Splini-uri

$f:\mathbb{R}\to\mathbb{R}$, $f(x)=\cos\frac{\pi}{2}x$, $\Delta: x_1=-1,\ x_2=0,\ x_3=1$.

$f(-1)=f(1)=0,\quad f(0)=1$

$P_1(x)=a_1+b_1(x+1)+c_1(x+1)^2+d_1(x+1)^3$, $P_1'(x)=b_1+2c_1(x+1)+3d_1(x+1)^2$

$P_2(x)=a_2+b_2x+c_2x^2+d_2x^3$, $P_2'(x)=b_2+2c_2x+3d_2x^2$

$P_1''(x)=2c_1+6d_1(x+1)$

$P_2''(x)=2c_2+6d_2x$

1) $P_1(-1)=f(-1)\Rightarrow a_1=0$ (1)
2) $P_1(0)=f(0)\Rightarrow b_1+c_1+d_1=1$ (2)
3) $P_2(0)=f(0)\Rightarrow a_2=1$ (3)
4) $P_2(1)=f(1)\Rightarrow 1+b_2+c_2+d_2=0$ (4)

I condiții meteolitole

$P_1'(0)=P_2'(0)\Rightarrow b_1+2c_1+3d_1=b_2$

$P_1''(0)=P_2''(0)\Rightarrow 2c_1+6d_1=2c_2$

II condiții naturale

$P_1''(-1)=0\Rightarrow 2c_1=0$

$P_2''(1)=0\Rightarrow$

$b_1+c_1+d_1=1$
$b_2+c_2+d_2=-1$
$b_1+2c_1+3d_1=b_2$
$2c_1+6d_1=2c_2\Rightarrow d_1=\frac{c_2}{3}$
$2c_1=0\Rightarrow \boxed{c_1=0}$
$2c_2+6d_2=0$

sol sistem: $b_1=\frac{3}{2}$, $b_2=0$, $c_1=0$, $c_2=-\frac{3}{2}$
$d_1=-\frac{1}{2}$, $d_2=\frac{1}{2}$

$b_1+d_1=1$
$b_2+d_2=-1$
$b_1+3d_1-b_2=0$
$6d_1-2c_2=0$
$2c_2+6d_2=0$

Ex2: $f:\mathbb{R}\to\mathbb{R}$, $f(x)=\sin\frac{\pi}{2}x$, $x_1=-1$, $y_2=0$, $x_3=1$

$f(-1)=-1$ $\quad f(1)=1$

$f(0)=0$ $\Rightarrow$ $(-1,-1)$, $(0,0)$, $(1,1)$

$P_1(x)=a_1+b_1(x+1)+c_1(x+1)^2+d_1(x+1)^3$

$P_2(x)=a_2+b_2x+c_2x^2+d_2x^3$

Condiții: $P_1(-1)=f(-1)\Rightarrow \boxed{a_1=-1}$

$P_1(0)=f(0)\Rightarrow a_1+b_1+c_1+d_1=0\Rightarrow b_1+c_1+d_1=1$ (1)

$P_2(0)=f(0)\Rightarrow \boxed{a_2=0}$

$P_2(1)=f(1)\Rightarrow a_2+b_2+c_2+d_2=1\Rightarrow \boxed{b_2+c_2+d_2=1}$ (2)

$P_1'(x)=b_1+2c_1(x+1)+3d_1(x+1)^2$ $\quad P_1''(x)=2c_1+6d_1(x+1)$

$P_2'(x)=b_2+2c_2x+3d_2x^2$ $\quad P_2''(x)=2c_2+6d_2x$

Condiții mețiune

$P_1'(0)=P_2'(0)\Rightarrow b_1+2c_1+3d_1=b_2$ (3)

$P_1''(0)=P_2''(0)\Rightarrow 2c_1+6d_1=2c_2$ (4)

Condiții capăt

$P_1'(-1)=f'(-1)$ $\quad f'(x)=\frac{\pi}{2}\cdot\cos\left(\frac{\pi}{2}x\right)$

$P_2'(1)=f'(1)\Rightarrow b_1=0$ (5)

$b_2+2c_2+3d_2=0$ (6)

$$\Rightarrow\begin{cases} b_1+c_1+d_1=1\\ b_2+c_2+d_2=1\\ \boxed{b_1=0}\\ b_2+2c_2+3d_2=0\\ 2c_1+6d_1=2c_2\\ b_1+2c_1+3d_1=b_2\end{cases}\Rightarrow\quad\begin{cases} c_1+d_1=1\\ b_2+c_2+d_2=1\\ b_2+2c_2+3d_2=0\\ 2c_1+6d_1=2c_2\\ 2c_1+3d_1=b_2\end{cases}$$

$b_1=x_1$ $\quad c_1=x_2$ $\quad d_1=x_3$

$b_2=x_4$ $\quad c_2=x_5$ $\quad d_2=x_6$

$$\begin{cases} 2c_1 + 3d_1 - b_2 = 0 \\ 2c_1 + 6d_1 - 2c_2 = 0 \quad \Big| \Rightarrow -3d_1 - b_2 + 2c_2 = 0 \\ b_2 + 2c_2 + 3d_2 = 0 \\ b_2 + c_2 + d_2 = 1 \quad \Big| \Rightarrow c_2 + 2d_2 = -1 \\ c_1 + d_1 = 1 \end{cases}$$

$$\Rightarrow b_1 = 0, \quad c_1 = \frac{3}{2} \quad d_1 = -\frac{1}{2}$$
$$b_2 = \frac{3}{2} \quad c_2 = 0 \quad d_2 = -\frac{1}{2}$$

$$\Rightarrow p(x) = \begin{cases} -1 + \frac{3}{2}(x+1)^2 - \frac{1}{2}(x+1)^3, & x \in [-1, 0] \\ \frac{3}{2}t - \frac{1}{2}t^3, & x \in (0, 1]. \end{cases}$$