ALO for LASSO with Intercept through Generalized LASSO

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1 The Dual of LASSO with intercept

For the generalized LASSO problem $\min_{\beta} \frac{1}{2}||y - X\beta|| + \lambda||D\beta||_1$, the dual problem is derived as:

$$\min_{u} \frac{1}{2}||y - \theta||_2^2$$
$$\theta \in \{X^\tau\theta = D^\tau u, ||u||_\infty \leq \lambda\}$$

The dual problem could be written in a proximal approach such that:

$$\hat{u} = \mathbf{prox}_R(y)$$
$$R(u) = \begin{cases} 0 & \{X^\tau\theta = D^\tau u, ||u||_\infty \leq \lambda\} \\ +\infty & otherwise \end{cases}$$

Denote J as the jacobian of the proximal operator at the full data problem y, then the ALO estimator could be obtained as:

$$y^{/i} = y_i - \frac{\hat{u}_i}{J_{ii}}$$

For the case of LASSO with an intercept, we could expand the X with a column of ones in the first column, expand β with another dimension and choose $D = [\mathbf{0}, I]$.

Denote $E := \{j : |X_i^\tau\theta| = \lambda\}$ to be the active set. Then the Jacobian is locally given as the projection onto the orthogonal complement of the span of X_E and the ones vector. Further denote $\tilde{X}_E = [\mathbf{1}, X_E]$, then the Jacobian is given as $I - \tilde{X_E}(\tilde{X_E}^\tau\tilde{X_E})\tilde{X_E}^\tau$.