# HEXAPOD KINEMATICS

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This document is a draft

## Part 1. Displacements of the platform joints $P_i$ relative to the servo pivots $B_i$

Let's consider an orthogonal system of axis $xyz$ with origin in the middle of the moving platform. The coordinates of the platform joints at home position are called $P_i$. Note that $Pz_i = 0$ and $i$ = motor index = 0..5.

$$P_i = \{Px_i, Py_i, 0, 1\}$$

Movements of the platform: the platform has six degrees of freedom, three rotations and three translations with the following transformation matrices:

$$Rx = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos A & \sin A & 0 \\ 0 & -\sin A & \cos A & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad Ry = \begin{pmatrix} \cos B & 0 & -\sin B & 0 \\ 0 & 1 & 0 & 0 \\ \sin B & 0 & \cos B & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$Rz = \begin{pmatrix} \cos C & \sin C & 0 & 0 \\ -\sin C & \cos C & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad Txyz = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ Tx & Ty & Tz & 1 \end{pmatrix}$$

The servo pivots have the following coordinates. Note that $Zhome$ is negative.

$$B_i = \{Bx_i, By_i, Zhome, 1\}$$

Position of platform joints relative to servo pivots:

$$TPB = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -Bx & -By & -Zhome & 1 \end{pmatrix}$$

The distances between the servo pivots $B_i$ and the platform joints $P0_i$ after movement are called $P1_i$ and can be found by calculating the following dot product:

$$PB_i = P_i \cdot Rx \cdot Ry \cdot Rz \cdot Txyz \cdot TPB =$$

$$\begin{pmatrix} PBx_i \\ PBy_i \\ PBz_i \\ 1 \end{pmatrix}^T = \begin{pmatrix} Px_i \cos B \cos C & + & Py_i (\sin A \sin B \cos C - \cos A \sin C) & + & Tx & - & Bx_i \\ Px_i \cos B \sin C & + & Py_i (\sin A \sin B \sin C + \cos A \cos C) & + & Ty & - & By_i \\ -Px_i \sin B & + & Py_i \sin A \cos B & + & Tz & - & Zhome \\ & & 1 & & & & \end{pmatrix}^T$$