Multistatic Passive Radar using GCCA

KONSTANTY BIALKOWSKI, VAUGHAN CLARKSON

School of Information Technology & Electrical Engineering
The University of Queensland, Qld., 4072
AUSTRALIA
ksb@itee.uq.edu.au,v.clarkson@uq.edu.au

Passive Sensing Workshop, 24th May, 2013

Overview

- Radar Detection and Coherence.
- Multistatic Passive Radar.
- Generalised Canonical Correlation Analysis (GCCA).
  - Single Tx, Multiple Rx.
  - Testbed Development.
  - Multiple Tx (Single Frequency Networks).
- Detection Statistics for GCCA.
  - Comparison with Generalized Coherence.
  - Surveillance Over a Wide Area.
- Stop Press.

Coherence

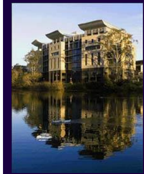
Coherence exists when we we have a common but unknown signal on two or more noisy data channels.

- Applications include seismology, communications, data mining, radio astronomy, biomedical signal processing.
- We are interested in its application to *passive radar*.
- This is natural since an *illuminator of opportunity* emits signals which are unknown to the radar receivers *a priori*.
- Coherence is to be discovered at one or many different time and Doppler shifts and each receiver.
  - This forms the basis of *detection*.

Multistatic Passive Radar

We are interested in a system with multiple (opportunistic) transmitters and/or receivers at known locations.

- Best accuracy is achieved when the elements are widely separated.
- Multistatic radar has a history of development going back to the 1960s if not earlier.
- Multistatic *passive* radar has only recently become feasible, since it requires:
  - appropriate signal processing,
  - accurate synchronisation between receivers and
  - sufficient bandwidth to collect and process the received signals.



Signal Model

We will assume N transmitters and M receivers.

- Each transmitter emits a signal $s_n(t)$, $n = 1, \ldots, N$, unknown to the receivers.
- The received signals are of the form

$$x_m(t) = \sum_{\ell=0}^{L} \sum_{n=1}^{N} \mu_{\ell,m,n} s_n(t - \tau_{\ell,m,n}) e^{j\omega_{\ell,m,n} t} + \xi_m(t).$$

  - There are $L + 1$ paths (including the *direct path*, indexed 0) between each transmitter-receiver pair.
    - $*$ Apart from the direct path, we will assume each is due to a simple reflection from a point scatterer (target or clutter).
  - $\mu_{\ell,m,n}$ is the received signal strength.
  - $\tau_{\ell,m,n}$ is the time delay.
  - $\omega_{\ell,m,n}$ is the Doppler shift.
  - $\xi_m(t)$ is the received noise.

Multistatic Detection

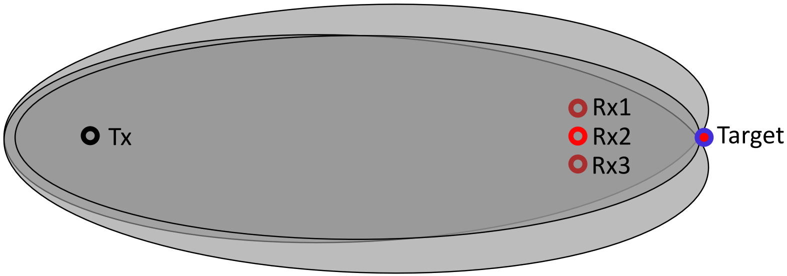
We will approach the problem of multistatic by postulating the position and velocity of a target (as we will call all scatterers/paths).

- Suppose target ℓ lies at position $\mathbf{p}_\ell$ and has velocity $\mathbf{v}$.
- This would induce time- and Doppler-shifts

$$\tau_{\ell,m,n} = \frac{\|\mathbf{t}_n - \mathbf{p}_\ell\| + \|\mathbf{p}_\ell - \mathbf{r}_m\|}{c}$$

and $$\omega_{\ell,m,n} = \frac{\omega_c}{c}\mathbf{v}^T\left(\frac{\mathbf{p}_\ell - \mathbf{r}_m}{\|\mathbf{p}_\ell - \mathbf{r}_m\|} - \frac{\mathbf{t}_n - \mathbf{p}_\ell}{\|\mathbf{t}_n - \mathbf{p}_\ell\|}\right)$$

where $\mathbf{t}_n$ and $\mathbf{r}_m$ are the positions of the transmitters and receivers.



Coherence in Detection

We 'correct' the received signals for the postulated time- and Doppler-shifts, so that

$$\tilde{x}_m(t) = x_m(t + \tau_{\ell,m,n})e^{-j\omega_{\ell,m,n}t} = \mu_{\ell,m,n}s_n(t) + \text{interference} + \text{noise}$$

where the interference arises from correcting all other paths by the same amount and the noise has likewise been corrected.

- With a single transmitter and an '*over-the-shoulder*' configuration where receivers are shielded from the direct path, there is no interference.
- If the postulated position and velocity corresponds to a target, there is coherence between the $\tilde{x}_m(t)$.
- Sample the $\tilde{x}_m(t)$ to form $\tilde{\mathbf{x}}_m$.
- With no interference, a single transmitter (so dropping the ℓ and m subscripts) and in AWGN, the log likelihood is

$$f(\mu_1, \ldots, \mu_M, \mathbf{s} \mid \tilde{\mathbf{x}}_1, \ldots, \tilde{\mathbf{x}}_M) = -\|\tilde{\mathbf{x}}_1 - \mu_1\mathbf{s}\|^2 - \cdots - \|\tilde{\mathbf{x}}_M - \mu_M\mathbf{s}\|^2.$$

Generalised Canonical Correlation

Conditioned on **s**, the likelihood is maximised when

$$\hat{\mu}_m = \frac{\mathbf{s}^H \tilde{\mathbf{x}}_m}{\|\mathbf{s}\|^2}.$$

- The log-likelihood, substituting the maximising $\hat{\mu}_m$ and jettisoning constant terms, is

$$f(\mathbf{s} \mid \tilde{\mathbf{x}}_1, \ldots, \tilde{\mathbf{x}}_m) = \frac{\mathbf{s}^H \mathbf{F} \mathbf{s}}{\mathbf{s}^H \mathbf{s}}$$

  where $\mathbf{F} = \tilde{\mathbf{X}} \tilde{\mathbf{X}}^H$ is the *frame matrix* formed from $\tilde{\mathbf{X}} = (\tilde{\mathbf{x}}_1, \ldots, \tilde{\mathbf{x}}_m)$.

- The maximiser, $\hat{\mathbf{s}}$, is the eigenvector corresponding to the largest eigenvalue of **F**.

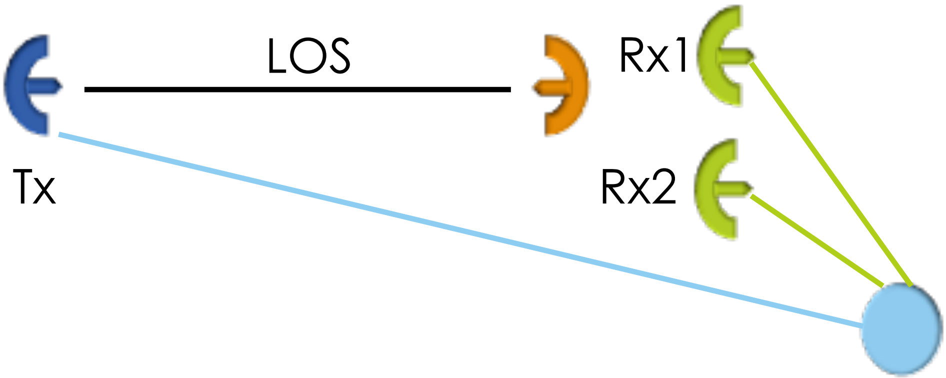
- This is *generalised canonical correlation analysis* (*GCCA*) to find the μ_m and **s**—it is *maximum likelihood*.

The Gram Matrix in GCCA

$\mathbf{F}$ shares its non-zero eigenvalues with the *Gram matrix* $\mathbf{G} = \tilde{\mathbf{X}}^H\tilde{\mathbf{X}}$, and this is generally a much smaller matrix.

⇒ We use $\mathbf{G}$ instead to find the maximum eigenvalue.

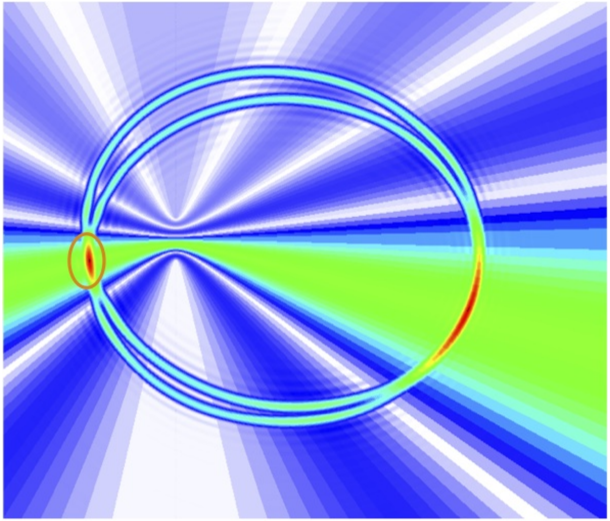
- The approach can be adapted to the case where one (directional) receiver is trained to pick up the direct path signal from the transmitter.



GCCA as a Detector

It follows that a *generalised likelihood ratio text* (*GLRT*) can be constructed from GCCA.

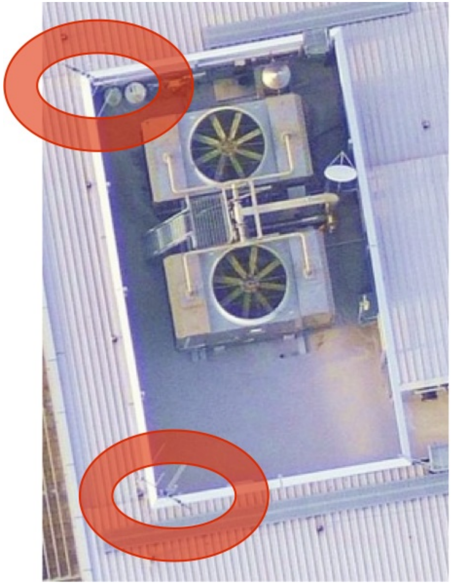
- The statistic is simply $\|\tilde{\mathbf{X}}\|_2^2$, the (squared) spectral norm of the matrix of (time-and-Doppler-corrected) received signals.
- Calculating this statistic on a grid of postulated target positions and velocities, we arrive a *plan position indication* (*PPI*) map.

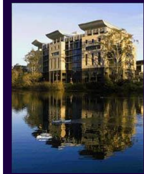


Testbed Development

A modest testbed has been developed at the School of ITEE, UQ.

- Two Yagis atop our building with about 8 m separation.
- ~ 8 km from Mt. Coot-tha, where the main terrestrial TV broadcast antennas are located.





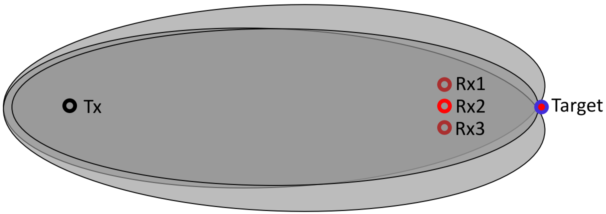
Baseband Components

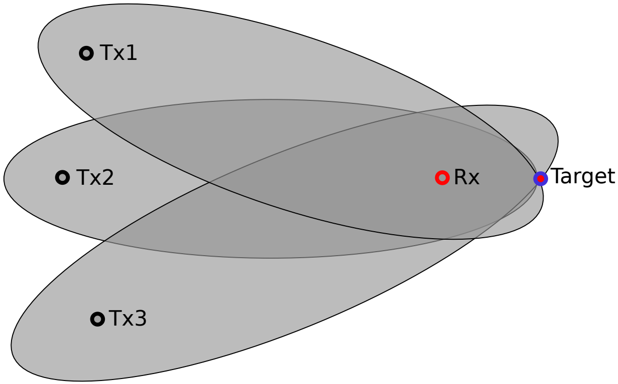
- Various off-the-shelf components are used:
  - $2\times$ USRP N210 (and older USRP devices).
  - Clocktamer GPS clock (for synchronisation).
  - DBSRX2 daughterboards.
  - Capable of 2 channels at 8 MSps I & Q.
  - Altera Cyclone III FPGA with $4 \times$ 14-bit 100 Msps DACs with custom firmware for capturing and communicating.
- 'Ground truth' data is obtained from ADS-B using a $10 DAB-T tuner stick.

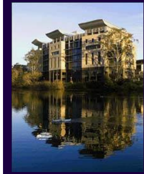
Exploiting Single-Frequency Networks

The ability to localise targets increases when the receivers are more widely separated.

- However, large separation causes synchronisation and communication problems.
- Instead, we can exploit a feature of large terrestrial digital TV broadcast operations, namely *single-frequency networks* (*SFNs*).
- A single antenna can be used with GCCA (although introduces problems of self-interference and noise correlation).

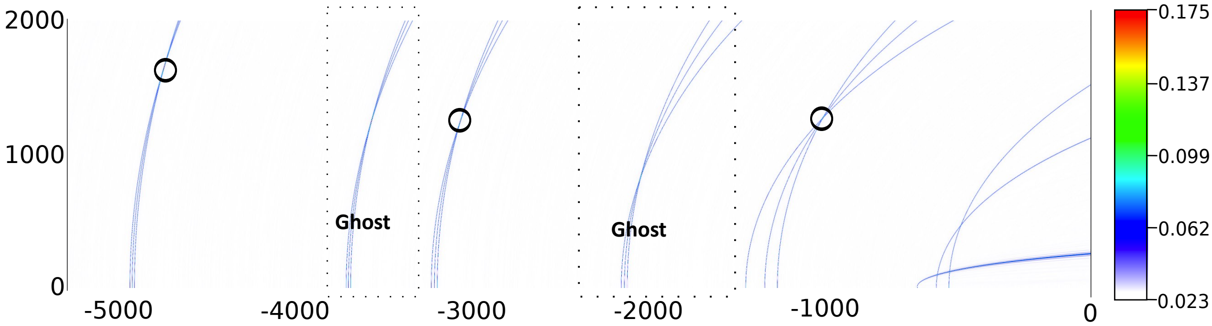




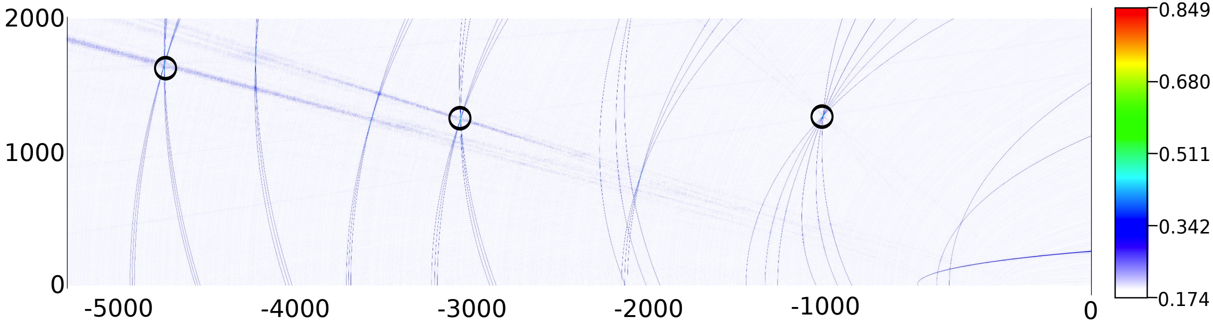


SFN-Aware Algorithm

- The self-interference between multiple transmitters can introduce ‘ghost’ targets if the SFN is not accounted for.

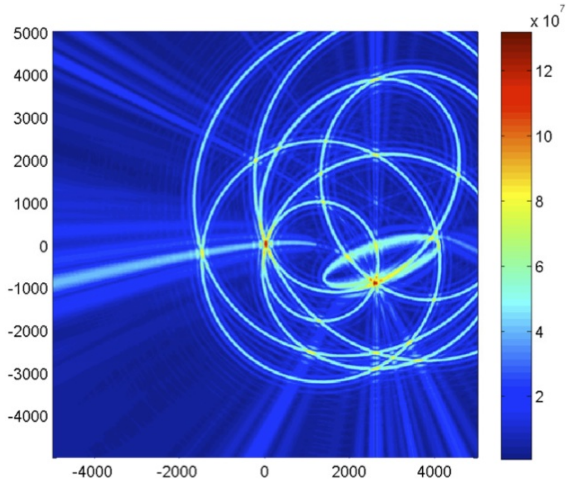


- With SFN Txs modelled, the ghosts are relatively less prominent.

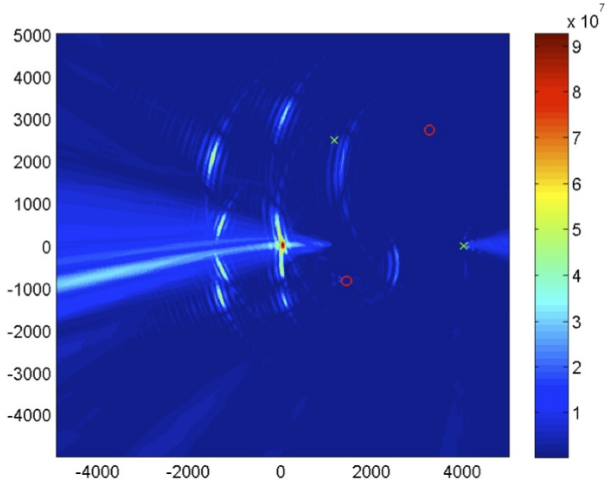


SFN and Moving Targets

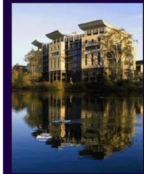
Consider a scenario with two transmitters and two receivers and a target at the origin.



Stationary target



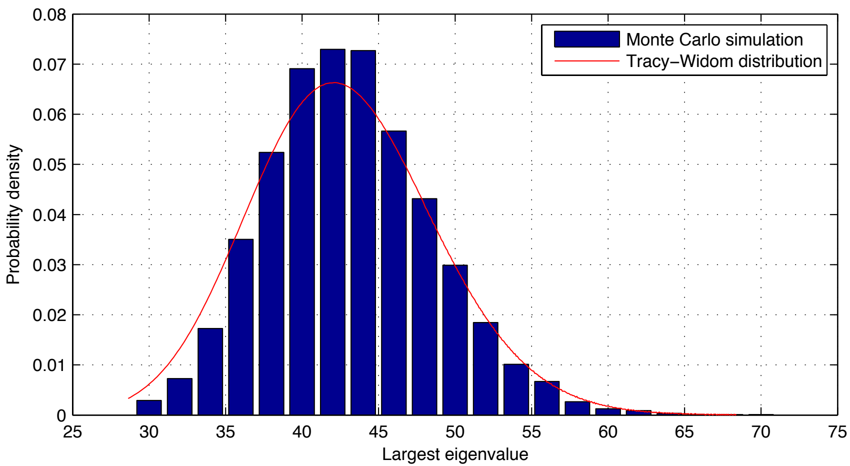
Moving target



GCCA and Tracy-Widom

A disadvantage to the use of the GCCA is the difficulty in working with the distribution of the detection statistic.

- The Gram matrix has complex Wishart distribution.
  - Central in H_0, non-central in H_1.
- The distribution of the largest eigenvalue therefore converges to the Tracy-Widom distribution $F_2(x)$.
- Tail decays like $e^{-\frac{2}{3}x^{3/2}}$.

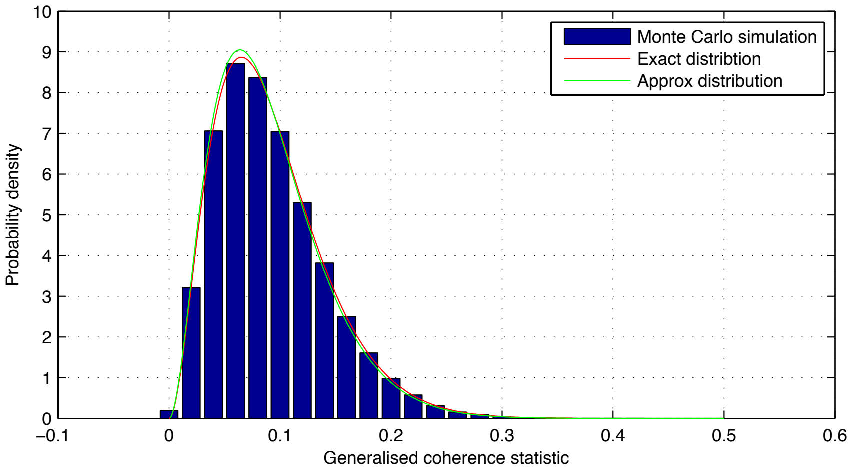


Generalised Coherence

An alternative statistic is *generalised coherence*:

$$\mathrm{GC} = 1 - \frac{\det \mathbf{G}}{\prod_{m=1}^{M} g_{mm}}.$$

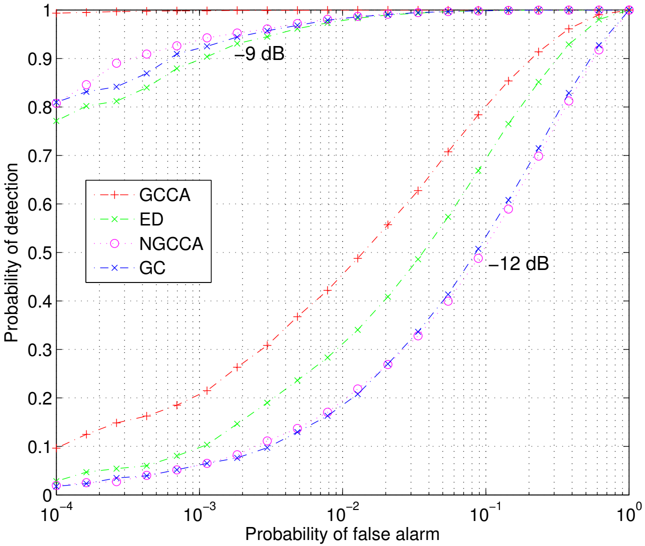
- Advantages: simple to compute, good approximations to H_0 and H_1 distributions.



Power of the Statistics

We compare the GCCA and GC detectors with two others:

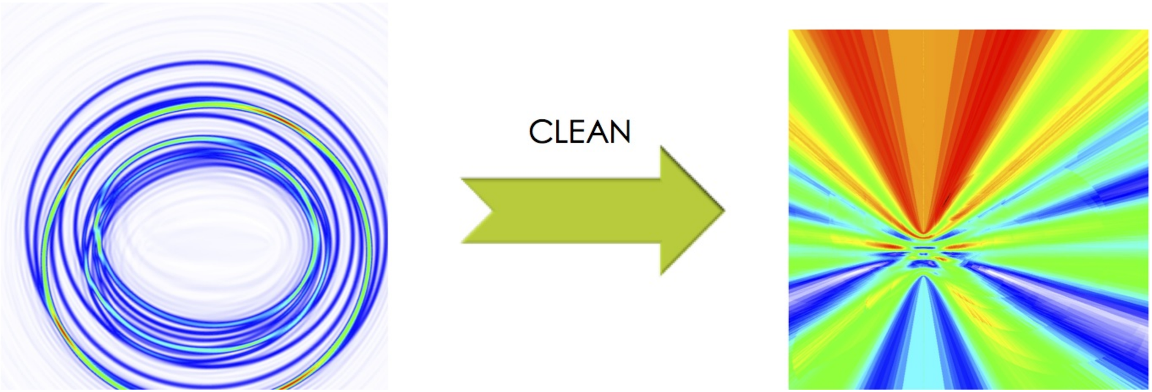
- the *energy detector* (*ED*) and
- the *normalised GCCA* (*NGCCA*) for which the statistic is $\|\mathbf{G}\|_2 / \operatorname{tr} \mathbf{G}$.



Successive Cancellation

The current approach to multi-target detection is successive cancellation.

- That is, from PPI, determine the most strongly localised return.
- Carefully determine the parameters of target and then subtract it from the return.
- Iterate until what remains is indistinguishable from noise.
- This is the *CLEAN* approach.



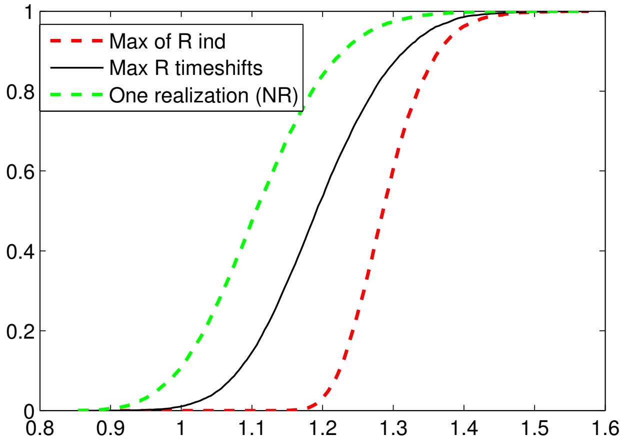
Wide-Area Surveillance

The statistics examined so far have been applied to a single position/velocity postulate.

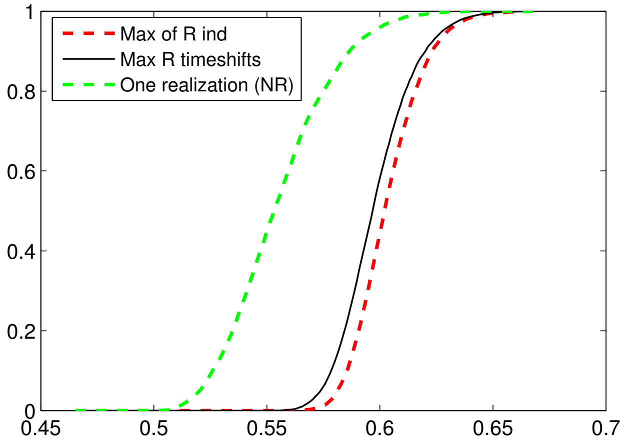
- In keeping an area under surveillance, we would like to test a single statistic for many such postulates at once.
- An obvious approach is to take the maximum of the statistics for a number, K, of discrete postulates.
- How should the threshold be set?
- If the statistics were independent r.v.s, we could simply use the order statistics, $F_{\mathrm{wa}}(x) = F^{K}_{\mathrm{GCCA}}(x)$.
- But the statistics are not independent.
- How quickly do the statistics decorrelate across an area?

Simulation Results

Monte Carlo simulations are performed for the GCCA and NGCCA detectors with one transmitter, two receivers and the maximum taken over 20 time-shifts.

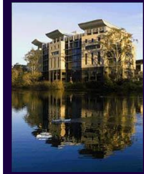


GCCA



NGCCA

- Observe that NGCCA under time-shifting behaves much more like independent r.v.s than GCCA.



School of Information Technology & Electrical Engineering

The University Of Queensland

Overnight Idea

A common theme in passive-radar detectors is that they assume that the receivers are divided into those that obtain a reference signal (direct path) and those that sense the environment (indirect path).

- This is the ‘over-the-shoulder’ configuration.
- Another option is that, when time- and Doppler-corrected, the direct path is treated as interference.
- Let’s assume instead that all receivers receive both direct path and, if present, target paths.
- We test, as with other detectors, the hypothesis that a single target is present at a postulated position and velocity.

Signal Model

The signal model is that, at receiver m,

$$\mathbf{x}_m = \sum_{k=1}^{K} \mu_{m,k} \mathbf{D}_{m,k} \mathbf{s} + \xi_m.$$

- $\mu_{m,k}$ is the strength of the signal received via the kth path.
- $\mathbf{D}_{m.k}$ is a matrix that delays and Doppler-shifts for the kth path.
- The scenario described above has $K = 2$, with path $k = 1$ being the direct path.
- The signal vector $\mathbf{s}$ has slightly greater dimension than the $\mathbf{x}_m$, to allow for the range of possible delays.
- We can adapt this directly to SFNs where $K = 2N$: one direct and indirect path for each transmitter.
- For convenience, we define the matrix

$$\mathbf{D}_m(\mathbf{s}) = (\mathbf{D}_{m,1}\mathbf{s}, \ldots, \mathbf{D}_{m,K}\mathbf{s}).$$

- Our model is then $\mathbf{x}_m = \mathbf{D}_m(\mathbf{s})\mu_m + \xi_m$.

World's Worst™ Passive-Radar Target Detector

The log-likelihood function under AWGN (or non-linear least-squares objective function) is

$$L(\mu, \mathbf{s} \mid \mathbf{x}_1, \ldots, \mathbf{x}_M) = -\sum_{m=1}^{M} \|\mathbf{x}_m - \mathbf{D}_m(\mathbf{s})\mu_m\|^2.$$

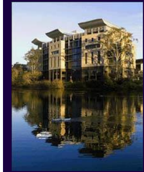
- It follows that

$$\hat{\mu}_m = \frac{\mathbf{x}_m^H \mathbf{D}_m(\mathbf{s})}{\mathbf{D}_m^H(\mathbf{s})\mathbf{D}_m(\mathbf{s})}$$

- Substituting $\hat{\mu}_m$, we have (neglecting constant terms)

$$L(\mathbf{s} \mid \mathbf{x}_1, \ldots, \mathbf{x}_M) = -\sum_{m=1}^{M} \|\mathbf{P}^{\perp}_{\mathbf{D}_m(\mathbf{s})}\mathbf{x}_m\|^2 = \sum_{m=1}^{M} \|\mathbf{P}_{\mathbf{D}_m(\mathbf{s})}\mathbf{x}_m\|^2 + \text{const.}$$

where $\mathbf{P}_\mathbf{Z}$ is the orthogonal projector onto the subspace spanned by the columns of $\mathbf{Z}$ (and $\perp$ denotes the complementary projector).



School of Information Technology & Electrical Engineering

The University Of Queensland

Alternative Formulation

Alternatively, we can write

$$L(\mathbf{s} \mid \mathbf{x}_1, \ldots, \mathbf{x}_M) = \sum_{m=1}^{M} \mathbf{s}^H \mathbf{A}_m^H [\mathbf{D}_m^H(\mathbf{s}) \mathbf{D}_m(\mathbf{s})]^{-1} \mathbf{A}_m \mathbf{s}$$

where

$$\mathbf{A}_m = (\mathbf{x}_m^H \mathbf{D}_{m,1}, \ldots, \mathbf{x}_m^H \mathbf{D}_{m,K}).$$

- In this form, the maximisation resembles a *generalised eigenvalue problem*.
- This connects the result to the earlier one for the GCCA.
- The detector can be expanded to multiple transmitters—a higher-rank problem.
- Optimisation could be by the method of GOLUB & PEREYRA.
- The optimisation could be seeded with estimates obtained by GCCA (or higher-rank alternatives).
- The detector is the difference in residuals between $K = 1$ and $K = 2$.

School of Information Technology & Electrical Engineering

THE UNIVERSITY OF QUEENSLAND