Fourier Transforms for Circuit and LTI Systems Analysis

Scope and Background Reading

This session we will apply what we have learned about Fourier transforms to some typical cicuit problems. After a short introduction, this session will be an examples class.

The material in this presentation and notes is based on Chapter 8 (Starting at Section 8.8) of Steven T. Karris, Signals and Systems: with Matlab Computation and Simulink Modelling, 5th Edition. (http://site.ebrary.com/lib/swansea/reader.action?docID=10547416&ppg=271) from the **Required Reading List**. I also used Chapter 5 of Benoit Boulet, Fundamentals of Signals and Systems (http://site.ebrary.com/lib/swansea/reader.action?docID=10228195&ppg=194) from the **Recommended Reading List**.

Agenda

- The system function
- Examples

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The System Function

System response from system impulse response

Recall that the convolution integral of a system with impulse response $h(t)$ and input $u(t)$ is

$$h(t) * u(t) = \int_{-\infty}^{\infty} h(t-\tau)u(\tau)\,d\tau.$$

We let

$$g(t) = h(t) * u(t)$$

Then by the time convolution property

$$h(t) * u(t) = g(t) \Leftrightarrow G(\omega) = H(\omega).\,U(\omega)$$

The System Function

We call $H(\omega)$ the *system function*.

We note that the system function $H(\omega)$ and the impulse response $h(t) form the Fourier transform pair

$$h(t) \Leftrightarrow H(\omega)$$

Obtaining system response

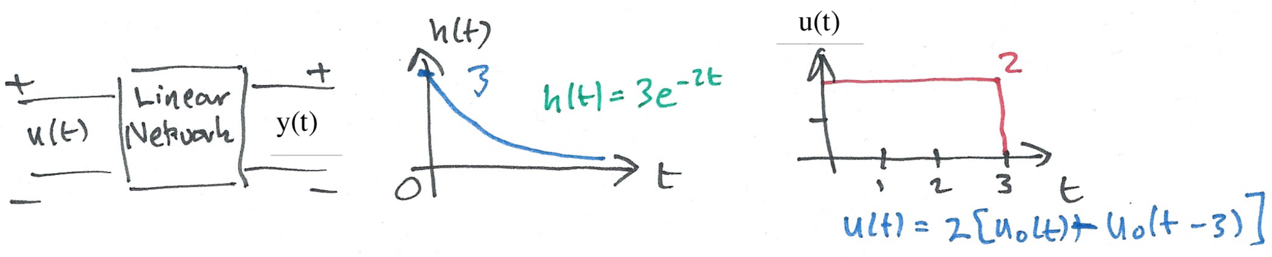
If we know the impulse resonse $h(t)$, we can compute the system response $g(t)$ of any input $u(t)$ by multiplying the Fourier transforms of $H(\omega)$ and $U(\omega)$ to obtain $G(\omega)$. Then we take the inverse Fourier transform of $G(\omega)$ to obtain the response $g(t)$.

1. Transform $h(t) \rightarrow H(\omega)$
2. Transform $u(t) \rightarrow U(\omega)$
3. Compute $G(\omega) = H(\omega).U(\omega)$
4. Find $\mathcal{F}^{-1}\left\{G(\omega)\right\} \rightarrow g(t)$

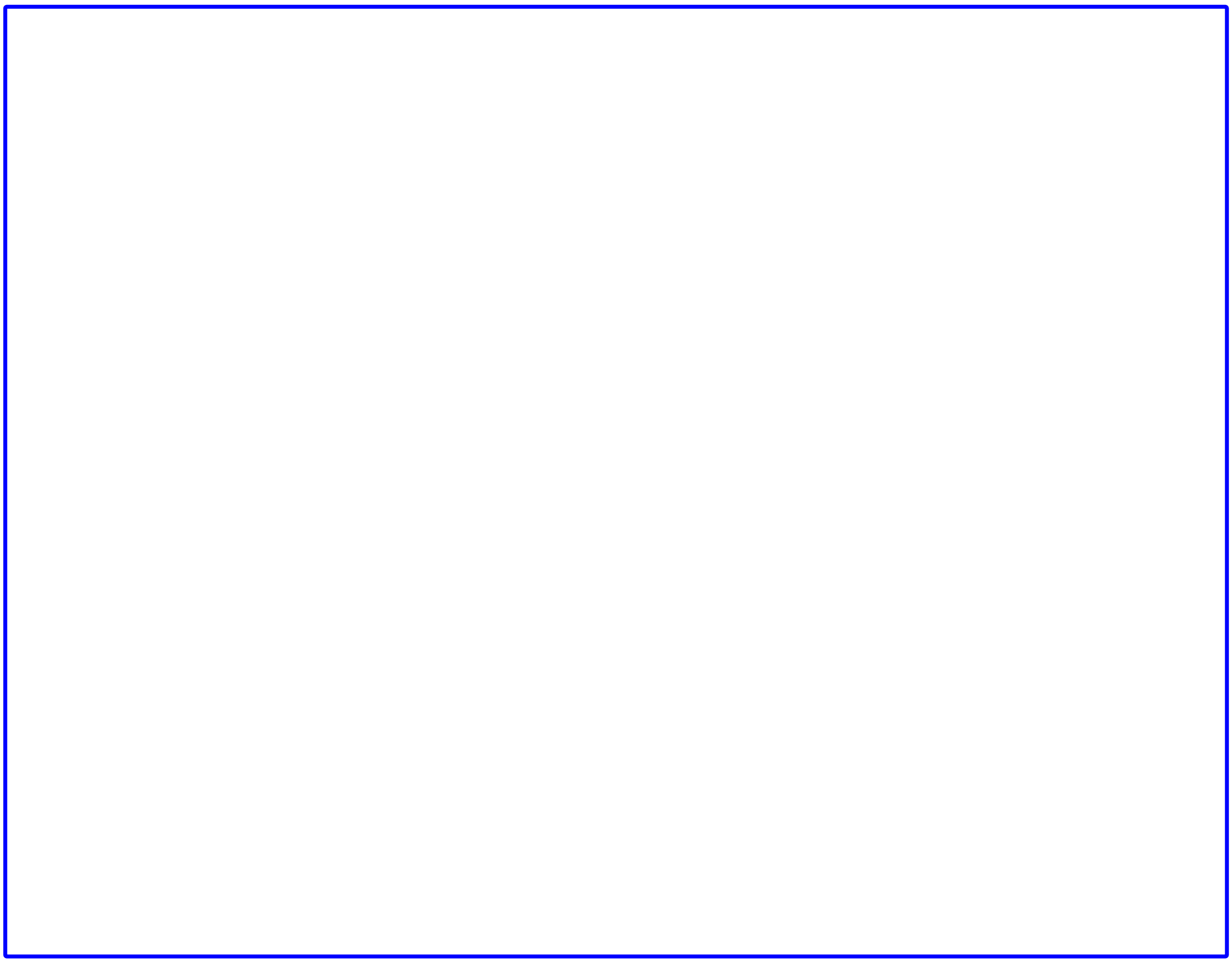
Examples

Example 1

Karris example 8.8: for the linear network shown below, the impulse response is $h(t) = 3e^{-2t}$. Use the Fourier transform to compute the response $y(t)$ when the input $u(t) = 2[u_0(t) - u_0(t-3)]$. Verify the result with Matlab.



Solution



Matlab verification

See ft3_ex1.m (matlab/ft3_ex1.m)

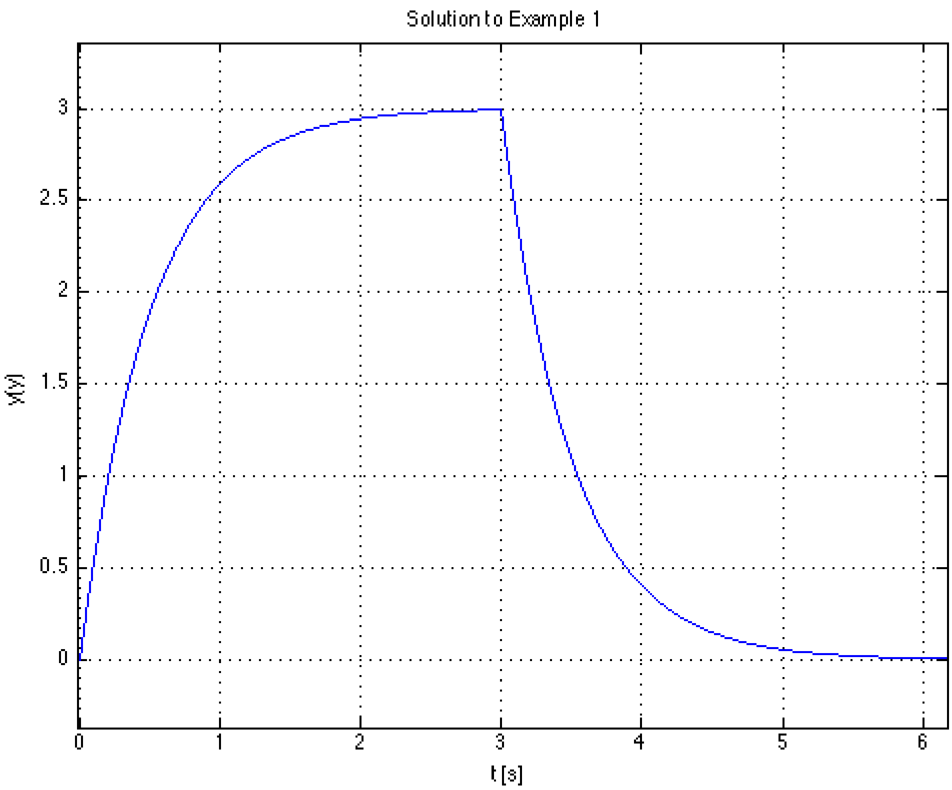
Result:

```
y = 3*heaviside(t) - 3*heaviside(t - 3) + 3*heaviside(t - 3)*exp(6 - 2*t)
 - 3*exp(-2*t)*heaviside(t)
```

Which after gathering terms gives

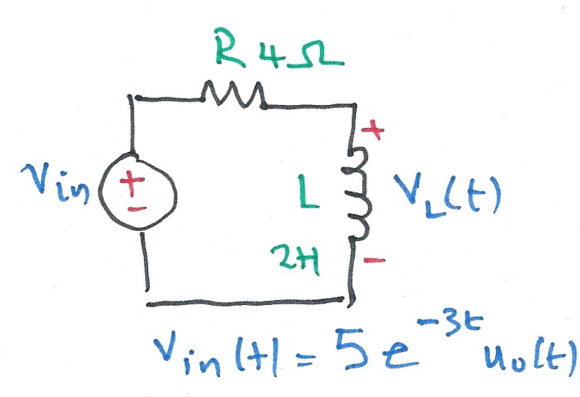
$$y(t) = 3(1 - 3e^{-2t})u_0(t) - 3(1 - 3e^{-2(t-3)})u_0(t-3)$$

And here's a plot:



Example 2

Karris example 8.9: for the circuit shown below, use the Fourier transfrom method, and the system function $H(\omega)$ to compute $V_L(t)$. Assume $i_L(0^-) = 0$. Verify the result with Matlab.



Solution

Matlab verification

See ft3_ex2.m (matlab/ft3_ex2.m)

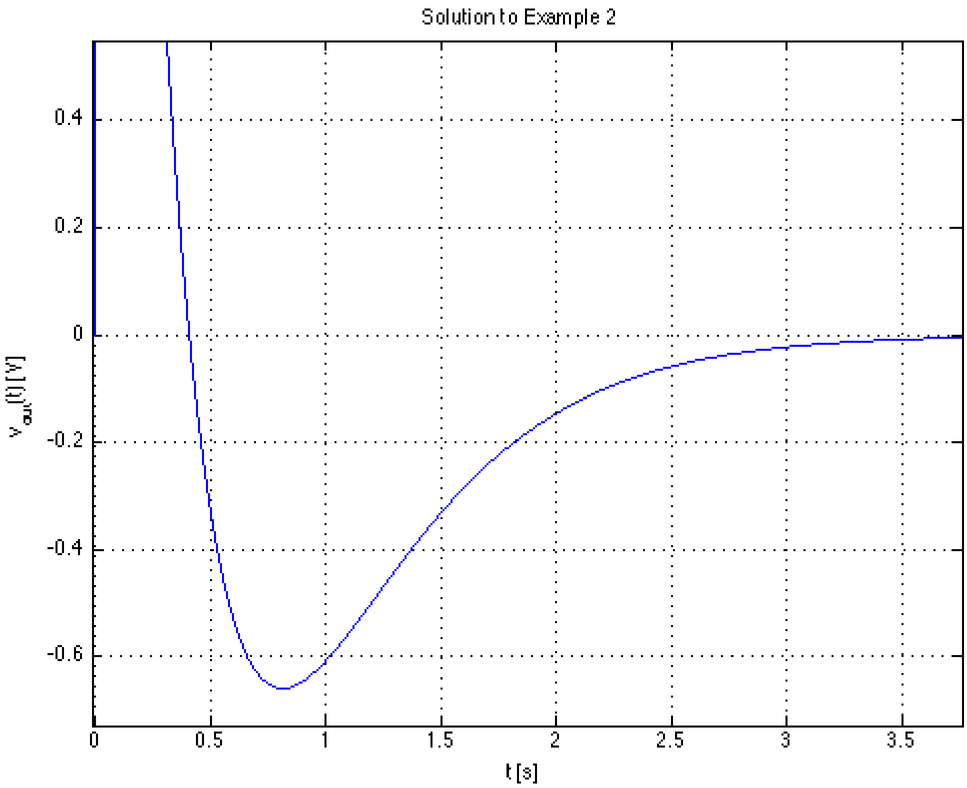
Result:

```
vout = -5*exp(-3*t)*heaviside(t)*(2*exp(t) - 3)
```

Which after gathering terms gives

$$v_{\mathrm{out}} = 5\left(3e^{-3t} - 2e^{-2t}\right) u_0(t)$$

And here's a plot:

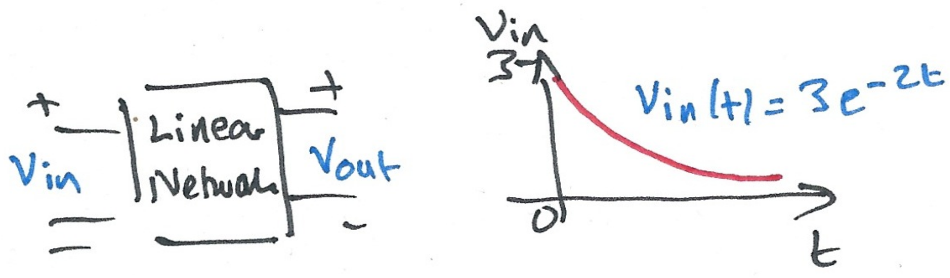


Example 3

Karris example 8.10: for the linear network shown below, the input-output relationship is:

$$\frac{d}{dt} v_{\text{out}} + 4 v_{\text{out}} = 10 v_{\text{in}}$$

where $v_{\text{in}} = 3e^{-2t}$. Use the Fourier transform method, and the system function $H(\omega)$ to compute the output v_{out}. Verify the result with Matlab.



Solution

Matlab verification

See ft3_ex3.m (matlab/ft3_ex3.m)

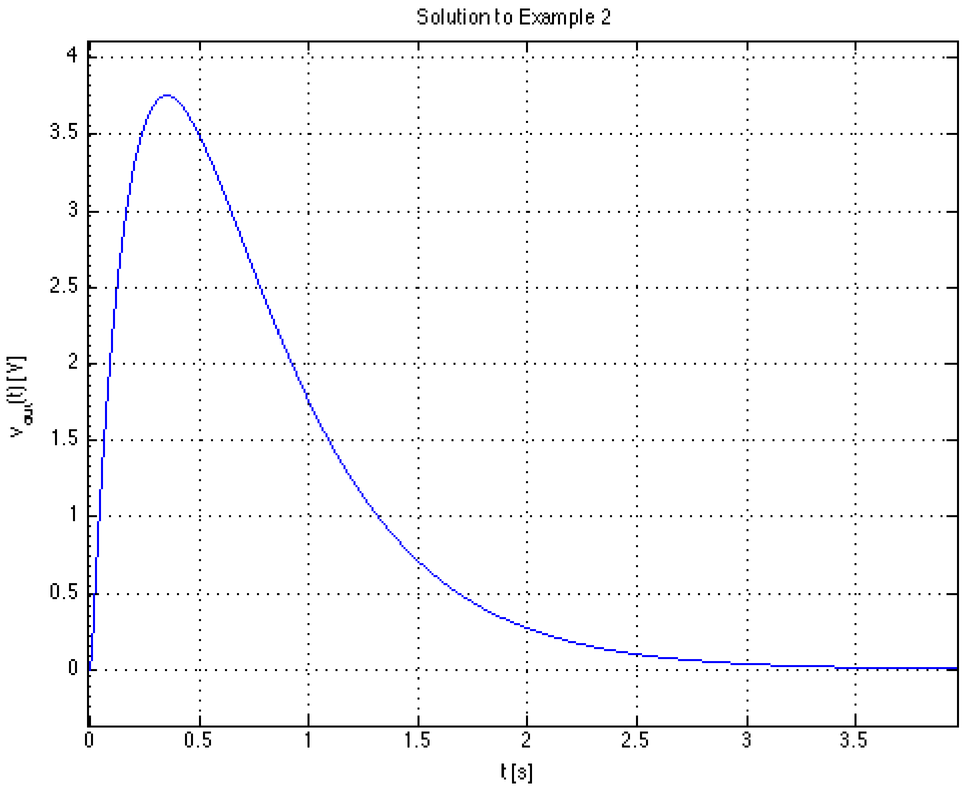
Result:

```
15*exp(-4*t)*heaviside(t)*(exp(2*t) - 1)
```

Which after gathering terms gives

$$v_{\text{out}}(t) = 15\left(e^{-2t}) - e^{-4t}\right) u_0(t)$$

And here's a plot:

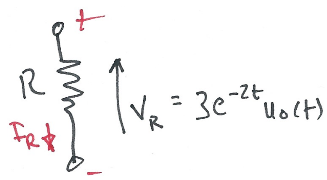


Example 4

Karris example 8.11: the voltage across a 1 Ω resistor is known to be $V_R(t) = 3e^{-2t}u_0(t)$. Compute the energy dissipated in the resistor for $0 < t < \infty$, and verify the result using Parseval's theorem. Verify the result with Matlab.

Note from tables of integrals (http://en.wikipedia.org/wiki/Lists_of_integrals)

$$\int \frac{1}{a^2 + x^2}\, dx = \frac{1}{a}\arctan\frac{x}{a} + C.$$



Solution

Matlab verification

See ft3_ex4.m (matlab/ft3_ex4.m)

Result:

```
Wr = (5160745025300393l*pi)/72057594037927936 = 2.25
```