Contents

1 Introduction

Hook

Our complex reality reveals itself as a limit for our compartmentalisation of knowledge, that is why I believe we should increase communication between different fields of science. Moreover, in an interdisciplinarity context, physics as a lot to offer. This work is a personal exercise towards this philosophy.

Research Aim

What is the impact on the transport of nitrates to non-motile phytoplankton cells (diatoms) in the presence of chemotactic bacteria?

Objectives

- Quantifying the impact of bacterial chemotaxis on the diatom's nutrient availability.
- Modelling the nutrient transport dynamics: Diffusion Equation with a Consumption term.
- Implementing a numerical solution for the model.
- Analytical results validating the numeric's.
- Qualify which symbiotic relationship exists between bacterium and the diatom.

Hook w Background

2 Literature Review

2.1 Biological Background

Overview of Phytoplankton and Their Role in Ecosystems

- Role of phytoplankton in aquatic ecosystems (Contribution to the food chain, O2 production and CO2 absorption).
- Importance of Nitrate Nutrients (Phytoplankton growth and nutrient uptake mechanisms, Emergence of competition due to nutrient limitation).

Bacterium: Competitors in Nutrient Uptake

- Chemotaxis mechanisms (Foraging strategy for marine (and soil) microorganisms)
- Competition (Chemotactic bacteria are attracted to and consume nitrates, competing with diatoms for nutrient availability).

Interaction with Bacteria: Competition for Nitrates and Symbiosis

- Diatoms secrete chemicals that attract bacteria.
- ¿Optimization of bacterial chemotaxis by diatoms?

3 Methodology

3.1 From Discreet to Continuum

Chemotaxis at an Individual Level

- Discreet and stochastic model.
- Individual trajectory, $x_j(t)$.

Chemotaxis at a Population Level

- Transition from individual to population-level, discrete to continuum approaches.
- Continuum and deterministic model, diffusion equation with consumption.
- Concentration, $c(t)$.

3.2 Non-Dimensionalisation

We start from the physical equation, where we have capitalised its magnitudes:

$$\frac{\partial N(\vec{R},T)}{\partial T} = D\,\nabla^2_{\vec{R}}\,N(\vec{R},T) - \mathrm{A}\,N(\vec{R},T)\,C(\vec{R},T) \tag{1}$$

These quantities represent the real, laboratory measures: $\vec{R}$ is the position vector relative to the diatom, T is time, D is the diffusion coefficient, N & C are the nutrient and bacterial concentration respectively, and A is the bacterial consumption rate. Now, we introduce the following variables changes, to obtain a dimensionless equation:

$$\left\{ \vec{R} = L\,\vec{r},\; T = \frac{L^2}{D}\,t, \quad N = \frac{n}{L^3}, \quad C = \frac{c}{L^3}, \quad \mathrm{A} = DL\,\alpha \right\} \tag{2}$$

Where the set of lowercase variables (like n or α) are the dimensionless equivalent of the capitalised variables. Whereas L represents the characteristic length of the system (for example, the system's or diatom's size), $\frac{L^2}{D}$ is the time unit of the system, meaning the time it takes for a nutrient to diffuse through L. Replacing these (2) into the equation (1), we obtain the new equation (3) which is ideal for numerical simulations.

$$\frac{\partial n(\vec{r},t)}{\partial t} = \nabla^2_{\vec{r}}\, n(\vec{r},t) - \alpha\, n(\vec{r},t)\, c(\vec{r},t) \tag{3}$$

4 Results

4.1 One-Dimensional Model

4.1.1 Diffusion With Consumption

Different Bacterial Concentrations

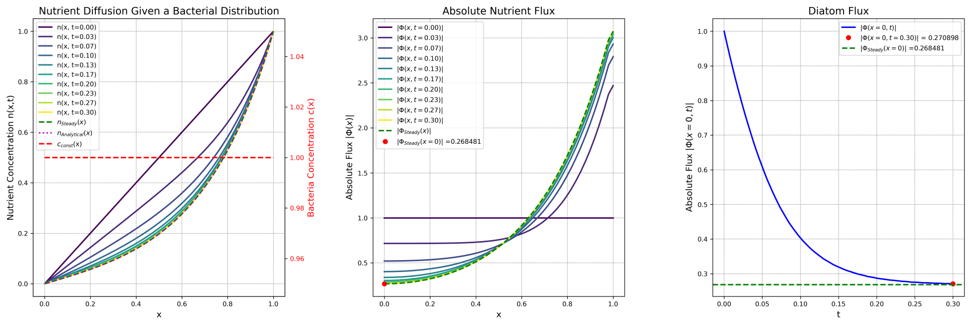

Figure 1: Constant Bacterial Concentration Profile, with Analytical Solution.

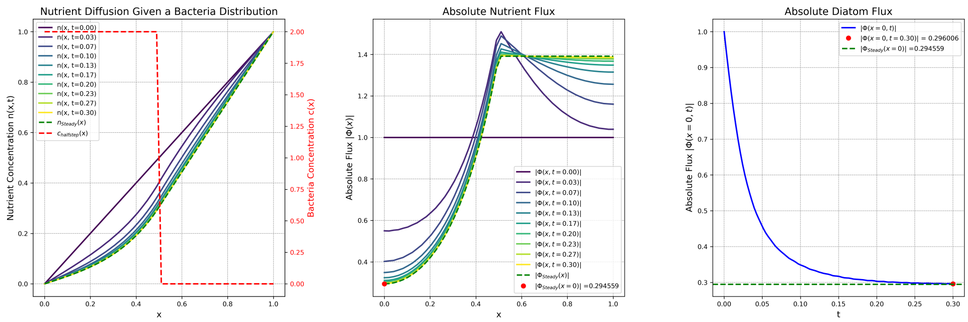

Figure 2: Half-Step Bacterial Concentration Profile.

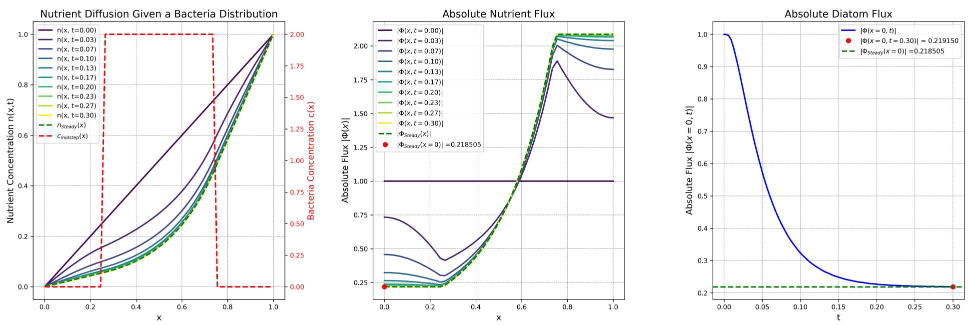

Figure 3: Mid-Step Bacterial Concentration Profile.

¿Different Diffusion Coefficients?

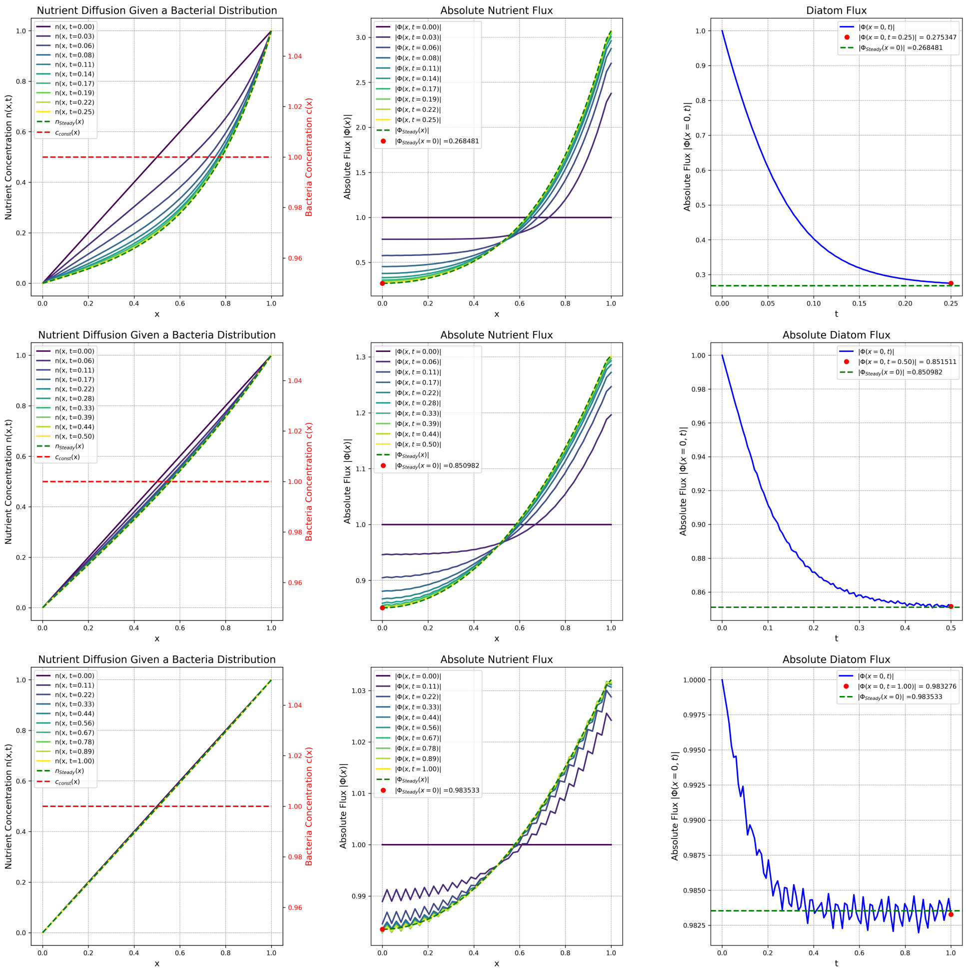

Figure 4: Comparing Different Diffusion-Consumption Ratios.

Diatom's FluxMaps, Generalising Results for Step & Exponential Concentrations

Concentration profile described by a midstep with varying starting positions (x_0) and lenghts (l), with the restriction $x_0 + l \leq L$.

$$c_{\text{midstep}}(x; x_0, l) = \begin{cases} \frac{1}{l} & \text{if } x_0 \leq x \leq x_0 + l, \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

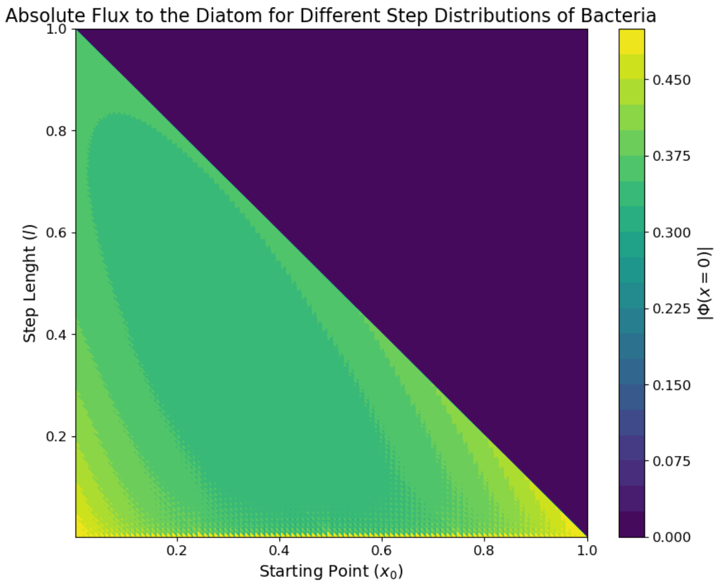

Figure 5: Flux Map for Midstep Concentration Profile.

Concentration profile described by an exponential with varying slope.

$$c_{\text{exp}}(x;\delta) = \frac{1}{\delta L} \frac{\mathrm{e}^{-\frac{x}{\delta L}}}{1 - \mathrm{e}^{-\frac{1}{\delta}}} \tag{5}$$

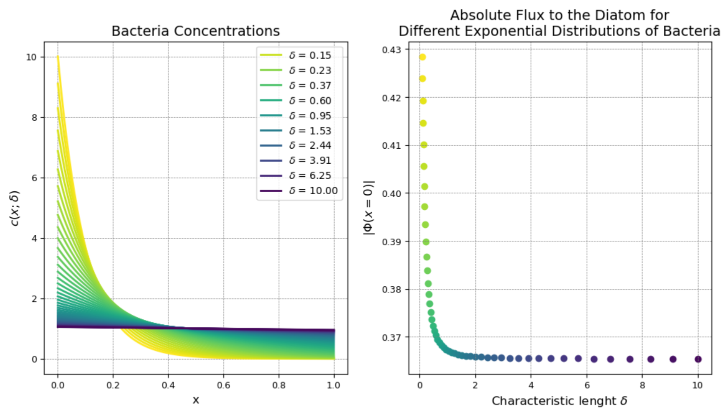

Figure 6: Flux Map for Exponential Concentration Profile.

4.1.2 Absorption of Brownian Particles

4.2 Tree-Dimensional Model

4.2.1 Numerical Solution

4.2.2 Analytical Solution

5 Conclusion

A Coding Appendix

A.1 Numerical Approach

B Mathematical Derivations