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Generalized Canonical Correlation for Passive Multistatic Radar Detection

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***Abstract*—In this paper, we consider the problem of target detection in passive multistatic radar. In passive radar, we make use of illuminators of opportunity. As the illuminators are not under our direct control, the illuminating signal itself is unknown. We propose a signal model which reflects this. In deriving a maximum-likelihood estimator for the unknown parameters, including the illumination, we find that the maximum value of the likelihood is a monotonic function of the largest eigenvalue of the Gram matrix of the received signals. The generalised likelihood ratio test turns out to be equivalent to comparison of the largest eigenvalue against a threshold, so we propose its use as a target detection statistic. The proposed detector is similar to generalised canonical correlation in multivariate statistics. The benefit of using this statistic over others such as generalised variance is demonstrated through numerical simulations in the context of passive radar using DVB-T signals.**

I. Introduction

A problem of interest in many disciplines is to determine the presence of a common but unknown signal in two or more signal sources in the presence of noise. One measure of such a relationship is the coherence. This term is well studied in a large number of fields, including communications [1], data mining [2], neural networks [3], radar and sonar [4] and radio-astronomy [5].

Measures of coherence were studied in detail in the area of multivariate statistics [6], [7]. Here, coherence is known as covariance [8] or, when the data has zero mean, simply correlation. For communications and signal processing, the signals are usually assumed to be complex-valued. Whether real or complex, there exist noise and signal components and therefore the measure of coherence is the same. For multivariate statistics, the first method is called *generalized canonical correlation analysis (GCCA)*, and the second is called *generalized variance* [7]. The former measures the level of correlation between different signals by finding the common signal components with maximum power. The latter finds the variance between the signals by minimizing the power of the non-signal components. The latter method requires normalization and subtraction from unity to be used as a measure of coherence. When there are only two channels, both of these methods provide the same answer, and this result is known as *magnitude-squared coherence (MSC)* [9].

MSC was extended to generalized coherence in [10] and analysed further in [11]. By using the Gram determinant (determinant of the covariance matrix) and subsequently normalizing and subtracting from unity, a measure of similarity is obtained which is equivalent to the MSC estimate when only two channels are considered. This method is equivalent to the generalized variance from statistics mentioned above.

In this paper, we find that another generalisation of coherence arises when the application is target detection in passive multistatic radar. In this application, the target or targets are illuminated by an unknown signal transmitted by an *illuminator of opportunity*, that is, a radiative source whose intended purpose is not for radar but to provide other services such as telecommunications or geolocation. The received signals, when corrected for the time delay and Doppler shift of a specific target, exhibit a coherence which it is our aim to detect.

Passive radar has been an active research topic due to its advantages over active radar technology [12], [13]. Because the transmitter is provided opportunistically, there is a lower system cost and ability to operate stealthily. In some areas, such as urban areas, there is also a wide choice of illuminators of opportunity. Together with the use of multiple receive antennas, the multistatic nature of the radar yields additional benefits arising from waveform, geographic diversity and from the fact that it may be more difficult for the target to minimise its radar cross section. As with any radar illuminator, it is preferable that such sources have high transmission power and a narrowly-peaked waveform ambiguity function in order to more reliably detect coherence and therefore detect and resolve targets.

In Section II, we introduce a simple signal model for passive multi-static radar. A number of parameters in this model are unknown, including the illuminating signal itself. Their values are estimated using maximum likelihood, as described in Section III. We find that the maximum value of the likelihood is a monotonic function of the largest eigenvalue of the Gram or covariance matrix of the received signals. In Section IV, we discuss the method in which maximum likelihood can be used as a detection statistic for multiple targets. This comes about through construction of the generalised likelihood ratio test. The test turns out to be equivalent to comparison of the largest eigenvalue against a threshold. For this reason, the largest eigenvalue can be regarded as a detection statistic and our approach is therefore, in this sense, closely related to GCCA.

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The results of a simulation study are reported in Section V. The study is designed to simulate a multi-target, multi-receiver passive radar scenario in which a DVB-T digital television signal is used as the illuminator. The proposed detector is compared with generalised coherence and the proposed detector is found to be superior in this application.

II. Signal Model

In multi-static passive radar, there are signals received by m geographically distributed sensors or *receivers* shown in Figure 1. In the scenario considered in this paper, there is only one transmitter but multiple receivers. The transmitted signal $s(t)$ originates at the point marked '*Tx*'. Importantly, the signal is transmitted by an *illuminator of opportunity*, not under our direct control, so $s(t)$ is unknown at the receivers. The purpose of multiple receivers, located on the left-hand side of the diagram, is to be able to resolve the location of the target in space and in velocity. The complex signals received at each receiver are $x_1(t), x_2(t), \ldots, x_m(t)$, respectively. Because of the positions and velocities of targets, each receiver receives a linear combination of time-delayed and frequency-shifted copies of the signal from the transmitter.

Each target echo at the receiver is delayed by a certain amount with respect to the direct-path signal. With respect to any particular receiver, the additional time delay in the echo allows us to deduce that the target lies on an ellipse whose foci are the known transmitter and receiver locations, as further illustrated in Figure 1.

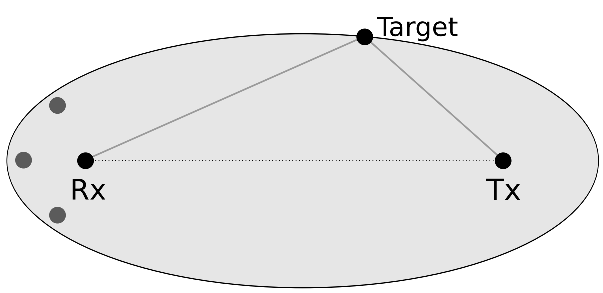


Fig. 1. Bistatic passive radar scenario

We will assume, initially, that the antennas are operated in an 'over-the-shoulder' arrangement in which the receiver antennas are directional and pick up signals only via the indirect target path rather than a direct path. In this arrangement, optionally, one of the antennas may be configured to be directional and pointed so as to pick up only the direct-path signal and not target reflections. Later, we will discuss how these assumptions may be relaxed.

Under these assumptions, the received signals are

$$\begin{aligned} x_1(t) &= \mu_1 s(t-\tau_1)e^{j\omega_1 t} + n_1(t), \\ x_2(t) &= \mu_2 s(t-\tau_2)e^{j\omega_2 t} + n_2(t), \\ &\vdots \\ x_m(t) &= \mu_m s(t-\tau_m)e^{j\omega_m t} + n_m(t), \end{aligned} \tag{1}$$

where the μ_i represent the amplitudes, the τ_i the time delays and the ω_i the frequency shifts in the received copy of transmitted signal. Additive noise is represented by the signals $n_i(t)$.

For a particular hypothesised set of time delays and frequency shifts, the received signals can be corrected so that

$$\begin{aligned} \tilde{x}_1(t) &= x_1(t+\tau_1)e^{-j\omega_1 t} = \mu_1 s(t) + \xi_1(t), \\ \tilde{x}_2(t) &= x_2(t+\tau_2)e^{-j\omega_2 t} = \mu_2 s(t) + \xi_2(t), \\ &\vdots \\ \tilde{x}_m(t) &= x_m(t+\tau_m)e^{-j\omega_m t} = \mu_m s(t) + \xi_m(t), \end{aligned} \tag{2}$$

where the $\xi_i(t)$ are the similarly modified noise components.

The signals are sampled at some appropriate sampling frequency, to yield the discrete-time signals $\tilde{x}_i[n]$. Observe that if the noise is assumed to be additive, white, Gaussian and mutually independent in the signals $x_i(t)$ then it retains this property in discrete time in $\tilde{x}_i[n]$, regardless of whether the correction described in (2) is carried out before or after sampling. If a number of samples, N, are taken at each receiver, we can form (column) vectors $\tilde{\mathbf{x}}_i$. Similarly, we let $\mathbf{s}$ be the vector of samples of $s[n]$ over the same time interval.

III. Maximum-Likelihood Parameter Estimation

The signal model contains several unknown parameters. To analyse an estimation procedure, let us first consider the case of two receivers or *channels*. In the two-channel case, only the relative time delays $\tau_1 - \tau_2$ and frequency shifts $\omega_1 - \omega_2$ are relevant to the signal model.

Given noise variance σ^2, the likelihood is

$$\begin{aligned} &L(\mu_1, \mu_2, \mathbf{s} \mid \tilde{\mathbf{x}}_1, \tilde{\mathbf{x}}_2) \\ &= \frac{1}{(2\pi\sigma^2)^N} \exp\left(- \frac{\|\tilde{\mathbf{x}}_1 - \mu_1\mathbf{s}\|^2 + \|\tilde{\mathbf{x}}_2 - \mu_2\mathbf{s}\|^2}{2\sigma^2} \right). \end{aligned} \tag{3}$$

For the purposes of maximisation, the likelihood can be replaced by the log-likelihood which, neglecting constant terms, can be written

$$\ell(\mu_1, \mu_2, \mathbf{s} \mid \tilde{\mathbf{x}}_1, \tilde{\mathbf{x}}_2) = -\|\tilde{\mathbf{x}}_1 - \mu_1\mathbf{s}\|^2 - \|\tilde{\mathbf{x}}_2 - \mu_2\mathbf{s}\|^2. \tag{4}$$

Conditioned on $\mathbf{s}$, ℓ is maximised by setting

$$\mu_1 = \frac{\mathbf{s}^H\tilde{\mathbf{x}}_1}{\|\mathbf{s}\|^2} \quad \text{and} \quad \mu_2 = \frac{\mathbf{s}^H\tilde{\mathbf{x}}_2}{\|\mathbf{s}\|^2}, \tag{5}$$

where $\cdot^H$ denotes the Hermitian conjugate.

Substituting into ℓ, we have

$$\ell(\mathbf{s} \mid \tilde{\mathbf{x}}_1, \tilde{\mathbf{x}}_2) = -\|\tilde{\mathbf{x}}_1 - \frac{\mathbf{s}\mathbf{s}^H}{\mathbf{s}^H\mathbf{s}}\tilde{\mathbf{x}}_1\|^2 - \|\tilde{\mathbf{x}}_2 - \frac{\mathbf{s}\mathbf{s}^H}{\mathbf{s}^H\mathbf{s}}\tilde{\mathbf{x}}_2\|^2. \tag{6}$$

Expanding and rearranging terms, we find that

$$\ell(\mathbf{s} \mid \tilde{\mathbf{x}}_1, \tilde{\mathbf{x}}_2) = -\tilde{\mathbf{x}}_1^H\tilde{\mathbf{x}}_1 - \tilde{\mathbf{x}}_2^H\tilde{\mathbf{x}}_2 + \frac{\mathbf{s}^H\mathbf{F}\mathbf{s}}{\mathbf{s}^H\mathbf{s}}. \tag{7}$$

where

$$\mathbf{F} = \tilde{\mathbf{x}}_1\tilde{\mathbf{x}}_1^H + \tilde{\mathbf{x}}_2\tilde{\mathbf{x}}_2^H = \mathbf{\Phi}\mathbf{\Phi}^H$$

and $\mathbf{\Phi} = (\tilde{\mathbf{x}}_1\,\tilde{\mathbf{x}}_2)$. The first two terms on the right-hand side are constants, so maximisation of ℓ is equivalent to maximising the Rayleigh quotient in the rightmost term. It is well known that the maximum value of the quotient is the maximum eigenvalue of $\mathbf{F}$. The maximising value of $\mathbf{s}$ is the corresponding eigenvector.

Now, it is also true that the largest eigenvalue of $\mathbf{F} = \mathbf{\Phi}\mathbf{\Phi}^H$ is equal to the largest eigenvalue of $\mathbf{G} = \mathbf{\Phi}^H\mathbf{\Phi}$, the *Gram matrix* or *covariance matrix* of the received signals.

From a computational point of view, calculation of the eigenvalues of the Gram matrix $\mathbf{G}$, as opposed to that of the (frame) matrix $\mathbf{F}$ is much to be preferred, as the former is a 2×2 matrix whereas the latter is $N \times N$.

The foregoing analysis for two channels can be extended to more channels in a straightforward way, by taking the time- and frequency-corrected signals in (2) and arranging them into the matrix $\mathbf{\Phi} = (\tilde{\mathbf{x}}_1 \cdots \tilde{\mathbf{x}}_m)$. The largest eigenvalue of the $m \times m$ Gram matrix formed from $\mathbf{\Phi}$ is of course equal to the square of the largest singular value of $\mathbf{\Phi}$ itself.

IV. Target Detection

In detecting a target at a specified position and velocity, we test a composite hypotheses. The null hypothesis is that the received signals consist only of noise. The alternative hypothesis is that the received signals can be modelled in the form described in (1). We propose a generalised likelihood ratio test. The detection statistic in this case is the likelihood ratio or, equivalently, the difference of log-likelihoods [14].

The log-likelihood for the null hypothesis can be derived from that for the alternative hypothesis by setting the μ_i to zero. That is, the log-likelihood is the negative sum of the squared magnitudes of the $\tilde{\mathbf{x}}_i$. It follows that the difference of the log-likelihoods—our proposed detection statistic—is the maximised value of the Rayleigh quotient in (7), that is, the largest eigenvalue of the Gram matrix of the time- and frequency-corrected received signals.

The approach to detection is to repeatedly calculate this statistic on a (multi-dimensional) grid of positions and velocities. Each postulated position on the grid corresponds to a pattern of time and frequency shifts in the received signals, for which the Gram matrix and its largest eigenvalue are calculated.

In the case of multiple targets, or where the receivers receive a direct-path signal in addition to target returns, the signal model is no longer strictly valid. However, if the illuminator is chosen in the expectation that the peak of its ambiguity is sufficiently narrow, then signals at time delays and frequency shifts other than those postulated can be treated as noise.

It will be observed that the detection statistic used here, the largest eigenvalue of the Gram matrix, has been used in other contexts as a measure of coherence. In [11], the determinant of this Gram matrix was the indicator of coherence. The level of the determinant is dominated by the smallest eigenvalue of the Gram matrix.

In both cases, the determinant and maximum eigenvalue derivations, with normalization are equivalent on maximum squared coherence (MSC) [9]. However, for multi-channel coherence, the properties differ greatly for partially correlated channels. With multiple signals, if two of the signals are highly correlated, the determinant of the Gram matrix will approach 0, giving a high detection statistic. For the maximum eigenvalue, the detection statistic will depend on the number of channels with correlated data. This is a preferable property in passive radar.

V. Simulation Results

We now report the results of a simulation study for multistatic radar. The simulation scenario uses parameters similar to those of DVB-T signals in Brisbane, Australia. The distance from the transmitter to the receiver array is 10 km and each of the receivers are separated by a distance of 50 m. The selected frequency band is 220 MHz, which is in the VHF band.

To test this detector, three targets are simulated. One target is moving, the other two are stationary. Five receivers are simulated. Noise is added to give a SNR of 10 dB. The two stationary objects are placed at $(-4500, 800)$ and $(-4000, 1500)$, whereas the moving target is at $(-3000, -1700)$ and assigned velocity vector of $(0, -100)$. Here, the units are metres for position and metres per second for velocities.

Target returns and direct-path signals are present in all received signals.

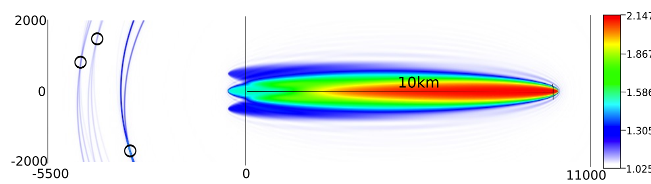


Fig. 2. Scenario using $\lambda_{\max}$ detection with LOS signal present

The result is shown as a location map in Figure 2. Each point represents a location in 2D space relative to the first receiver which is placed at the origin of the coordinate system. The postulated velocity at each point on the map is identically zero *i.e.*, it is designed to highlight stationary objects. As we have discussed, the intensity of the targets represent how strong the coherence is at this point in space. This type of detection is only possible with multiple receivers. The line-of-sight (LOS) signal is dominant in this figure, as it travels a shorter distance and without reflections, and so is received more strongly. The maximum point in this plot is the exact position of the transmitter, the two stationary targets are faintly visible in the correct locations, and the moving target is also visible. It can be seen that all three targets exist at the convergence of several faint ellipses. The ellipses arise from the geometry described earlier in Figure 1. The moving target is visible due to the high frequency ambiguity in the VHF band (220 MHz) as well as the closer proximity to the receivers, which means there is less path loss. This is clear from the range-Doppler map calculated from two of the receivers, as shown in Figure 3.

To detect the target signals more strongly, the line-of-sight signal can be removed. This is done here by first estimating $\mathbf{s}$ at the known position of the transmitter. The values of the μ_i

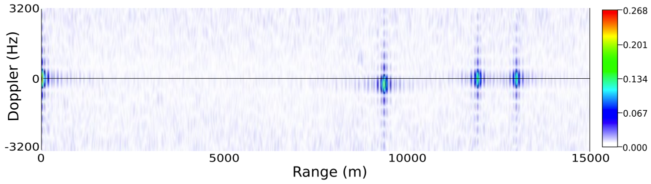
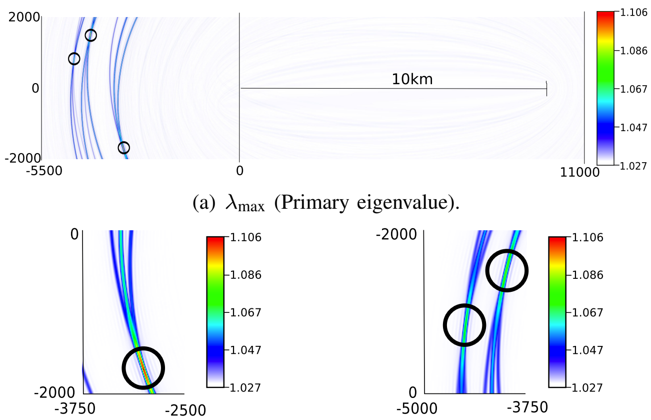


Fig. 3. Scenario using only 2 receivers in the range and Doppler domain

are then calculated using (5) and μ_is is subtracted from each received signal.

Once the direct-path signal has been removed, target detection can be performed again as shown in Figure 4.

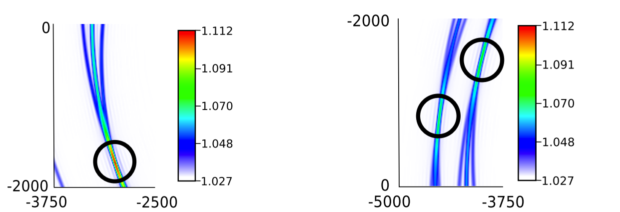


(a) $\lambda_{\max}$ (Primary eigenvalue).

(b) $\lambda_{\max}$ - moving target region. (c) $\lambda_{\max}$ - stationary targets region.

Fig. 4. Three targets using λmax detection with LOS signal removed.

In Figure 4, the two stationary targets and one moving target are highly prominent in the location space, when using the largest eigenvalue. As the moving target is closer, it has a higher coherence than the two stationary targets. However, if the correct velocity vector is also assumed (277 m/s south) as seen in Figure 5, the moving target's coherence increases from 1.106 to 1.112 and the stationary targets' coherence decrease from 1.081 and 1.075 to 1.079 and 1.074 respectively.



(a) $\lambda_{\max}$ - moving target region. (b) $\lambda_{\max}$ - stationary targets region.

Fig. 5. Scenario with three targets using $\lambda_{\max}$ detection assuming velocity of 277 m/s south with LOS signal removed.

By way of comparison, the same scenario was analysed using generalized coherence. This is shown in Figure 6. Broadly speaking, generalized coherence, being based on generalized variance, is better at detecting dissimilarity rather than similarity. There are not only many local maxima along the respective target ellipses, but we also observe an effect whereby a target tends to cast a shadow onto other ellipses.

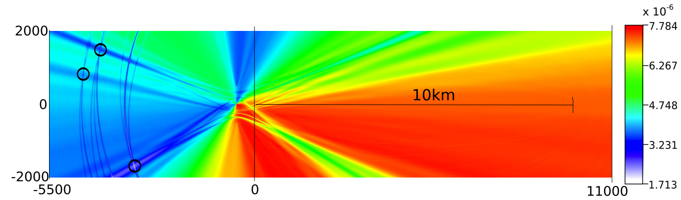


Fig. 6. Scenario using Generalized Coherence detection with LOS signal removed.

VI. Conclusion

In this paper, a new coherence detector is proposed. The result arises from maximum-likelihood estimation of parameters in a signal model for passive multistatic radar. When a generalised likelihood ratio test is considered for target detection, the detection statistic turns out to be the largest eigenvalue of the Gram matrix of the time- and frequency-corrected received signals. It is therefore closely related to generalised canonical correlation analysis in multivariate statistics.

Typical DVB-T signals and transmitter parameters are simulated for the purposes of evaluating the proposed coherence detector. We compare the detector with generalised coherence, which uses the determinant of the Gram matrix. In a multiple receiver scenario, we observe an improvement in detection performance using the proposed detector.

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