Transfer Functions

Second Hour's Agenda

- Transfer Functions
- A Couple of Examples
- Circuit Analysis Using MATLAB LTI Transfer Function Block
- Circuit Simulation Using Simulink Transfer Function Block

Transfer Functions for Circuits

When doing circuit analysis with components defined in the complex frequency domain, the ratio of the output voltage $V_{\rm out}(s)$ ro the input voltage $V_{\rm in}(s)$ *under zero initial conditions* is of great interest. This ratio is known as the *voltage transfer function* denoted $G_v(s)$:

$$G_v(s) = \frac{V_{\rm out}(s)}{V_{\rm in}(s)}$$

Similarly, the ratio of the output current $I_{\rm out}(s)$ to the input current $I_{\rm in}(s)$ *under zero initial conditions*, is called the *cuurent transfer function* denoted $G_i(s)$:

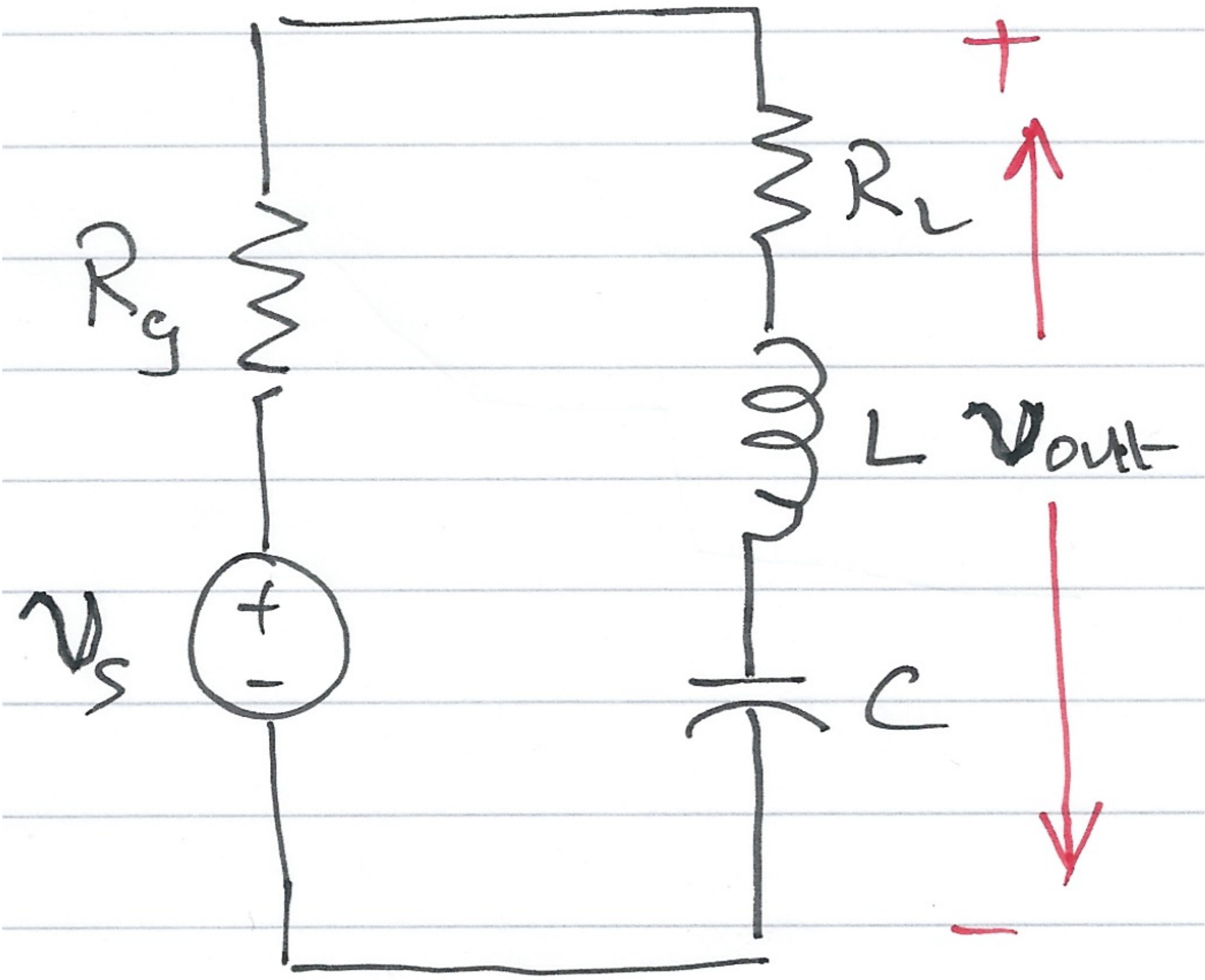
$$G_i(s) = \frac{I_{\rm out}(s)}{I_{\rm in}(s)}$$

In practice, the current transfer function is rarely used, so we will use the voltage transfer function denoted:

$$G(s) = \frac{V_{\rm out}(s)}{V_{\rm in}(s)}$$

Example 6

Derive an expression for the transfer function $G(s)$ for the circuit below. In this circuit R_g represents the internal resistance of the applied (voltage) source v_s, and R_L represents the resistance of the load that consists of R_L, L and C.



Sketch of Solution

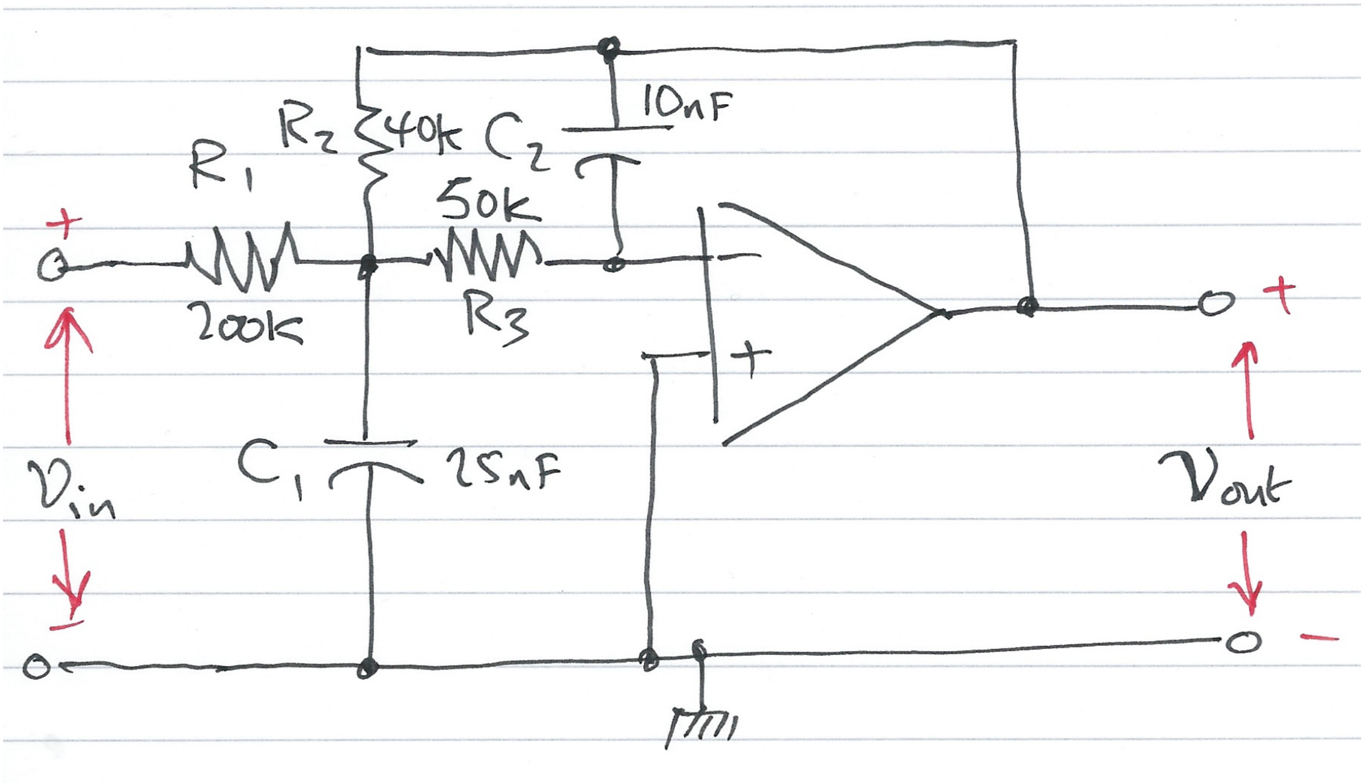
- Replace $v_s(t)$, R_g, R_L, L and C by their transformed (*complex frequency*) equivalents: $V_s(s)$, R_g, R_L, sL and $1/(sC)$
- Use the *Voltage Divider Rule* to determine $V_{\mathrm{out}}(s)$ as a function of $V_s(s)$
- Form $G(s)$ by writing down the ratio $V_{\mathrm{out}}(s)/V_s(s)$

Answer

$$G(s) = \frac{V_{\mathrm{out}}(s)}{V_s(s)} = \frac{R_L + sL + 1/sC}{R_g + R_L + sL + 1/sC}.$$

Example 7

Compute the transfer function for the op-amp circuit shown below in terms of the circuit constants R_1, R_2, R_3, C_1 and C_2. Then replace the complex variable s with $j\omega$, and the circuit constants with their numerical values and plot the magnitude $|G(s)| = |V_{\text{out}}(s)/V_{\text{in}}(s)|$ versus radian frequency ω.



Sketch of Solution

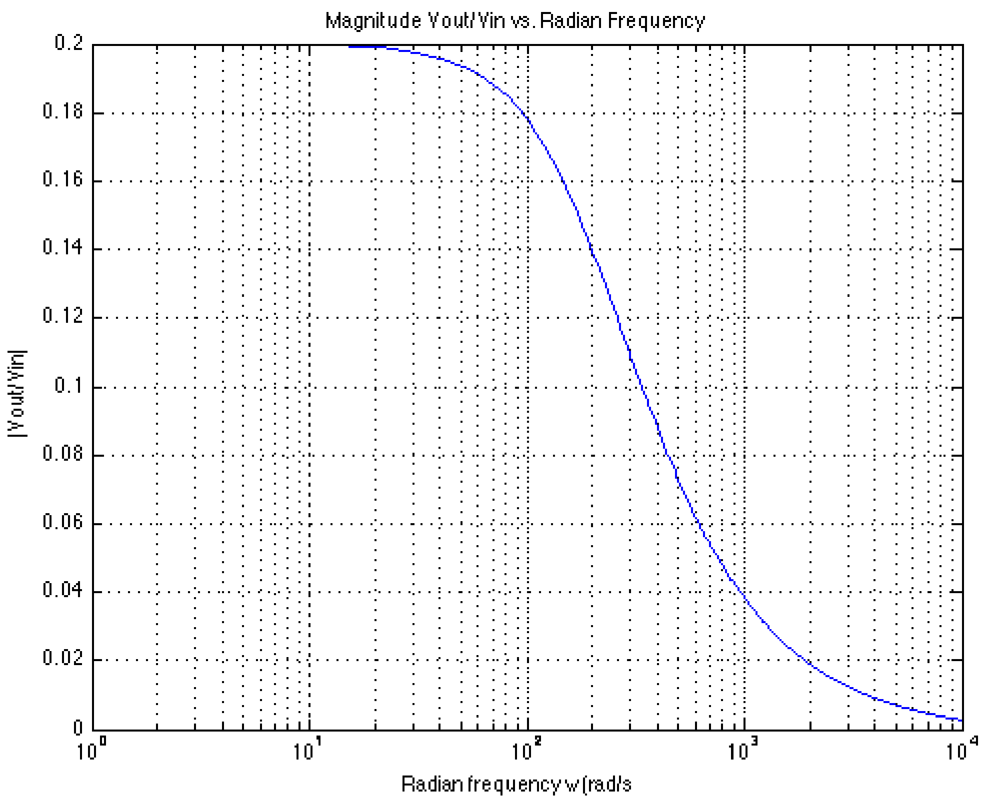
- Replace the components and voltages in the circuit diagram with their complex frequency equivalents
- Use nodal analysis to determine the voltages at the nodes either side of the 50K resistor R_3
- Note that the voltage at the input to the op-amp is a virtual ground
- Solve for $V_{\text{out}}(s)$ as a function of $V_{\text{in}}(s)$
- Form the reciprocal $G(s) = V_{\text{out}}(s)/V_{\text{in}}(s)$
- Use MATLAB to calculate the component values, then replace s by $j\omega$.
- Plot $|G(j\omega)|$ on log-linear "paper"

Answer

$$G(s) = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{-1}{R_1\left(\left(1/R_1 + 1/R_2 + 1/R_3 + sC_1\right)\left(sC_2R_3\right) + 1/R_2\right)}.$$

The Matlab Bit

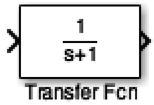
See attached script: solution7.m (matlab/solution7.m).



Using Transfer Functions in Matlab for System Analysis

Please use the file tf_matlab.m (matlab/tf_matlab.m) to explore the Transfer Function features provide by Matlab. Use the *publish* option to generate a nicely formatted document.

Using Transfer Functions in Simulink for System Simulation



The Simulink transfer function (`Transfer Fcn`) block shown above implements a transfer function representing a general input output function

$$G(s) = \frac{N(s)}{D(s)}$$

that it is not specific nor restricted to circuit analysis. It can, however be used in modelling and simulation studies.

Example

Recast Example 7 as a Matlab problem using the LTI Transfer Function block. For simplicity use parameters $R_1 = R_2 = R_3 = 1\ \Omega$, and $C_1 = C_2 = 1\ \mathrm{F}$. Calculate the step response using the LTI functions.

The Matlab solution: example8.m (matlab/example8.m)

Verify the result with Simulink.