Defining Transfer Functions in Matlab

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There are two forms of polynomial representation in Matlab. The most obvious is the **expanded polynomial form** where the numerator and denominator of a transfer function would be entered as two row vectors with the polynomial coefficients entered in the order of **descending** powers of s.

For example, if:

$$G(s) = \frac{a(s)}{b(s)} = \frac{s^2+2s+3}{s^3+4s^2+5s+6}$$

The numerator and denominator are entered in Matlab as

```
b = [1, 2, 3];
a = [1, 4, 5, 6];
```

Missing coefficients, must be entered as zero: so $q(s) = s^2 + 2s$ and $r(s) = s^4 + s^2 + 1$ are entered as

```
q = [1, 2, 0];
r = [1, 0, 2, 0, 1];
```

Polynomials - Factorised form¶

$$G(s) = \frac{(s+1)(s+3)}{s(s+2)(s+4)}$$

The advantage of this formulation is that the **zeros** of the numerator and denominator polynomials are obvious by inspection. So it is often used in the preliminary analysis of the performance of a dynamic system. The poles of this transfer function are $s = 0,\ -1\ -4$ and the zeros are $s = -1,\ -3$

In Matlab, this form of transfer function is specified by a column vector of the zeros and a column vector of the poles:

```
z = [-1; -3];
```

```
p = [0; -2; -4];
```

A third parameter, the overall gain K, completes the definition of the so called **pole-zero-gain** form of transfer function. In this case $K = 1$

```
K = 1;
%
```

The Linear Time Invariant System Object

A few years ago, the Mathworks introduced a new data object for the creation and manipulation of system transfer functions. This object is called the ***Linear Time Invariant (LTI) System Object***. It is used to gather the components of a transfer function (or state-space model -- see Week 4) into a single variable which can then easily be combined with other LTI system objects and passed to system analysis functions.

To create a LTI system object representing a factored transfer function the following command is issued:

```
G = zpk(z,p,K)
```

```
G =

   (s+1) (s+3)
  -------------
  s (s+2) (s+4)

Continuous-time zero/pole/gain model.
```

The expanded numerator and denominator form of the transfer function is readily obtained by using a â##data extractionâ## function.

```
[num,den]=tfdata(G,'v')
```

```
num =

     0     1     4     3


den =

     1     6     8     0
```

LTI system objects can also be created from the expanded form of a transfer function directly:

```
G2=tf(num,den)
```

```
G2 =

    s^2 + 4 s + 3
  -----------------
  s^3 + 6 s^2 + 8 s
```

```
Continuous-time transfer function.
```

and the zeros and poles similarly extracted:

```
[zeros,poles,gain]=zpkdata(G2,'v')


zeros =

    -3
    -1


poles =

     0
    -4
    -2


gain =

     1
```

LTI system objects can also be created from the expanded form of a transfer function directly:

```
G2=tf(num,den)


G2 =

    s^2 + 4 s + 3
  -----------------
  s^3 + 6 s^2 + 8 s

Continuous-time transfer function.
```

and the zeros and poles similarly extracted:

```
[zeros,poles,gain]=zpkdata(G2,'v')


zeros =

    -3
    -1


poles =

     0
    -4
    -2
```

```
gain =

     1
```

Setting LTI Properties

Numerous options are available to document the LTI system objects that you create. For example, suppose the transfer function `G` represents a servomechanism with input `'Voltage'` and output `'Angular Position'` We can add this information to the LTI system as follows:

```
set(G,'InputName','Voltage','OutputName','Angular Position')
```

Such documentary information is probably best added when the LTI system object is created, for example as:

```
G3=zpk(z,p,K,'InputName','Armature Voltage (V)',...
               'OutputName','Load Shaft Position (rad)',...
               'notes','An armature voltage controlled
 servomechanism')
```

```
G3 =

  From input "Armature Voltage (V)" to output "Load Shaft Position
 (rad)":
    (s+1) (s+3)
  -------------
  s (s+2) (s+4)

Continuous-time zero/pole/gain model.
```

Once the LTI object has been documented, the documentation can be extracted using commands like:

```
get(G3,'notes')
```

```
ans =

  1×1 cell array

    {'An armature voltage controlled servomechanism'}
```

One can also access the documentation using an â##object referenceâ## notation

```
in=G3.InputName, out=G3.OutputName
```

```
in =

  1×1 cell array
```

```
    {'Armature Voltage (V)'}


out =

  1×1 cell array

    {'Load Shaft Position (rad)'}
```

All the documentation available on an LTI system object may be extracted with a single command:

```
get(G3)

                Z: {[2×1 double]}
                P: {[3×1 double]}
                K: 1
    DisplayFormat: 'roots'
         Variable: 's'
          IODelay: 0
       InputDelay: 0
      OutputDelay: 0
               Ts: 0
         TimeUnit: 'seconds'
        InputName: {'Armature Voltage (V)'}
        InputUnit: {''}
       InputGroup: [1×1 struct]
       OutputName: {'Load Shaft Position (rad)'}
       OutputUnit: {''}
      OutputGroup: [1×1 struct]
            Notes: {'An armature voltage controlled servomechanism'}
         UserData: []
             Name: ''
     SamplingGrid: [1×1 struct]
```

There are numerous other documentation features provided for LTI system objects. Please consult the on-line help for full details.

System Transformations

Matlab supports the easy transformation of LTI system objects between expanded and factored forms. For example to convert a transfer function from **expanded** form to pole-zero-gain form the following command is used:

```
G4 = zpk(G2)


G4 =

   (s+3) (s+1)
  -------------
  s (s+4) (s+2)

Continuous-time zero/pole/gain model.
```

To convert from zero-pole-gain form to expanded form we use the function `tf`

```
G5 = tf(G)
```

```
G5 =

  From input "Voltage" to output "Angular Position":
    s^2 + 4 s + 3
  -----------------
  s^3 + 6 s^2 + 8 s

Continuous-time transfer function.
```

Please note that these transformations are merely a convenience that allow you to work with your preferred form of representation. Most of the tools that deal with LTI system objects will work with any form. Furthermore, you can always use the data extraction functions `zpdata` and `tfdata` to extract the zero-pole-gain and numerator-denominator parameters from a LTI system, no matter in which form it was originally defined, without the need for an explicit conversion.

Combining LTI System Objects

A powerful feature of the LTI system object representation is the ease with which LTI objects can be combined. For example, suppose we have two transfer functions

$$G_1(s) = \frac{s+1}{s+3}$$

and

$$G_2(s) = \frac{10}{s(s+2)}$$

then the series combination of the two transfer functions

$$G_s(s) = G_1(s)G_2(s)$$

is obtained using the `*`Â# (multiplication) operator:

```
G1=tf([1 1],[1 3]);
G2=tf(10,conv([1 0],[1 2])); % conv is polynomial multiplication
Gs=G1*G2 % series connection of two LTI objects
```

```
Gs =

      10 s + 10
  -----------------
  s^3 + 5 s^2 + 6 s

Continuous-time transfer function.
```

```
[zeros,poles,K]=zpkdata(Gs,'v')
%
```

```
zeros =

    -1


poles =

         0
   -3.0000
   -2.0000


K =

    10
```

The parallel connection of two LTI system objects corresponds to addition

$$G_p(s) = G_1(s) + G_2(s)$$

```
Gp = G1 + G2
```

```
Gp =

  s^3 + 3 s^2 + 12 s + 30
  -----------------------
     s^3 + 5 s^2 + 6 s

Continuous-time transfer function.
```

The feedback connection of two LTI system objects is also supported. The function feedback is used for this. Let

$$G(s) = \frac{2s^2 + 5s + 1}{s^2 + 2s + 3}$$

be the forward transfer function of a closed-loop system and

$$H(s) = \frac{5(s+2)}{(s+10)}$$

be the feedback network. Then the closed-loop transfer function (assuming negative feedback) is

$$G_c(s) = \frac{G(s)}{1 + G(s)H(s)}$$

In matlab:

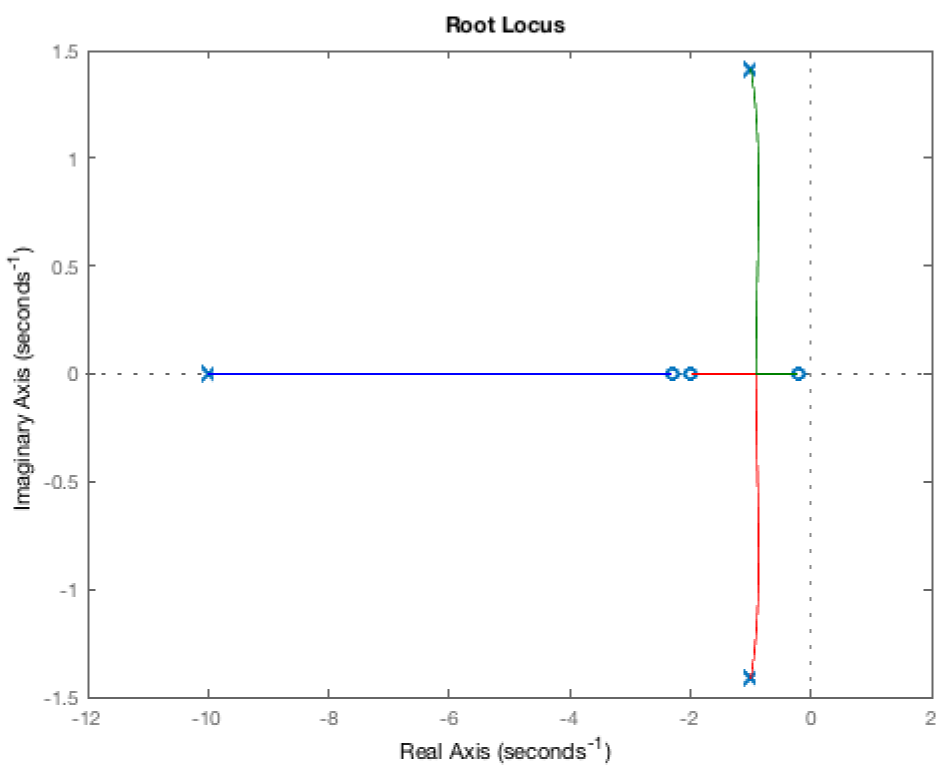
```
G = tf([2 5 1],[1 2 3],'inputname','torque',...
                       'outputname','velocity');
H = zpk(-2,-10,5);
Gc = feedback(G,H) % negative feedback assumed
%


Gc =

  From input "torque" to output "velocity":
  0.18182 (s+10) (s+2.281) (s+0.2192)
  -----------------------------------
   (s+3.419) (s^2 + 1.763s + 1.064)

Continuous-time zero/pole/gain model.
```

The Analysis of LTI System Objects

Matlab uses the LTI system objects as parameters for the analysis tools such as `impulse`, `step`, `nyquist`, `bode` and `rlocus`. As an example of their use try each of following:

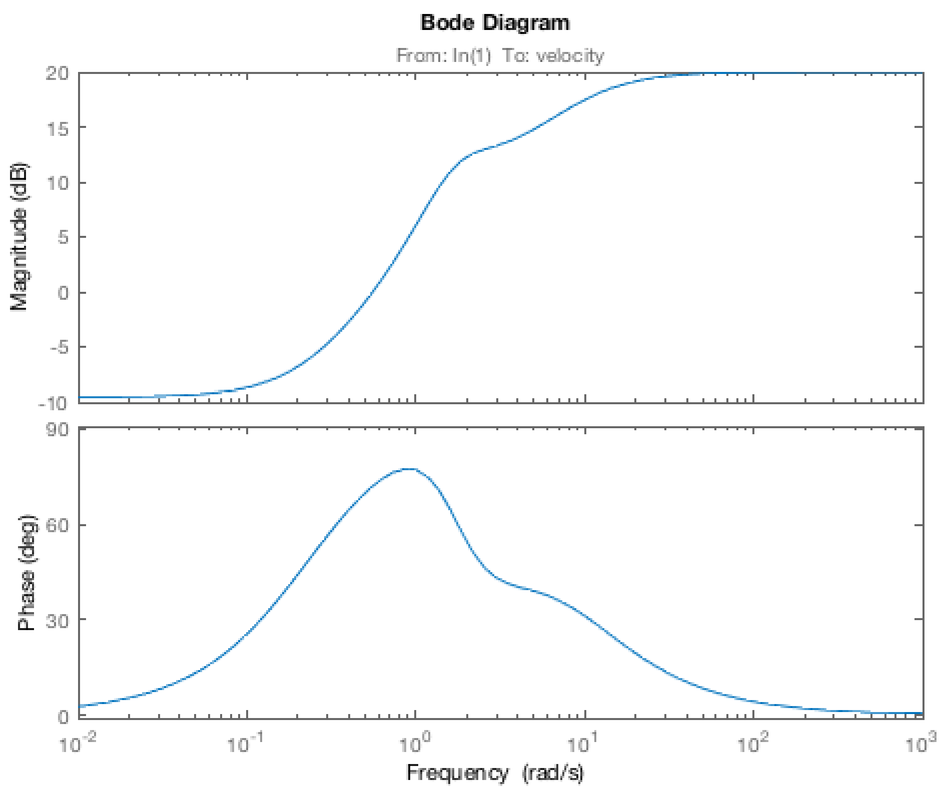
Root locus

```
rlocus(G*H)
```



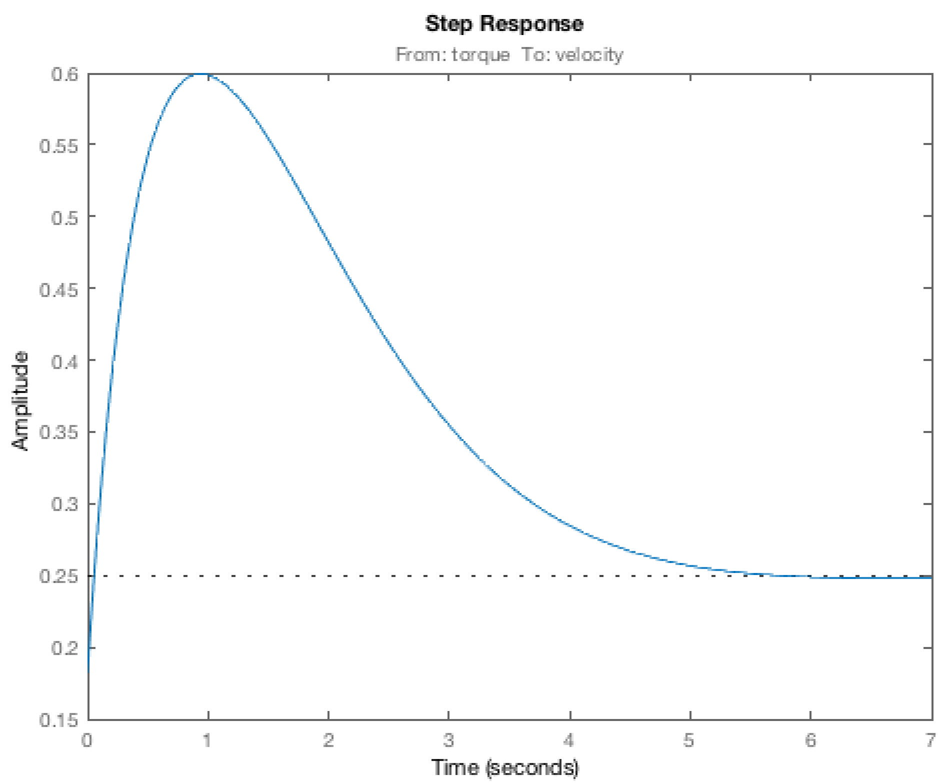
open-loop frequency response

```
bode(G*H)
```



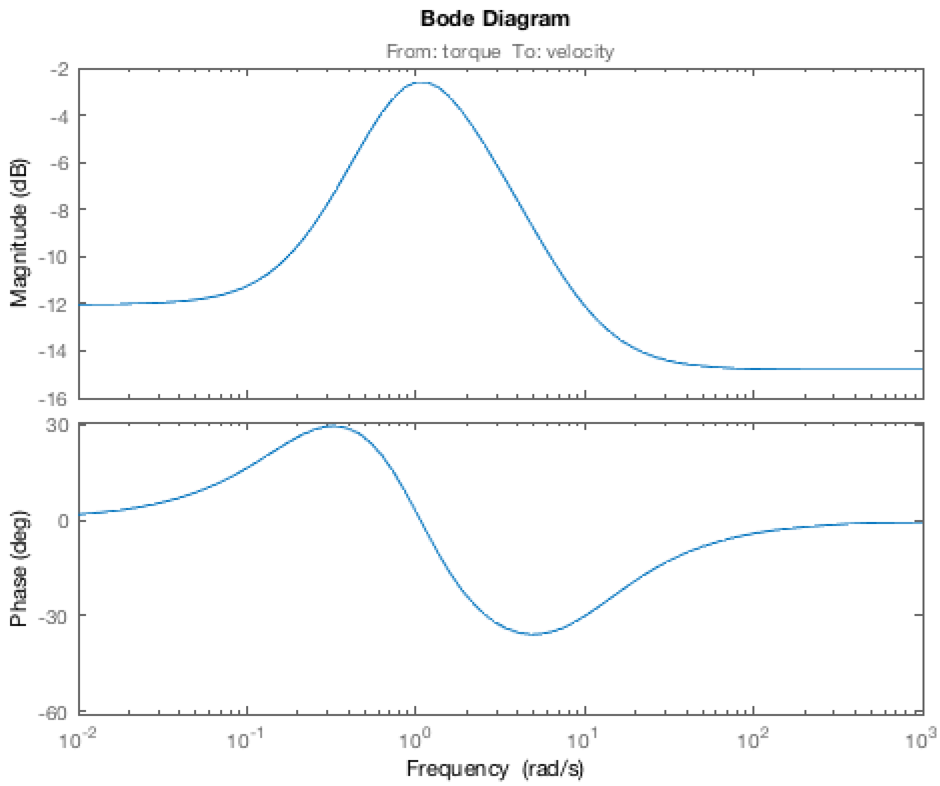
closed-loop step response

```
step(Gc)
```



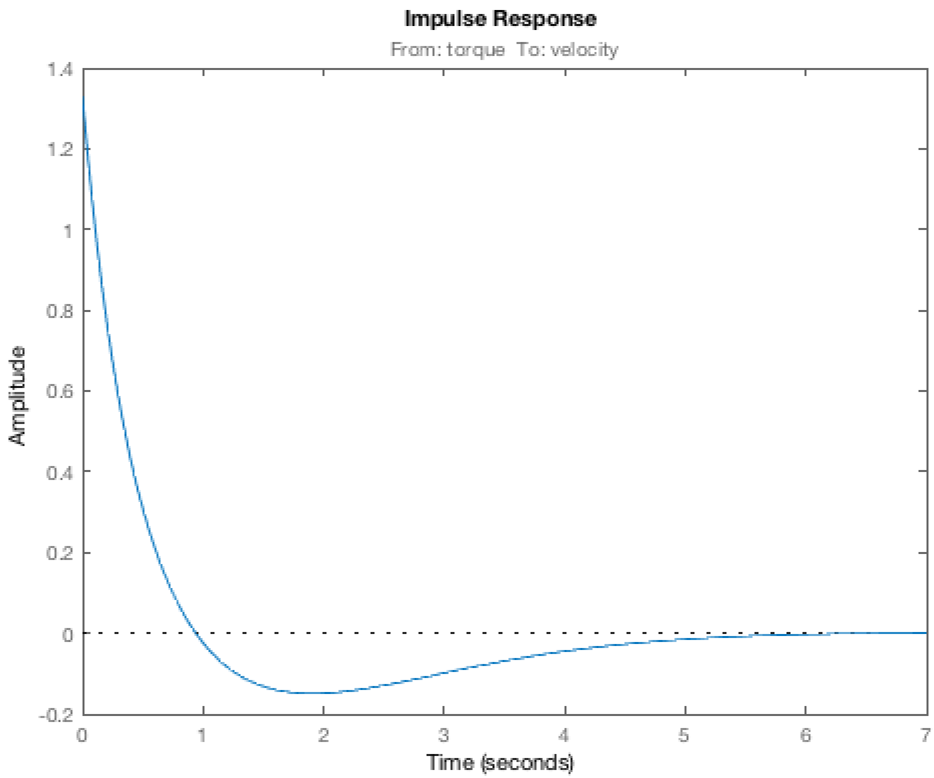
closed-loop frequency response

```
bode(Gc)
```



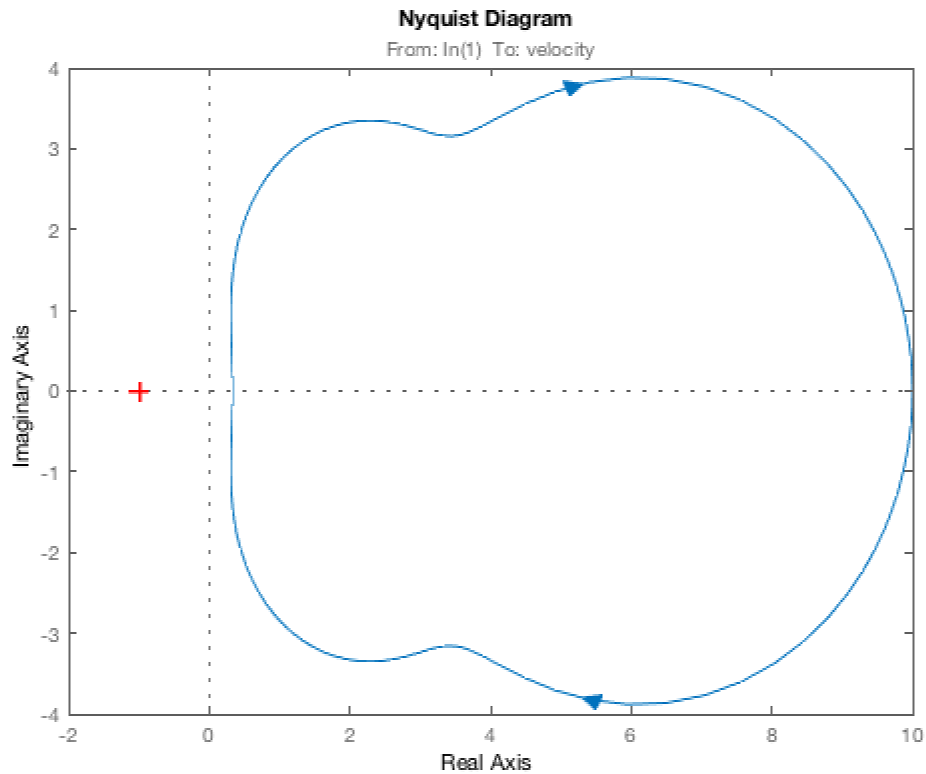
closed-loop impulse response

```
impulse(Gc)
```



Nyquist diagram

```
nyquist(G*H)
```



Matlab also provides two interactive graphical tools that work with LTI system objects. * `ltiview` is a graphical tool that can be used to analyze systems defined by LTI objects. It provides easy access to LTI objects and time and frequency response analysis tools. * `rltool` is an interactive tool for designing controllers using the root locus method.

Control engineers will find `sisotool` useful.

You are encouraged to experiment with these tools.

Partial Fraction Expansions

Matlab provides a command called `residue` that returns the partial fraction expansion of a transfer function. That is, given

$$G(s) = \frac{s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots a_1 s + a_0}$$

it returns

$$\frac{r_1}{s+p_1} + \frac{r_2}{s+p_2} + \cdots + \frac{R_n}{s+p_n} + K(s)$$

where p_i are the poles of the transfer function, r_i are the coefficients of the partial fraction terms (called the **residues of the poles**) and $K(s)$ is a remainder polynomial which is usually empty.

To use this, the starting point must (rather perversely) be the expanded form of the transfer function in polynomial form.

Thus given

$$C(s) = \frac{5(s+2)}{s(s+3)(s+10)}$$

we obtain the partial fraction expansion using the Matlab command sequence:

```
k = 5; z = [-2]; p = [0; -3; -10]; % zero-pole-gain form
C = zpk(z,p,k);
[num,den] = tfdata(C,'v')
%
% (Note that the leading terms in num are zero).


num =

     0     0     5    10


den =

     1    13    30     0


[r,p,k] = residue(num,den)


r =

   -0.5714
    0.2381
    0.3333


p =

   -10
    -3
     0


k =

     []
```

which we interpret to mean

$$C(s) = \frac{0.3333}{s} + \frac{0.2381}{s+3} - \frac{0.5714}{s+5}.$$

If $C(s)$ represents the step response of the system

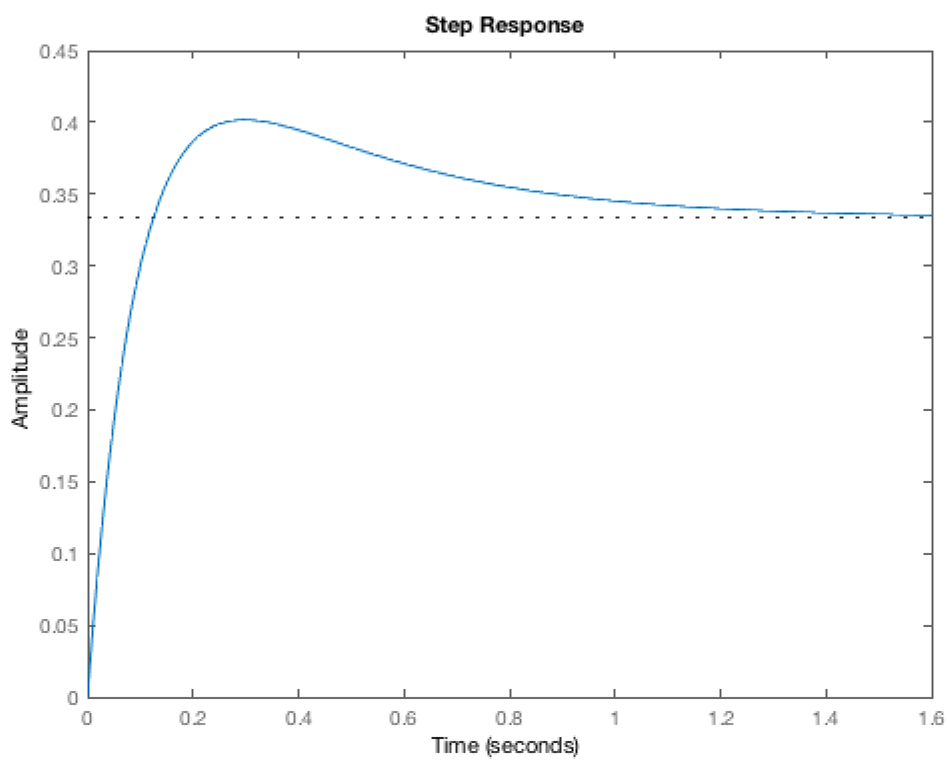
$$G(s) = \frac{5(s+2)}{(s+3)(s+10)}$$

then the step response is, by **Inverse Laplace Transform**:

$$y(t) = \left(0.3333 + 0.2381e^{-3t} - 0.5714e^{-10t}\right) u_0(t).$$

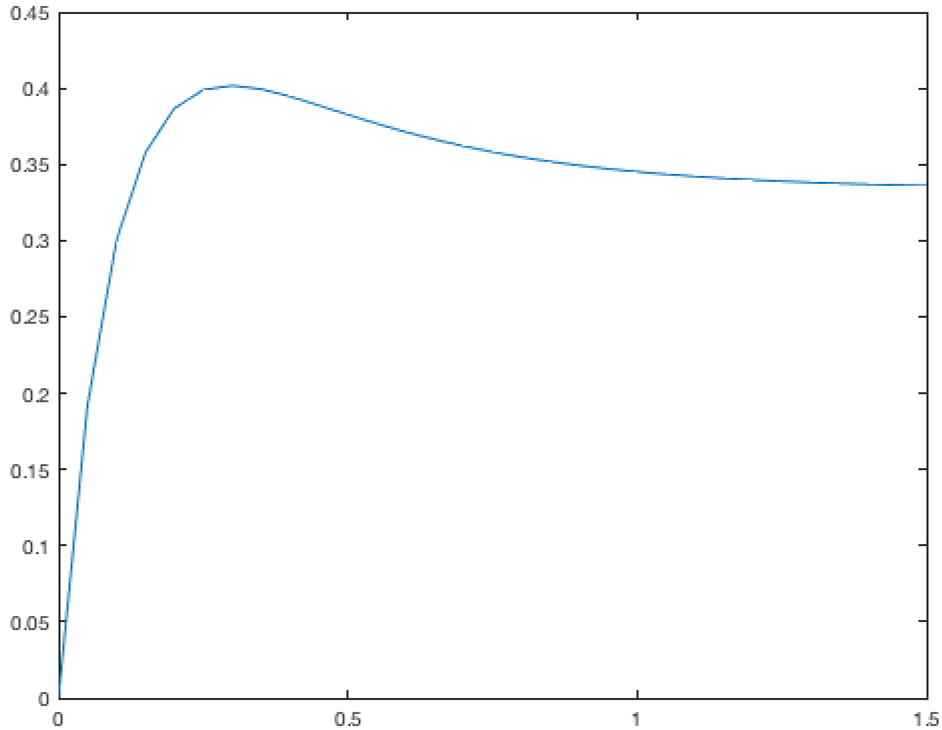
You can check this with the commands:

```
newC = tf([5, 10],[1, 13, 30])
step(newC) % provides $u_0(t)$


newC =

     5 s + 10
  ---------------
  s^2 + 13 s + 30

Continuous-time transfer function.
```




```
t = 0:.05:1.5; % time vector
c = 0.3333 + 0.2381 * exp(-3*t) - 0.5714 * exp(-10*t);
```

```
plot(t,c)
```



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