Forecasting Implied Volatility using Nelson Siegel

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Abstract

This paper develops an algorithmic trading strategy based on the statistical properties of variance, particularly its autocorrelation. We apply the Nelson-Siegel model, typically used for interest rate term structures, to estimate the term structure of implied volatility derived from American option prices. Using time-series models, we forecast the evolution of volatility and devise a systematic trading approach based on volatility predictions. The strategy is designed for trading VIX futures or implementing delta-hedging techniques.

1 Methodology

The Black-Scholes model provides a theoretical framework for option pricing under the assumption of constant volatility. The price C of a European call option is given by:

$$C = S_0 N(d_1) - Ke^{-rT} N(d_2), \tag{1}$$

where

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}. \tag{2}$$

Solving numerically for σ, given market-observed option prices, yields the implied volatility σ_{impl}, which is considered the best available estimate of future market volatility.

The implied volatility term structure captures how σ_{impl} varies across different option expirations. By taking progressive expirations (e.g., today, tomorrow, in two days, etc.), we expect an increasing term structure due to rising market uncertainty over time.

The Nelson-Siegel model is adapted here to model $\sigma(\tau)$, where τ represents time to maturity:

$$\sigma(\tau) = \beta_0 + \beta_1 \frac{1 - e^{-\tau/\lambda}}{\tau/\lambda} + \beta_2 \left(\frac{1 - e^{-\tau/\lambda}}{\tau/\lambda} - e^{-\tau/\lambda} \right). \tag{3}$$

The parameters $\beta_0, \beta_1, \beta_2$ have distinct interpretations:

- β_0: Long-term level of volatility.
- β_1: Short-term deviation.
- β_2: Curvature component, capturing medium-term dynamics.

The parameter λ governs the decay of these factors.

For each trading day, implied volatility is estimated across different expirations using market option prices. The Nelson-Siegel parameters are then estimated by minimizing the sum of squared errors:

$$\min_{\lambda} \sum_i \left(\sigma_i - X(\tau_i)\boldsymbol{\beta} \right)^2, \tag{4}$$

where $X(\tau)$ is the design matrix:

$$X(\tau) = \begin{bmatrix} 1 & \frac{1-e^{-\tau/\lambda}}{\tau/\lambda} & \left(\frac{1-e^{-\tau/\lambda}}{\tau/\lambda} - e^{-\tau/\lambda}\right) \end{bmatrix}. \tag{5}$$

Since the model is linear in β, Ordinary Least Squares (OLS) is used for estimation. The optimal λ is determined via numerical methods such as grid search.

Given the estimated time series of $\boldsymbol{\beta}_t = (\beta_0, \beta_1, \beta_2)$, we use autoregressive models to forecast $\boldsymbol{\beta}_{t+1}$. A simple AR(1) model is given by:

$$\boldsymbol{\beta}_{t+1} = \boldsymbol{\alpha} + \boldsymbol{\rho}\boldsymbol{\beta}_t + \boldsymbol{\epsilon}_t, \quad \boldsymbol{\epsilon}_t \sim \mathcal{N}(0, \Sigma). \tag{6}$$

Given the autocorrelation property of implied variance, forecasting $\boldsymbol{\beta}_{t+1}$ enables the estimation of future implied volatility, forming the basis of our trading strategy.

2 Trading Strategy

With the predicted implied volatility $\hat{\sigma}_{t+1}$, trading decisions are made as follows:

- Buy VIX futures if $\hat{\sigma}_{t+1} > 1.05\sigma_t$.
- Short VIX futures if $\hat{\sigma}_{t+1} < 0.95\sigma_t$.
- Otherwise, hold.

A stricter condition can be imposed, executing a trade only if $\hat{\sigma}_{t+i} > \sigma_t$ for all $i = 1, 2, ..., T$, ensuring higher signal quality.

An alternative strategy is *delta hedging*, where a trader adjusts positions to maintain a *delta-neutral* portfolio. The delta, denoted as Δ, represents the sensitivity of an option's price to small changes in the price of the underlying asset, mathematically expressed as $\Delta = \frac{\partial V}{\partial S}$, where V is the option price and S is the price of the underlying asset. To hedge, a trader must hold $-\Delta$ units of the underlying asset, continuously adjusting this position based on volatility forecasts to minimize risk exposure.

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