

Hexapod kinematics

This document is a draft

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The coordinates of the platform joints at home position are called $P0_i$.

Note that $P0z_i = 0$ and $i = \text{motor index} = 0..5$.

$$P0_i = \{P0x_i, P0y_i, 0, 1\}$$

The platform has six degrees of freedom, three rotations and three translations with the following transformation matrices :

$$R_x = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos A & \sin A & 0 \\ 0 & -\sin A & \cos A & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} R_y = \begin{pmatrix} \cos B & 0 & -\sin B & 0 \\ 0 & 1 & 0 & 0 \\ \sin B & 0 & \cos B & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$R_z = \begin{pmatrix} \cos C & \sin C & 0 & 0 \\ -\sin C & \cos C & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} T_{xyz} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ T_x & T_y & T_z & 1 \end{pmatrix}$$

The coordinates of the platform joints after movement are called $P1_i$ and can be found by calculating the following dot product :

$$P1_i = (P0_i \cdot R_x \cdot R_y \cdot R_z \cdot T_{xyz}) =$$

$$\begin{pmatrix} P1x_i \\ P1y_i \\ P1z_i \\ 1 \end{pmatrix}^T = \begin{pmatrix} P0x_i \cos B \cos C + P0y_i(\sin A \sin B \cos C - \cos A \sin C) + T_x \\ P0x_i \cos B \sin C + P0y_i(\sin A \sin B \sin C + \cos A \cos C) + T_y \\ -P0x_i \sin B + P0y_i \sin A \cos B + T_z \\ 1 \end{pmatrix}^T$$

We can now calculate the distances between the servo pivots B_i and the platform joints $P1_i$:

$$dPB_i = (P1_i - B_i) = (P1_i - \{Bx_i, By_i, -Z_{home}, 1\}) =$$

$$\begin{pmatrix} dPBx_i \\ dPBy_i \\ dPBz_i \\ 1 \end{pmatrix}^T = \begin{pmatrix} P0x_i \cos B \cos C + P0y_i(\sin A \sin B \cos C - \cos A \sin C) + T_x - Bx_i \\ P0x_i \cos B \sin C + P0y_i(\sin A \sin B \sin C + \cos A \cos C) + T_y - By_i \\ -P0x_i \sin B + P0y_i \sin A \cos B + T_z + Z_{home} \\ 1 \end{pmatrix}^T \quad (\text{eq 1})$$

And the squares of the length of the vectors are :

$$d_i^2 = dPBx_i^2 + dPBy_i^2 + dPBz_i^2 \quad (\text{eq 2})$$

For a platform using linear motors, the calculation can be done with eq 1 and eq 2. In our case we have rotational motors so we need to continue. Note that it is not a good idea to compute the square root of d_i^2 because it is a computer intensive operation and we don't need the value of d .

Now, we need to check if the arm of length a and the rod of length s are long enough to actually go to the target position :

$$d_i^2 \leq (a + s)^2 \quad (\text{eq 3})$$

TO BE CONTINUED...