

Pentru:

$$\begin{bmatrix} 5 & 5 & 2 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Solutia este:

$$[x_1, x_2, x_3, x_4, x_5]$$

$$x_1 + x_5 + 1 = 0$$

$$x_2 + x_1 + 0 = 0$$

$$x_3 + x_2 + 0 = 0$$

$$x_4 + x_3 + 0 = 0$$

$$x_5 + x_4 + 0 = 0$$

$$x_1 + x_5 = 4$$

$$x_2 + x_1 = 0$$

$$x_3 + x_2 = 0$$

$$x_4 + x_3 = 0$$

$$x_5 + x_4 = 0$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Folosind inversul matricei in modulo 5:

$$\begin{bmatrix} 3 & 3 & 2 & 3 & 2 \\ 2 & 3 & 3 & 2 & 3 \\ 3 & 2 & 3 & 3 & 2 \\ 2 & 3 & 2 & 3 & 3 \\ 3 & 2 & 3 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 2 & 3 & 2 \\ 2 & 3 & 3 & 2 & 3 \\ 3 & 2 & 3 & 3 & 2 \\ 2 & 3 & 2 & 3 & 3 \\ 3 & 2 & 3 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 2 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 2 \\ -2 \\ 2 \end{bmatrix}, \text{ pentru ca e mai eficient sa scadem 2 decat sa adunam 3. Deci 10 operatii}$$

Dar pentru ca nu tot timpul avem o matrice inversabila, cel mai probabil pentru ca sunt mai multe solutii. Eu am folosit Gauss-Jordan Elimination in modul.

De exemplu pentru:

5 9 3

8 1 4 5 0

Ajungi la

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -4 \\ 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 7 \\ -3 \\ 0 \\ 3 \end{bmatrix}$$

daca rezolvi $-3 \cdot x_5 = 3$ ajungi la singurele solutii valide.

Ori gasesti regula de construire a matricii inverse de care ai nevoie, ori faci Gauss-Jordan Elimination foarte rapid :) Eu nu am reusit in python (desi nu am folosit numpy lists, nu cred ca poti face ceva destul de rapid nici cu alea).