3-D GEOLOGIC MODELING OF FAULTED GEOLOGIC STRUCTURE

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ABSTRACT

There is a definite rule of the formative process of geologic structure formed through sedimentation and erosion corresponds with the logical model of geologic structure. Concerning the faulting we define the rule which suggests the surface of fault divides a 3-D geologic unit and the open space into two areas, and the geologic structures of each area can be preserved. Therefore, faulting can be reasonably included into the recursive definition, which leads logical model of geologic structures formed through the sedimentation and the erosion, and the faulted geologic structure can be expressed as recursive definition. In addition, this recursive definition can lead a logical model of geologic structure cut by plural faults. With introducing a logical model of faulted geologic structure, we propose the faulted geologic map can be generated without any changes of the existent processing system based on a logical model of geologic structure.

1. BASIC THEORY

1.1 Logical model of geologic structure

Let an objective 3-D space Ω be a survey area and suppose that the space Ω is divided into two subspaces on surface S. Where S^+ and S^- give subspaces that lie above and below the surface S, respectively. The surface S is contained in subspace S^-, it has the relation of the next way.

$$S^+ \cup S^- = \Omega \,, \qquad S^+ \cap S^- = \varnothing \,. \tag{1}$$

The space Ω is composed of n geologic units (b_1, …, b_n) including open space b_0 (air). Figure 1 shows that the simple geologic structures are expressed as the vertical section. The geologic unit b_1 represent basement rock. When there are sedimentation and erosion, the geologic unit b_2 is formed. Surface S_1 is a geologic boundary surface, and surface S_2 is a topographic surface. The distribution of geologic unit b_1, b_2 and b_0 are expressed as follow (see Table 1);

$$b_1 = S_1^{\,-} \cap S_2^{\,-} \,, \qquad b_2 = S_1^{\,+} \cap S_2^{\,-} \,, \qquad b_0 = S_2^{\,+} \,. \tag{2}$$

The distribution of geologic units b_0, …, b_n are defined by surfaces S_1, … , S_p. The logical relation between the distribution of geologic unit and the surface are termed *a logical*

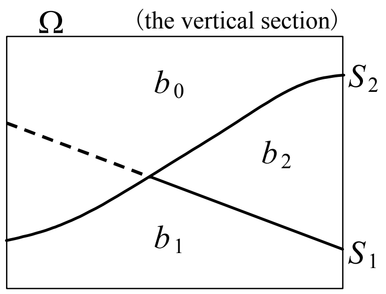


Figure 1. The simple geologic structures.

Table 1. The value of function g_1.

b_0		+
	S_1	S_2
b_1	−	−
b_2	+	−

model of geologic structure (Shiono *et al.*, 1994, 1998).

1.2 Geologic function

As the geologic units b_0, b_1, … , b_n are defined by surfaces, they can be expressed in *minset* (Gill, 1976). The *minset* is a minimum subspace that is divided by the surfaces S_1, … , S_p in the space Ω. Each minset defined by;

$$m_{d1d2\ldots dp} = X_1 \cap X_2 \cap \ldots \cap X_p \,, \tag{3}$$

$$X_k = \begin{cases} S_k^- \,;\, d_k = 0 \\ S_k^+ \,;\, d_k = 1 \end{cases} \quad (k = 1, 2, \ldots, p)\,.$$

Four *minsets* can be defined by two surfaces S_1 and S_2 as follows;

$$m_{00} = S_1^- \cap S_2^- \,, \quad m_{01} = S_1^- \cap S_2^+ \,, \quad m_{10} = S_1^+ \cap S_2^- \,, \quad m_{11} = S_1^+ \cap S_2^+ \,. \tag{4}$$

When it was provided the logical model of geologic structure, the distribution of geologic units b_0, … , b_n can be expressed the union of minset is generated by the surfaces S_1, … , S_p. In the case of Figure 2, *minset* can be derived for the geologic units as follows;

$$\begin{cases} b_0 = (S_1^+ \cup S_1^-) \cap S_2^+ = m_{01} \cup m_{11} \,, \\ b_1 = m_{00} \,, \\ b_2 = m_{10} \,. \end{cases} \tag{5}$$

There is the relation between the set of minset $M = \{m_{00}, m_{01}, m_{10}, m_{11}\}$ and the set of geologic unit $B = \{b_0, b_1, b_2\}$ as shown below;

$$m_{00} \subset b_1 \,, \qquad m_{01} = b_0 \,, \qquad m_{10} = b_2 \,, \qquad m_{11} = b_0 \,. \tag{6}$$

When let a function g_1: $M \to B$ corresponds to the geologic units including their *minset*, a value of geologic function g_1 in Figure 1 is expressed in Table 2.

Further, for a point P (x, y, z) in a space Ω, a *minset* $m_{d1d2\ldots dp}$ can be assigned a value of $d_k = 0$ or $d_k = 1$ depending on whether P (x, y, z) lies S_k^+ or S_k^-, respectively. This correspondence between every point in the space Ω and *minset* is expressed by a function g_2: $\Omega \to M$.

Therefore, the function g: $\Omega \to B$ is expressed to compound the function g_1 and function g_2 as follows;

Table 2. The value of function g_1.

M	$g_1(m)$
m_{00}	b_1
m_{01}	b_0
m_{10}	b_2
m_{11}	b_0

$$g(x, y, z) = g_1(g_2(x, y, z)) \,. \tag{7}$$

The function g is termed *the geologic function* (Masumoto *et al.*, 1997), assigns a unique geologic unit to every point in the space Ω.

2. LOGICAL MODEL OF FAULTED GEOLOGIC STRUCTURE

2.1 Geologic structure formed by faulting

It is assumed here that the fault plane is a single plane, and the internal structures on both sides of the plane are preserved during and after faulting. The fault plane divides filled space β into two subspaces; hanging wall and foot wall. Further, in order to formulate the effect of faulting, we assume that fault plane F also divides the open space α into two subspaces. Then, the space Ω can be classified into four subspaces by the fault plane F and the topographic surface as shown in Figure 3. The open space in the lower side and the upper side of the fault plane F can be given by $\alpha \cap F^-$ and $\alpha \cap F^+$ respectively. The filled space in the lower side and the upper side of the fault plane F can be given by $\beta \cap F^-$ and $\beta \cap F^+$ respectively. After fault movement, these subspaces change into $\alpha' \cap F^-$, $\beta' \cap F^-$, $\alpha'' \cap F^+$ and $\beta'' \cap F^+$ respectively. Such an event is termed a faulting event, and is denoted by "event f" (Yonezawa *et al.*, 2006).

Let us consider the effect of the event f after the initial state. At the initial state, a surface S_1 is a boundary surface between the open space b_0 and the geologic unit b_1. Let b_0', b_1' and S_1' be the open space, the geologic unit and the surface in foot wall respectively and b_0'', b_1'' and S_1'' the open space, the geologic unit and the surface in hanging wall respectively.

$$\begin{cases} b_1' = S_1'^- \cap F^- \,, \\ b_1'' = S_1''^- \cap F^+ \,, \\ b_0 = b_0' \cup b_0'' = (S_1'^+ \cap F^-) \cup (S_1''^+ \cap F^+) \,. \end{cases} \tag{8}$$

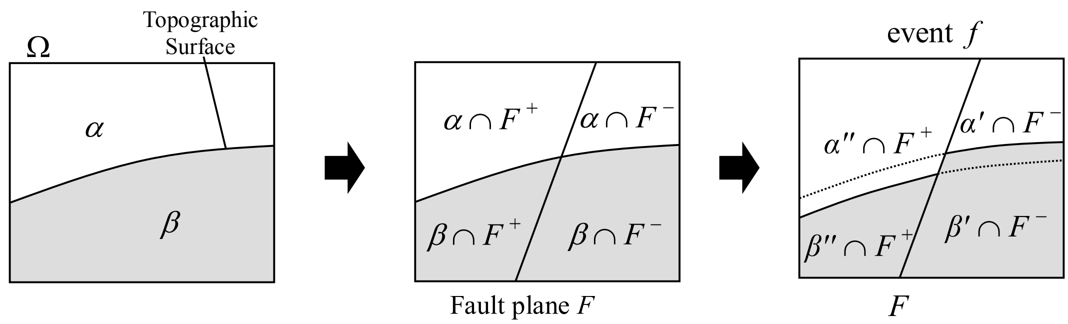


Figure 3. Concept of event f.

2.2 Recursive definition of logical model of faulted geologic structure

In order to embed the event f, we substitute $b_0, \ldots, b_{2m}$ for $b_0', \ldots, b_m''$ and $S_1, \ldots, S_{2p}$ for $S_1', \ldots, S_p''$ and further S_{2p+1} for F. Then the recursive definition can be represented as follows:

(1) The initial state ($k = 1$),

$$\begin{cases} b_1^{(1)} = B_1^{(1)}(S_1) = S_1^-, \\ b_0^{(1)} = B_0^{(1)}(S_1) = S_1^+. \end{cases} \tag{9}$$

(2) Let $\{b_0^{(k)}, b_1^{(k)}, \ldots, b_m^{(k)}\}$ be the geologic structure formed by a sequence of events (v_1, $v_2, \ldots, v_k$) like the sedimentation, erosion and faulting. If each geologic unit $b_i^{(k)}$ is expressed by surfaces $S_1, S_2, \ldots, S_p$;

$$b_i^{(k)} = B_i^{(k)}(S_1, S_2, \ldots, S_p), \qquad (i = 0, \ldots, m) \tag{10}$$

then the geologic structure is changed by an event v_{k+1} as follows:

If $v_{k+1} = f$,

$$\begin{cases} b_i^{(k+1)} = B_i^{(k)}(S_1, S_2, \ldots, S_p) \cap S_{2p+1}^-, & (i = 1, \ldots, m) \\ b_{m+i}^{(k+1)} = B_i^{(k)}(S_{p+1}, S_{p+2}, \ldots, S_{2p}) \cap S_{2p+1}^+, & (i = 1, \ldots, m) \\ b_0^{(k+1)} = \{B_0^{(k)}(S_1, S_2, \ldots, S_p) \cap S_{2p+1}^-\} & \\ \qquad\qquad \cup \{B_0^{(k)}(S_{p+1}, S_{p+2}, \ldots, S_{2p}) \cap S_{2p+1}^+\}. & \end{cases} \tag{11}$$

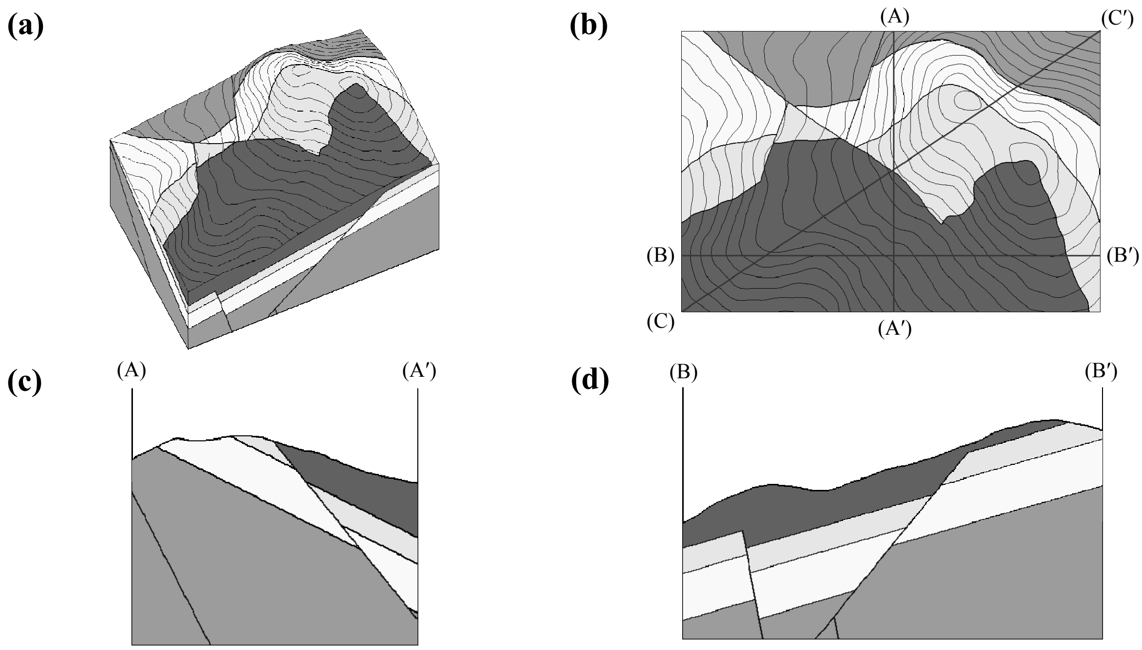
3. CASE STUDY

Visual Basic program Geomodel2003 was developed in order to visualize the geologic map (Yonezawa *et al.*, 2004). Example of faulted geologic structure is shown in Figure 4(a) by using Geomodel2003. The surface is given in a form of a grid composed of elevations, which is known as DEM (Digital Elevation Model). The data of the objective surface and the boundary surfaces are given from Table 3. They were generated by Visual Basic program Terramod2001 (Sakamoto *et al.*, 2001).

This geologic structure is formed by $V_6 = (v_1, c, c, f, f, c^*)$. v_1 is the initial state. c is the sedimentation event, c^* is the complex event of erosion and sedimentation event. The logical model of geologic structure is shown in Table 4. Figure 4(b) is the 2-D geologic map. Figures 4(c) – (e) represent the vertical geologic section map.

4. CONCLUSION

The recursive definition of faulted geologic structure is integrated consistently in our previous computer algorithm for 3-D modeling of geologic structure. It is expected that the newly developed theory and algorithm would advance the computer processing of the faulted geologic structure on to the next stage, and there will be more applications of such geologic information in various other fields.



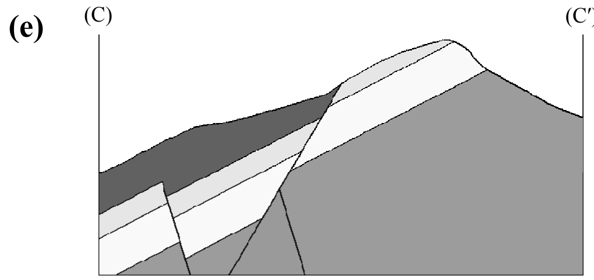


Figure 4. Example of visualization of geologic structure formed by $V_6 = (v_1, c, c, f, f, c^*)$.

Table 3. Data of surfaces (Foot wall and Hanging wall) for $V_6 = (v_1, c, c, f, f, c^*)$.

Foot wall for S_{15}

Boundary upper / lower	Surface	Strike and dip of surface	Linear equation of plane	Position
b_2/b_1	S_1	N60°W / 30°S	$-0.250x - 0.433y + 0.866z = 383.5$	Foot wall
b_3/b_2	S_2	N60°W / 30°S	$-0.250x - 0.433y + 0.866z = 412.6$	Foot wall
b_0/b_3	S_3	N60°W / 30°S	$-0.250x - 0.433y + 0.866z = 428.1$	Foot wall
Fault	S_7	N20°E / 80°S	$0.925x - 0.337y + 0.174z = 118.0$	–
b_5/b_4	S_4	N60°W / 30°S	$-0.250x - 0.433y + 0.866z = 371.7$	Hanging wall
b_6/b_5	S_5	N60°W / 30°S	$-0.250x - 0.433y + 0.866z = 400.8$	Hanging wall
b_0/b_6	S_6	N60°W / 30°S	$-0.250x - 0.433y + 0.866z = 416.3$	Hanging wall

Hanging wall for S_{15}

Boundary upper / lower	Surface	Strike and dip of surface	Linear equation of plane	Position
b_8/b_7	S_8	N60°W / 30°S	$-0.250x - 0.433y + 0.866z = 383.5$	Foot wall
b_9/b_8	S_9	N60°W / 30°S	$-0.250x - 0.433y + 0.866z = 412.6$	Foot wall
b_0/b_9	S_{10}	N60°W / 30°S	$-0.250x - 0.433y + 0.866z = 428.1$	Foot wall
Fault	S_{14}	N20°E / 80°S	$0.925x - 0.337y + 0.174z = 118.0$	–
b_{11}/b_{10}	S_{11}	N60°W / 30°S	$-0.250x - 0.433y + 0.866z = 371.7$	Hanging wall
b_{12}/b_{11}	S_{12}	N60°W / 30°S	$-0.250x - 0.433y + 0.866z = 400.8$	Hanging wall
b_0/b_{12}	S_{13}	N60°W / 30°S	$-0.250x - 0.433y + 0.866z = 416.3$	Hanging wall

Table 4. Tabular form of logical model of geologic structure for $V_6 = (v_1, c, c, f, f, c^*)$.

b_0																+
	S_1	S_2	S_3	S_7	S_4	S_5	S_6	S_{15}	S_8	S_9	S_{10}	S_{14}	S_{11}	S_{12}	S_{13}	S_{16}
b_1	−			−				−								−
b_2	+	−		−				−								−
b_3	+	+	−	−				−								−
b_4				+	−			−								−
b_5				+	+	−		−								−
b_6				+	+	+	−	−								−
b_7								+	−			−				−
b_8								+	+	−		−				−
b_9								+	+	+	−	−				−
b_{10}								+				+	−			−
b_{11}								+				+	+	−		−
b_{12}								+				+	+	+	−	−
b_{13}	+	+	+	−				−								−
b_{13}				+	+	+	+	−								−
b_{13}								+	+	+	+	−				−
b_{13}								+				+	+	+	+	−

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