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Acoustic multi - scale full waveform inversion of time domain based on FCG

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Summary

Full waveform inversion(FWI) reconstructs the structure of stratum using the parameters of seismic wave field kinematic and dynamic, like amplitude, travel time and phase, to get high precision and high resolution velocity model. The accuracy of inversion an be order of magnitude as the wavelength. And it is an advanced method to improve the accuracy of seismic velocity modeling and migration. The FWI has many methods, the steepest descent method, the conjugate gradient method, Gauss Newton method, etc...In this paper, a fast conjugate gradient method (FCG) is applied to the full waveform inversion of multi - scale time domain acoustic wave, which is greatly reduce computation time compared with the traditional conjugate gradient method. And last, the paper uses complex Marmousi model to test the method showed the effective of the method.

Introduction

The full waveform inversion is based on the wave equation, and uses the amplitude and phase information of the seismic record to match the different types of earthquakes. Compared with the traditional inversion method, the full waveform inversion has many advantages. With the development of petroleum industry and the development of exploration, the full waveform inversion method has been developed rapidly.

In 1980s, Lailly and Tarantola (1984) established the inverse theory framework of seismic wave field in time domain through fitting of the forward and the actual observation data to inverse the parameters of underground medium. Mora (1998) proposed a time domain elastic wave and Pratt (2004) proposed a hybrid inversion method based on the advantage of forward of time domain and frequency domain.

The most commonly used methods for the full waveform inversion is Newton method and gradient method. Necedal (1980) first put forward the BFGS method, however, due to the computation needed to calculate the Hessian matrix. It needs large storage and large computational quantity. In order to solve this problem, Patrick Lailly studied the conjugate gradient method with limited memory. Necedal (1989), proposes the L-BFGS method to avoid calculating Hessian matrix directly, it can greatly reduce the amount of storage and computation. In this paper, a fast conjugate gradient method is applied to the acoustic multi-scale full waveform inversion of time domain, which greatly accelerates the calculation speed and gets better results.

Theory of time domain full waveform inversion

The full waveform inversion searches the best model parameters that makes simulation data and observation data in good agreement through matching the seismic data obtained by wave equation forward modeling with the seismic data of field observation.

Two dimensional constant density first order velocity pressure acoustic wave equation can be written as:

$$\frac{1}{k(x,z)}\frac{\partial p(x,z,t)}{\partial t} = \frac{\partial v_x(x,z,t)}{\partial x} + \frac{\partial v_z(x,z,t)}{\partial z} \tag{1}$$

$$\rho(x,z)\frac{\partial v_x(x,z,t)}{\partial t} = \frac{\partial p(x,z,t)}{\partial x}$$

$$\rho(x,z)\frac{\partial v_z(x,z,t)}{\partial t} = \frac{\partial p(x,z,t)}{\partial z}$$

Set the speed of the model parameters v , point coordinates of the source x_s , coordinate of geophone stations x_r , observation to the field artillery set wave field $d_{obs}(t,x_r,x_s)$, the whole wave field by simulation $u(t,x,x_s;v)=L(v,x_s)$. Use R to express the receivers position, then data residuals can be express as $Ru(t,x,x_s;v)=u(t,x=x_r,x_s;v)$. The full waveform error functional can be written using the least square as:

$$E(v) = \frac{1}{2}\sum_{x_s}\sum_{x_r}\sum_{t}(Ru(t,x,x_s;v) - d_{obs}(t,x_r,x_s))^2 \tag{2}$$

The goal of the full waveform inversion is to obtain the reasonable model parameters to make the wave field residuals minimum. In mathematics it's an optimization problem of nonlinear, unconstrained and multi-parameter. It can be solve by gradient method. The iteration formula is obtained as :

$$v^{(k+1)} = v^{(k)} - \alpha^k g^k \tag{3}$$

In the formula, α is iterative step, g is gradient, k is iteration number. The direction of the iteration is the opposite direction of the objective function gradient. The velocity gradient of the full waveform inversion is based on the conjugate gradient method can be express as.

$$g(v) = \frac{\delta E}{\delta v} = \frac{2}{v^3}\sum_{x_s}\sum_{t}\frac{\partial u}{\partial t}p_1 \tag{4}$$

where p_1 is a backward propagated wave field of residual data.

The optimal step size can be obtained by the parabolic interpolation method. Given global test step α_0 , we can get pseudo renewal speed model v_p . Then the optimal step can be expressed as:

$$\alpha = \frac{2\alpha_0^2\|g(v)\|^2}{2(E_p(v_p)-E(v)+\alpha_0\|g(v)\|^2)} \tag{5}$$

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FCG algorithm

Conjugate gradient method revises the direction of the gradient through the previous iteration gradient direction modified so that the iterative results are better. Different gradient formulas are obtained through different conjugate gradient orientation correction factors. One of the most commonly used is the following form:

$$v^{(k+1)} = v^{(k)} - \alpha^k \psi^k \tag{6}$$

$$\psi^k = g^k + \beta^k \psi^{k-1} \tag{7}$$

$$\beta^k = \frac{(g^k - g^{k-1}, g^k)}{\left\|g^{k-1}\right\|_2^2} \tag{8}$$

where ψ is optimization direction, β is correction factor.

The fast conjugate gradient method not only use the direction of the gradient of the previous iteration, but also use the previous iterative correction factor to modify the gradient direction, greatly accelerate the convergence speed of iteration. Using optimization coefficient y and β, FCG can be expressed as follow:

$$y^1 = v^0 \tag{9}$$

$$\beta^1 = 1 \tag{10}$$

$$v^k = y^k - \alpha^k g^k \tag{11}$$

$$\beta^{k+1} = \frac{1 + \sqrt{1 + 4\left(\beta^k\right)^2}}{2} \tag{12}$$

$$y^{k+1} = v^k + \frac{\beta^{k-1}}{\beta^{k+1}}\left(v^k - v^{k-1}\right) \tag{13}$$

Examples

In order to verify the validity of the fast conjugate gradient method, we apply it to the acoustic multi - scale full waveform inversion of time domain to inversion velocity. The Marmousi model is used to measure the result. The velocity model has a finite difference grid dimension of 260130, with the grid interval of 10m. The acquisition

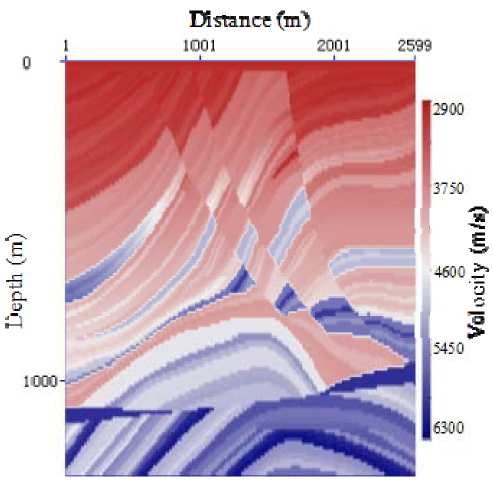

Figure 1: *Marmousi model*

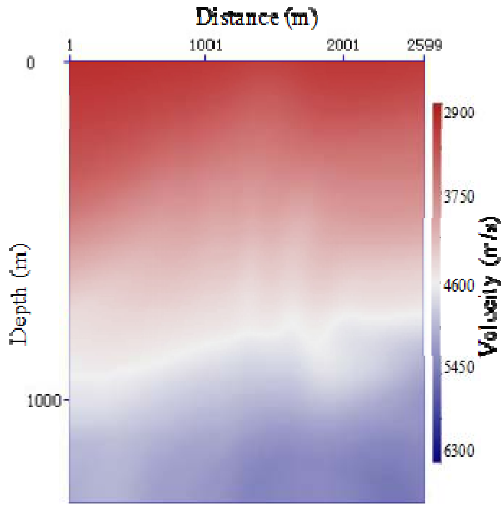

Figure 2: *Initial model*

system consists of a line of 38 sources and recorded by a coincident line of 260 receivers. The sources are located at the surface every 70m while the receivers are located at the surface every 10 m. The source is the Ricker wavelet with a dominant frequency of 30 Hz. Figure 1 is the true Marmousi velocity model while Figure 2 is the initial velocity for inversion. The multi - scale full waveform inversion of time domain can take full advantage of low-frequency information and high-frequency information respectively to recover important structures and improve resolution which improves the accuracy of the inversion results of velocity field. Therefore, the multi-scale full waveform inversion of time domain is applied to inverse velocity. The traditional conjugate gradient method and FCG method are used to get velocity results individually to demonstrate our method. We set the same termination condition as follow:

$$\left|\frac{V_n - V_{n-1}}{V_{n-1}}\right| < \varepsilon \tag{14}$$

and get inversion results (Figure 3 and Figure 4).

Compared with the Figures 3 and Figure 4, it can be found that the FCG can obtain same iteration result as the traditional CG. The vertical profiles of velocity in horizontal position of 900m and 1300m as shown in Figure

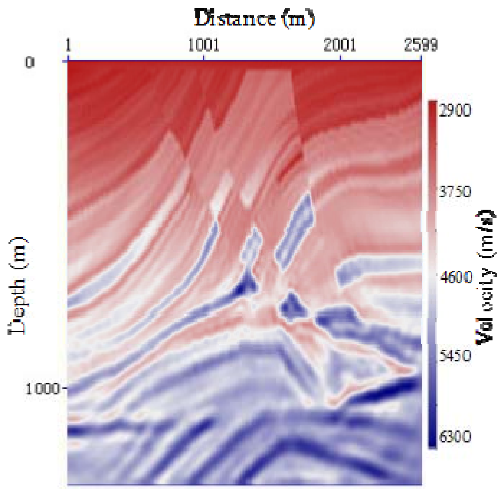

Figure 3: *The inversion result by multi-scale time domain acoustic full waveform inversion with traditional CG*

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5 make a clear demonstration that the reconstructed result by the acoustic multi-scale full waveform inversion is similar to the true velocity and the velocity result inversed by FCG is similar to the velocity result inversed by traditional CG, even better. Form the misfit convergence curve as shown in Figure 6, we can get that these two methods obtain the same convergence results, but the number of iteration used FCG is two-thirds as much as the number of iteration used traditional CG method. It also illustrates that the FCG method is effective in improving the convergence speed of inversion.

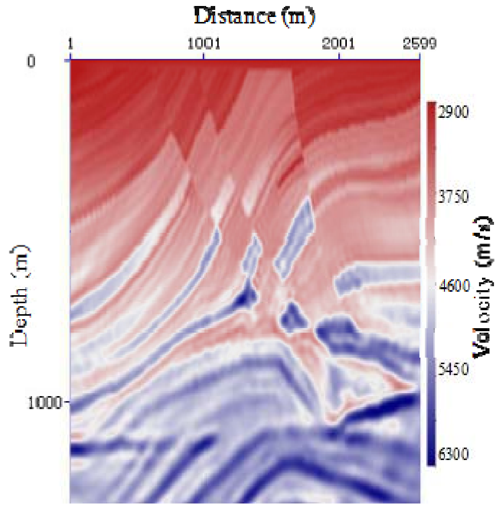

Figure 4: *The inversion result by multi-scale time domain acoustic full waveform inversion with FCG*

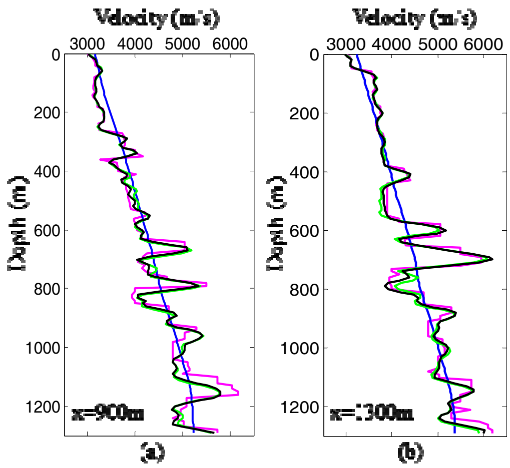

Figure 5: *Vertical velocity*

(a) is vertical velocity at the distance of 900 m of the model and (b) is vertical velocity at the distance of 1300 m. The red line indicates the true model, the blue line indicates the initial model, the green line denotes the reconstructed model by the traditional and the black line denotes the reconstructed model by FCG.

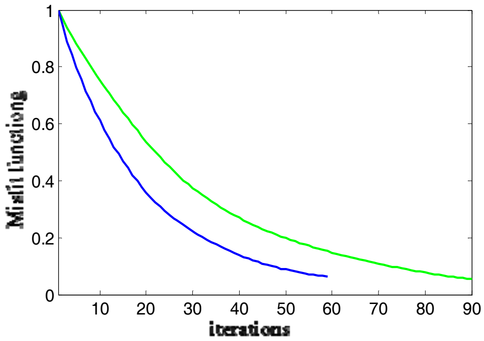

Figure 6: *Misfit function convergence curve*

The blue line indicates the misfit function convergence curve of FCG, the green line indicates the misfit function convergence curve of traditional CG.

Conclusions

Time domain acoustic full waveform inversion is able to objectively show the law of seismic wave propagation, and the accurate characterization of the details of the model. It gets more accurate inversion results, which has great development potential. However, there are still many limitations in the application of the actual data. Iterative efficiency directly affects the speed of inversion, but also affects the computation and storage in the inversion process. In this paper, based on the Marmousi model test results, we can obtain that the traditional CG method and FCG method would get the same inversion results and convergence effect, but the convergence rate of FCG method is significantly faster than traditional CG method. The FCG method could save the time of FWI, the amount of computations and storage space. It has great advantages in the inversion process.

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