Justification

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Definition 1 (Justified Proposition)**.** A proposition Q is justified when for some proposition P, P and $P \implies Q$ are justified.

Definition 2. We call a set S of propositions **self-justifying** if for every $Q \in S$, P and $(P \implies Q)$ are elements of S. That is, every element of S is justified by some elements of S.

Corollary 0.1. *If S_1, S_2 are self-justifying sets of propositions, $S_1 \cup S_2$ is self-justifying.*

Theorem 0.2. *For any $n \in \mathbb{N}$, there exists a self-justifying set S of propositions with $|S| = n$.*

Proof. In the case $n = 0$, there are no elements of S, so the definition self-justifying technically holds true. Otherwise, for $1 \leq i \leq n$, call $P_i := P_i \implies P_i$, and let $S = \{P_i : 1 \leq i \leq n\}$. Now let $Q \in S$. Then $Q \in S$, so $Q = Q \implies Q$ and in turn $(Q \implies Q) \in S$. Hence Q and $Q \implies Q$ are elements of S and Q is justified. Thus all elements of S are justified. $\square$

Theorem 0.3. *Every finite self-justifying set S of propositions consists entirely of elements of the form $P := Q \implies R$ for some $Q, R \in S$.*

Proof. Since every element of S is justified, there exists $|S|$ elements in S of the form $Q \implies R$ for some $Q, R \in S$. But that is all of the elements in S, so every element must be of the form $Q \implies R$ for some $Q, R \in S$. $\square$