THESIS PROPOSAL:

PASSIVE RADAR DETECTION

Lewis Chambers

Electrical Engineering Undergraduate

The University of Queensland

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Contents

Part I

Background Work

1 Introduction

This document will act as a proposal for an undergraduate thesis project. It contains the proposed topic definition and the relevance to the student's area of study. A cursory review of the current literature in the field on the topic is provided in this proposal, as well as the student's own introductory work and results. This is followed by a detailed project plan and list of milestones for the project. To this end, the project will also act as a reference document for the progress of the student. Finally, this report shall include and occupational health and safety analysis risk analysis for all work to be carried out in this project.

2 Topic, goal and relevance

2.1 Topic and relevance

Passive radar (or passive coherent location) detection is the problem of locating unknown targets in an area of interest using only an array of static radar receivers. Using a numbers of spatially separated radar receivers, the goal is to resolve the location of target objects nearby. The challenge lies in the fact that an active signal is not broadcast by the receivers, so they must make use of signals already permeating the air, so called illuminators of opportunity. As these signals are not generated by the receivers, they are unknown ahead of time, so must somehow be resolved from noise and reflected signals received by the radars to determine the position of targets in the area of interest.

With the advent of digital television broadcast, this technique of passive radar detection has become more viable as there are many high strength, high frequency signals being broadcast in most areas. Other examples of illuminators of opportunity include cellular phone base stations, navigations satellites and air surveillance radars [1]. Many of these transmissions are at VHF and UHF, which is optimal for radar detection. Another factor contributing to the recent feasibility of Multistatic passive radar is the requirement of suitable digital signal processing technologies, as there is large amount of data to be processed and it is a computationally significant load.

Passive radar has a number of advantages over active radar. Firstly, a passive radar system consists of only receivers, so the initial cost of implementation is lower as transmitters are not required. Secondly, many implementations of passive radar schemes require only static or non-directional receivers, so receivers need not be prohibitively expensive. Thirdly, as mentioned above, many illuminators are at VHF and UHF frequencies, where stealth treatment of targets may be less effective [2]. Finally, and most importantly, passive radar allows the covert surveillance of an area, as it is undetectable due to the absence of a transmitter. This means that passive radar could have a variety of applications in military where a low-cost, covert, radar system is required.

2.2 Project goals

This project will test many different schemes for solving the problem, by attempting to recreate results given in various papers, using their various methods. These will all be implemented by the student in MATLAB and will simulate a variety of scenarios to test the veracity of the solution. These different schemes will then be compared, for efficiency, applicability, correctness, ease of implementation and scalability. The goal of this analysis is to determine which, if any, of the solutions are applicable to a real-time, passive, radar tracking system.

The student (with the assistance of the supervisor) will then also develop some new, unpublished techniques that will attempt to solve the problem of passive radar detection. These techniques will be compared to previous methods on the same attributes mentioned above.

The student will also investigate techniques to improve the performance of passive radar methods. These will involve methods for speeding up implementations of passive radar solutions. This will incorporate many areas including looking at the following:

- Data storage techniques
- Approximations for large matrices and matrix multiplication
- Approximations for eigenvalues and eigenvectors

- Selective area scanning (i.e. reducing the number of points to check, therefor reducing number of computations)
    - Adaptive schemes for recursively scanning areas of high target likelihood
    - Applying probability models to areas or frequencies of scanning
- Implementation language

Through looking at these, the student hopes to be able to simulate a real world system in real time, using one of the above methods, preferably their new technique for passive radar detection.

The project will contribute to the topic of passive radar detection by provided a summary and analysis of a variety of the current literature on the topic. It will also introduce a new method to the field and introduce some techniques of approximation to decrease computational time of passive radar system. It is the hope of the student that the project will push the subject of passive radar towards a more commercially realisable focus.

3 Review of background material and related work

3.1 History of radar and background material

Radar is a system of detection that utilises pulses of radio waves to determine the distance, altitude or speed of an object. The term radar is an acronym for radio detection and ranging. The basic principle of operation of radar is fairly simple. The set-up consists of a transmitter and receiver. The transmitter generates short pulses of radio waves in the microwave spectrum, which are broadcast in the direction of interest. The receiver picks up any reflected waves that return. Computing the distance to the target can be achieved in many ways, with the most common being timing total transit time of the pulse. The distance to the target is equal to one half of the distance that light travels in the time of transit of the pulse. Other methods that can determine a target’s distance or speed involve frequency modulation of the transmitted pulse, and Doppler processing of the received signal.

The earliest radar systems were bistatic, that means that the transmitter and receiver were in different locations [3]. With the advent of duplexing in radar technology, the splitting of transmit and receive operations in time over the same channel became possible, meaning that the bistatic systems could become monostatic; that is, it became possible to transmit and receive through the same antenna. Although monostatic radar has become the standard, bistatic and passive radar technologies are beginning to emerge as a topic of interest in the industry, especially in military applications [4].

The earliest passive radar systems can be found as far back as 1935, with Sir Robert Watson-Watt detecting a Heyford bomber aircraft 8km away using the illumination signal from the shortwave BBC empire transmitter [5, 6]. The interest in passive radar continued on the German side from 1943, with the 'Klein Heidelberg' receivers used to detect British incoming aircraft. Interest continued to grow in the following years and many projects followed, including Lockheed Martin's *'Silent Sentry,'* an all-weather passive surveillance technology [7]. The passive system uses transmissions from multiple commercial FM radio stations to detect airborne targets in real-time. Around this time the awareness and interest in this technology started growing rapidly, and for good reason.

Passive radar technology was starting to come to the forefront, and a number of technological factors were helping. First, the appropriate signal processing power was becoming available to make the system commercially viable. Another contributing factor was the increase in high gain, high signal strength digital broadcasting frequencies, such as digital TV [8]. These have helped passive radar stay on the forefront of radar technologies.

As well as these factors, passive radar systems have many advantages over active systems. The main distinguishing factor between passive and active radar is the presence (or lack of in the case of passive), of a transmitting antenna. This leads us to the advantage that passive radar has over active radar, its ability to detect covertly. Not only is this technology covert, but it also is a cost effective counter to stealth. A system of passive receivers can track, detect and target stealth operations, as well as providing information for anti-air systems [4]. The system is also more immune to electronic counter measures (ECM), such as jamming from opposition transmitters [6]. With all of these features considered, passive radar technology has shown to have very strong military applications. Passive radar has also been used for civilian applications. PARASOL is a project sponsored by the German ministry of environmental affairs that uses passive radar for collision warning of wind power plants [5]. It employs a passive radar system that uses the DVB-T illuminators to detect aircraft approaching wind farms and also to reduce the collision of birds.

It has been shown that the predicted detection ranges for a variety of illuminators of opportunity, including analogue FM radio, cellular phone base stations and digital audio broadcast (DAB) is in the order of 'several tens of kilometres'[9]. The signals of interest in latest works on the topic have been focused on DVB-T (Digital Video Broadcasting - Terrestrial) signals [10-13], with a higher transmitter gain and signal power, although the exact medium of the illuminating signal does not have a direct effect on the method of resolving target position and velocity.

3.2 Related Work

Many authors have proposed different methods of solving the problem of resolving the speed and locations of targets using a passive radar system. All system models consist of at least one passive receiver (will be denoted Rx), at least one non-cooperative transmitter or illuminator of opportunity (will be denoted Tx) and at least one target that may be stationary or moving. While these factors remain constant between methods, there are a number of choices that can be made in deriving a model. These include:

- Single receiver or multiple receiver
- Using a single illuminator or multi-band/channel illuminators
- Position of transmission source known vs unknown
- Noise variance known vs unknown
- Method for calculating position involves:
    - Correlation between each Tx-Rx pair for computation
    - Correlation between each Rx-Rx pair for computation
    - A combination of both

While these are not the only choices to be made when designing a passive radar system, they are the most common considerations when employing a passive methodology. The big point of difference between many techniques is the algorithm used to detect and track target objects.

Many of the techniques follow similar steps to obtain the positions and speeds of targets. Initially a signal model is generated, and received signals are measured for coherence and time delay estimation [14]. To do this, signals need to be time and frequency shifted so they are in the same frame of reference. Doing this requires knowledge of the geometric location of the receivers. If a transmitter generates a signal $s(t)$, the Rx receives a time and frequency shifted version of that signal, namely

$$x(t) \approx \mu s(t-\tau)e^{j\omega t}, \tag{1}$$

where μ is the signal amplitude, τ is the time delay and ω is the frequency shift. This signal can then be time and frequency adjusted, and sampled appropriately to produce a vector

$$\begin{aligned} \bar{x}(t) &= x(t+\tau)e^{-j\omega t} = \bar{\mu}s(t) \\ \bar{x}[n] &= \bar{\mu}s[n] \end{aligned} \tag{2}$$

The signal $\bar{x}[n]$ is the signal of interest that analysis will be performed on. In practice, we do not know ahead of time what the correcting factors τ or ω will be, so we must generate the signals $\bar{x}[n]$ for analysis for each possible time and frequency delay in a multi-dimensional grid. We now need to manipulate these signals to perform some detection analysis, by finding the likelihood that any of the scenarios is true. This is done by comparing two hypotheses,

- the null hypothesis, where the received signals are not correlated more that random white Gaussian noise
- the alternative hypothesis, where the received signals are correlated

These are compared by a metric known as the generalised likelihood ratio test (GLRT). This is the real distinguishing factor between methods, and there are a number of approaches of interest to this project, including:

- Generalised canonical correlation analysis (GCCA) for single transmitter [10]
- Bayesian detection for signals of know rank with known and unknown noise variance [15]
- Source localisation for each Rx-Rx pair and each Tx-Rx pair [16]

The GLRT is calculated over an array of position (giving us the time correction factor τ) and frequency (giving us ω). This gives us a physical view of the likelihood of a target occurring at each position in an image known as the plan position indicator (PPI) map (Examples of a PPI map can be seen in figures 1,2 in chapter 4). Some methods also include post analysis on the PPI map, for a sharpened location of targets.

Without going into lengthy detail of the contents of these methods, there are similarities in each of them. They each manipulate matrices related to the vectors $\bar{x}[n]$, and they each need to computes eigenvalues of these matrices to calculate their respective GLRT's [10, 15, 16]. Since these tests need to be performed on a large array of positions and frequencies, calculating the eigenvalues of these matrices must be done as efficiently as possible.

Many different algorithms exist for computing eigenvalues of matrices. The algorithms of interest would need different properties depending on which passive radar method they were applied to. All would need to have a fast convergence rate to be considered appropriate for implementation in a real-time system. There are a number of iterative methods that could be employed, and have been employed for a variety of applications including Power iteration, Rayleigh-quotient iteration and the Lanczos algorithm – for computing eigenvalues [17-19].

Passive radar has a long history and many practical applications for the future in both military and civilian scenarios. Many factors coming together including the rise of suitable signal processing technologies and proliferation of wide band high signal strength commercial transmissions. It has been shown that passive radar is a strong candidate for a radar technology because of its covert nature and stealth-finding abilities. There are many variables in passive radar systems that need to be considered, including number of receivers, non-cooperative transmitters and channels/frequencies. These considerations influence the analysis that is performed in the way of shaping how the GLRT is calculated for a particular situation. Many different derivations for a GLRT exist, and it is the goal of this proposal to analyse a few and propose a new GLRT for passive radar detection systems.

Part II

Current Work and Proposal

4 Current Work

I have begun the analysis and re-creation of the first paper of interest, *‘Generalized Canonical Correlation for Passive Multistatic Radar Detection’* [10]. Very initial results using GCCA with somewhat inaccurate simulations have been completed with very positive results so far.

The simulated situation is as follows

- 5 non-directional receiver located at the origin and 50m North, South, East and West of the origin
- 1 single frequency transmitter located 1000m East of the origin

A target can be simulated at any given position. From these constraints, signals are generated from the transmitter and broadcast to the Rx points so that each Rx receives a line of sight (LOS) signal and a reflected signal off the target, both with the appropriate time delay. At this point in time, there is no signal attenuation accounted for in this model, so received signals are at 100% of broadcast signal strength for their entire path. This is inaccurate and will be changed for future simulations. Another inaccuracy at this point is that the simulated signals are purely real, in future they will be simulated as complex baseband signals. There is also no signal noise added into this situation to match real world sampling and interference. This processing is done in the MATLAB file *signal_gen.m,* which will appear in appendix A. This function outputs the signals that each Rx receives from the same time reference.

This information, along with the search area of interest is then fed into a different function, *target_detect.m* (found in appendix B), which attempts to do the processing as described in the paper. This function determines the largest eigenvalue of the time-adjusted Gram Matrix, G, as per the GLRT described in the paper. At this stage, the function only processes the spatial components, no effort has been put into a Doppler-Range computation, and the processing only considers the time difference between the Rx, the Tx is not considered. There is also no attempt to remove the LOS signal from the received signals at each position. This function outputs a plan position indictor (PPI) map, which is then handled and graphed by a parent function, *test_canon.m* (appendix C), to output the information in an acceptable form. The results from simulating a target at position (-100,-100) and testing the area of 400m by 400m are shown in Figure 1 below.

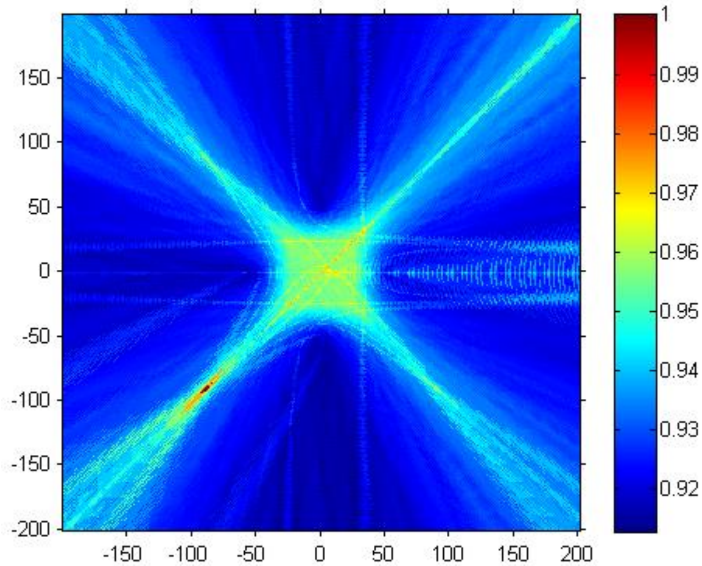


Figure 1: PPI map of detection of a single target at (-100,-100) using 5 receivers and GCCA method

As can be seen, the analysis is detecting our target at the correct location. While these results are not very robust or insightful, they do show progress on the topic and are a good start on the topic. One thing to note is how the Tx signal affects the results in the PPI map in figure 2. Processing to remove the LOS signal could be used to remove this interference from the results, as is discussed in the paper.

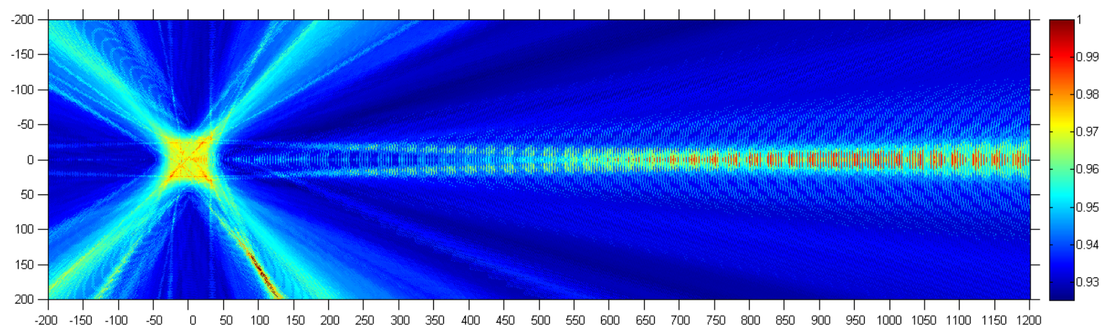
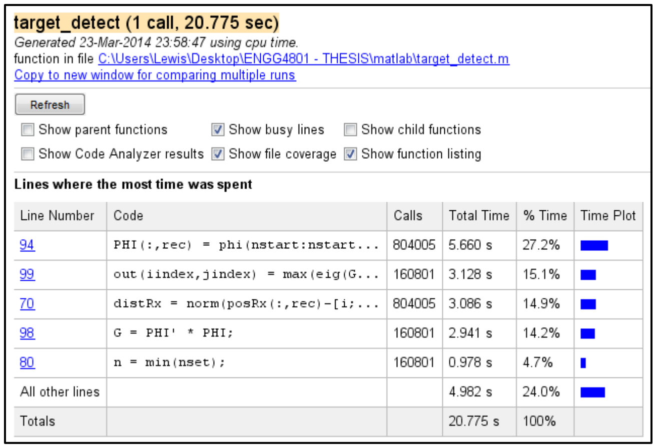


Figure 2: PPI map of detection of a single target at (100,-150) using 5 receivers, showing the effect of transmitter on results

Analysis of the time profiles of functions led to awareness of which calls were slowing down performance. The MALTAB profiler tool allowed me to see which lines were slowing the system. From there, I considered alternative implementations of those lines, and wrote test modules for them, to determine if a speed improvement could be had. This was the case for two such lines. The total computation time for my test scenario decreased from 21.8s down to 16.8 seconds, an improvement of 23%.



Line Number	Code	Calls	Total Time	% Time	Time Plot
94	PHI(:,rec) = phi(nstart:nstart...	804005	5.660 s	27.2%	
99	out(iindex,jindex) = max(eig(G...	160801	3.128 s	15.1%	
70	distRx = norm(posRx(:,rec)-[i;...	804005	3.086 s	14.9%	
98	G = PHI' * PHI;	160801	2.941 s	14.2%	
80	n = min(nset);	160801	0.978 s	4.7%	
All other lines			4.982 s	24.0%	
Totals			20.775 s	100%	

Figure 3: MATLAB profile of function *target_detect.m* before editing

5 Project plan

The project will be split into two halves, a *review and prior works* section, and a *new techniques and approaches* section. By using this approach, I will be quite grounded in the previous works that have been done before I try and extend to some new techniques.

The first half shall consist of reproducing and recreating results from previously published papers. This will involve simulating results from at least 3 papers with different approaches to the problem. This will provide a solid hands-on background of the current literature by actually implementing the proposed solutions and allowing me to get more in depth understanding of the challenges by actually coding solutions. I will then compare these different methods based on their efficiency, ease of implementation and correctness (how well they perform).

The second half of the project shall focus on developing new techniques for passive radar detection. This will have two main focuses. Firstly, a new passive radar detection scheme will be formulated in its entirety. The development of the signal model and detector algorithm will be derived in full. This solution will then be implemented in MATLAB and analysed and compared to the previous methods. The second focus will be on increasing the efficiency of the algorithm, with the aim of making it possible for real-time computation in a system. This could be achieved in many ways, and I will utilize techniques involving computational efficiency, algorithm modification (by either making approximations for some values of interest, or implementing more efficient algorithms for some steps) [20].

Table 1: Milestone list for '*Review and Prior works'*

Task Name	Description	Allotted time (Expected Completion date)
Reproduce results 1	Recreate results from paper: Generalized Canonical Correlation for Passive Multistatic Radar Detection [10]	3 weeks (11-Apr)
Reproduce results 2	Recreate results from paper: MULTIPLE-CHANNEL DETECTION OF SIGNALS HAVING KNOWN RANK [15]	3 weeks (2-May)
Reproduce results 3	Recreate results from paper: On the Applicability of Source Localization Techniques to Passive Multistatic Radar [16]	3 weeks (23-May)
Seminar preparation	Compile current work and prepare seminar for presentation	3 Weeks (28-Apr to 19-May)
Compare Methods	Write up comparison between the three compared approaches detailing their: • Efficiency • Implementation • Accuracy of results	3 weeks (13-June)
Reproduce results 4(+)	If time permits before returning after semester break, test approaches found in other research papers and incorporate them into comparisons	4 weeks (28-July)

Table 2: Milestone list for *'New techniques and approaches'*

Task Name	Description	Allotted time (Expected Completion date)
Develop new passive radar scheme	Develop mathematical models for the signals	1-2 weeks (18-Aug)
	Develop mathematical model and reasoning for the algorithm to locate targets	1-2 weeks (18-Aug)
	Implement in MATLAB	2 weeks (1-Sep)
	Work on increasing efficiency of algorithm by decreasing computation time	2 weeks (15-Sep)

	Analyse results of new scheme and compare to previously proposed scheme	2 weeks (15-Sep)
Incorporate results	Begin writing about work covered in second half of thesis plan	3 weeks (6-Oct)
Demonstration Preparation	Assemble all work and simulations achieved into presentation and demonstration of project	3 Weeks (20-Oct)
Submit Thesis	Compile all results and previously written sections into final thesis form	5 Weeks(10-Nov)

6 OHS Risk Assessment

All work for this thesis is simulation performed on computers. As such, work will be completed in a Low risk laboratory, covered by general OHS laboratory rules. Potential risks involved in the project are computer use based and include:

- Carpal tunnel syndrome
- Computer vision syndrome
- Musculoskeletal problems cause by awkward posture
- Risks associated with using a laptop instead of a desktop computer

As such, the following will be implemented by the student to reduce risks/ injuries that could be cause by the above:

- The student will take regular breaks while using the computer
    - Use the 20/20/20 rule. 20 seconds of break staring at objects 20m away every 20 minutes on the computer
    - Work on the computer will be limited to a maximum of 2hours at a time
- Repetitive tasks will be broken up and interspaced so the same action is not consistently repeated for prolonged periods of time
- When using a laptop, it will always be placed on a desk or table, never on the student's lap
- The keyboard/mouse will be set up so that they are at a comfortable distance when sitting upright
- F.lux will be installed on devices used by the student to reduce strain on eyes at night

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Appendix A. MATLAB file: *signal_gen.m*

```
function phi = signal_gen(posRx, posTx, posTarg, freq, N)
%A function that generates random signal, then distributes it as well as
%off 1 target
%
%ALL DISTANCES ARE IN METRES
%
%INPUTS:
%   posRx - the position of each receiver so that posRx(1,i) and posRx(2,i)
%         are the x- and y-positions of receiver i
%   posTx - the position of the transmitter so that posTx = [xpos; ypos]
%   posTarg - the positions of the targets so that
%           posTarg(:,i) = [xpos taget i; ypos target i]
%   freq - The sampling frequency of the sampled signals
%   N - The number of samples we want generated
%
%OUTPUTS:
%   phi - An array of inputs from receivers in the form [X1, X2,...,Xm]
%         where Xi is a column array of values. Xi can be recovered by
%         Xi = phi(:,i)

%CONSTANTS
%C = Speed of light (in m/s)
C = 299792458;
%alpha = Signal degredation (/m or m^-1) Range: 0-1;
%0 is full signal degredation and 1 is none
%alpha = 1;


%DERIVED CONSTANTS
%m = number of receivers
m = size(posRx,2);
%dt = time between samples (seconds)
dt = 1/freq;
%DTxTarg = Distance from transmitter to target
DTxTarg = norm(posTx-posTarg);
%DTargRx = Array of distances where D(i) = distance from target to Rxi
DTargRx = zeros(m,1);
for rec = 1:m
    DTargRx(rec) = norm(posTarg-posRx(:,rec));
end
%DTxRx = Array of distances where D(i) = distance from Tx to Rxi
DTxRx = zeros(m,1);
for rec = 1:m
    DTxRx(rec) = norm(posTx-posRx(:,rec));
end

%Calculate maximum travel time of the signal and the equivelant number of
%samples that it would be delaed by
MAX_DIST = max(DTargRx) + DTxTarg;
MAX_TIME = MAX_DIST / C;
MAX_SAMP = round(MAX_TIME / dt)+1;
```

```matlab
%Number of generated samples
Ngen = N+MAX_SAMP;
S = rand(Ngen,1);

%Initialize phi (will truncate at end)
phi = zeros(Ngen,m);

% Give each receiver the target signal
for rec = 1:m
    DIST = DTxTarg + DTargRx(rec);
    TIME = DIST / C;
    SAMP = round(TIME / dt);
    phi(SAMP:end,rec) = S(1:Ngen-SAMP+1);
end

%Give each receiver the original signal
for rec = 1:m
    DIST = DTxRx(rec);
    TIME = DIST / C;
    SAMP = round(TIME / dt);
    phi(SAMP:end,rec) = phi(SAMP:end,rec)+S(1:Ngen-SAMP+1);
end

phi = phi(Ngen-N+1:end,:);
```

Appendix B. MATLAB file: *target_detect.m*

```
function out = target_detect(phi, posRx, posTx, freq, xb, yb, tick)
%A function that returns array of log-likelihood for input phi
%
%ALL DISTANCES ARE IN METRES
%
%INPUTS:
%   phi - An array of inputs from receivers in the form [X1, X2,...,Xm]
%         where Xi is a column array of values. Xi can be recovered by
%         Xi = phi(:,i)
%   posRx - the position of each receiver so that posRx(1,i) and posRx(2,i)
%         are the x- and y-positions of receiver i
%   posTx - the position of the transmitter so that posTx = [xpos; ypos]
%   freq - The sampling frequency of the sampled signals
%
%OPTIONAL INPUTS:
%   xb - the x-boundary. xb = [xmin, xmax]. will iterate over this boundary
%   yb - the y-boundary. yb = [ymin, ymax]. will iterate over this boundary
%   tick - the resolution of the iteration in space
%
%OUTPUTS:
%   out - an array of dimensions [diff(yb), diff(xb)] with log likelihood
%       of each position being a target
%function out

%NOTES:
%   - In code all arrays are row/col == i/j == x/y
%   - At the end out is transposed to get in the correct orientation

%Sort out optional inputs defaults
if nargin < 4
    xb = [-5500,11000];
    yb = [-2000,2000];
    tick = 10;
end

%CONSTANTS
%C = Speed of light (in m/s)
C = 299792458;

%DERIVED CONSTANTS
%m = number of receivers
m = size(posRx,2);
%num_samp = number of samples of each signal taken
num_samp = size(phi,1);
%dt = time between samples (seconds)
dt = 1/freq;
%

%The possible xy positions to check
x = xb(1):tick:xb(2);
y = yb(1):tick:yb(2);
```

```
out = zeros(numel(x),numel(y));

%Iterate through each xy position and receiver
iindex = 0;
for i = x
    iindex = iindex + 1;
    jindex = 0;
    for j = y
        jindex = jindex+1;

        %Iterate through each receiver to get the time delay and calculate
        %the number of samples delay to put in nset - the set of sample
delays
        nset = zeros(m,1);
        for rec = 1:m
            %Calculate distance along Target - Rx and time taken
            %Can ignore distance from Tx to ij as is constant for each
            %receiver and ij.
            distRx = norm(posRx(:,rec)-[i;j]);
            travel_time = distRx/C;

            %calculate n, number of samples delayed
            nset(rec) = round(travel_time / dt);
        end

        %time adjusted signal x'(t) and put into PHI
        %Need to set maxn
        %Shortest delay will be our starting point
        n = min(nset);
        nset = nset - n;
        %Longest delay will determine how many signals we store for each
        nmax = max(nset);
        nstore = num_samp - nmax;

        %Initialise the PHI from paper. PHI = [X1' X2' ... Xm']
        %Note: here X1' does not denote transpose
        PHI = zeros(nstore,m);

        for rec = 1:m
            %So we can throw away the first nset samples of each signal to
            %get all signal from the same time
            nstart = nset(rec)+1;
            PHI(:,rec) = phi(nstart:nstart+nstore-1,rec);
        end
        %Here we have PHI for the position (i,j)
        %Calculate Gram Matrix G:
        G = PHI' * PHI;
        out(iindex,jindex) = max(eig(G));
    end
end
out = out';
out = flipud(out);
```

Appendix C. MATLAB file: *test_canon.m*

```
%test_canon.m
%Test Generalised canonical correllation

%There will initially be 5 receivers spaced 50m apart located at origin
%(0,0), (0,50), (0,-50), (-50,0), (50,0)
posRx = [0, 0, 0, -50, 50;
         0,50,-50, 0,  0];
%The transmitter will be located 1km East of the receivers
posTx = [1000;0];
%There will be one stationary target located ~141m NW of the origin Rx
posTarg = [100; -150];
%Sampling Frequency of 220MHz
freq = 220e6;
%Will start with 1000 samples
N = 1000;

clc;

%Generate the
phi = signal_gen(posRx,posTx,posTarg,freq,N);

xb = [-200,200];
yb = [-200,200];
tick = 1;

out = target_detect2(phi, posRx, posTx, freq, xb, yb, tick);

imshow(out);
colormap(jet);

out2 = out/max(max(out));
figure;
imshow(out2);
colormap(jet);
caxis([min(min(out2)) 1]);
hold on;
h = colorbar;
set(h, 'ylim', [min(min(out2)) 1]);

%Relabel the axes correctly
ttk=50;
set(gca,'YTick',1:ttk:diff(yb)+1);
%set(gca,'YTickLabel','200|150|100|50|0|-50|-100|-150|-200')
set(gca,'YtickLabel',num2str(str2num(get(gca,'YTickLabel'))-1+yb(1)));

set(gca,'XTick',1:ttk:diff(xb)+1);
%set(gca,'XTickLabel','-200|-150|-100|-50|0|50|100|150|200')
set(gca,'XtickLabel',num2str(str2num(get(gca,'XTickLabel'))-1+yb(1)));
%TAKES 21.8 seconds
```