The Application of Physical Potential Theory to the Geographic Information Science
- An Example to the Population Density -

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ABSTRACT

The potential theory, a concept in physics, is applied to the population density distribution on maps in this study. Based on some assumptions, using this potential theory, virtual force to population density can be imaged. It helps to understand, how the population density distribution is determined, where is the most popular place in a region, and so on. The potential distribution is numerically solved by using Poisson equation. From the numerical results of models about Japan, Vietnam, and China, properties of potential distributions for these models are identified. In model of Japan, The huge concentration to Tokyo, Nagoya, and Osaka regions is appeared quantitatively. The bipolar structure of the potential in the model of Vietnam and large scale one dish like structure in the model of China are also derived. The investigation of properties of the potential must be effective to consider problems about population and other geographical studies.

1. INTRODUCTION

Geographical Information System (GIS) now becomes one of the useful methods for several academic and practical researches including humanities or social studies. So developing new GIS scheme is important for new analyses in such researches. Several geographical data distributed on maps, for example, population, culture, ecosystems, and so on, can be compared their common properties, distributions on maps, with physical data, such as the spatial distributions of mass or electric charge. From this similarity, we can understand that the potential theory used in physics can be applied to geographical data analysis under some conditions.

The potential is a kind of physical concept to describe specification of the fields, such as gravity field, electric field, and so on. In such physical fields, the gradient of potential means the force driven by those fields. In case of "geographical potential," we can assume similar virtual force by taking the gradient of the potential. So we can know the reason why such distribution appears or what results are there on maps by considering the potential.

Here after, based on such point of view, author shows basics of the application of the potential theory to geoinformatics, and as an important applicative analysis, also shows the analysis of population density distributions for three countries, Japan, Vietnam, and China. In case of the potential in population density, gradient of the potential distribution works as like the force which moves populations.

In the next chapter, basics of the potential theory and its numerical style are explained. In continuous chapters, models and results of the numerical method will be shown. We also consider about the summary of our results in the final chapter.

2. APPLICATION OF THE POTENTIAL TO GEOINFORMATICS

From the theory of the gravity field in physics, its specifications can be written by the spatial distribution of the potential. It formulates some basic field equations, and finally, the Poisson equation is taken as the most primitive expression. The Poisson equation for the gravity field is:

$$\Delta\varphi = 4\pi G\rho, \tag{1}$$

where Δ, φ, G, and ρ mean Laplacian, gravitational potential, gravitational constant, and mass density, respectively. Laplacian is an operator written as:

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \tag{2}$$

in two dimensional Cartesian coordinate.

In this case, constants are determined from mechanics exactly. However in the application for the geoinformatics, to determine constants, we assume some conditions based on observations. Here we use temporal constant b as the coefficient of right side of the Poisson equation. So the Poisson equation for population density now becomes:

$$\Delta\varphi = b\rho, \tag{3}$$

where φ, b, and ρ here mean the potential about population density field, assumed constant, and population density. Here after, constant b is assumed to 1.

From the comparison of equation (3) with (1), we understand the first derivative of φ means a kind of the force acting to the population density. This virtual force concentrates population to the higher density region. With the assumption, static equilibrium of the population density, we determine the unique relation among the population density, the potential, and the virtual force.

3. NUMERICAL METHOD

Solving equation (3) is the main method of this work. However generally, differential equation can not be solved analytically, so we must apply numerical method to the finite differential form of the equation (3). The equation (3) is the second order differential equation which has boundary value problem. So the finite differential form of the equation (3) becomes the form of large scale linear simultaneous equations. The ICCG Scheme (Meijerink and van der Vorst, 1977; van der Volst, 1981) is employed to solve these linear simultaneous equations in the Poisson solver. The program is written in Fortran90 by GNU Fortran90 compiler (GNU) on the Linux system.

The detail of practical explanation about the numerical method is described in appendix of Umekawa (2000).

4. MODEL

The models of three countries, Japan (Model 1), Vietnam (Model 2), and China (Model

3) are analyzed. Parameters for the models are summarized in table 1. The global population density data in 2000, published from Socioeconomic Data Analysis Center (SEDAC, 2007), are employed. They all are grid data on maps. The size of grid is 2.5' for the direction of both longitude and latitude. The number of grid in the table mean two dimensional values in directions for longitude times latitude. The "Population number" column is describing approximated population numbers based on the population statistics come from United Nations (SEDAC, 2007).

For simplicity, we assume the two dimensional Cartesian coordinate in numerical calculations. Boundary conditions for solving Poisson equation are zero fixed condition. Regions of sea are regarded as zero density grids. In real fluctuation, these conditions are not exact, but under the assumption of equilibrium, they are acceptable because the time evolution is not supposed in this work.

Table 1. Models

Model	Country	Grid size	# of Grids (Long. x Lat.)	Population Number	Year
1	Japan	2.5'	840x600	127,096,000	2000
2	Vietnam	2.5'	216x432	77,402,000	2000
3	China	2.5'	1632x960	1,275,133,000	2000

5. RESULT

The results of three models will be shown in this chapter. In all models, 0.0 on boundary of the regions is the maximum potential value from the boundary conditions, and the minimum of each models are -2,709,445 in Model 1 (Japan), -638,060 in Model 2 (Vietnam), and -14,906,282 in Model 3 (China), respectively. These values should be used to analyses from the point of relative view but absolute.

5.1 Model 1 (Japan)

Figure 1 shows contours of the potential distribution, vector field of the virtual force for population density, and color contour plots of the population density in model 1. Contours are drawn for every forty equal division of values. In color contours, blue means no data and 0 region, red is the highest value, and white region means data saturated. Arrows in vector field are normalized as the length of forty grids is corresponding to the maximum value. These subjects are shown as the same normalized scales in all figures here after.

From the figure, we can understand Tokyo, Nagoya, and Osaka regions become one large scale valley of the potential. This means these three cities have weight much larger than other regions in Japan. This behavior has been qualitatively understood from other data such as the population distribution already. However, it is clearly shown with the quantitative evidence in this study.

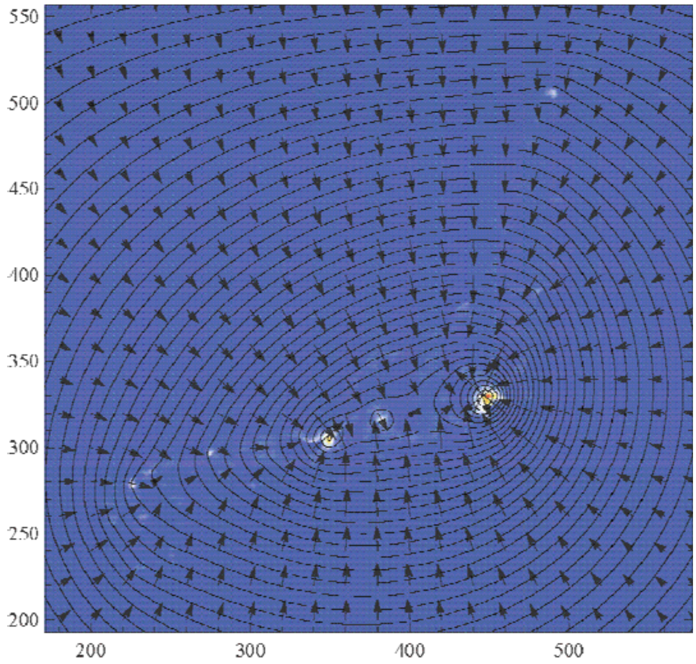

Figure 1. The result of model 1 (Japan).

To investigate details of this large scale deep potential valley on three cities, the close up picture is shown in figure 2.

Figure 2 clearly shows that the potential valley made by Tokyo is much deeper than those made by Nagoya and Osaka. Depth of the potential at Tokyo is corresponding to six contour curves. On the other hand, they are one in Nagoya and two in Osaka. This means that Tokyo is dominant not only number of population but also a kind of magnetism. It can be understood by contours of the potential and vector field of the force.

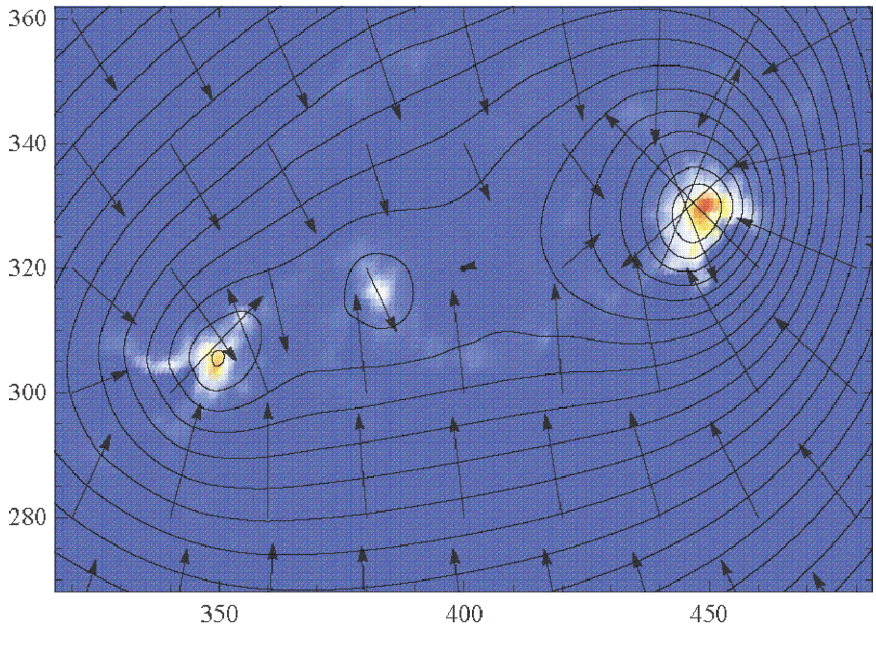

Figure 2. Close up picture of model 1, Tokyo, Nagoya, and Osaka regions.

5.2 Model 2 (Vietnam)

The result of model 2 (Vietnam) is shown in figure 3. In this model, two deep potential valleys at Hanoi and Ho Chi Minh regions are characteristic. Between two potential valleys, small potential valleys are in line. The bipolar structure leads to conserve dispersion of the population.

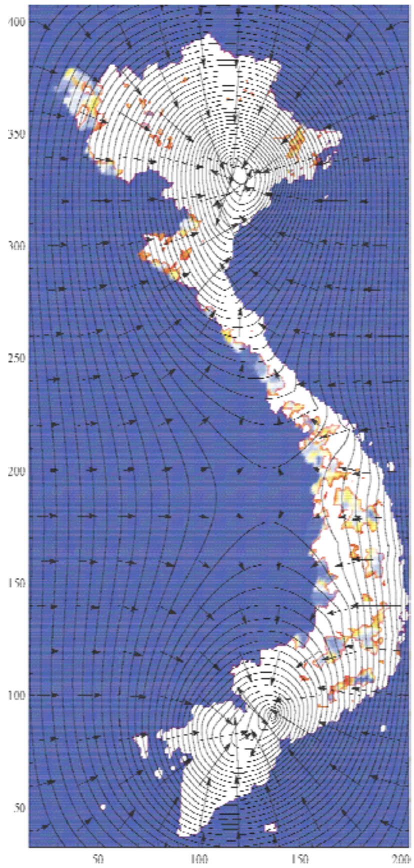

Figure 3. The result of model 2 (Vietnam).

5.3 Model 3 (China)

Because of a technical reason, the figure about model 3 (China) is not shown. In model 3, large scale dish like structure of the potential is shown. This means that the concentration of population is weak and its dispersion is kept at a certain level relatively. The population structure in China is generally "flat" in whole scale of plain region.

6. SUMMARY

The population density potential is defined in this study. Based on the theory, three models, in Japan, Vietnam, and China are investigated. In the models of Japan and Vietnam, the figures about contours of the potential and vector field of the virtual force come from gradient of the potential are shown.

In the model of Japan, the concentration of the population to Tokyo, Nagoya, and Osaka regions are shown. This is the fact unnecessary to discuss. However in this work, we can understand quantitatively how large the concentration of the population density is and how much these regions have the power to attract the population. To be able to get the quantitative result about properties of the population densities is one of the important conclusions of this study. When we consider details of the structure of the potential around these three cities, standing out of Tokyo is clearly shown. The depth of the potential in Tokyo is three times larger than Osaka and six times larger than Nagoya. This numerically exact result is first described by using the potential theory of the population density.

The properties in each country are also shown. They are one large valley of the potential in the model of Japan, bipolar structure in the model of Vietnam, and dish like structure in the model of China. By the comparison of such properties of each country, we can derive methods to apply this potential theory to solve other population problems. Additionally, the potential theory itself is expected applying to other geographical quantities such as distributions of culture, vegetation, and so on, in future works.

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