PHYS222 ASsignment Due on 9/9/25

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Problem 1: Analytical part

solution: The recursive formula for finding the Lagrange polynomials for Aitken/Nevill interpolation method is given by:

$$P_{i,i+m,}(x) = \frac{(x - x_{i+m})P_{i,i+m-1}(x) + (x_i - x)P_{i+1,i+m}(x)}{x_i - x_{i+m}}$$

We need to find the polynomial of 2nd order with 3 datapoints:$P_{1,3}(x)$. we use the original forumla to get:

$$P_{1,3}(x) = \frac{(x - x_3)P_{1,2}(x) + (x_1 - x)P_{2,3}(x)}{x_1 - x_3}$$

now for $P_{1,2}(x)$ and $P_{2,3}(x)$

- Note that $P_{1,1} = y_1$

$$P_{1,2}(x) = \frac{(x - x_2)y_1 + (x_1 - x)y_2}{x_1 - x_2}$$

- Note that $P_{2,2} = y_2$

$$P_{2,3}(x) = \frac{(x - x_3)y_2 + (x_2 - x)y_3}{x_2 - x_3}$$

the final form of the polynimial becomes:

$$P_{1,3}(x) = \frac{(x - x_3)\frac{(x-x_2)y_1+(x_1-x)y_2}{x_1-x_2} + (x_1 - x)\frac{(x-x_3)y_2+(x_2-x)y_3}{x_2-x_3}}{x_1 - x_3}$$

After simplifying we get:

$$P_{1,3}(x) = \frac{(x - x_2)(x - x_3)}{(x_1 - x_2)(x_1 - x_3)}y_1 + \frac{(x - x_1)(x - x_3)}{(x_2 - x_1)(x_2 - x_3)}y_2 + \frac{(x - x_1)(x - x_2)}{(x_3 - x_1)(x_3 - x_2)}y_3$$

Now to find the polynomial of 3rd order with 4 datapoints we repeat the process similarly.

$$P_{1,4}(x) = \frac{(x - x_4)P_{1,3}(x) + (x_1 - x)P_{2,4}(x)}{x_1 - x_4}$$

We have already found $P_{1,3}(x)$. we have to find $P_{2,4}(x)$

$$P_{2,4}(x) = \frac{(x - x_4)P_{2,3} + (x_2 - x)P_{3,4}}{x_2 - x_3}$$

- $P_{2,3}$ has already been found in the first part
- we need to find $P_{3,4}$

$$P_{3,4}(x) = \frac{(x - x_4)y_3 + (x_3 - x)y_4}{x_3 - x_4}$$

$P_{2,4}$ becomes

$$P_{2,4}(x) = y_2 \frac{(x - x_3)(x - x_4)}{(x_2 - x_3)(x_2 - x_4)} + y_3 \frac{(x - x_2)(x - x_4)}{(x_3 - x_2)(x_3 - x_4)} + y_4 \frac{(x - x_2)(x - x_3)}{(x_4 - x_2)(x_4 - x_3)}.$$

After replacing $P_{1,3}$ and $P_{2,4}$ in $P_{1,4}$ we get:

$$P_{1,4}(x) = y_1 \frac{(x - x_2)(x - x_3)(x - x_4)}{(x_1 - x_2)(x_1 - x_3)(x_1 - x_4)} + y_2 \frac{(x - x_1)(x - x_3)(x - x_4)}{(x_2 - x_1)(x_2 - x_3)(x_2 - x_4)} + y_3 \frac{(x - x_1)(x - x_2)(x - x_4)}{(x_3 - x_1)(x_3 - x_2)(x_3 - x_4)} + y_4$$