Curs - rezolvarea sistemelor liniare

Matrici speciale:

- transpusa conjugată: A^*
- normală: $A^*A = A \cdot A^*$
- unitară: $AA^* = A^*A = I$
- ortogonală: $A \cdot A^T = A^T A = I$, A - reală
- hermitiană: $A^* = A$
- simetrică: $A^T = A$, A - reală

Eigen - values
- vectors

$A \in \mathbb{C}^{n \times n}$, $\lambda = \{\lambda_1, \lambda_2 \dots \lambda_n\}$

λ_i - s.n. eigen value $\Leftrightarrow$ $A \cdot x = \lambda_i x$, x - vector

$Ax - \lambda_i x I_n = 0 \Rightarrow x(A - \lambda_i I_n) = 0$. $x \neq 0 \Rightarrow$ det

$\Rightarrow \det(A - \lambda_i I_n) = 0 \dots$

Raza spectrală $\rho(A) = \max\{|\lambda_1|, |\lambda_2|, \dots, |\lambda_n|\}$.

Dacă A - nesingulară ($\det A \neq 0$) $\Rightarrow$ $\text{cond}(A) = \|A\| \cdot \|A^{-1}\|$, dacă e - singulară

$\Rightarrow \text{cond}(A) = \infty$

$\boxed{\text{Eliminare gaussiană cu pivotare}}$

$$\begin{pmatrix} 10 & -7 & 0 \\ -3 & 2 & 6 \\ 5 & -1 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 7 \\ 4 \\ 6 \end{pmatrix}$$

$\Downarrow$ $L_1 \cdot 0.3 + L_2 \to L_2$
$-0.5 \cdot L_1 + L_3 \to L_3$

$$\begin{pmatrix} 10 & -7 & 0 \\ 0 & -0.1 & 6 \\ 0 & 2.5 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 7 \\ 6.1 \\ 2.5 \end{pmatrix}$$

$\Downarrow$ $L_3 \leftrightarrow L_2$

$$\begin{pmatrix} 10 & -7 & 0 \\ 0 & 2.5 & 5 \\ 0 & -0.1 & 6.2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 7 \\ 2.5 \\ 6.1 \end{pmatrix}$$

$$\Downarrow \; 0.04 \cdot E_2 + E_3 \to E_3$$

$$\begin{pmatrix} 10 & -7 & 0 \\ 0 & 2.5 & 5 \\ 0 & 0 & 6.2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 7 \\ 2.5 \\ 6.2 \end{pmatrix} \Rightarrow$$

$6.2 x_3 = 6.2 \Rightarrow \underline{x_3 = 1}$

$\Rightarrow x_2 = -1$

$\Rightarrow x_1 = 0$

$\Rightarrow x = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

<u>Backsubst</u>

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ 6 \end{pmatrix}$$

$x_3 = \frac{6}{1}$; $2x_2 + 4 \cdot x_3 = 5 \Rightarrow x_2 = \frac{5 - 4 \cdot x_3}{2}$

$1 \cdot x_1 + 2 x_2 + 3 \cdot x_3 = 2 \Rightarrow x_1 = 2 -$

Avem: <u>$PA = LU$</u>

$A \cdot x = b \mid P \cdot \Rightarrow \left. \begin{array}{l} PAx = P \cdot b \\ PA = L \cdot U \end{array} \right| \Rightarrow$

$LUx = P \cdot b \Rightarrow$

$\Rightarrow Ux = \underbrace{L^{-1} \cdot P \cdot b}_{y}$ (2)

$y = L^{-1} \cdot P \cdot b \Leftrightarrow \boxed{Ly = P \cdot b}$ Alg rez sist Lower triang $\Rightarrow$ (2)

$Ux = y \Rightarrow \ldots$