Interpolare polinomială

I interpolare Lagrange

$$(L_m f)(x) = \sum_{j=0}^{m} f(x_i)\cdot l_i(x) \quad , \quad l_i(x) = \prod_{\substack{j=0 \\ j\neq i}}^{m} \frac{x-x_j}{x_i-x_j}$$

$$f(x) = 6x+1 \quad , \quad \begin{aligned} x_0 = 1 &\Rightarrow f(x) = 7 \\ x_1 = 2 &\Rightarrow f(x) = 13 \end{aligned}$$

$$(L_m f)(x) = f(x_0)\cdot l_0(x) + f(x_1)\cdot l_1(x) \Rightarrow$$

$$\Rightarrow 7\, l_0(x) + 13\, l_1(x)$$

$$L_0(x) = \frac{x-2}{-1} \cdot \quad L_1(x) = \frac{x-1}{1} \Rightarrow f(x) = -7x+14+13x-13 \Rightarrow$$

$$\Rightarrow \boxed{f(x) = 6x+1}$$

$$P_m = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \qquad a = \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} \qquad b = \begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$\overbrace{\begin{pmatrix} 1 & x_0 & x_0^2 & x_0^3 \\ 1 & x_1 & x_1^2 & x_1^3 \\ 1 & x_2 & x_2^2 & x_2^3 \\ 1 & x_3 & x_3^2 & x_3^3 \end{pmatrix}}^{V} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} \Rightarrow Va = b$$

$$\Rightarrow V^{-1} = U^T, \quad U = (u_{ij}), \quad l_i(x) = \sum_{k=0}^{m} u_{ik}\, x^k .$$

Interpolare Newton

$(x, y) = \{(0,1), (1,4), (2,9), (3,16)\}$.

x	y (b_0)	b_1	b_2	b_3
0	1	$\frac{4-1}{1-0} = 3$	$\frac{5-3}{2-0} = 1$	$\frac{1-1}{3-0} = \frac{0}{3} = 0$
1	4	$\frac{9-4}{2-1} = 5$	$\frac{7-5}{3-1} = 1$	
2	9	$\frac{7}{1} = 7$		
3	16			

$$f(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1) + \dots$$

$$f(x) = 1 + 3(x-0) + 1\cdot(x-0)(x-1) + 0(x-0)(x-1)(x-2)(x-3)$$

$$= 1 + 3x + x^2 - x = x^2 + 2x + 1.$$

$f(x) = x^3$. $\Rightarrow x_k = k,\ k = 0 \dots 3$

x	$f(x)$ (b_0)	b_1	b_2	b_3
0	0	$\frac{1-0}{1-0} = 1$	$\frac{7-1}{2-0} = 3$	$\frac{6-3}{3-0} = 1$
1	1	$\frac{8-1}{2-1} = 7$	$\frac{19-7}{3-1} = \frac{12}{2} = 6$	
2	8	$\frac{27-8}{1} = 19$		
3	27			

$$f(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1) + b_3(x - x_0)(x - x_1)(x - x_2) \Rightarrow$$

$$\Rightarrow f(x) = 0 + 1(x - 0) + 3(x-0)(x-1) + (x-0)(x-1)(x-2)$$

$$= 0 + x + 3x^2 - 3x + (x^2 - x)(x-2) = 0 + \cancel{x} + \cancel{3x^2} - \cancel{3x} + x^3 - \cancel{2x^2} - \cancel{x^2} + \cancel{2x}$$

$$= x^3$$

Hermite

$f \in C^4[-1,1]$. $\quad x_0=-1$ simplu, $x_1 = 0$ - dublu, $x_2 = 1$ - simplu

$$
\begin{array}{cccccc}
z_0 & -1 & f(-1) & \frac{f(0)-f(-1)}{0-(-1)} & \frac{f'(0) - f(0)+f(-1)}{0+1} & \frac{f(1)}{2} - f'(0) + \frac{f(-1)}{2} \\
z_1 & 0 & f(0) & f'(0) & \frac{f(1)-f(0)-f'(0)}{1-0} & \\
z_2 & 0 & f(0) & \frac{f(1)-f(0)}{1-0} & & \\
z_3 & 1 & f(1) & & &
\end{array}
$$

$$\frac{f(1) - \cancel{f(0)} - f'(0) - f'(0) + \cancel{f(0)} - f(-1)}{1+1} =$$

$$= \frac{f(1)}{2} - f'(0) + \frac{f(-1)}{2}$$

$$(H_3 f)(x) = f(-1) + \big(f(0) - f(-1)\big)(x+1) + \big(f'(0) - f(0) + f(-1)\big)(x+1)x +$$
$$+ \left(\frac{f(1)}{2} - f'(0) + \frac{f(-1)}{2}\right)(x+1)\cdot x^2$$

$$\int_{-1}^{1} H_3 f(x)\,dx = \int_{-1}^{1} f(-1)\,dx + \big(f(0) - f(-1)\big)\int_{-1}^{1} x+1\,dx +$$

$$+ \big(f'(0) - f(0) + f(-1)\big)\int_{-1}^{1} x^2 + x\,dx + \left(\frac{f(1)}{2} - f'(0) + \frac{f(-1)}{2}\right)\int_{-1}^{1} x^3 + x^2\,dx$$

$$= f(-1)\cdot x\Big|_{-1}^{1} + \big(f(0) - f(-1)\big)\,\frac{x^2}{2} + x\Big|_{-1}^{1} +$$

$$\big(f'(0) - f(0) + f(-1)\big)\cdot\frac{x^3}{3} + \frac{x^2}{2}\Big|_{-1}^{1} + \left(\frac{f(1)}{2} - f'(0) + \frac{f(-1)}{2}\right)\cdot\frac{x^4}{4} + \frac{x^3}{3}\Big|_{-1}^{1}$$

$$= \cancel{2f(-1)} + \overset{3}{\cancel{2}}\,f(0) - \cancel{2f(-1)} + \cancel{\frac{2}{3}f'(0)} - \frac{2}{3}f(0) + \frac{2}{3}f(-1)$$

$$+ \frac{f(1)}{3} - \cancel{\frac{2f'(0)}{3}} + \frac{f(-1)}{3} = \frac{1}{3}f(-1) + \frac{4}{3}f(0) + \frac{1}{3}f(1)$$

$$(R_3 f)(x) = \frac{1}{4!}(x+1)\cdot f^{(4)}(\xi(x))$$

$\Rightarrow$ integrala $\int_{-1}^{1} A_3 f(x) = \frac{1}{3} f(-1) + \frac{4}{3} f(0) + \frac{1}{3} f(1) - \int_{-1}^{1} R_3 f(x) =$

$$= \frac{1}{3} f(-1) + \frac{4}{3} f(0) + \frac{1}{3} f(1) - \frac{1}{90}\cdot f^{(4)}(\xi(x))$$

s.v. $x = \frac{b-a}{2} t + \frac{a+b}{2} \Rightarrow dx = \frac{b-a}{2} dt$

$$\int_a^b f(x)\,dx = \frac{b-a}{2}\int_{-1}^{1} f\left(\frac{b-a}{2} t + \frac{a+b}{2}\right) dt =$$

$$= \frac{b-a}{2}\left[\frac{1}{3} f(a) + \frac{4}{3} f\left(\frac{a+b}{2}\right) + \frac{1}{3} f(b) + \left(\frac{b-a}{2}\right)^4 \frac{1}{90} f^{(4)}(\xi)\right]$$

$$= \frac{b-a}{6}\left[f(a) + 4 f\left(\frac{a+b}{2}\right) + f(b) + \frac{(b-a)^5}{32\cdot 90}\cdot f^{4}(\xi)\right].$$

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1) $x_0 = -1$, $x_1 = 0$ - dublu, $x_2 = 1$

$X = \{x_1, x_2 \dots x_m\}$ $\qquad P(x_i) = f(x_i)$

$y = \{f(x_1), \dots f(x_m)\}$

. . . .

$\frac{\partial f}{\partial x^n} = \{f^n(x_1), \dots f^n(x_m)\}$ $\qquad \frac{\partial P}{\partial x^n} = f^n(x_i)$

$x_0 = -1, \quad x_1 = 0$ - dublu, $\quad x_2 = 1$

	x_i	$f(x_i)$ (b_0)	b_1	b_2	b_3
z_0	-1	$f(-1)$	$\frac{f(0)-f(-1)}{0-(-1)}$	$\frac{f'(0)-f(0)+f(-1)}{0-(-1)}$	$\frac{f(-1)}{2} - f'(0) - \frac{f(1)}{2}$
z_1	0	$f(0)$	$f'(0)$	$\frac{f(1)-f(0)-f'(0)}{1-0}$	
z_2	0	$f(0)$	$\frac{f(1)-f(0)}{1-0}$		
z_3	1	$f(1)$			

$$(H_3 f)(x) = b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1) + b_3(x-x_0)(x-x_1)(x-x_2)$$

$$(H_3 f)(x) = f(-1) + (f(0)-f(-1))(x+1) + (f'(0)-f(0)+f(-1))(x+1)\cdot x +$$
$$+ \left(\frac{f(-1)}{2} + f'(0) - \frac{f(1)}{2}\right)(x+1)\cdot x^2 \qquad a)$$

$$\int_{-1}^{1} (H_3 f)(x)\,dx = f(-1)\cdot \underbrace{\int_{-1}^{1} dx}_{2} + (f(0)-f(-1)) \overbrace{\int_{-1}^{1} \frac{x+1}{1}\,dx}^{2} + (f'(0)-f(0)+f(-1)) \underbrace{\int_{-1}^{1} x^2+x\,dx}_{2/3}$$
$$+ \left(\frac{f(-1)}{2} - f'(0) - \frac{f(1)}{2}\right) \underbrace{\int_{-1}^{1} x^3+x^2\,dx}_{2/3} =$$

$$2\cancel{f(-1)} + {}^{3}\!/2\, f(0) - 2\cancel{f(-1)} + \cancel{\tfrac{2}{3}f'(0)} - \frac{2}{3}f(0) + \frac{2}{3}f(-1) + \frac{f(-1)}{3} - \cancel{\tfrac{2}{3}f'(0)}$$
$$- \frac{f(1)}{3} = \frac{1}{3}f(-1) + \frac{4}{3}f(0) + \frac{1}{3}f(+1)$$

$$(R_m f)(x) = \frac{u(x)}{(m+1)!}\cdot f^{(m+1)}(\xi), \quad m = 3$$

r_i - grad multiplicitate

$$u(x) = (x-x_0)^{r_0+1} \cdots (x-x_m)^{r_m+1}$$

$$u(x) = (x+1)^1 \cdot (x-0)^2 \cdot (x-1)^1$$

$$(R_3 f)(x) = \frac{(x+1)\cdot x^2 \cdot (x-1)}{4!} \cdot f^{(4)}(\xi(x))$$

$$\int_{-1}^{1} f(x)\,dx = \frac{1}{3} f(-1) + \frac{4}{3} f(0) + \frac{1}{3} f(1) - f^{(4)}(\xi) \cdot \int_{-1}^{1} \underbrace{\frac{(x+1)\cdot x^2 \cdot (x-1)}{4!}}_{\frac{1}{90}} \cdot dx$$

$$\int_{-1}^{1} f(x)\,dx = \frac{1}{3} f(-1) + \frac{4}{3} f(0) + \frac{1}{3} f(1) - \frac{1}{90} \cdot f(\xi) \quad b)$$

$T(x) = \alpha x + \beta$. $\quad T(-1) = a$

$T(1) = b$

$$\alpha + \beta = b$$
$$-\alpha + \beta = a$$
$$2\beta = a + b \Rightarrow \beta = \frac{a+b}{2}$$

$$\alpha = b - \frac{a+b}{2} \Rightarrow \alpha = \frac{2b - a - b}{2} = \frac{b-a}{2}$$

$$x = \frac{b-a}{2} t + \frac{a+b}{2} \Rightarrow dx = \frac{b-a}{2} dt$$

$$\int_a^b f(x)\,dx = \frac{b-a}{2} \cdot \int_{-1}^{1} f\left(\frac{b-a}{2} t + \frac{a+b}{2}\right) dt =$$

$$= \frac{b-a}{2} \cdot \left[\frac{1}{3} f(a) + \frac{4}{3} f\left(\frac{a+b}{2}\right) + \frac{1}{3} f(b) - \frac{(b-a)^4}{90 \cdot 2^4} \cdot f^{(4)}(\xi)\right] =$$

$$\underbrace{\frac{b-a}{6}\left[f(a) + 4 f\left(\frac{a+b}{2}\right) + f(b)\right]}_{\text{formula lui Simpson}} - \frac{(b-a)^5}{90 \cdot 32} \cdot f^{(4)}(\xi) \quad c)$$