Hexapod kinematics

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The coordinates of the platform joints at home position are called $P0_i$.

Note that $P0z_i = 0$ and i = motor index = 0..5.

$P0_i = \{P0x_i, P0y_i, 0, 1\}$

The platform has six degrees of freedom, three rotations and three translations with the following transformation matrices :

$$

$$\text{Rx} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(A) & \sin(A) & 0 \\ 0 & -\sin(A) & \cos(A) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{Rz} = \begin{pmatrix} \cos(C) & \sin(C) & 0 & 0 \\ -\sin(C) & \cos(C) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{Ry} = \begin{pmatrix} \cos(B) & 0 & -\sin(B) & 0 \\ 0 & 1 & 0 & 0 \\ \sin(B) & 0 & \cos(B) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{Txyz} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \text{Tx} & \text{Ty} & \text{Tz} & 1 \end{pmatrix}$$

$$

The coordinates of the platform joints after movement are called $P1_i$ and can be found by calculating the following dot product :

$\left(P1_i = \left(P0_i \cdot Rx \cdot Ry \cdot Rz \cdot Txyz\right) =\right.$

$$ (\begin{array}{c} _i\ _i\ _i\ 1 \end{array})^T =

(\begin{array}{c} P0x_i B C + P0y_i (A B C-A C) + \ P0x_iB C+ P0y_i (A C + A B C) + \ -P0x_i B + A B y + \ 1 \ \end{array})^T $$

We can now calculate the distances between the servo pivots B_i and the platform joints $P1_i$:

$dPB_i = \left(P1_i - B_i\right) = \left(P1_i - \{Bx_i,\ By_i,\ -Z_{home},\ 1\}\right) =$

$$ (\begin{array}{c} i\ i\ i\ 1 \end{array})^T = (\begin{array}{c} P0x_i B C + P0y_i (A B C-A C) + -{i}\ P0x_i B C + P0y_i (A C + A B C) + - {i}\ -P0x_i B + A B y + + {home}\ 1 \ \end{array})^T

$$

And the squares of the length of the vectors are :

$d_i^2 = dDPx_i^2 + dDPy_i^2 + dDPz_i^2 \quad \text{(eq 2)}$

For a platform using linear motors, the calculation can be done with eq 1 and eq 2. In our case we have rotational motors so we need to continue. Note that it is not a good idea to compute the square root of d_i^2 because it is a computer intensive operation and we don't need the value of d.

Now, we need to check if the arm of length a and the rod of length s are long enoug to actualy go to the target position :

$d_i^2 \leq (a + s)^2 \quad \text{(eq 3)}$