Topic 29

Tucker Circles

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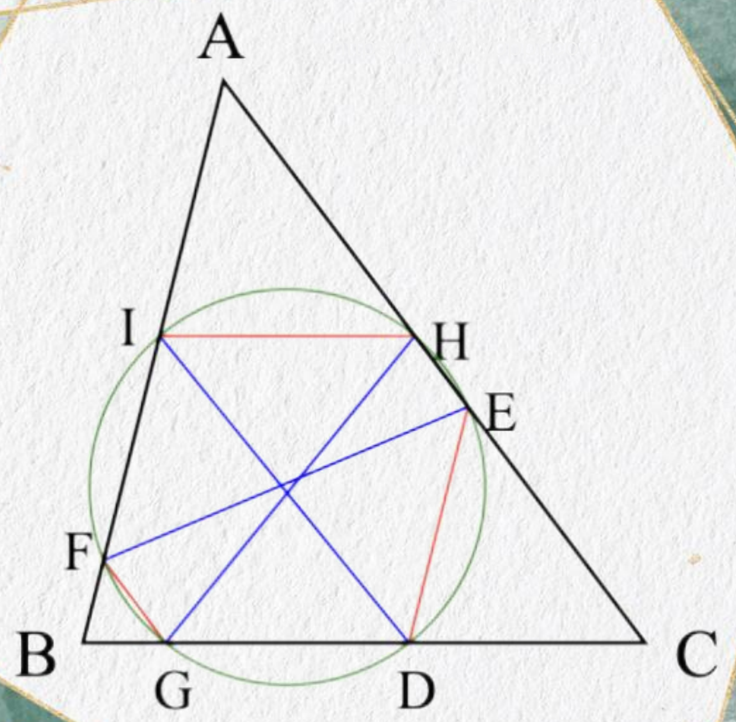
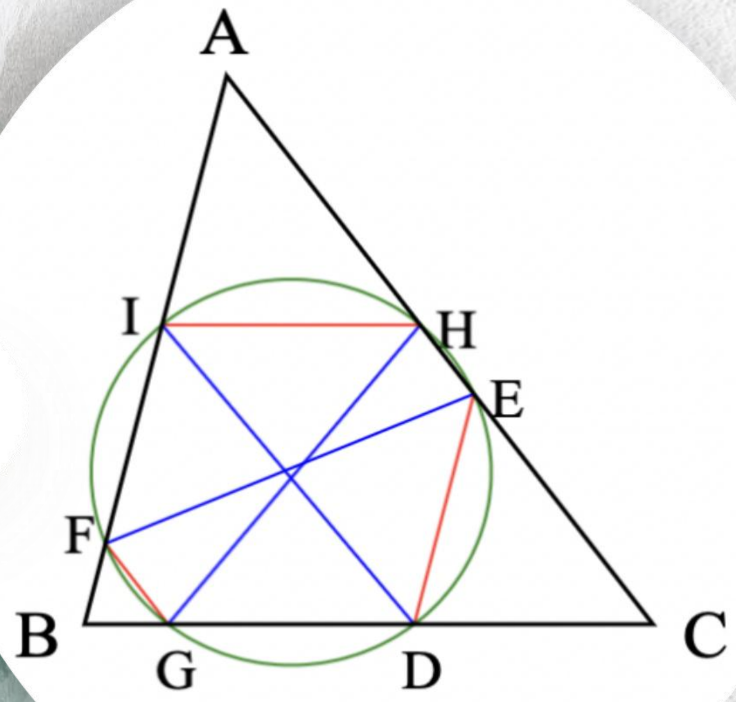


Introduction

Background Info.

Tucker Circles

Special Cases



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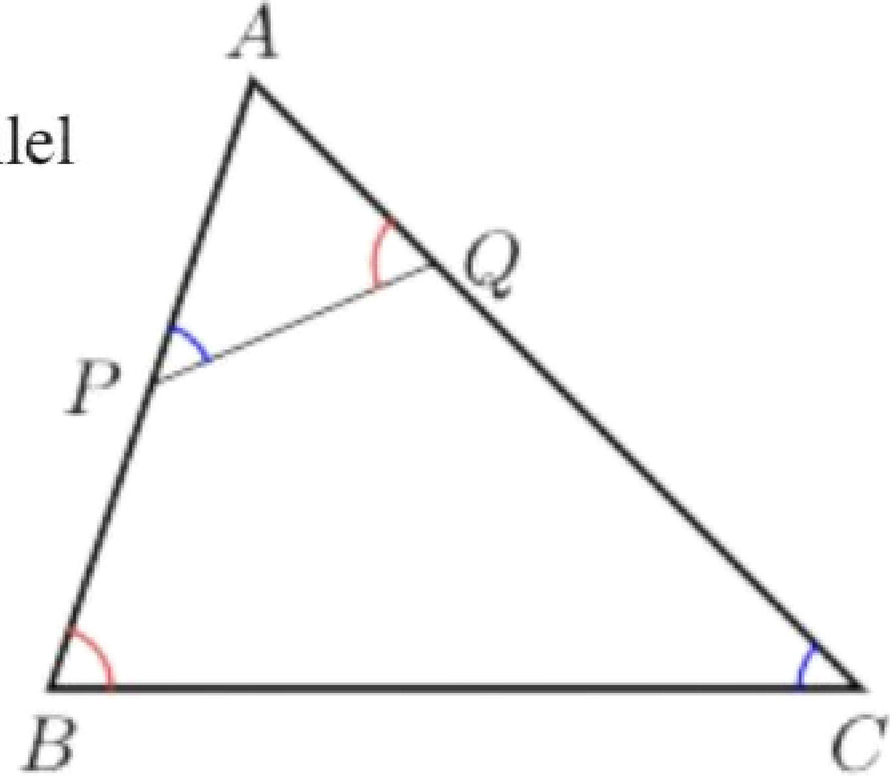
Introduction

Tucker circles is a family of circles obtained by parallel displacing sides of corresponding Cosine or Lemoine hexagons, which contains the **Cosine Circle** and the First **Lemoine Circle** as special cases.

Background Info.

Two lines, PQ and BC, are antiparallel if $\angle APQ = \angle C$ and $\angle AQP = \angle B$.



ANTIPARALLEL

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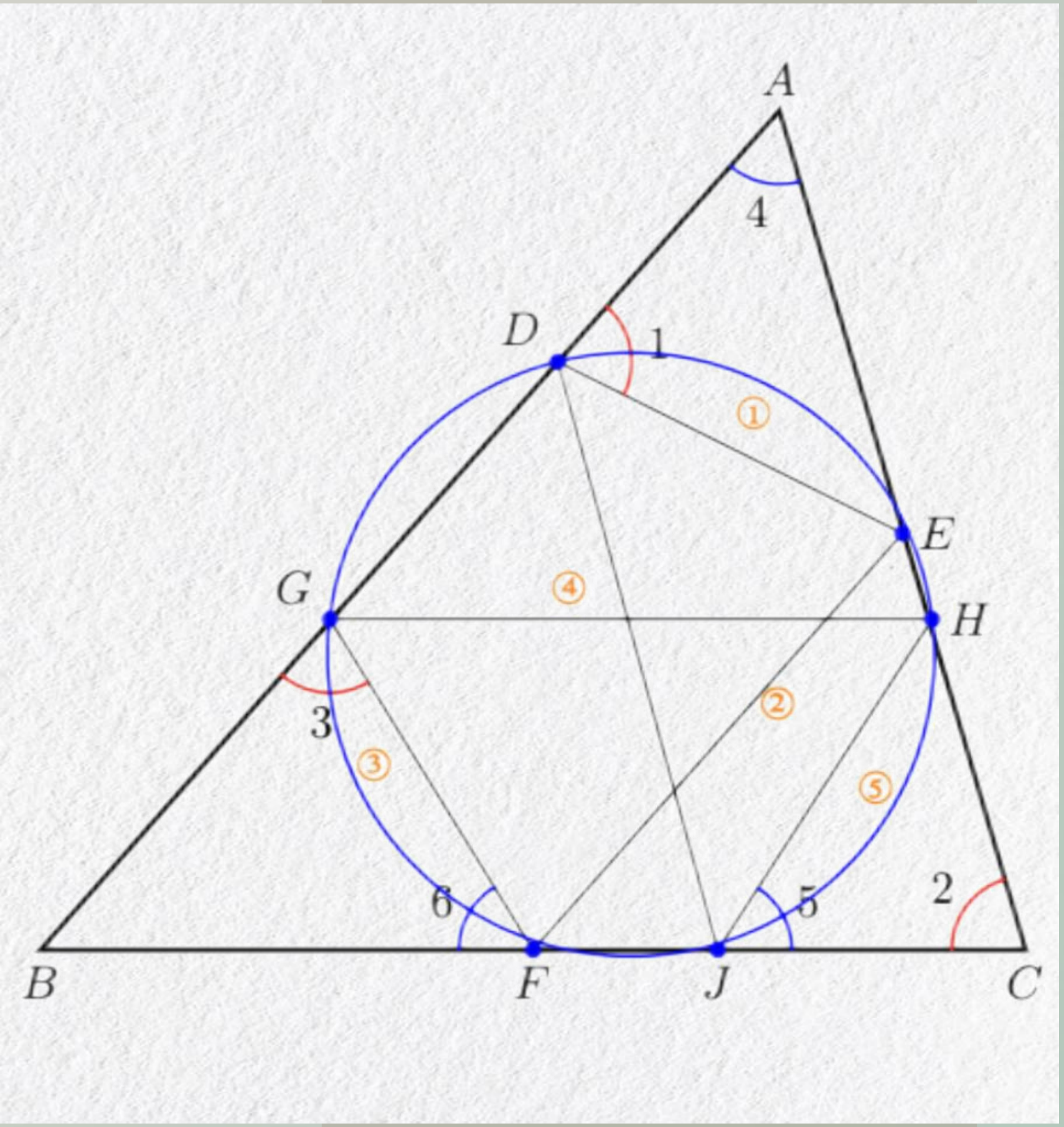
Therefore, this circle is called the Tucker Circle of △ABC.



Set point D on one side of △ABC.

Draw a segment from point D antiparallelled to BC, intersecting with side AC at point E.

Draw a segment from point E || AB, intersecting with side BC at point F.

Similarly, we will get six points on these 3 sides: D, E, F, G, H, and J, which are cocircular.

Tucker Circles

Proof ①

Since $\overline{DE}$ is antiparallel to $\overline{BC}$ and $\overline{FG}$ is antiparallel to $\overline{AC}$,

$\angle 1 = \angle 2$,

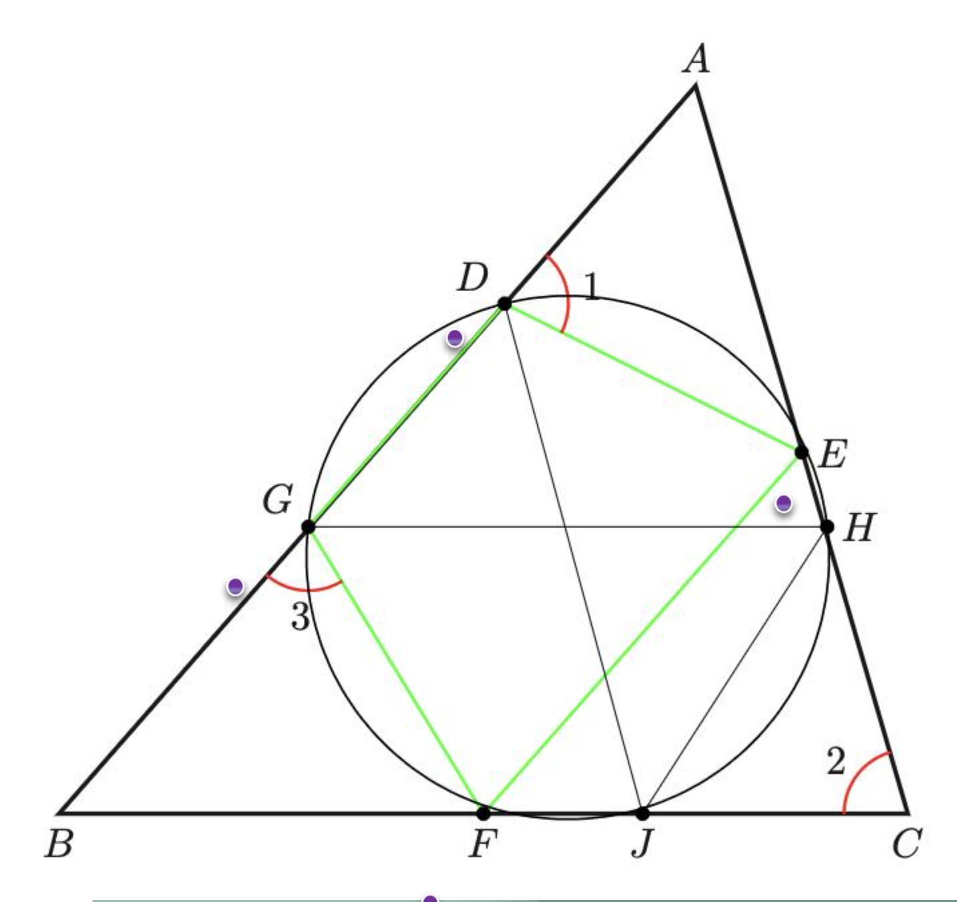
And $\angle 2 = \angle 3$.

Thus, $\angle 1 = \angle 3$.

Because $\overline{EF}$ is parallel to $\overline{AB}$, DEFG is an isoceles trapezoid.

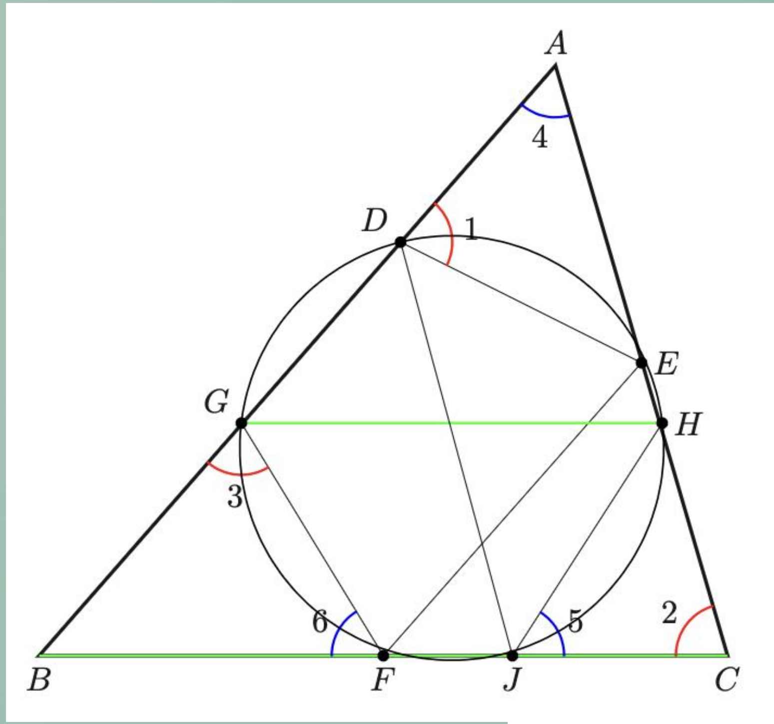
Therefore, points D, E, F, and G are cocircular.

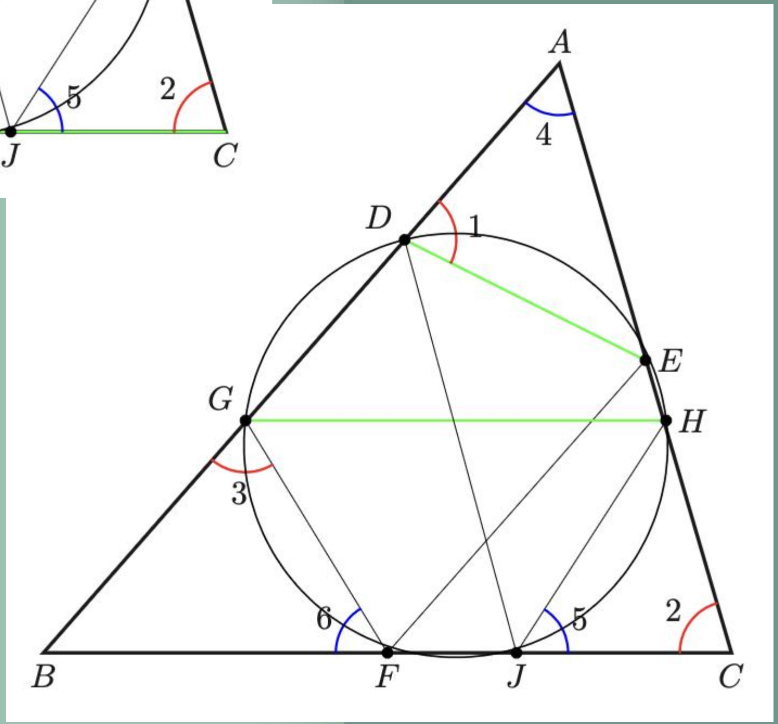
We set this circle as O_1.



Tucker Circles

Proof ②





Because $\overline{\mathrm{GH}}$ is parallel to $\overline{\mathrm{BC}}$,

$\overline{\mathrm{DE}}$ is antiparallel to $\overline{\mathrm{GH}}$.

Therefore, points D, E, H, and G are cocircular.

We set this circle as O_2.

Tucker Circles

Proof ③

Since circle O_1 and circle O_2 both pass through point D, point E and point G,

And the circle that can pass through 3 points is unique.

Thus, circle O_1 and circle O_2 is the same circle.

Therefore, points D, E, F, G, and H are cocircular.

Tucker Circles

Proof ④

Similar to ①, GHJF is another isosceles trapezoid.

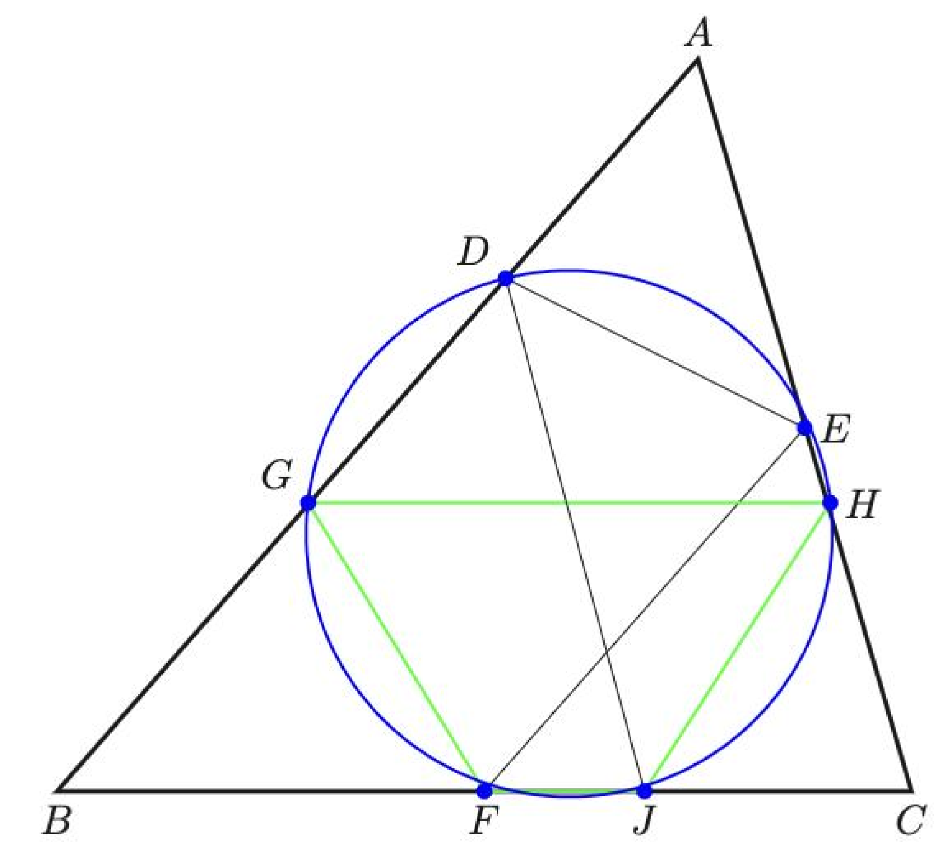
Thus, points F, G, H, and J are cocircular.

Therefore, point J is on the circle that passes through points F, G, and H

That circle is the circle in ③.

Finally, points D, E, F, G, H, and J are cocircular,

Which is called the Tucker Circle of $\triangle ABC$.



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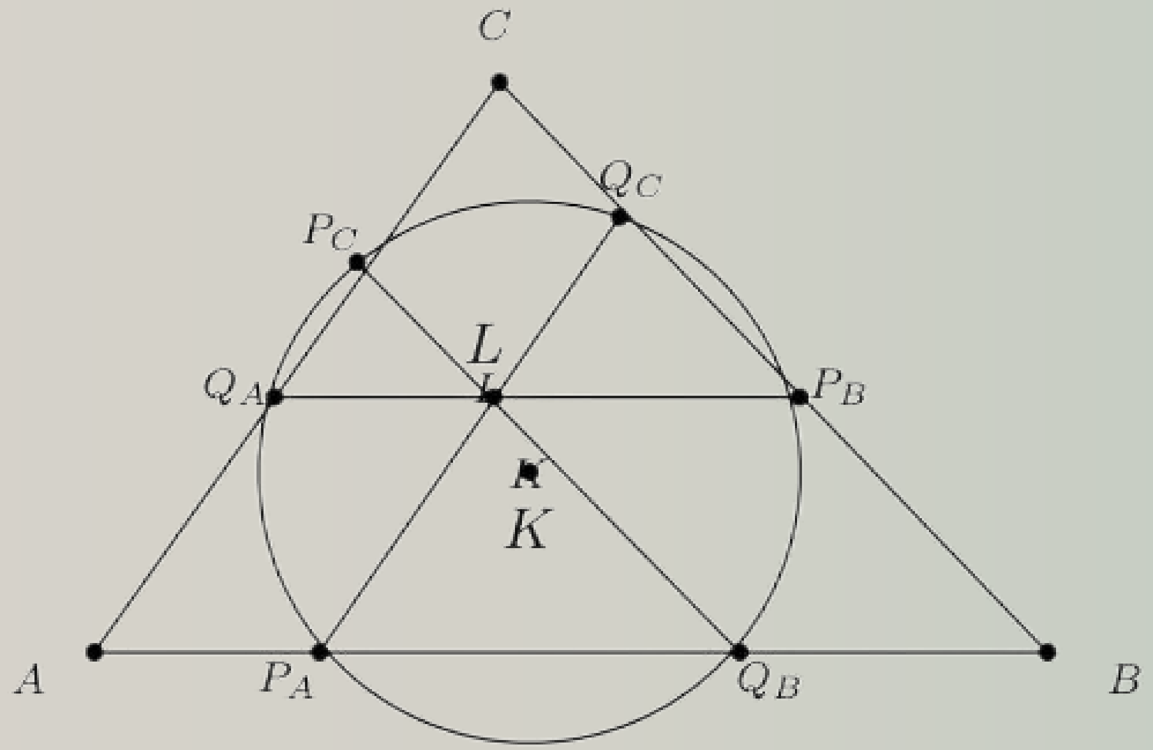
Special Cases - First Lemoine Circle



Let L be the symmedian point, and draw 3 lines through L parallel to the sides of △ABC, respectively. If points PA, QA, PB, QB, PC, and QC are cocircular, then this circle is called the First Lemoine circle.

Let K be the circumcenter, and we can find the center of the circle by the midpoint of LK.

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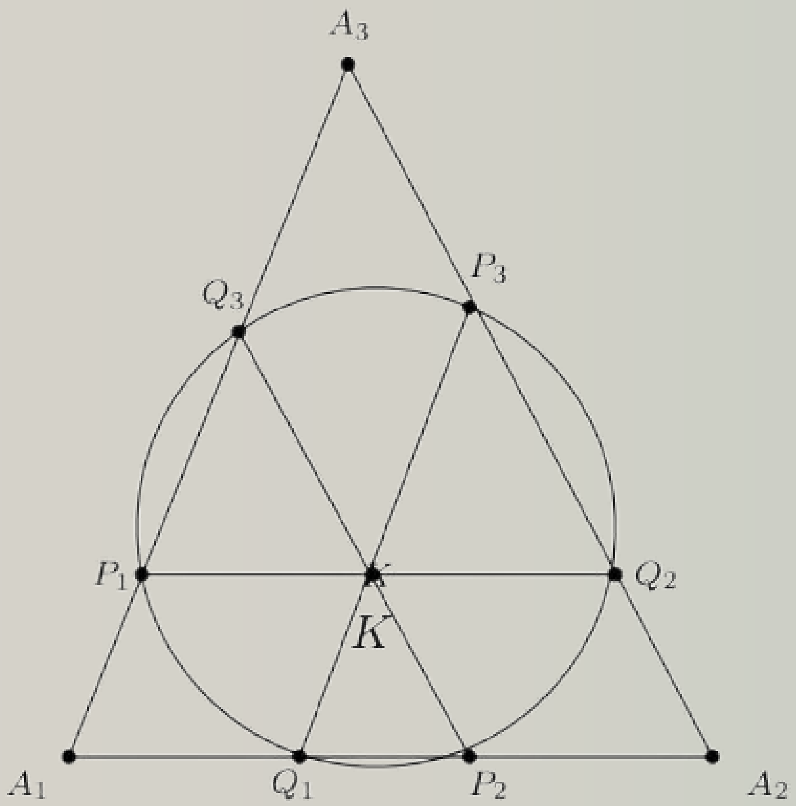
Special Cases - Cosine Circle



Let K be the symmedian point, and draw 3 lines through K antiparallel to the sides of △ABC, respectively. If points P1, P2, P3, Q1, Q2, and Q3 are cocircular, this circle is called the Cosine Circle, and sometimes Second Lemoine Circle

Thank You