or $\mathbf{n}_2 = \langle -8, 0, 1\rangle$.

The angle between two adjacent sides is given by

$$\cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\,\|\mathbf{n}_2\|} = \frac{1}{\sqrt{65}\sqrt{65}} = \frac{1}{65} \implies \theta \approx 89.12°.$$

49. False. They might be skew lines, such as:

L_1: $x = t$, $y = 0$, $z = 0$ (x-axis)

and L_2: $x = 0$, $y = t$, $z = 1$

50. True

51. The lines are parallel:

$-\frac{3}{2}\langle 10, -18, 20\rangle = \langle -15, 27, -30\rangle$

52. (a) Sphere: $(x - 4)^2 + (y + 1)^2 + (z - 1)^2 = 4$

(b) Two planes parallel to given plane. Let $Q = (x, y, z)$ be a point on one of these planes, and pick $P = (0, 0, 10)$ on the given plane. By the distance formula,

$$2 = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{|\langle x, y, z - 10\rangle \cdot \langle 4, -3, 1\rangle|}{\sqrt{26}}$$

$$\pm 2\sqrt{26} = 4x - 3y + z - 10$$

$4x - 3y + z = 10 \pm 2\sqrt{26}$ (Two planes parallel to given plane)

53. $x^2 + y^2 = 10^2 = 100$

54. $\theta = \dfrac{3\pi}{4} \implies \tan\theta = -1 = \dfrac{y}{x} \implies y = -x$ (line)

55.

$$r = 3\cos\theta$$
$$r^2 = 3r\cos\theta$$
$$x^2 + y^2 = 3x$$

56. $r = \dfrac{1}{2 - \cos\theta} \implies 2r - r\cos\theta = 1 \implies 2\sqrt{x^2 + y^2} - x = 1$

$\implies 2\sqrt{x^2 + y^2} = x + 1 \implies 4(x^2 + y^2) = x^2 + 2x + 1 \implies 3x^2 + 4y^2 = 2x + 1$

57. $r^2 = 49$

$r = 7$

58. $x^2 + y^2 - 4x = 0$

$r^2 - 4r\cos\theta = 0$

$r - 4\cos\theta = 0 \implies r = 4\cos\theta$

59. $y = 5$

$r\sin\theta = 5$

$r = 5\csc\theta$

60. $2x - y + 1 = 0$

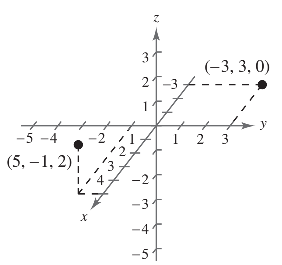
$2r\cos\theta - r\sin\theta = -1$

$r(2\cos\theta - \sin\theta) = -1$

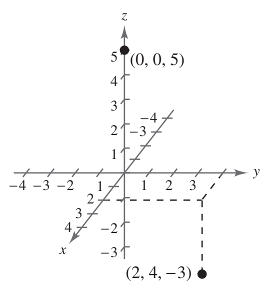
$$r = \frac{1}{\sin\theta - 2\cos\theta}$$

Review Exercises for Chapter 11

1. (a) and (b)



2.



3. $(-5, 4, 0)$

4. y-axis $\implies x = z = 0$

$(0, -7, 0)$

5. $d = \sqrt{(5-4)^2 + (2-0)^2 + (1-7)^2}$

$= \sqrt{1 + 4 + 36}$

$= \sqrt{41}$

6. $d = \sqrt{(2-(-1))^2 + (3-(-3))^2 + (-4-0)^2}$

$= \sqrt{9 + 36 + 16}$

$= \sqrt{61}$

7. $d_1 = \sqrt{(3-0)^2 + (-2-3)^2 + (0-2)^2} = \sqrt{9 + 25 + 4} = \sqrt{38}$

$d_2 = \sqrt{(0-0)^2 + (5-3)^2 + (-3-2)^2} = \sqrt{4 + 25} = \sqrt{29}$

$d_3 = \sqrt{(0-3)^2 + (5-(-2))^2 + (-3-0)^2} = \sqrt{9 + 49 + 9} = \sqrt{67}$

${d_1}^2 + {d_2}^2 = 38 + 29 = 67 = {d_3}^2$

8. $d_1 = \sqrt{(4-0)^2 + (3-0)^2 + (2-4)^2} = \sqrt{16 + 9 + 4} = \sqrt{29}$

$d_2 = \sqrt{(4-4)^2 + (5-3)^2 + (5-2)^2} = \sqrt{4 + 9} = \sqrt{13}$

$d_3 = \sqrt{(4-0)^2 + (5-0)^2 + (5-4)^2} = \sqrt{16 + 25 + 1} = \sqrt{42}$

$d_1^2 + d_2^2 = d_3^2 = 42$

9. Midpoint: $\left(\frac{8+5}{2}, \frac{-2+6}{2}, \frac{3+7}{2}\right) = \left(\frac{13}{2}, 2, 5\right)$

10. Midpoint: $\left(\frac{7+1}{2}, \frac{1-1}{2}, \frac{-4+2}{2}\right) = (4, 0, -1)$

11. Midpoint: $\left(\frac{10-8}{2}, \frac{6-2}{2}, \frac{-12-6}{2}\right) = (1, 2, -9)$

12. Midpoint: $\left(\frac{-5-7}{2}, \frac{-3-9}{2}, \frac{1-5}{2}\right) = (-6, -6, -2)$

13. $(x-2)^2 + (y-3)^2 + (z-5)^2 = 1$

14. $(x-3)^2 + (y+2)^2 + (z-4)^2 = 16$

15. Radius: 6

$(x-1)^2 + (y-5)^2 + (z-2)^2 = 36$

16. Radius $= \frac{15}{2}$

$x^2 + (y-4)^2 + (z+1)^2 = \frac{225}{4}$

17. $(x^2 - 4x + 4) + (y^2 - 6y + 9) + z^2 = -4 + 4 + 9$

$(x-2)^2 + (y-3)^2 + z^2 = 9$

Center: $(2, 3, 0)$

Radius: 3

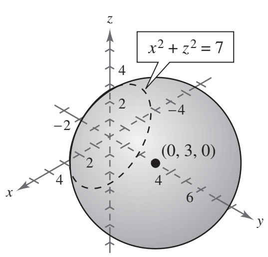
18. $(x^2 - 10x + 25) + (y^2 + 6y + 9) + (z^2 - 4z + 4) = -34 + 25 + 9 + 4$

$(x-5)^2 + (y+3)^2 + (z-2)^2 = 4$

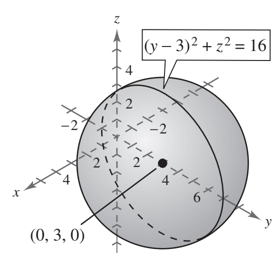
Center: $(5, -3, 2)$

Radius: 2

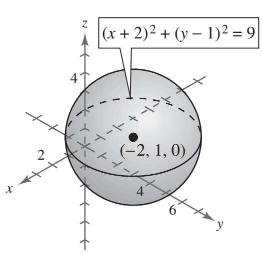
19. (a) xz-trace $(y = 0)$: $x^2 + z^2 = 7$, circle



(b) yz-trace $(x = 0)$: $(y-3)^2 + z^2 = 16$, circle

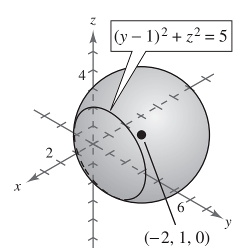


20. (a) xy-trace $(z = 0)$: $(x+2)^2 + (y-1)^2 = 9$ circle



(b) yz-trace $(x = 0)$: $4 + (y-1)^2 + z^2 = 9$

$(y-1)^2 + z^2 = 5$ circle



21. Initial point: $(2, -1, 4)$

Terminal point: $(3, 3, 0)$

(a) $\mathbf{v} = \langle 3-2, 3-(-1), 0-4\rangle = \langle 1, 4, -4\rangle$

(b) $\|\mathbf{v}\| = \sqrt{(1)^2 + (4)^2 + (-4)^2} = \sqrt{33}$

(c) $\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \left\langle \frac{1}{\sqrt{33}}, \frac{4}{\sqrt{33}}, -\frac{4}{\sqrt{33}}\right\rangle$

22. (a) $\overrightarrow{PQ} = \langle -3-2, 2-(-1), 3-2\rangle = \langle -5, 3, 1\rangle$

(b) $\|\overrightarrow{PQ}\| = \sqrt{35}$

(c) Unit vector: $\frac{1}{\sqrt{35}}\langle -5, 3, 1\rangle = \frac{\sqrt{35}}{35}\langle -5, 3, 1\rangle$

23. Initial point: $(7, -4, 3)$

Terminal point: $(-3, 2, 10)$

(a) $\mathbf{v} = \langle -3 - 7, 2 - (-4), 10 - 3 \rangle = \langle -10, 6, 7 \rangle$

(b) $\|\mathbf{v}\| = \sqrt{(-10)^2 + (6)^2 + (7)^2} = \sqrt{185}$

(c) $\mathbf{u} = \dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \left\langle -\dfrac{10}{\sqrt{185}}, \dfrac{6}{\sqrt{185}}, \dfrac{7}{\sqrt{185}} \right\rangle$

24. (a) $\overrightarrow{PQ} = \langle 5 - 0, -8 - 3, 6 - (-1) \rangle = \langle 5, -11, 7 \rangle$

(b) $\|\overrightarrow{PQ}\| = \sqrt{195}$

(c) Unit vector: $\dfrac{1}{\sqrt{195}}\langle 5, -11, 7 \rangle = \dfrac{\sqrt{195}}{195}\langle 5, -11, 7 \rangle$

25. $\mathbf{u} \cdot \mathbf{v} = -1(0) + 4(-6) + 3(5) = -9$

26. $\mathbf{u} \cdot \mathbf{v} = 8(2) - 4(5) + 2(2) = 0$

27. $\mathbf{u} \cdot \mathbf{v} = 2(1) - 1(0) + 1(-1) = 1$

28. $\mathbf{u} \cdot \mathbf{v} = 2(1) + 1(-3) - 2(2) = -5$

29. Since $\mathbf{u} \cdot \mathbf{v} = 0$, the angle is $90°$.

30. $\cos\theta = \dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \dfrac{12 + 5 - 2}{\sqrt{11}\sqrt{45}}$

$$= \frac{15}{\sqrt{11}\sqrt{45}} \implies \theta \approx 47.61°$$

31. Since $-\frac{2}{3}\langle 39, -12, 21 \rangle = \langle -26, 8, -14 \rangle$, the vectors are parallel.

32. $\mathbf{u} \cdot \mathbf{v} = \langle 8, 5, -8 \rangle \cdot \left\langle -2, 4, \frac{1}{2} \right\rangle$

$= -16 + 20 - 4 = 0$

Orthogonal

33. First two points: $\mathbf{u} = \langle -3, 4, 1 \rangle$

Last two points: $\mathbf{v} = \langle 0, -2, 6 \rangle$

Since $\mathbf{u} \neq c\mathbf{v}$, the points are not collinear.

34. First two points: $\langle -1, 5, 4 \rangle$

Last two points: $\langle 2, -10, -8 \rangle$

Since, $\langle 2, -10, -8 \rangle = -2\langle -1, 5, 4 \rangle$, the 3 points are collinear.

35. Let **a**, **b**, and **c** be the three force vectors determined by $A(0, 10, 10)$, $B(-4, -6, 10)$ and $C(4, -6, 10)$.

$$\mathbf{a} = \|\mathbf{a}\|\frac{\langle 0, 10, 10 \rangle}{10\sqrt{2}} = \|\mathbf{a}\|\left\langle 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$

$$\mathbf{b} = \|\mathbf{b}\|\frac{\langle -4, -6, 10 \rangle}{\sqrt{152}} = \|\mathbf{b}\|\left\langle \frac{-2}{\sqrt{38}}, \frac{-3}{\sqrt{38}}, \frac{5}{\sqrt{38}} \right\rangle$$

$$\mathbf{c} = \|\mathbf{c}\|\frac{\langle 4, -6, 10 \rangle}{\sqrt{152}} = \|\mathbf{c}\|\left\langle \frac{2}{\sqrt{38}}, \frac{-3}{\sqrt{38}}, \frac{5}{\sqrt{38}} \right\rangle$$

Must have $\mathbf{a} + \mathbf{b} + \mathbf{c} = 300\mathbf{k}$. Thus:

$$\frac{-2}{\sqrt{38}}\|\mathbf{b}\| + \frac{2}{\sqrt{38}}\|\mathbf{c}\| = 0$$

$$\frac{1}{\sqrt{2}}\|\mathbf{a}\| - \frac{3}{\sqrt{38}}\|\mathbf{b}\| - \frac{3}{\sqrt{38}}\|\mathbf{c}\| = 0$$

$$\frac{1}{\sqrt{2}}\|\mathbf{a}\| + \frac{5}{\sqrt{38}}\|\mathbf{b}\| + \frac{5}{\sqrt{38}}\|\mathbf{c}\| = 300.$$

—CONTINUED—

35. —CONTINUED—

From the first equation $\|\mathbf{b}\| = \|\mathbf{c}\|$. From the second equation, $\dfrac{1}{\sqrt{2}}\|\mathbf{a}\| = \dfrac{6}{\sqrt{38}}\|\mathbf{b}\|$.

From the third equation, $\dfrac{1}{\sqrt{2}}\|\mathbf{a}\| = 300 - \dfrac{10}{\sqrt{38}}\|\mathbf{b}\|$. Thus,

$$\frac{6}{\sqrt{38}}\|\mathbf{b}\| = 300 - \frac{10}{\sqrt{38}}\|\mathbf{b}\| \implies \frac{16}{\sqrt{38}}\|\mathbf{b}\| = 300 \text{ and } \|\mathbf{b}\| = \|\mathbf{c}\| = \frac{75\sqrt{38}}{4} \approx 115.58.$$

Finally, $\|\mathbf{a}\| = \sqrt{2}\left(\dfrac{6}{\sqrt{38}}\right)\left(\dfrac{75\sqrt{38}}{4}\right) = \dfrac{225\sqrt{2}}{2} \approx 159.10.$

36. Let $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ be the three force vectors determined by $A(0, 10, 10)$, $B(-4, -6, 10)$ and $C(4, -6, 10)$.

$$\mathbf{a} = \|\mathbf{a}\|\,\langle 0, 10, 10\rangle/10\sqrt{2} = \|\mathbf{a}\|\left\langle 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\rangle$$

$$\mathbf{b} = \|\mathbf{b}\|\,\langle -4, -6, 10\rangle/\sqrt{152} = \|\mathbf{b}\|\left\langle \frac{-2}{\sqrt{38}}, \frac{-3}{\sqrt{38}}, \frac{5}{\sqrt{38}}\right\rangle$$

$$\mathbf{c} = \|\mathbf{c}\|\,\langle 4, -6, 10\rangle/\sqrt{152} = \|\mathbf{c}\|\left\langle \frac{2}{\sqrt{38}}, \frac{-3}{\sqrt{38}}, \frac{5}{\sqrt{38}}\right\rangle$$

We must have $\mathbf{a} + \mathbf{b} + \mathbf{c} = 200\mathbf{k}$. Thus,

$$\frac{-2}{\sqrt{38}}\|\mathbf{b}\| + \frac{2}{\sqrt{38}}\|\mathbf{c}\| = 0$$

$$\frac{1}{\sqrt{2}}\|\mathbf{a}\| - \frac{3}{\sqrt{38}}\|\mathbf{b}\| - \frac{3}{\sqrt{38}}\|\mathbf{c}\| = 0$$

$$\frac{1}{\sqrt{2}}\|\mathbf{a}\| + \frac{5}{\sqrt{38}}\|\mathbf{b}\| + \frac{5}{\sqrt{38}}\|\mathbf{c}\| = 200$$

Solving this system, $\|\mathbf{a}\| \approx 106.1$, $\|\mathbf{b}\| = \|\mathbf{c}\| = 77.1$.

Thus, the tensions are 106.1, 77.1 and 77.1 pounds.

37. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 8 & 2 \\ 1 & 1 & -1 \end{vmatrix} = \langle -10, 0, -10\rangle$

38. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 10 & 15 & 5 \\ 5 & -3 & 0 \end{vmatrix} = \langle 15, 25, -105\rangle$

39. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 2 & -5 \\ 10 & -15 & 2 \end{vmatrix} = \langle -71, -44, 25\rangle$

$\|\mathbf{u} \times \mathbf{v}\| = \sqrt{7602}$

Unit vector: $\dfrac{1}{\sqrt{7602}}\langle -71, -44, 25\rangle$

40. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 4 \\ 1 & 0 & 12 \end{vmatrix} = 4\mathbf{j} \implies$ unit vector: $\mathbf{j} = \langle 0, 1, 0\rangle$

41. First two points: $\langle 3, 2, 3\rangle$

Last two points: $\langle 3, 2, 3\rangle$

First and third points: $\langle -2, 2, 0\rangle$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & 3 \\ -2 & 2 & 0 \end{vmatrix} = \langle -6, -6, 10\rangle$$

$$\text{Area} = |\langle -6, -6, 10\rangle| = \sqrt{36 + 36 + 100}$$
$$= \sqrt{172} = 2\sqrt{43} \text{ square units}$$

42. $\mathbf{u} = \langle 1, 0, 1\rangle$, $\mathbf{v} = \langle 1, 0, 1\rangle$ opposite sides parallel and equal length.

Adjacent sides: $\mathbf{u} = \langle 1, 0, 1\rangle$, $\mathbf{w} = \langle 0, 2, 0\rangle$

$$\mathbf{u} \times \mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 1 \\ 0 & 2 & 0 \end{vmatrix} = \langle -2, 0, 2\rangle$$

$$\text{Area} = \|\mathbf{u} \times \mathbf{w}\| = \sqrt{4 + 4} = 2\sqrt{2} \text{ square units}$$

43. The parallelogram is determined by the three vectors with initial point $(0, 0, 0)$.

$\mathbf{u} = \langle 3, 0, 0\rangle$, $\mathbf{v} = \langle 2, 0, 5\rangle$, $\mathbf{w} = \langle 0, 5, 1\rangle$

$$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 3 & 0 & 0 \\ 2 & 0 & 5 \\ 0 & 5 & 1 \end{vmatrix} = -75$$

$$\text{Volume} = |-75| = 75 \text{ cubic units}$$

44. $\mathbf{u} = \langle 2, 0. 0\rangle$, $\mathbf{v} = \langle 0, 4, 0\rangle$, $\mathbf{w} = \langle 0, 0, 6\rangle$

$$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \end{vmatrix} = 48$$

$$\text{Volume} = |\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})| = 48 \text{ cubic units}$$

45. $\mathbf{v} = \langle 3 + 1, 6 - 3, -1 - 5\rangle = \langle 4, 3, -6\rangle$, point: $(-1, 3, 5)$

(a) Parametric equations: $x = -1 + 4t, y = 3 + 3t, z = 5 - 6t$

(b) Symmetric equations: $\dfrac{x + 1}{4} = \dfrac{y - 3}{3} = \dfrac{z - 5}{-6}$

46. (a) $\mathbf{v} = \langle 5, 20, -3\rangle$

$x = 5t,\ y = -10 + 20t,\ z = 3 - 3t$

(b) $\dfrac{x}{5} = \dfrac{y + 10}{20} = \dfrac{z - 3}{-3}$

47. Use $2\mathbf{v} = \langle -4, 5, 2\rangle$, point: $(0, 0, 0)$.

(a) Parametric equations: $x = -4t, y = 5t, z = 2t$

(b) Symmetric equations: $\dfrac{x}{-4} = \dfrac{y}{5} = \dfrac{z}{2}$

48. (a) $\mathbf{v} = \langle 1, 1, 1\rangle$

$x = 3 + t, y = 2 + t, z = 1 + t$

(b) $\dfrac{x - 3}{1} = \dfrac{y - 2}{1} = \dfrac{z - 1}{1}$ or

$x - 3 = y - 2 = z - 1$

49. $\mathbf{u} = \langle 5, 0, 2\rangle$, $\mathbf{v} = \langle 2, 3, 8\rangle$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 0 & 2 \\ 2 & 3 & 8 \end{vmatrix} = \langle -6, -36, 15\rangle$$

$\mathbf{n} = \langle 2, 12, -5\rangle$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$
$$2(x - 0) + 12(y - 0) - 5(z - 0) = 0$$
$$2x + 12y - 5z = 0$$

50. $\mathbf{u} = \langle 5, -5, -2\rangle$, $\mathbf{v} = \langle 3, 5, 2\rangle$

$$\mathbf{n} = \mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & -5 & -2 \\ 3 & 5 & 2 \end{vmatrix} = \langle 0, -16, 40\rangle$$

$$\text{Plane: } 0(x + 1) - 16(y - 3) + 40(z - 4) = 0$$
$$-2(y - 3) + 5(z - 4) = 0$$
$$-2y + 5z - 14 = 0$$

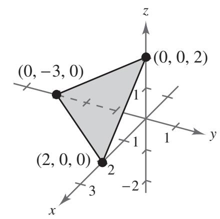
51. $\mathbf{n} = \mathbf{k}$, normal vector

$$\text{Plane: } 0(x - 5) + 0(y - 3) + 1(z - 2) = 0$$
$$z - 2 = 0$$

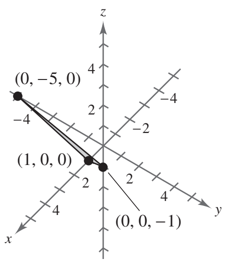
52. $\mathbf{n} = \langle -1, 1, -2 \rangle$, point: $(0, 0, 6)$

$$-1(x - 0) + 1(y - 0) - 2(z - 6) = 0$$
$$-x + y - 2z + 12 = 0$$
$$x - y + 2z - 12 = 0$$

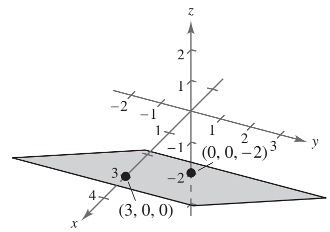
53. $3x - 2y + 3z = 6$



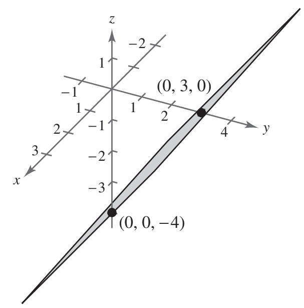
54. $5x - y - 5z = 5$



55. $2x - 3z = 6$



56. $4y - 3z = 12$



57. $\mathbf{n} = \langle 2, -20, 6 \rangle$, $P = (0, 0, 1)$ in plane, $Q = (2, 3, 10)$, $\overrightarrow{PQ} = \langle 2, 3, 9 \rangle$

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{|-2|}{\sqrt{440}} = \frac{1}{\sqrt{110}} = \frac{\sqrt{110}}{110} \approx 0.0953$$

58. $D = \dfrac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|}$

$Q = (1, 2, 3)$, $P = (2, 0, 0)$ on plane. $\overrightarrow{PQ} = \langle -1, 2, 3 \rangle$, $\mathbf{n} = \langle 2, -1, 1 \rangle$

$$D = \frac{|\langle -1, 2, 3 \rangle \cdot \langle 2, -1, 1 \rangle|}{\sqrt{6}} = \frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{6}$$

59. $\mathbf{n} = \langle 1, -10, 3 \rangle$, $P = (2, 0, 0)$ in plane, $Q = (0, 0, 0)$, $\overrightarrow{PQ} = \langle -2, 0, 0 \rangle$

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{|-2|}{\sqrt{1 + 100 + 9}} = \frac{2}{\sqrt{110}} = \frac{2\sqrt{110}}{110} = \frac{\sqrt{110}}{55} \approx 0.191$$

60. $D = \dfrac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|}$

$Q = (0, 0, 0)$, $P = (0, 0, 12)$ on plane. $\overrightarrow{PQ} = \langle 0, 0, -12 \rangle$, $\mathbf{n} = \langle 2, 3, 1 \rangle$

$$D = \frac{|\langle 0, 0, -12 \rangle \cdot \langle 2, 3, 1 \rangle|}{\sqrt{14}} = \frac{12}{\sqrt{14}} = \frac{6\sqrt{14}}{7}$$

61. False. $\mathbf{a} \times \mathbf{b} = -(\mathbf{b} \times \mathbf{a})$

62. True. See page 831.

63.
$$\begin{aligned}\mathbf{u} \cdot \mathbf{u} &= \langle 3, -2, 1 \rangle \cdot \langle 3, -2, 1 \rangle \\ &= 9 + 4 + 1 \\ &= 14 \\ &= \|\mathbf{u}\|^2\end{aligned}$$

64. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -2 & 1 \\ 2 & -4 & -3 \end{vmatrix} = \langle 10, 11, -8 \rangle$

$\mathbf{v} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -4 & -3 \\ 3 & -2 & 1 \end{vmatrix} = \langle -10, -11, 8 \rangle$

Thus, $\mathbf{u} \times \mathbf{v} = -(\mathbf{v} \times \mathbf{u})$.

65. $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \langle 3, -2, 1 \rangle \cdot \langle 1, -2, -1 \rangle = 6$

$\mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w} = 11 + (-5) = 6$

66. $\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = \mathbf{u} \times \langle 1, -2, -1 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -2 & 1 \\ 1 & -2 & -1 \end{vmatrix} = \langle 4, 4, -4 \rangle$

$\mathbf{u} \times \mathbf{v} = \langle 10, 11, -8 \rangle$ (Exercise 64)

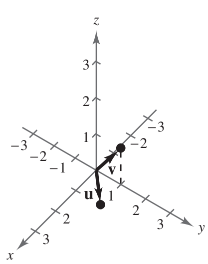
$\mathbf{u} \times \mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -2 & 1 \\ -1 & 2 & 2 \end{vmatrix} = \langle -6, -7, 4 \rangle$

$(\mathbf{u} \times \mathbf{v}) + (\mathbf{u} \times \mathbf{w}) = \langle 10, 11, -8 \rangle + \langle -6, -7, 4 \rangle = \langle 4, 4, -4 \rangle$

$= \mathbf{u} \times (\mathbf{v} + \mathbf{w})$

Problem Solving for Chapter 11

1. (a)



(b) $\mathbf{w} = a\mathbf{u} + b\mathbf{v} = a\langle 1, 1, 0 \rangle + b\langle 0, 1, 1 \rangle$

$\mathbf{0} = \langle a, a + b, b \rangle \Longrightarrow a = b = 0$

(c) $\mathbf{w} = \langle 1, 2, 1 \rangle = a\langle 1, 1, 0 \rangle + b\langle 0, 1, 1 \rangle$

$1 = a$

$2 = a + b$

$1 = b$

Hence, $a = b = 1$.

(d) $\mathbf{w} = \langle 1, 2, 3 \rangle = a\langle 1, 1, 0 \rangle + b\langle 0, 1, 1 \rangle$

$1 = a$

$2 = a + b$

$3 = b$

Impossible

2. This set is a sphere:

$(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 = 16$

3. Programs will vary. See online website.

4. (a) $\mathbf{u} + \mathbf{v} = \langle 4, 7.5, -2 \rangle$

(b) $\|\mathbf{u} + \mathbf{v}\| \approx 8.7321$

(c) $\|\mathbf{u}\| = \sqrt{26} \approx 5.0990$

(d) $\|\mathbf{v}\| \approx 9.0139$

5. The largest angle in a triangle is always opposite the longest side of the triangle. First, determine the lengths of the three sides. Then, once the largest angle has been identified, use the fact that $\cos\theta = \dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}$, where $\mathbf{u}$ and $\mathbf{v}$ are defined to be the vectors that form θ. If $\mathbf{u} \cdot \mathbf{v} = 0$, the angle is a right angle. If $\mathbf{u} \cdot \mathbf{v} > 0$, the angle is acute. If $\mathbf{u} \cdot \mathbf{v} < 0$, the angle is obtuse.

(a) A: $(1, 2, 0)$

B: $(0, 0, 0)$

C: $(-2, 1, 0)$

$d(AB) = \sqrt{5}, d(AC) = \sqrt{10}, d(BC) = \sqrt{5}$

Angle B is largest.

$\overrightarrow{BA} = \langle 1, 2, 0\rangle, \overrightarrow{BC} = \langle -2, 1, 0\rangle$

$\overrightarrow{BA} \cdot \overrightarrow{BC} = 0 \implies$ The triangle is a right triangle.

(b) A: $(-3, 0, 0)$

B: $(0, 0, 0)$

C: $(1, 2, 3)$

$d(AB) = 3, d(AC) = \sqrt{29}, d(BC) = \sqrt{14}$

Angle B is largest.

$\overrightarrow{BA} = \langle -3, 0, 0\rangle, \overrightarrow{BC} = \langle 1, 2, 3\rangle$

$\overrightarrow{BA} \cdot \overrightarrow{BC} = -3 < 0 \implies$ The triangle is an obtuse triangle.

(c) A: $(2, -3, 4)$

B: $(0, 1, 2)$

C: $(-1, 2, 0)$

$d(AB) = \sqrt{24}, d(AC) = \sqrt{50}, d(BC) = \sqrt{6}$

Angle B is largest.

$\overrightarrow{BA} = \langle 2, -4, 2\rangle, \overrightarrow{BC} = \langle -1, 1, -2\rangle$

$\overrightarrow{BA} \cdot \overrightarrow{BC} = -10 < 0 \implies$ The triangle is an obtuse triangle.

(d) A: $(2, -7, 3)$

B: $(-1, 5, 8)$

C: $(4, 6, -1)$

$d(AB) = \sqrt{178}, d(AC) = \sqrt{189}, d(BC) = \sqrt{107}$

Angle B is largest.

$\overrightarrow{BA} = \langle 3, -12, -5\rangle, \overrightarrow{BC} = \langle 5, 1, -9\rangle$

$\overrightarrow{BA} \cdot \overrightarrow{BC} = 48 \implies$ The triangle is an acute triangle.

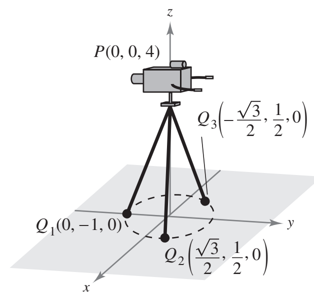
6. $\overrightarrow{PQ_1} = \langle 0 - 0, -1 - 0, 0 - 4\rangle = \langle 0, -1, -4\rangle$

$$\overrightarrow{PQ_2} = \left\langle \frac{\sqrt{3}}{2} - 0, \frac{1}{2} - 0, 0 - 4\right\rangle = \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2}, -4\right\rangle$$

$$\overrightarrow{PQ_3} = \left\langle -\frac{\sqrt{3}}{2} - 0, \frac{1}{2} - 0, 0 - 4\right\rangle = \left\langle -\frac{\sqrt{3}}{2}, \frac{1}{2}, -4\right\rangle$$

Note that $\|\overrightarrow{PQ_1}\| = \|\overrightarrow{PQ_2}\| = \|\overrightarrow{PQ_3}\| = \sqrt{17}$.

Unit vectors:



$$\mathbf{u}_1 = \frac{1}{\sqrt{17}}\langle 0, -1, -4\rangle$$

$$\mathbf{u}_2 = \frac{1}{\sqrt{17}}\left\langle \frac{\sqrt{3}}{2}, \frac{1}{2}, -4\right\rangle$$

$$\mathbf{u}_3 = \frac{1}{\sqrt{17}}\left\langle -\frac{\sqrt{3}}{2}, \frac{1}{2}, -4\right\rangle$$

The unit vectors $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$ give the directions of the force in each leg. Since the legs are the same length, the total weight is distributed equally among the legs. So,

$$\mathbf{F}_1 = \frac{120}{3}\mathbf{u}_1 = \frac{40}{\sqrt{17}}\langle 0, -1, -4\rangle$$

$$\mathbf{F}_2 = \frac{120}{3}\mathbf{u}_2 = \frac{40}{\sqrt{17}}\left\langle \frac{\sqrt{3}}{2}, \frac{1}{2}, -4\right\rangle$$

$$\mathbf{F}_3 = \frac{120}{3}\mathbf{u}_3 = \frac{40}{\sqrt{17}}\left\langle -\frac{\sqrt{3}}{2}, \frac{1}{2}, -4\right\rangle$$

7. Let A lie on the y-axis and the wall on the x-axis. Then, $A = (0, 10, 0)$, $B = (8, 0, 6)$, $C = (-10, 0, 6)$ and $\overrightarrow{AB} = \langle 8, -10, 6\rangle, \overrightarrow{AC} = \langle -10, -10, 6\rangle$.

$$\|\overrightarrow{AB}\| = \sqrt{8^2 + (-10)^2 + 6^2} = 10\sqrt{2}$$

$$\|\overrightarrow{AC}\| = \sqrt{(-10)^2 + (-10)^2 + 6^2} = 2\sqrt{59}$$

$$\mathbf{F}_1 = 420\frac{\overrightarrow{AB}}{\|\overrightarrow{AB}\|} = \frac{420}{10\sqrt{2}}\langle 8, -10, 6\rangle = \frac{84}{\sqrt{2}}\langle 4, -5, 3\rangle$$

$$\mathbf{F}_2 = 650\frac{\overrightarrow{AC}}{\|\overrightarrow{AC}\|} = \frac{650}{2\sqrt{59}}\langle -10, -10, 6\rangle = \frac{650}{\sqrt{59}}\langle -5, -5, 3\rangle$$

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \left\langle \frac{(4)(84)}{\sqrt{2}} + \frac{(-5)(650)}{\sqrt{59}}, \frac{(-5)(84)}{\sqrt{2}} + \frac{(-5)(650)}{\sqrt{59}}, \frac{(3)(84)}{\sqrt{2}} + \frac{(3)(650)}{\sqrt{59}}\right\rangle$$

$$\approx \langle -185.526, -720.099, 432.059\rangle$$

$\|\mathbf{F}\| \approx 860.0$ lb

8. Note that $\cos\theta = \dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}$. Therefore,

$$\|\mathbf{u}\|\,\|\mathbf{v}\|\sin\theta = \|\mathbf{u}\|\,\|\mathbf{v}\|\sqrt{1 - \cos^2\theta} = \|\mathbf{u}\|\,\|\mathbf{v}\|\sqrt{1 - \frac{(\mathbf{u}\cdot\mathbf{v})^2}{\|\mathbf{u}\|^2\|\mathbf{v}\|^2}} = \sqrt{\|\mathbf{u}\|^2\|\mathbf{v}\|^2 - (\mathbf{u}\cdot\mathbf{v})^2}$$

$$= \sqrt{(u_1^2 + u_2^2 + u_3^2)(v_1^2 + v_2^2 + v_3^2) - (u_1v_1 + u_2v_2 + u_3v_3)^2}$$

$$= \sqrt{(u_2v_3 - u_3v_2)^2 + (u_1v_3 - u_3v_1)^2 + (u_1v_2 - u_2v_1)^2}$$

$$\|\mathbf{u}\|\,\|\mathbf{v}\|\sin\theta = \|\mathbf{u}\times\mathbf{v}\|$$

Since $\mathbf{u}$ and $\mathbf{v}$ are orthogonal, $\theta = 90°$ and $\sin\theta = 1$. So, $\|\mathbf{u}\|\,\|\mathbf{v}\| = \|\mathbf{u}\times\mathbf{v}\|$, $\mathbf{u}, \mathbf{v}$ orthogonal.

9. $\mathbf{u} = \langle a_1, b_1, c_1\rangle, \mathbf{v} = \langle a_2, b_2, c_2\rangle, \mathbf{w} = \langle a_3, b_3, c_3\rangle$

$$\mathbf{v}\times\mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = (b_2c_3 - b_3c_2)\mathbf{i} - (a_2c_3 - a_3c_2)\mathbf{j} + (a_2b_3 - a_3b_2)\mathbf{k}$$

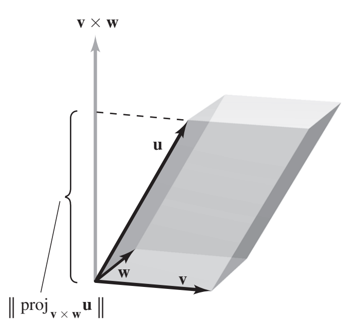
$$\mathbf{u}\times(\mathbf{v}\times\mathbf{w}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & b_1 & c_1 \\ (b_2c_3 - b_3c_2) & (a_3c_2 - a_2c_3) & (a_2b_3 - a_3b_2) \end{vmatrix}$$

$$\begin{aligned}\mathbf{u}\times(\mathbf{v}\times\mathbf{w}) &= [b_1(a_2b_3 - a_3b_2) - c_1(a_3c_2 - a_2c_3)]\mathbf{i} - [a_1(a_2b_3 - a_3b_2) - c_1(b_2c_3 - b_3c_2)]\mathbf{j} \\ &\qquad + [a_1(a_3c_2 - a_2c_3) - b_1(b_2c_3 - b_3c_2)]\mathbf{k} \\ &= [a_2(a_1a_3 + b_1b_3 + c_1c_3) - a_3(a_1a_2 + b_1b_2 + c_1c_2)]\mathbf{i} \\ &\qquad + [b_2(a_1a_3 + b_1b_3 + c_1c_3) - b_3(a_1a_2 + b_1b_2 + c_1c_2)]\mathbf{j} \\ &\qquad + [c_2(a_1a_3 + b_1b_3 + c_1c_3) - c_3(a_1a_2 + b_1b_2 + c_1c_2)]\mathbf{k} \\ &= (\mathbf{u}\cdot\mathbf{w})\mathbf{v} - (\mathbf{u}\cdot\mathbf{v})\mathbf{w}\end{aligned}$$

10. $$\begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} = u_1\begin{vmatrix} v_2 & v_3 \\ w_2 & w_3\end{vmatrix} - u_2\begin{vmatrix} v_1 & v_3 \\ w_1 & w_3\end{vmatrix} + u_3\begin{vmatrix} v_1 & v_2 \\ w_1 & w_2\end{vmatrix}$$

$$\begin{aligned} &= u_1\begin{vmatrix} v_2 & v_3 \\ w_2 & w_3\end{vmatrix}(\mathbf{i}\cdot\mathbf{i}) - u_2\begin{vmatrix} v_1 & v_3 \\ w_1 & w_3\end{vmatrix}(\mathbf{j}\cdot\mathbf{j}) + u_3\begin{vmatrix} v_1 & v_2 \\ w_1 & w_2\end{vmatrix}(\mathbf{k}\cdot\mathbf{k}) \\ &= (u_1\mathbf{i})\cdot\left[\begin{vmatrix} v_2 & v_3 \\ w_2 & w_3\end{vmatrix}\mathbf{i}\right] + (u_2\mathbf{j})\cdot\left[-\begin{vmatrix} v_1 & v_3 \\ w_1 & w_3\end{vmatrix}\mathbf{j}\right] + (u_3\mathbf{k})\cdot\left[\begin{vmatrix} v_1 & v_2 \\ w_1 & w_2\end{vmatrix}\mathbf{k}\right] \\ &= (u_1\mathbf{i} + u_2\mathbf{j} + u_3\mathbf{k})\cdot\left[\begin{vmatrix} v_2 & v_3 \\ w_2 & w_3\end{vmatrix}\mathbf{i} - \begin{vmatrix} v_1 & v_3 \\ w_1 & w_3\end{vmatrix}\mathbf{j} + \begin{vmatrix} v_1 & v_2 \\ w_1 & w_2\end{vmatrix}\mathbf{k}\right] \\ &= \mathbf{u}\cdot\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} = \mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})\end{aligned}$$

11.

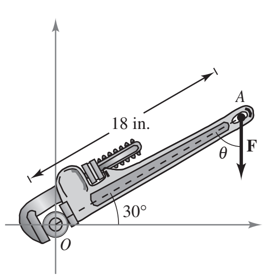


$\|\mathbf{v}\times\mathbf{w}\|$ = area of base and $\|\text{proj}_{\mathbf{v}\times\mathbf{w}}\mathbf{u}\|$ = height of parallelepiped

Therefore, the volume is

$$\begin{aligned} V = (\text{height})(\text{area of base}) &= \|\text{proj}_{\mathbf{v}\times\mathbf{w}}\mathbf{u}\|\,\|\mathbf{v}\times\mathbf{w}\| \\ &= \left|\frac{\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})}{\|\mathbf{v}\times\mathbf{w}\|}\right|\|\mathbf{v}\times\mathbf{w}\| \\ &= |\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})|.\end{aligned}$$

12.

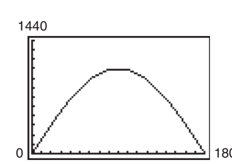


(a) $O = (0, 0, 0)$

$A = (18\cos 30°, 18\sin 30°, 0) = (9\sqrt{3}, 9, 0)$

$\overrightarrow{OA} = \langle 9\sqrt{3}, 9, 0\rangle$

$$\begin{aligned}\|\mathbf{M}\| = \|\overrightarrow{OA}\times\mathbf{F}\| &= \|\overrightarrow{OA}\|\,\|\mathbf{F}\|\sin\theta \\ &= \left(\sqrt{(9\sqrt{3})^2 + 9^2 + 0^2}\right)(60)\sin\theta \\ &= 1080\sin\theta\end{aligned}$$



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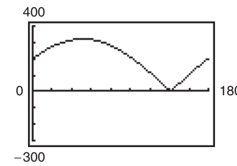
12. —CONTINUED—

(b) $\|\overrightarrow{M}\| = 1080 \sin(45°) = (1080)\left(\frac{\sqrt{2}}{2}\right) = 540\sqrt{2} \approx 763.7$ in-lb

(c) $\|\overrightarrow{M}\| = 1080 \sin \theta$ has its maximum value at $\theta = 90°$. In order to generate the maximum torque, the force should be applied in a direction perpendicular to the wrench handle.

13. (a) In inches: $\overrightarrow{AB} = -15\mathbf{j} + 12\mathbf{k}$



In feet: $\overrightarrow{AB} = -\frac{5}{4}\mathbf{j} + \mathbf{k}$

$\mathbf{F} = -200(\cos \theta\mathbf{j} + \sin \theta\mathbf{k})$

(b) $\overrightarrow{AB} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -\frac{5}{4} & 1 \\ 0 & -200 \cos \theta & -200 \sin \theta \end{vmatrix} = (250 \sin \theta + 200 \cos \theta)\mathbf{i}$

$\|\overrightarrow{AB} \times \mathbf{F}\| = |250 \sin \theta + 200 \cos \theta| = 25|10 \sin \theta + 8 \cos \theta|$

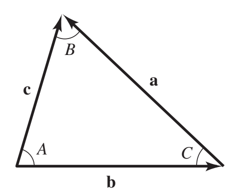
(c) When $\theta = 30°$: $\|\overrightarrow{AB} \times \mathbf{F}\| = 25\left[10\left(\frac{1}{2}\right) + 8\left(\frac{\sqrt{3}}{2}\right)\right] = 25(5 + 4\sqrt{3}) \approx 298.2$

(d) From the graph we see that the maximum value occurs when $\theta \approx 51.34°$.

(e) From the graph we see that the zero occurs when $\theta \approx 141.34°$, the angle making $\overrightarrow{AB}$ parallel to $\mathbf{F}$.

14. The area of the triangle is one-half of the area of any of the 3 parallelograms having the following adjacent sides:

$\mathbf{b}$ and $\mathbf{c}$, $-\mathbf{b}$ and $\mathbf{a}$, $-\mathbf{c}$ and $-\mathbf{a}$



So,

$$\text{Area} = \frac{\|\mathbf{b} \times \mathbf{c}\|}{2} = \frac{\|(-\mathbf{a}) \times (-\mathbf{c})\|}{2} = \frac{\|\mathbf{a} \times (-\mathbf{b})\|}{2}$$

$\|\mathbf{b} \times \mathbf{c}\| = \|(-\mathbf{a}) \times (-\mathbf{c})\| = \|\mathbf{a} \times (-\mathbf{b})\|$

$\|\mathbf{b}\|\,\|\mathbf{c}\| \sin A = \|\mathbf{a}\|\,\|\mathbf{c}\| \sin B = \|\mathbf{a}\|\,\|\mathbf{b}\| \sin C$

Divide by $\|\mathbf{a}\|\,\|\mathbf{b}\|\,\|\mathbf{c}\|$:

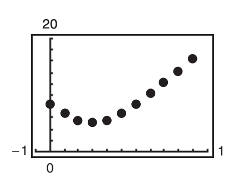
$\frac{\sin A}{\|\mathbf{a}\|} = \frac{\sin B}{\|\mathbf{b}\|} = \frac{\sin C}{\|\mathbf{c}\|}$

15. First insect: $x = 6 + t, y = 8 - t, z = 3 + t$

Second insect: $x = 1 + t, y = 2 + t, z = 2t$

(a) When $t = 0$ the first insect is located at $(6, 8, 3)$ and the second insect is located at $(1, 2, 0)$.

$d = \sqrt{(1 - 6)^2 + (2 - 8)^2 + (0 - 3)^2} = \sqrt{70}$ inches



(b) $d = \sqrt{[(1 + t) - (6 + t)]^2 + [(2 + t) - (8 - t)]^2 + [2t - (3 + t)]^2}$

$= \sqrt{(-5)^2 + (2t - 6)^2 + (t - 3)^2}$

$= \sqrt{5t^2 - 30t + 70}$

t	0	1	2	3	4	5	6	7	8	9	10
d	$\sqrt{70}$	$\sqrt{45}$	$\sqrt{30}$	5	$\sqrt{30}$	$\sqrt{45}$	$\sqrt{70}$	$\sqrt{105}$	$\sqrt{150}$	$\sqrt{205}$	$\sqrt{270}$

—CONTINUED—

15. —CONTINUED—

(c) The distance between the two insects appears to lessen in the first 3 seconds, but then begins to increase with time.

(d) When $t = 3$, the insects get within 5 inches of each other.

16. (a) $x = -2 + 4t, y = 3, z = 1 - t \implies$ direction vector $\mathbf{u} = \langle 4, 0, -1 \rangle$

Let P be the point on the line with $t = 0$:

$$P = (-2 + 4 \cdot 0, 3, 1 - 0) = (-2, 3, 1), Q = (1, 5, -2)$$

$$\overrightarrow{PQ} = \langle 1 - (-2), 5 - 3, -2 - 1 \rangle = \langle 3, 2, -3 \rangle$$

$$\overrightarrow{PQ} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & -3 \\ 4 & 0 & -1 \end{vmatrix} = \begin{vmatrix} 2 & -3 \\ 0 & -1 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 3 & -3 \\ 4 & -1 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 3 & 2 \\ 4 & 0 \end{vmatrix}\mathbf{k}$$

$$= -2\mathbf{i} - 9\mathbf{j} - 8\mathbf{k} = \langle -2, -9, -8 \rangle$$

$$\|\overrightarrow{PQ} \times \mathbf{u}\| = \sqrt{(-2)^2 + (-9)^2 + (-8)^2} = \sqrt{149}$$

$$\|\mathbf{u}\| = \sqrt{4^2 + 0^2 + (-1)^2} = \sqrt{17}$$

$$D = \frac{\|\overrightarrow{PQ} \times \mathbf{u}\|}{\|\mathbf{u}\|} = \frac{\sqrt{149}}{\sqrt{17}} = \frac{\sqrt{2533}}{17} \approx 2.9605$$

(b) $x = 2t, y = -3 + t, z = 2 + 2t \implies$ direction vector $\mathbf{u} = \langle 2, 1, 2 \rangle$

Let P be the point on the line with $t = 0$:

$$P = (2 \cdot 0, -3 + 0, 2 + 2 \cdot 0) = (0, -3, 2), Q = (1, -2, 4)$$

$$\overrightarrow{PQ} = \langle 1 - 0, -2 - (-3), 4 - 2 \rangle = \langle 1, 1, 2 \rangle$$

$$\overrightarrow{PQ} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 2 \\ 2 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}\mathbf{k}$$

$$= 0\mathbf{i} + 2\mathbf{j} - \mathbf{k} = \langle 0, 2, -1 \rangle$$

$$\|\overrightarrow{PQ} \times \mathbf{u}\| = \sqrt{0^2 + 2^2 + (-1)^2} = \sqrt{5}$$

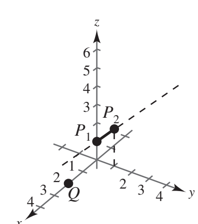
$$\|\mathbf{u}\| = \sqrt{2^2 + 1^2 + 2^2} = \sqrt{9} = 3$$

$$D = \frac{\|\overrightarrow{PQ} \times \mathbf{u}\|}{\|\mathbf{u}\|} = \frac{\sqrt{5}}{3} \approx 0.7454$$

17. (a) $\mathbf{u} = \langle 0, 1, 1 \rangle$ direction vector of line determined by P_1 and P_2

$$D = \frac{\|\overrightarrow{P_1Q} \times \mathbf{u}\|}{\|\mathbf{u}\|} = \frac{\|\langle 2, 0, -1 \rangle \times \langle 0, 1, 1 \rangle\|}{\sqrt{2}}$$

$$= \frac{\|\langle 1, -2, 2 \rangle\|}{\sqrt{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$



(b) The shortest distance to the line **segment** is $\|P_1Q\| = \|\langle 2, 0, -1 \rangle\| = \sqrt{5}$.

18. (a) $x = -t + 3, y = \frac{1}{2}t + 1, z = 2t - 1 \implies$ direction vector $\mathbf{u} = \left\langle -1, \frac{1}{2}, 2 \right\rangle$

Let $Q = (4, 3, s)$.

Let P be the point on the line corresponding to $t = 0$: $P = (3, 1, -1)$

$$\overrightarrow{PQ} = \langle 4 - 3, 3 - 1, s - (-1) \rangle = \langle 1, 2, s + 1 \rangle$$

$$\overrightarrow{PQ} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & s+1 \\ -1 & \frac{1}{2} & 2 \end{vmatrix} = \begin{vmatrix} 2 & s+1 \\ \frac{1}{2} & 2 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 1 & s+1 \\ -1 & 2 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 1 & 2 \\ -1 & \frac{1}{2} \end{vmatrix}\mathbf{k}$$

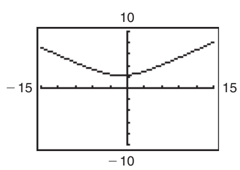
$$= \frac{7 - s}{2}\mathbf{i} - (s + 3)\mathbf{j} + \frac{5}{2}\mathbf{k} = \left\langle \frac{7 - s}{2}, -s - 3, \frac{5}{2} \right\rangle$$

$$\|\overrightarrow{PQ} \times \mathbf{u}\| = \sqrt{\left(\frac{7 - s}{2}\right)^2 + (-s - 3)^2 + \left(\frac{5}{2}\right)^2} = \frac{\sqrt{5s^2 + 10s + 110}}{2}$$

$$\|\mathbf{u}\| = \sqrt{(-1)^2 + \left(\frac{1}{2}\right)^2 + 2^2} = \frac{\sqrt{21}}{2}$$

$$D = \frac{\|\overrightarrow{PQ} \times \mathbf{u}\|}{\|\mathbf{u}\|} = \frac{\sqrt{5s^2 + 10s + 110}}{\sqrt{21}} = \frac{\sqrt{105s^2 + 210s + 2310}}{21}$$

(b)



Minimum distance is $D = 2.23607$ at $s = -1$.

(c) $D = \dfrac{\sqrt{105s^2 + 210s + 2310}}{21}$

As s approaches very large (very positive) or very small (very negative) values, the expression under the radical is dominated by the term $105s^2$:

$105s^2 + 210s + 2310 \approx 105s^2$, $|s| =$ "large"

So, at large values of $|s|$,

$$D \approx \frac{\sqrt{105s^2}}{21} = \pm\frac{\sqrt{105}}{21}s.$$

Asymptotes: $D = \pm\dfrac{\sqrt{105}}{21}(s + 1)$

Chapter 11 Practice Test

1. Find the lengths of the sides of the triangle with vertices $(0, 0, 0)$, $(1, 2, -4)$, and $(0, -2, -1)$. Show that the triangle is a right triangle.

2. Find the standard form of the equation of a sphere having center $(0, 4, 1)$ and radius 5.

3. Find the center and radius of the sphere $x^2 + y^2 + z^2 + 2x - 4z - 11 = 0$.

4. Find the vector $\mathbf{u} - 3\mathbf{v}$ given $\mathbf{u} = \langle 1, 0, -1\rangle$ and $\mathbf{v} = \langle 4, 3, -6\rangle$.

5. Find the length of $\frac{1}{2}\mathbf{v}$ if $\mathbf{v} = \langle 2, 4, -6\rangle$.

6. Find the dot product of $\mathbf{u} = \langle 2, 1, -3\rangle$ and $\mathbf{v} = \langle 1, 1, -2\rangle$.

7. Determine whether $\mathbf{u} = \langle 1, 1, -1\rangle$ and $\mathbf{v} = \langle -3, -3, 3\rangle$ are orthogonal, parallel, or neither.

8. Find the cross product of $\mathbf{u} = \langle -1, 0, 2\rangle$ and $\mathbf{v} = \langle 1, -1, 3\rangle$. What is $\mathbf{v} \times \mathbf{u}$?

9. Use the triple scalar product to find the volume of the parallelepiped having adjacent edges $\mathbf{u} = \langle 1, 1, 1\rangle$, $\mathbf{v} = \langle 0, -1, 1\rangle$, and $\mathbf{w} = \langle 1, 0, 4\rangle$.

10. Find a set of parametric equations for the line through the points $(0, -3, 3)$ and $(2, -3, 4)$.

11. Find an equation of the plane passing through $(1, 2, 3)$ and perpendicular to the vector $\mathbf{n} = \langle 1, -1, 0\rangle$.

12. Find an equation of the plane passing through the three points $A = (0, 0, 0)$, $B = (1, 1, 1)$, and $C = (1, 2, 3)$.

13. Determine whether the planes $x + y - z = 12$ and $3x - 4y - z = 9$ are parallel, orthogonal or neither.

14. Find the distance between the point $(1, 1, 1)$ and the plane $x + 2y + z = 6$.