Viviani’s Theorem

Yiding Huang
Math199b
Professor Lu
March 2023

Contents

Vincenzo Viviani

- Vincenzo Viviani (April 5, 1622 – September 22, 1703) was an Italian mathematician and scientist. He was a pupil of Torricelli and a disciple of Galileo.
- Assitant for Torricelli experiment

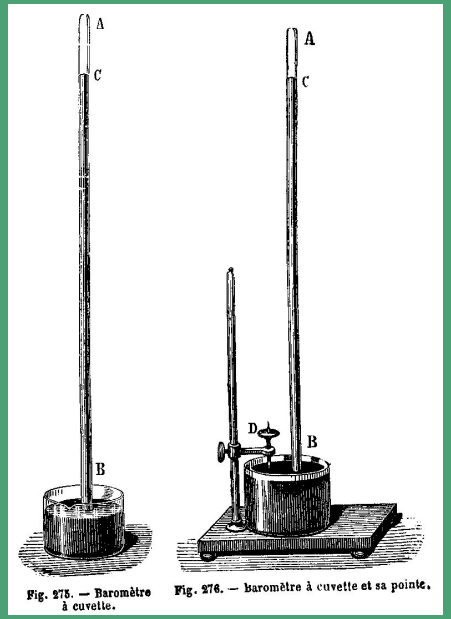
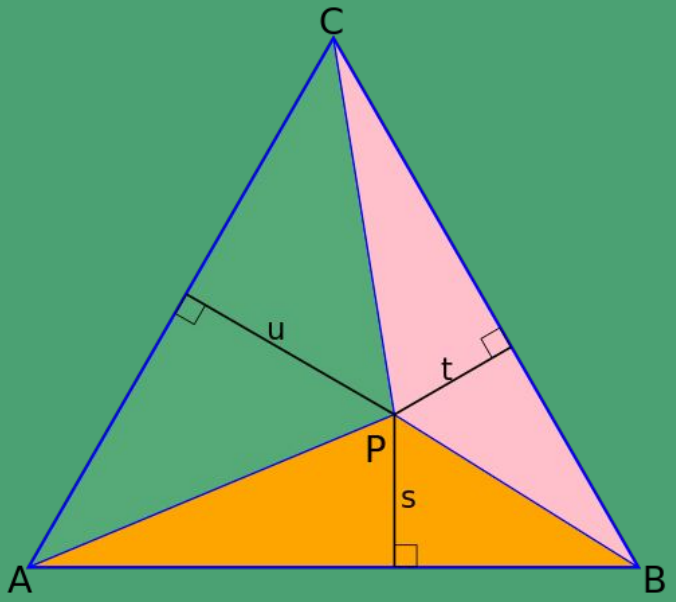


Fig. 275. — Baromètre à cuvette.

Fig. 276. — Baromètre à cuvette et sa pointe.

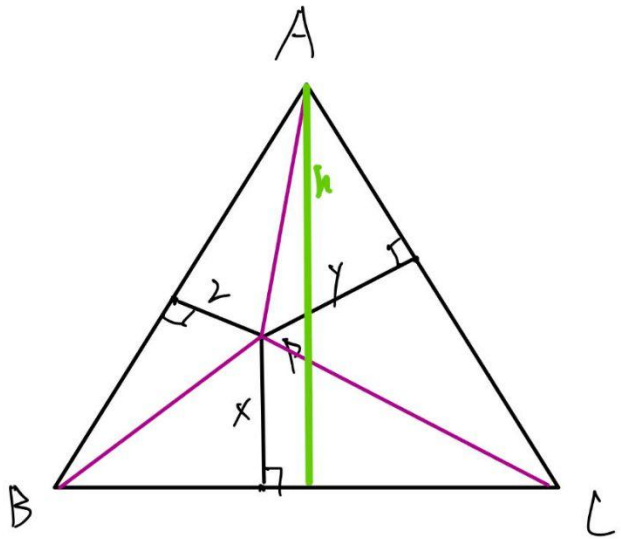
Viviani’s Theorem

-The sum of the distances from any interior point to the sides of an equilateral triangle equals the length of the triangle's altitude



u+t+s=h

Proof



① $\triangle ABC = \frac{AB \cdot z}{2} + \frac{AC \cdot y}{2} + \frac{BC \cdot x}{2}$

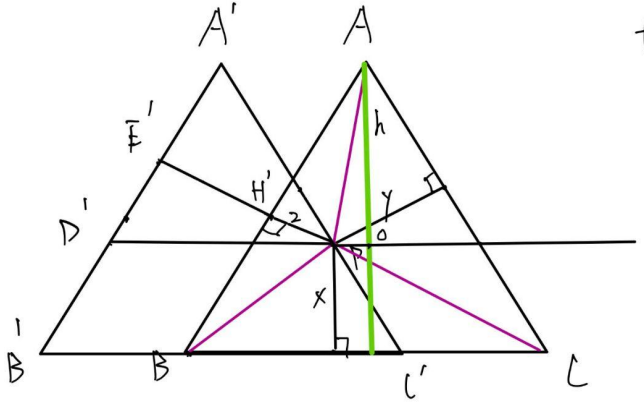
$= \frac{1}{2} \cdot BC \cdot (x+y+z)$

② $\triangle ABC = \frac{1}{2} BC \cdot h$

from ① and ②

$x+y+z = h$

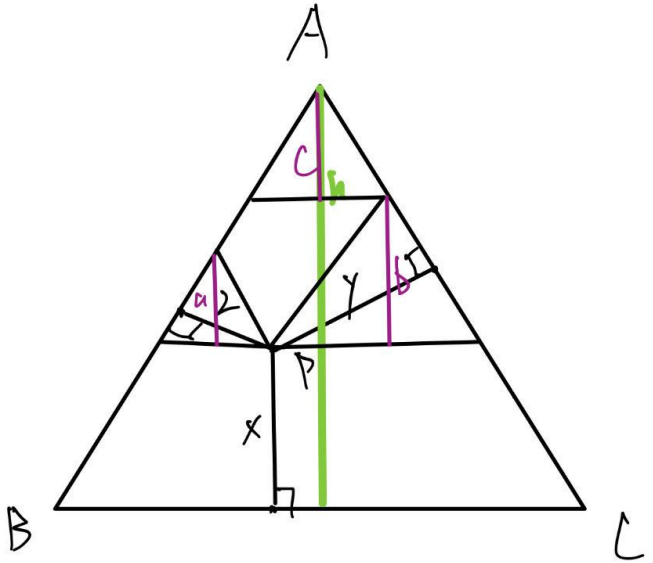
Proof



Let P be on $\triangle A'B'C'$ which is shifted from $\triangle ABC$. Extend PH' to $A'B'$ at E'.

$D'P \parallel B'C'$.

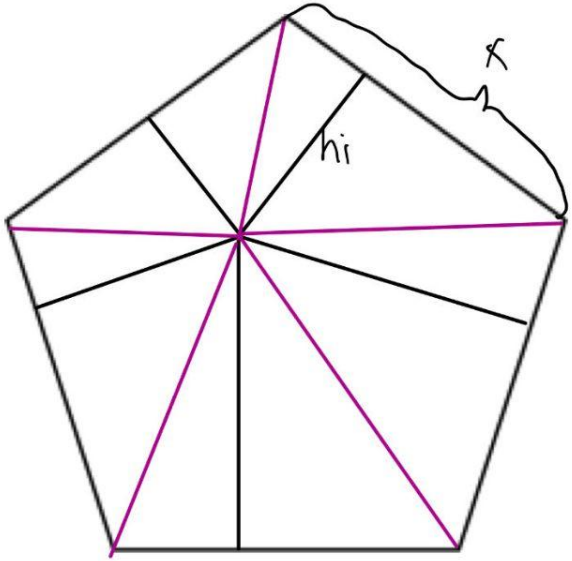
$$h = x + AO$$
$$= x + z + E'H'$$
$$= x + z + y$$



Generalization

-The sum of the distances from any point P inside a regular polygon to the sides of the polygon is independent of the location of P.

Proof



$$S = \sum \frac{1}{2} x \cdot h_i$$

$$= \frac{1}{2} x \sum h_i$$

Thanks