title: Hexapod kinematics
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Hexapod kinematics

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The coordinates of the platform joints at home position are called $\mathrm{P0}_i$.

Note that $\mathrm{P0z}_i = 0$ and $i = \text{motor index} = 0..5$.

$$\mathrm{P0}_i = \{P0x_i, P0y_i, 0, 1\}$$

The platform has six degrees of freedom, three rotations and three translations with the following transformation matrices :

$$\mathrm{Rx} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(A) & \sin(A) & 0 \\ 0 & -\sin(A) & \cos(A) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \mathrm{Ry} = \begin{pmatrix} \cos(B) & 0 & -\sin(B) & 0 \\ 0 & 1 & 0 & 0 \\ \sin(B) & 0 & \cos(B) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{Rz} = \begin{pmatrix} \cos(C) & \sin(C) & 0 & 0 \\ -\sin(C) & \cos(C) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \mathrm{Txyz} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \mathrm{Tx} & \mathrm{Ty} & \mathrm{Tz} & 1 \end{pmatrix}$$

The coordinates of the platform joints after movement are called $\mathrm{P1}_i$ and can be found by calculating the following dot product :

$$\mathrm{P1}_i = (\mathrm{P0}_i \cdot \mathrm{Rx} \cdot \mathrm{Ry} \cdot \mathrm{Rz} \cdot \mathrm{Txyz}) =$$

$$\begin{pmatrix} \mathrm{P1x}_i \\ \mathrm{P1y}_i \\ \mathrm{P1z}_i \\ 1 \end{pmatrix}^T = \begin{pmatrix} P0x_i \cos B \cos C + P0y_i(\sin A \sin B \cos C - \cos A \sin C) + \mathrm{Tx} \\ P0x_i \cos B \sin C + P0y_i(\cos A \cos C + \sin A \sin B \sin C) + \mathrm{Ty} \\ -P0x_i \sin B + \sin A \cos By + \mathrm{Tz} \\ 1 \end{pmatrix}^T$$

We can now calculate the distances between the servo pivots B_i and the platform joints $\mathrm{P1}_i$:

$$* * * * \mathrm{dPB}_i = (\mathrm{P1}_i - \mathrm{B}_i) = (\mathrm{P1}_i - \{Bx_i, By_i, -Z_{home}, 1\}) =$$

$$\begin{pmatrix} \mathrm{dPBx}_i \\ \mathrm{dPBy}_i \\ \mathrm{dPBz}_i \\ 1 \end{pmatrix}^T = \begin{pmatrix} P0x_i \cos B \cos C + P0y_i(\sin A \sin B \cos C - \cos A \sin C) + \mathrm{Tx} - \mathrm{Bx}_i \\ P0x_i \cos B \sin C + P0y_i(\cos A \cos C + \sin A \sin B \sin C) + \mathrm{Ty} - \mathrm{By}_i \\ -P0x_i \sin B + \sin A \cos By + \mathrm{Tz} + \mathrm{Z}_{home} \\ 1 \end{pmatrix}^T (eq1)$$

And the squares of the length of the vectors are :

$$d_i^2 = \mathrm{dDPx}_i^2 + \mathrm{dDPy}_i^2 + \mathrm{dDPz}_i^2 (eq2)$$

For a platform using linear motors, the calculation can be done with $\mathrm{e}q1$ and $\mathrm{e}q2$. In our case we have rotational motors so we need to continue. Note that it is not a good idea to compute the square root of d_i^2 because it is a computer intensive operation and we don't need the value of d.

Now, we need to check if the arm of length a and the rod of length s are long enoug to actualy go to the target position :

$$d_i^2 \le (a + s)^2 (eq3)$$