Dynamics: $x_1(t), \ldots, x_n(t)$

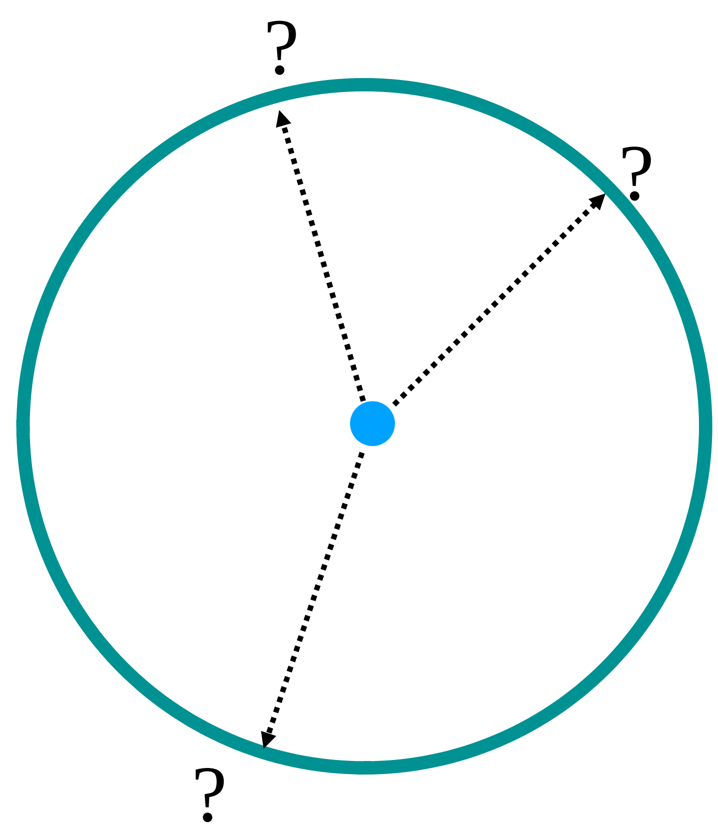
$$O\left(x_1(t), \ldots, x_n(t), \dot{x}_1(t), \ldots, \dot{x}_n(t), \ldots\right)$$

Classical dynamics: $\frac{dp_i}{dt} = -\frac{\partial H}{\partial x_i}, \quad \frac{dx_i}{dt} = \frac{\partial H}{\partial p_i}, \quad H \equiv H(x_i, p_i, t)$

State of a system: $(x_1, p_1, \ldots, x_n, p_n)$

Quantum dynamics?

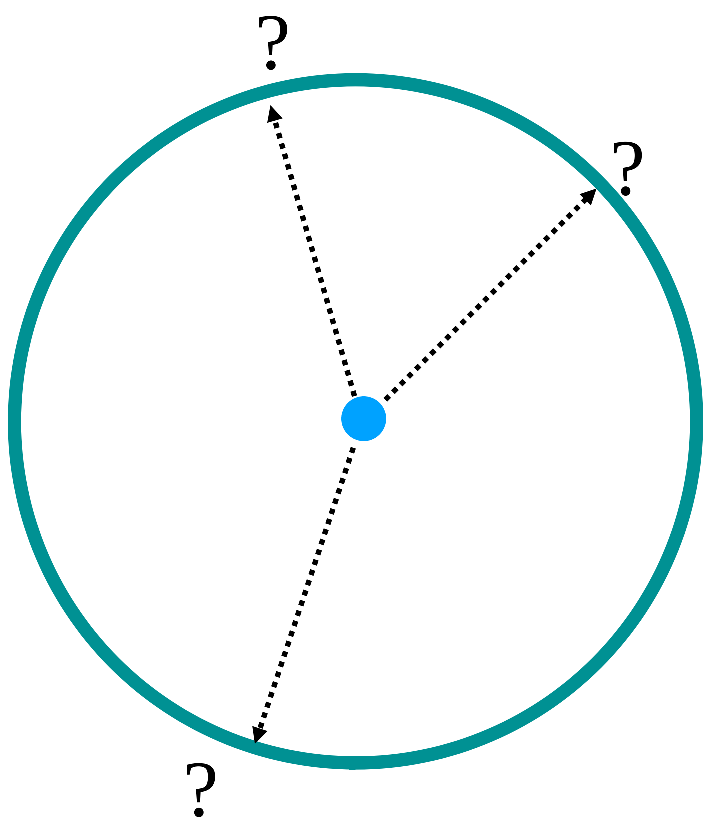
Gedanken experiment:



- The outcome is not deterministic
- The system has no memory (it does not matter how I prepare the initial state)
- The statistical distribution of the results does not change with time P(φ,t,t')=P(φ,t-t')
- The particle does not disappear

Examples of dynamical models

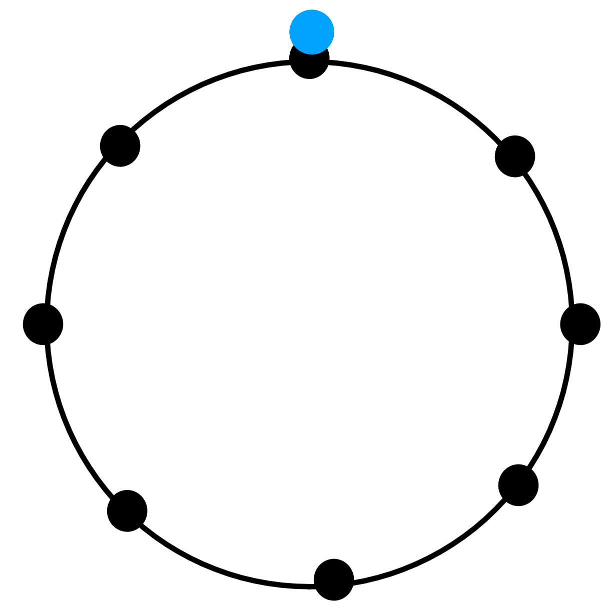
Gedanken experiment:



1. Markov process (Brownian motion)
2. Quantum mechanics: unitary evolution + measurement

Simpler Example: 1D N-chain with nn hopping

Discrete time - for convenience



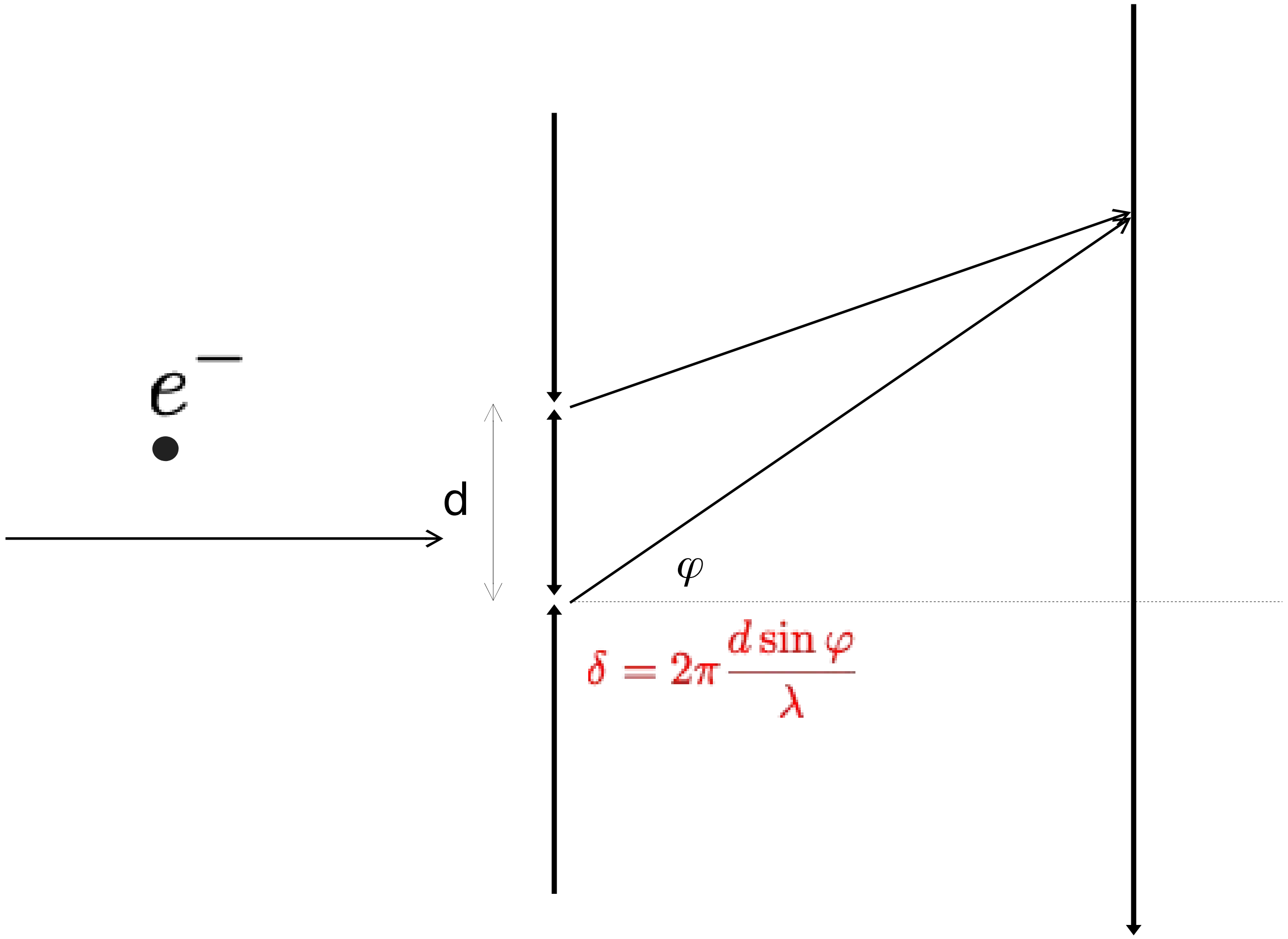
1. Markov process (Brownian motion)

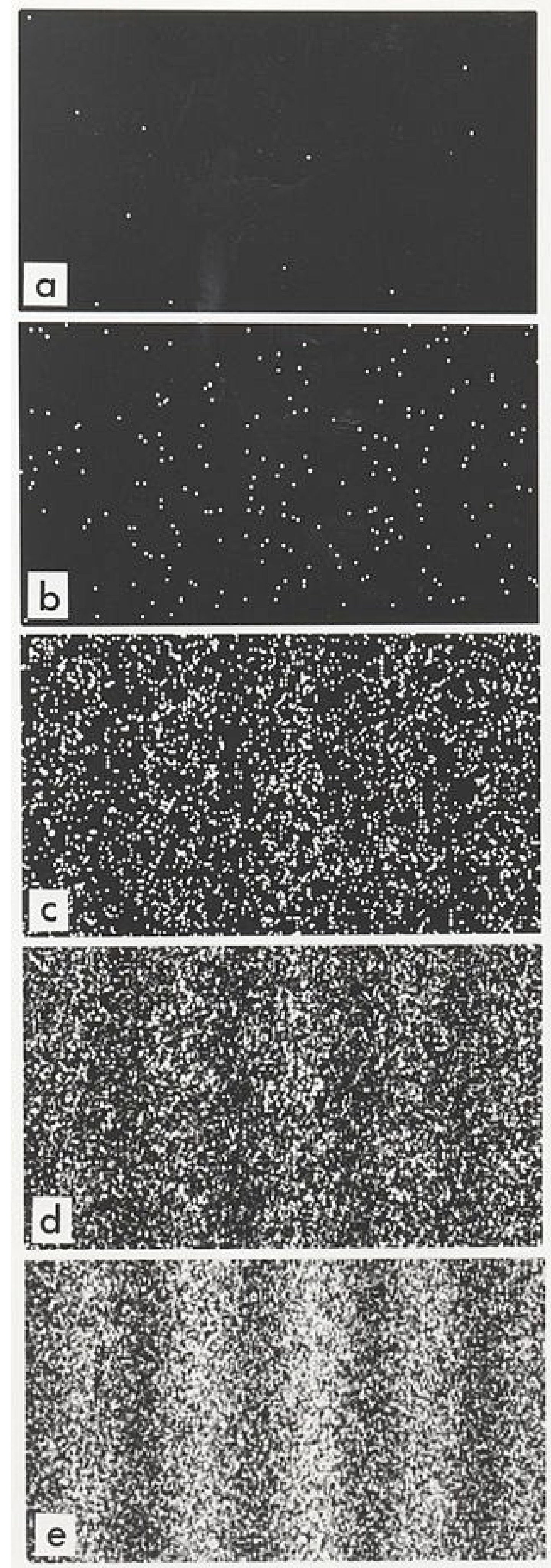
- Particle is always at a definite site
- At each time-step it can choose randomly between hopping left (x^2), hopping right (x^2), staying put ($1-2x^2$)
- We use a N-vector to describe the probability of finding the particle at any given site

1. Quantum mechanics: unitary evolution + measurement

- We use a complex N-vector (WF) to describe the state of the particle
- At each time-step the WF undergoes a unitary transformation reflecting hopping of the particle
- At time t we perform a measurement (compute the amplitude of WF at any given site)

Double slit experiment with electrons
(Wikipedia)





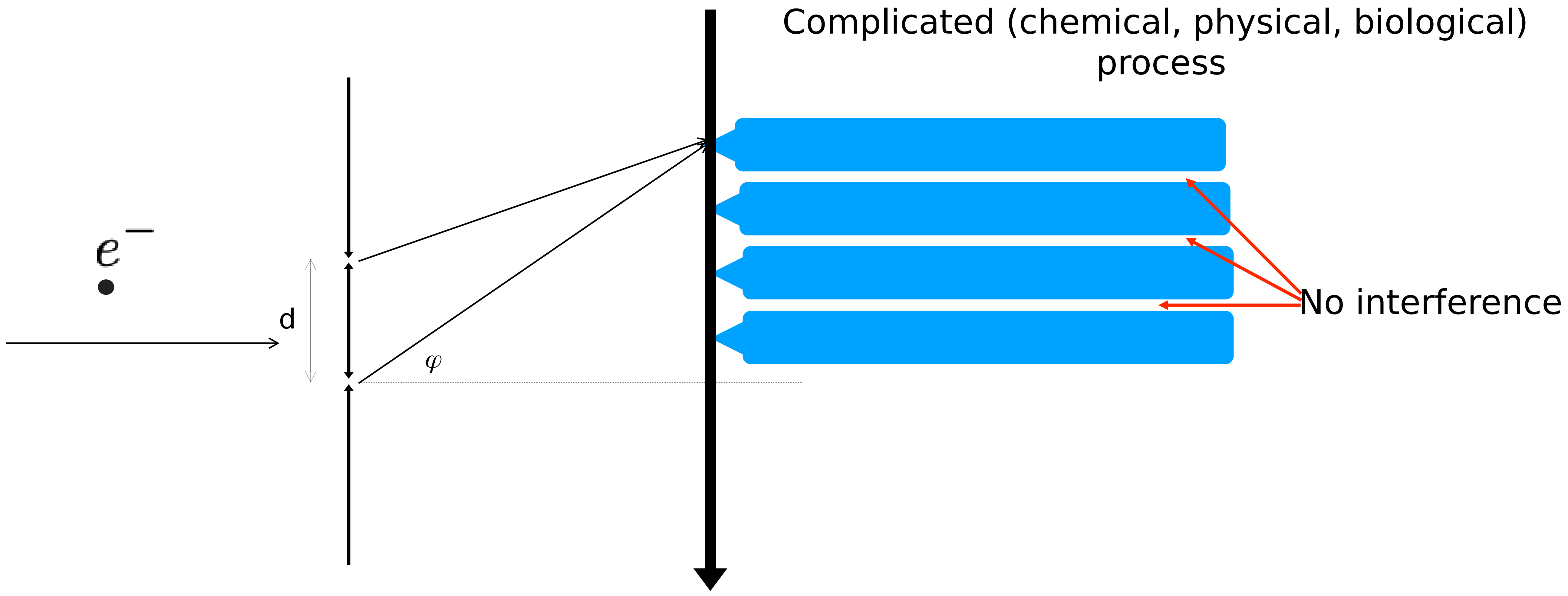
Electron has a wavelength

$$p = \frac{h}{\lambda} \equiv \hbar k$$

Impuls

Wellenlänge

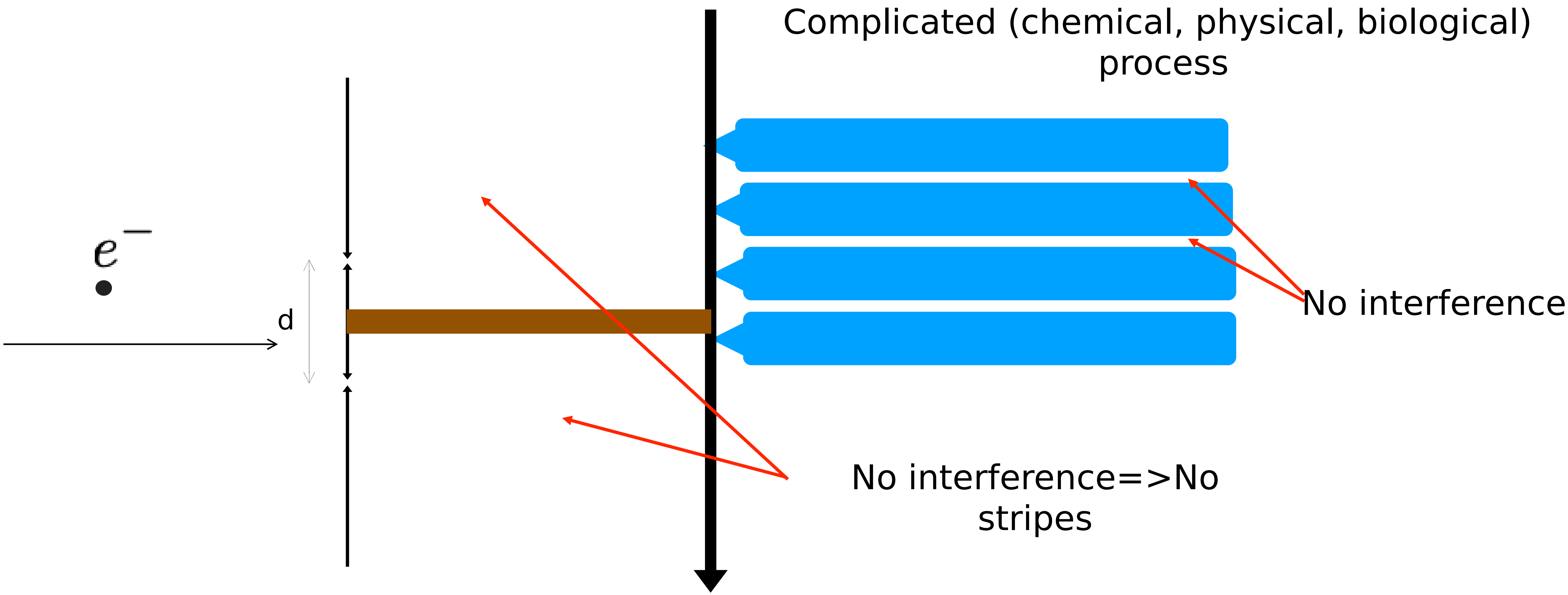
Wave function collapse



Measurement in QM:

- Results of measurement are randomly distributed
- The possible outcomes are the eigenvalues of a corresponding hermitian operator
- Probability of an outcome O_i is given by the weight $|\langle\phi_i|\Psi\rangle|^2$ of the corresponding eigenvector in the probed wave function
- In the process of measurement the system becomes entangled with the wave function of the macroscopic apparatus, which leads to collapse of the wave function system (is reduced to the eigenfunction of the measured eigenvalue).

Wave function collapse



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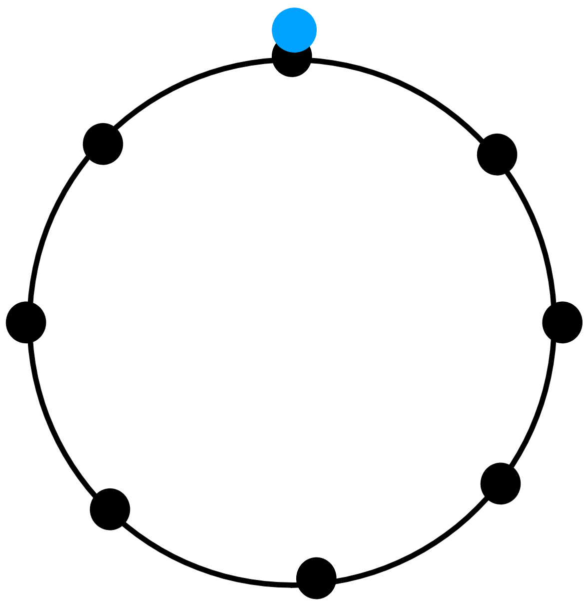
Schrodinger equation



Making time-step infinitely small

$$i\frac{d}{dt}\Psi_i = H_{ij}\Psi_j$$

H is constant in time

$$\Psi = \psi e^{-iEt}$$

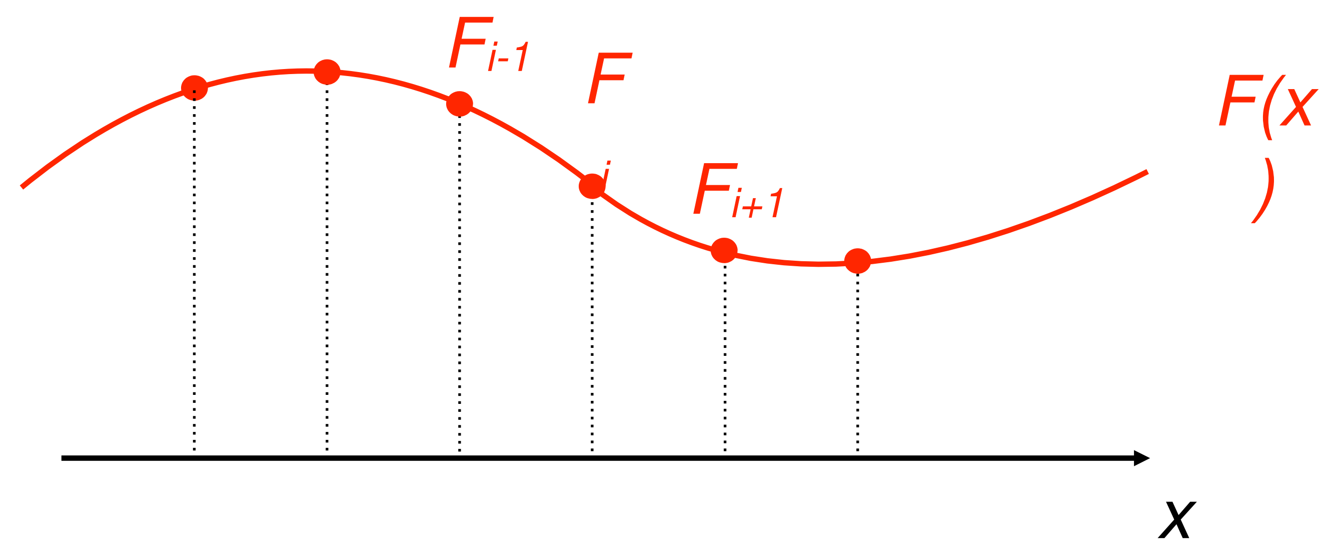
$$H\psi^\lambda = E^\lambda \psi^\lambda$$

$$\Psi = \sum_\lambda c^\lambda \psi^\lambda e^{-iE^\lambda t}$$

Schrodinger equation in continuum

Discretisation

$$F(x) \to F_i$$



$$-\frac{1}{2}\partial_x^2\psi(x) + V(x)\psi(x) = E\psi(x)$$

Derivatives replaced with differences

$$\partial_x \psi_i \approx \frac{\psi_{i+1} - \psi_i}{a}$$

$$\partial_x^2 \psi_i \approx \frac{\partial_x\psi_i - \partial_x\psi_{i-1}}{a} = \frac{\psi_{i+1} + \psi_{i-1} - 2\psi_i}{a^2}$$

$$E\cdot \to E\delta_{ij}$$ Identity matrix

$$V(x)\cdot \to V_i\delta_{ij}$$ Diagonal matrix

$$\partial_x^2 \to -2\delta_{ij} + \delta_{ij+1} + \delta_{ij-1}$$

Irrelevant constant shift

Nearest-neighbour hopping

Wave functions and expectation values

Wave function is not observable: The same dynamical system can be described by a different wave function (similar gauge freedom as with electromagnetic potentials)

Operators are also not unique:

$$\langle\psi|O|\psi\rangle$$

Observables - unique: