

Pathfinding Lists

- ❑ Here are some **possible implementation choices** for open/closed lists:
 - Unsorted arrays or linked lists
 - Sorted arrays
 - Sorted linked lists
 - Sorted **skip lists** (hierarchy of linked lists)
 - Indexed arrays
 - Hash tables
 - Binary heaps
 - Splay trees
 - HOT (Heap On Top) queues

Pathfinding Lists

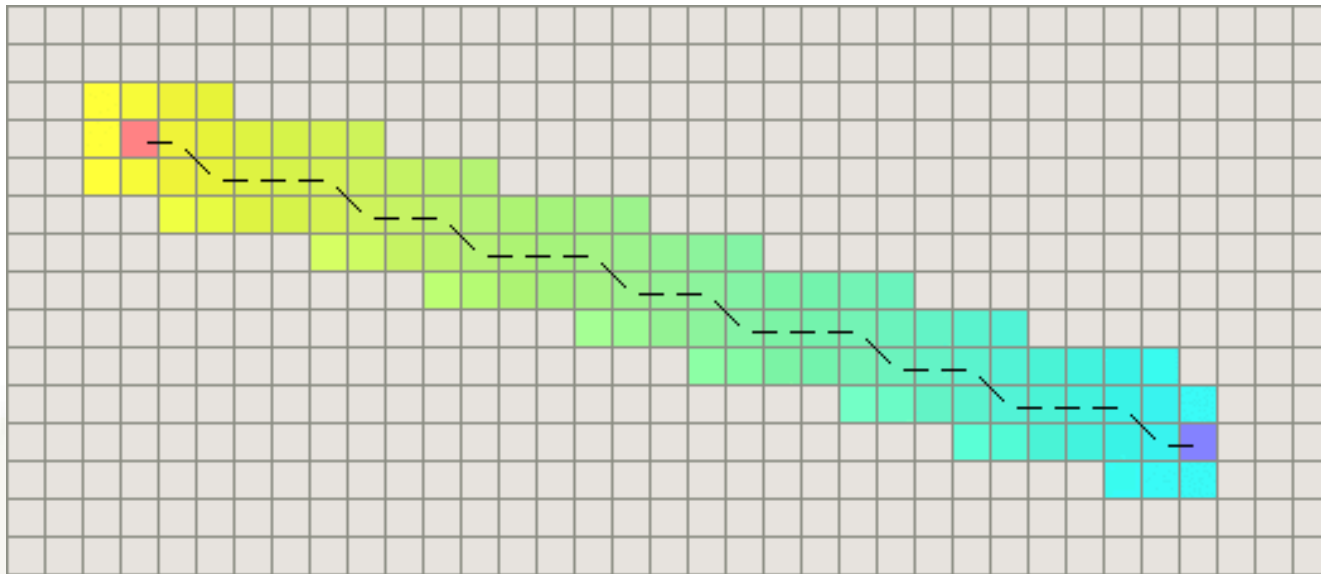
- ❑ To get the best performance, you will want a hybrid data structure.
- ❑ For example1, you can use an indexed array for $O(1)$ membership test and a binary (min-)heap for $O(\log l)$ insertion and $O(\log l)$ to find and remove the node with the smallest estimated-total-cost.
- ❑ For **cost-so-far updates**, use an indexed array for an $O(1)$ test whether an update is needed (by storing the cost-so-far value in the indexed array), and then use an $O(1)$ update (**rarely done**) on the binary heap. Better: you can also use the indexed array to store the location in the heap of each node; this would give you $O(\log l)$ for update.

Tampering with heuristic function

- ❑ A* can theoretically produce non-optimal results
- ❑ Choice of heuristic function can determine the outcome of results
 - Use a good heuristic function (with additional termination rules) to ensure better optimal results
 - Deliberately allow sub-optimal results (by using a mediocre heuristic) to give faster execution

Choosing a Heuristic

- ❑ The **more accurate the heuristic**, the less fill A* will have (i.e., a smaller l), the faster it will run
- ❑ **Perfect heuristic** → always gives correct answer
- ❑ In only a few cases will a practical heuristic be accurate.



Choosing a Heuristic

□ Underestimating heuristic

- Gives *cost less than or equal to actual cost*
- *Biased towards cost-so-far*
- *Increases running time*
- A* will prefer to examine nodes closer to the start node, rather than those closer to the goal
- It generates the *exact same path* that the *Dijkstra* algorithm would generate.
- Best if accuracy more important than performance

Choosing a Heuristic

❑ Overestimating heuristic

- Gives *cost greater than actual cost*
- *Biased towards heuristic value*
- *Faster search towards goal*
- The A* algorithm will pay *proportionally less attention to the cost-so-far* and will *tend to favour nodes that have less distance to go* (i.e., closer to the goal if heuristic reflects this).
- Definitely *sub-optimal*
- Best if *performance* and *speed* is *more important than accuracy*

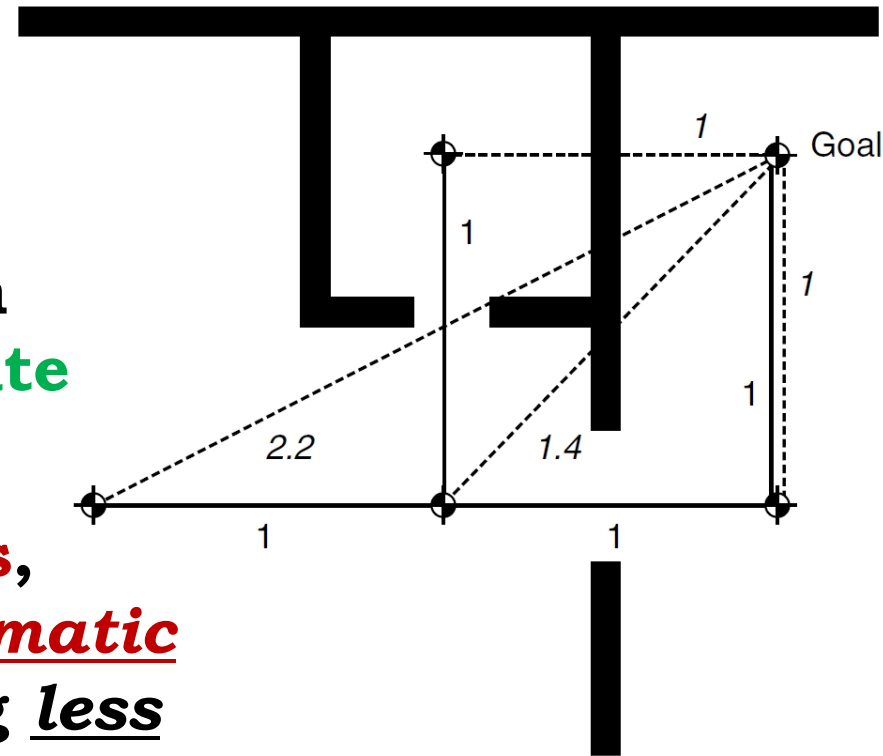
Heuristic – Euclidean Distance

- Or “as the crow flies” or “straight-line distance”

- Guaranteed to be underestimating

- In outdoor settings: with few constraints on movement: very accurate with fast pathfinding.

- In indoor environments, as shown: can be a dramatic underestimate, causing less than optimal pathfinding.



Key

----- Heuristic value

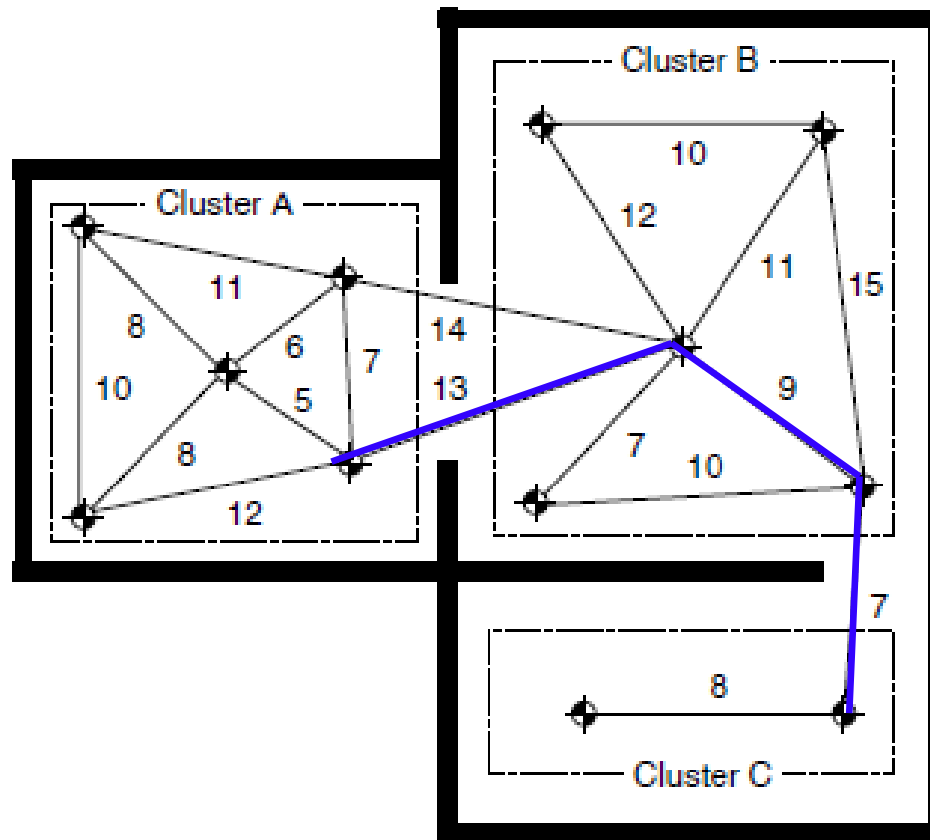
—— Connection cost

Heuristic – Cluster Heuristic

- ❑ Works by **grouping nodes together in clusters**
- ❑ Nodes in a cluster represent some region of level (room, corridor, building, etc.) that is **highly interconnected**
 - Clustering can be done by some graph clustering algorithms (**beyond the scope of this course**),
 - but often clustering is **manual** or a **by-product of the level design**
- ❑ Create a **lookup table** with the smallest path length between each pair of clusters. An offline step based on running lots of pathfinding trials between nodes in pairs of clusters.

Heuristic – Cluster Heuristic

- Intra-cluster Euclidean distances & **Cluster distance lookup table**



	A	B	C
A	x	13	29
B	13	x	7
C	29	7	x

Lookup table

Heuristic – Cluster Heuristic

□ In a game,

- If start and goal nodes are in same cluster, **Euclidean distance** (or some other simpler fallback such as a null heuristic – i.e., use Dijkstra within the cluster) is used to get result
- Otherwise, *look up the table for the distance between clusters* (this distance is the minimum distance between any node in the current cluster and any node in the destination cluster; e.g., the distance 29 on the previous slide).

Heuristic – Cluster Heuristic

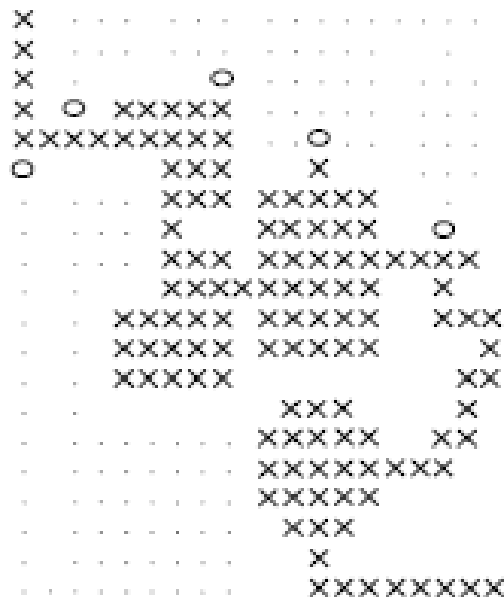
- ❑ Do you foresee any problem since all nodes in a cluster are given the same cluster heuristic value?
- ❑ What are the implications if the cluster sizes are
 - **Small** (i.e., using lots of clusters)?
 - **Large** (i.e., using only a few clusters)?
- ❑ There are ways to improve the cluster heuristic, but no accepted techniques for reliable improvement; essentially a matter of experimenting within the context of your game's level design.

Fill Patterns in A*

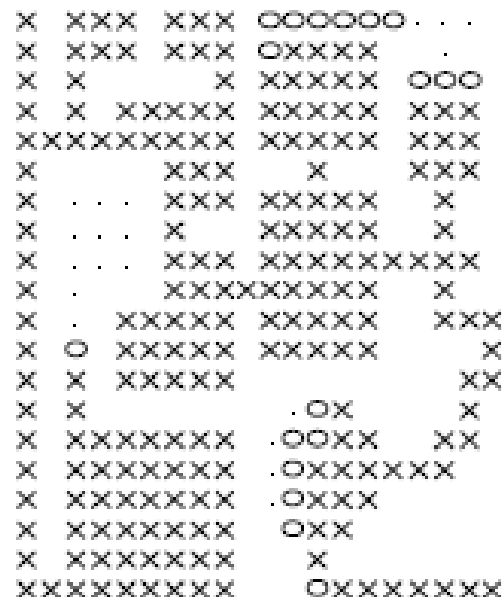
❑ Indoor fill patterns of a tile-based level

(Dijkstra)

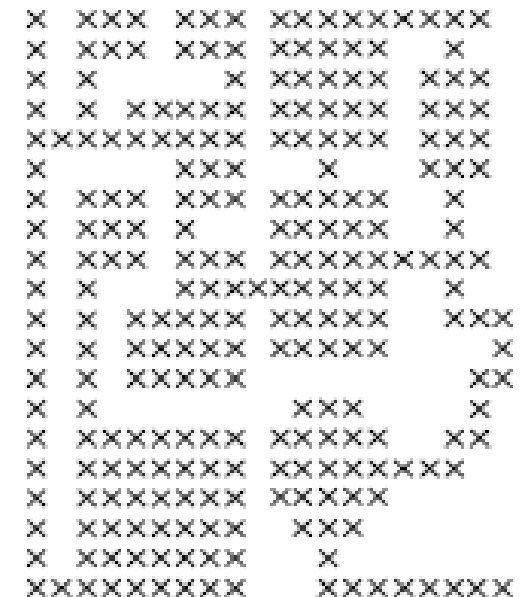
Cluster heuristic



Euclidean distance heuristic



Null heuristic



Key

- × Closed node
- Open node
- Unvisited node

Fill Patterns in A*

Outdoor fill patterns

Euclidean distance heuristic

[illegible]

Null heuristic

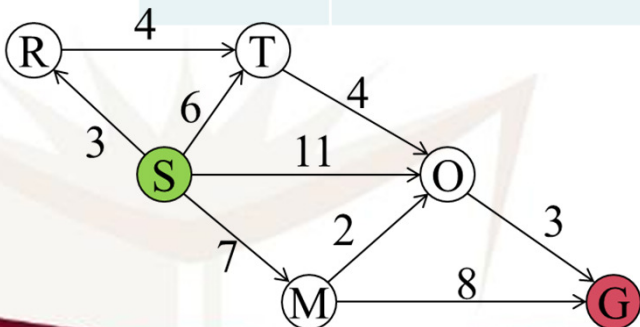
A 20x20 grid of 'x' marks. In the center, there is a 2x2 block of missing 'x' marks, creating a defect in the lattice. The missing marks are located at positions (row, column) (9, 9), (9, 10), (10, 9), and (10, 10), assuming the top-left corner is (0, 0).

Key

- × Closed node
- Open node
- Unvisited node

A* Example

Current Node	Open List, Format: (Node, Cost-so-far, Connection, Estimated-total-cost)	Closed List, Format: (Node, Cost-so-far, Connection)
S	(T,6,ST,6+6=12), (O,11,SO,11+3=14), (M,7,SM,7+8=15), (R,3,SR,3+15=18)	(S,0,-)
T	(O,10,ST,13) , (M,7,SM,15), (R,3,SR,18)	(S,0,-), (T,6,ST)
O	(M,7,SM,15), (R,3,SR,18), (G,13,OG,13)	(S,0,-), (T,6,ST), (O,10,TO)
G	(M,7,SM,15), (R,3,SR,18)	(S,0,-), (T,6,ST), (O,10,TO), (G,13,OG)
M	(R,3,SR,18), (O,9,MO,12)	(S,0,-), (T,6,ST), (G,13,OG), (M,7,SM)
O	(R,3,SR,18), (G,12,OG,12)	(S,0,-), (T,6,ST), (M,7,SM), (O,9,MO)
G	(R,3,SR,18)	(S,0,-), (T,6,ST), (M,7,SM), (O,9,MO), (G,12,OG)



Node	Heuristic
S	10
T	6
O	3
M	8
G	0
R	15

Quality of Heuristics

- ❑ Producing a heuristic is far more of an art than a science.
- ❑ Most AI developers just drop in a simple Euclidean distance heuristic without thought and hope for the best.
- ❑ The only sure-fire way to get a decent heuristic is to visualize the fill of your algorithm.
- ❑ Current research is exploring *automatically generating heuristics* based on examining the structure of the graph and its connections. But so far, the results have yet to prove compelling.

World Representations

- ❑ To squeeze your game level into the pathfinder you need to do some translation—from geometry of the map and the movement capabilities of your characters to the nodes and connections of the graph and the cost function that values them.
- ❑ For each pathfinding world representation, we will *divide the game level into linked regions* that correspond to nodes and connections.
- ❑ The different ways this can be achieved are called **division schemes**. Each division scheme has three important properties we'll consider in turn:
 - ❑ **quantization/localization, generation, and validity**

World Representations

Quantization:

- ❑ The process of converting *locations/positions* in the game into *nodes in the graph*

Localization:

- ❑ The conversion of *nodes in the pathfinder path* back into *game world locations*

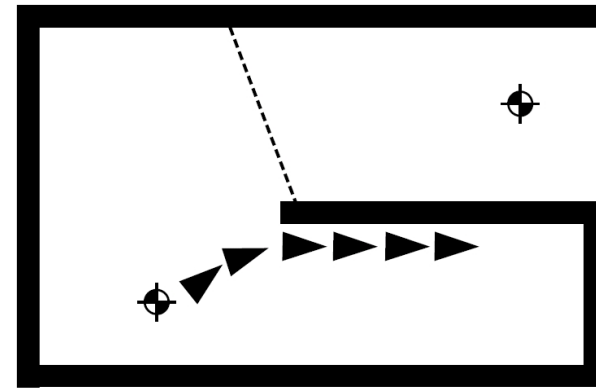
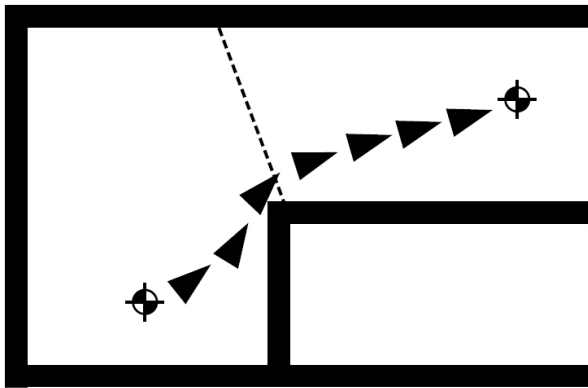
World Representations

Generation:

- ❑ There are many ways of dividing up a continuous space into regions and connections for pathfinding.
- ❑ Standard methods used regularly:
 - **Manual Techniques:**
 - ✓ Dirichlet domain
 - **Algorithmic Methods:**
 - ✓ Tile graphs,
 - ✓ Points of visibility, and
 - ✓ Navigation meshes (NavMeshes)

Validity

- ❑ If a plan tells a character to move along a connection from node A to node B, then the *character should be able to carry out that movement*.
- ❑ A division scheme is valid if all points in two connected regions can be reached from each other.
- ❑ In practice, most division schemes **don't enforce validity**.
- ❑ There can be different levels of validity:



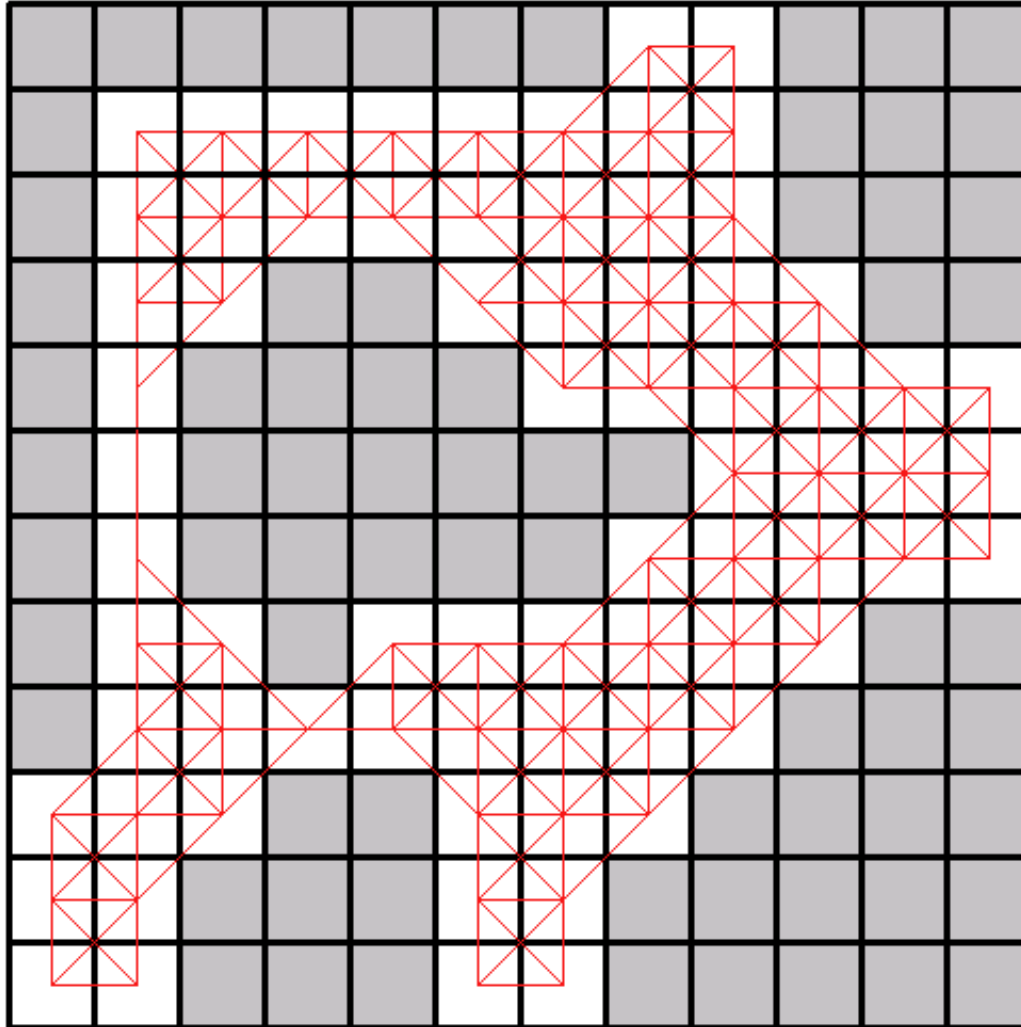
Tile Graphs

- ❑ Tile-based levels *rarely appear in mainstream games*
- ❑ A large number of games, though, use a regular grid in which they place 3D characters.
- ❑ This grid can be simply turned into a tile-based graph



- ❑ Many RTS games still use tile-based graphs
- ❑ Tile-based levels **split the whole world into regular (usually square) regions.**

Tile Graphs



- **Division Scheme:**
Nodes in the pathfinder's graph represent **tiles in the game world**. The connections between nodes **connect** to their **immediate** (obvious set of) **neighbours**.

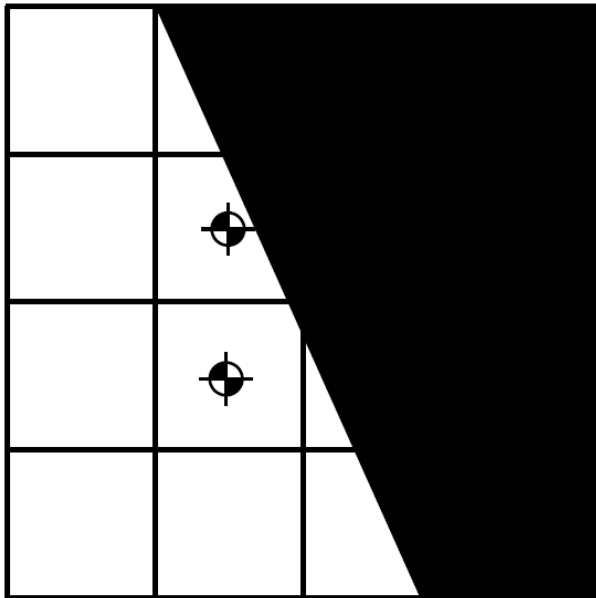
Image from <http://www.codeproject.com/Articles/14840/Artificial-Intelligence-in-Games>

Tile Graphs

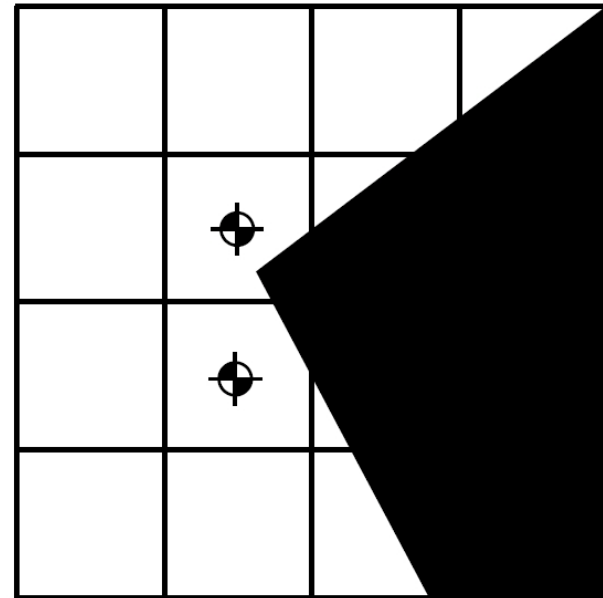
- ❑ **Quantization and Localization:** Simple with a regular grid.
- ❑ **Generation:** Tile-based graphs are generated automatically (even at runtime if needed). Connections can be generated as requested by the pathfinding (some *connections to blocked tiles will not be available*).
- ❑ For tile-based grids representing outdoor height fields (a rectangular grid of height values), the costs often depend on gradient (and distance).

Tile Graphs

Validity: if only *empty tiles* are connected, then the *graph* will be *guaranteed to be valid*. When a graph node is *only partially blocked*, then the *graph may not be valid*, depending on the shape of the blockage.



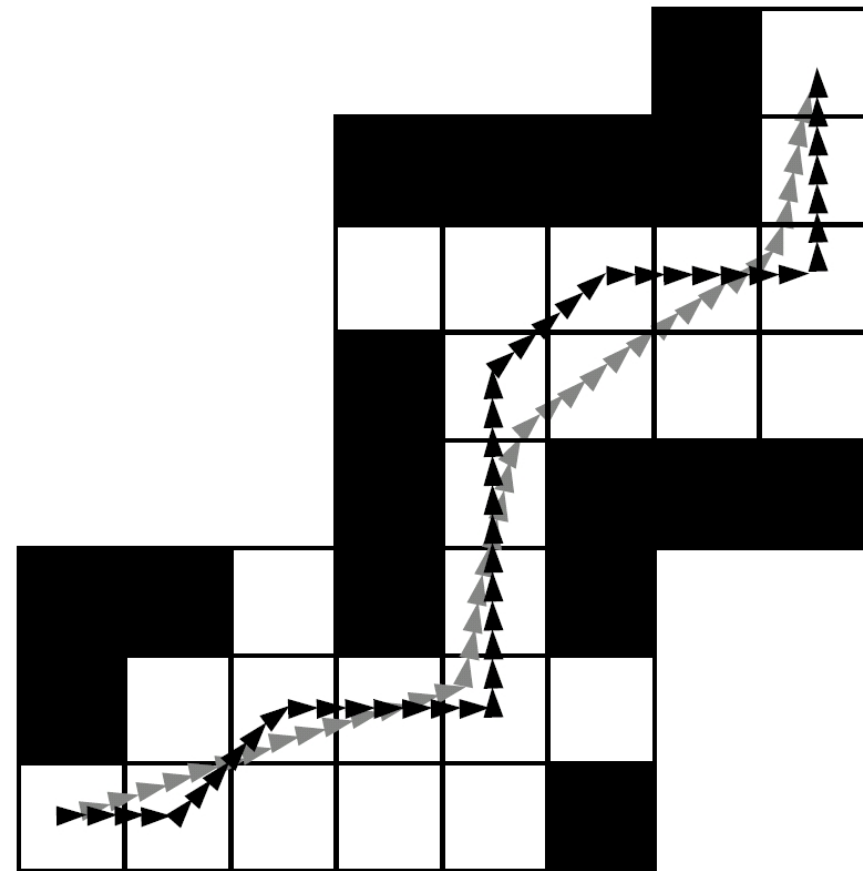
Valid partial blockage



Invalid partial blockage

Tile Graph Pathfinding

- ❑ When the *paths* returned by the *pathfinder* are drawn on the graph (using localization for each node in the plan), they can appear blocky and irregular.
- ❑ Modest 100 x 100 cell map: 10,000 nodes and 78,000 edges
- ❑ *can burden CPU and memory*, especially if multiple AI's calling in

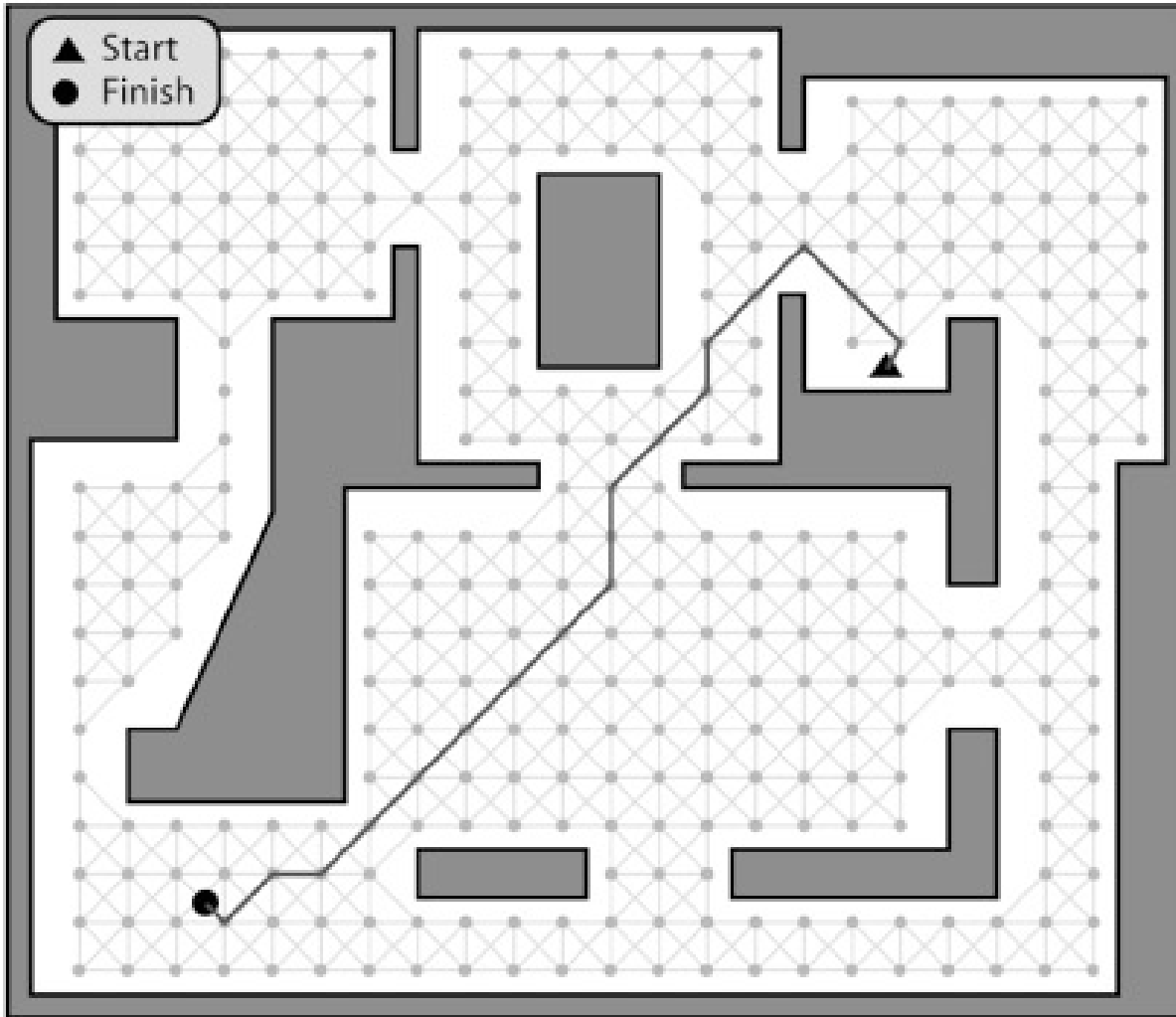


Key

Output blocky plan

Ideal direct plan

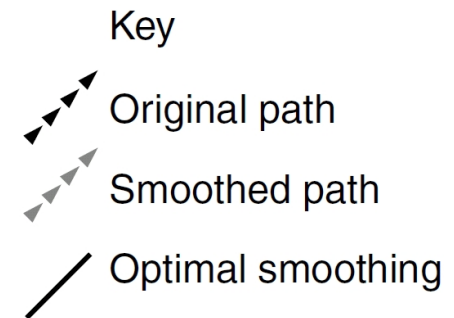
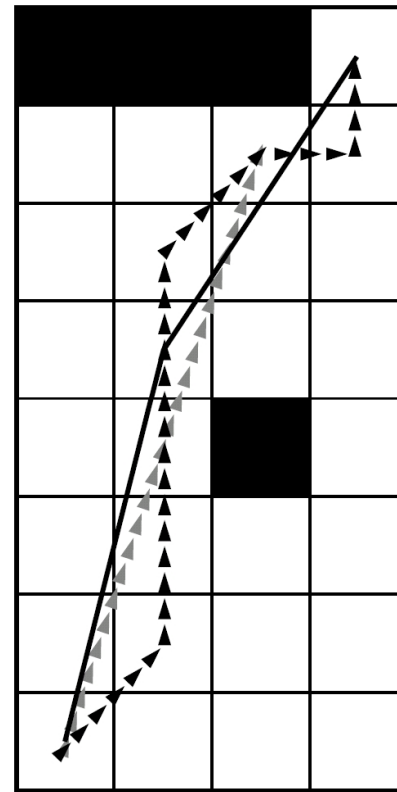
Tile Graph Pathfinding



- ❑ Can lead to “kinky paths”
- ❑ Solution: *path smoothing*

Path Smoothing

- ❑ A path that travels from node to node through a graph *can appear erratic*. Sensible node placing can give rise to very *odd looking paths*.
- ❑ The final appearance also depends on *how characters act on the path*. E.g., a path following steering behaviour will gently smooth out the path.
- ❑ Aka *string pulling* or *funnel algorithm*.

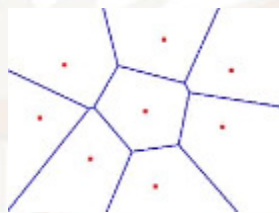


Path Smoothing

- ❑ Assume there is a clear route between adjacent nodes in the input path (i.e., subdivision is valid)
- ❑ **Add start node** ($i = 1$) to output path
- ❑ **Until the end node is reached** do:
 - Starting with the third node ($i = 3$) in the input path, cast a ray *from the last node of the output path to consecutive nodes* in the input list **until the ray fails with the i^{th} node**. Put $(i-1)^{\text{th}}$ node on output path
- ❑ Put end node on output path
- ❑ The result is a smooth path, but perhaps *not the (optimally) “smoothest”*.

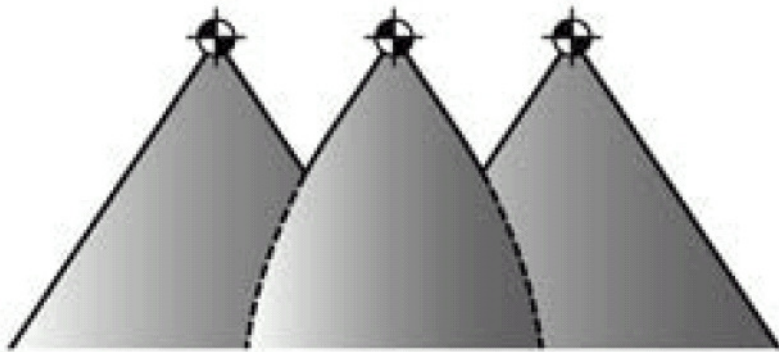
Dirichlet Domains

- ❑ A Dirichlet domain, also referred to as a Voronoi polygon in two dimensions, is a region around one of a finite set of source points whose interior consists of everywhere that is closer to that source point than any other.
- ❑ **Division Scheme:** Pathfinding nodes have an associated point in space called the *characteristic point*, and the quantization takes place by mapping all locations in the point's Dirichlet domain to the node.
- ❑ The set of characteristic points is usually specified by a **level designer** as part of the level data.

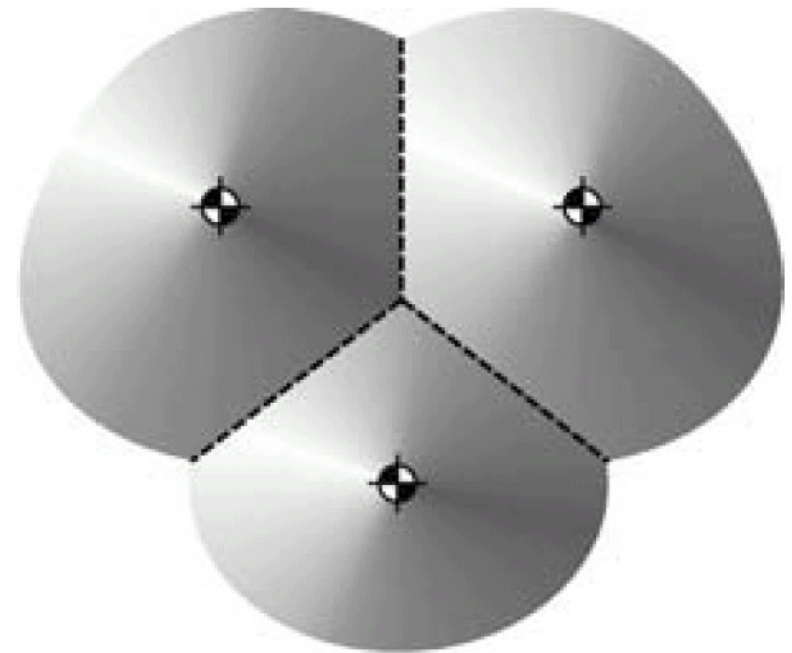


Dirichlet Domains

- **Division Scheme:** Can be viewed as being *cones originating from the characteristic points.*



Side view

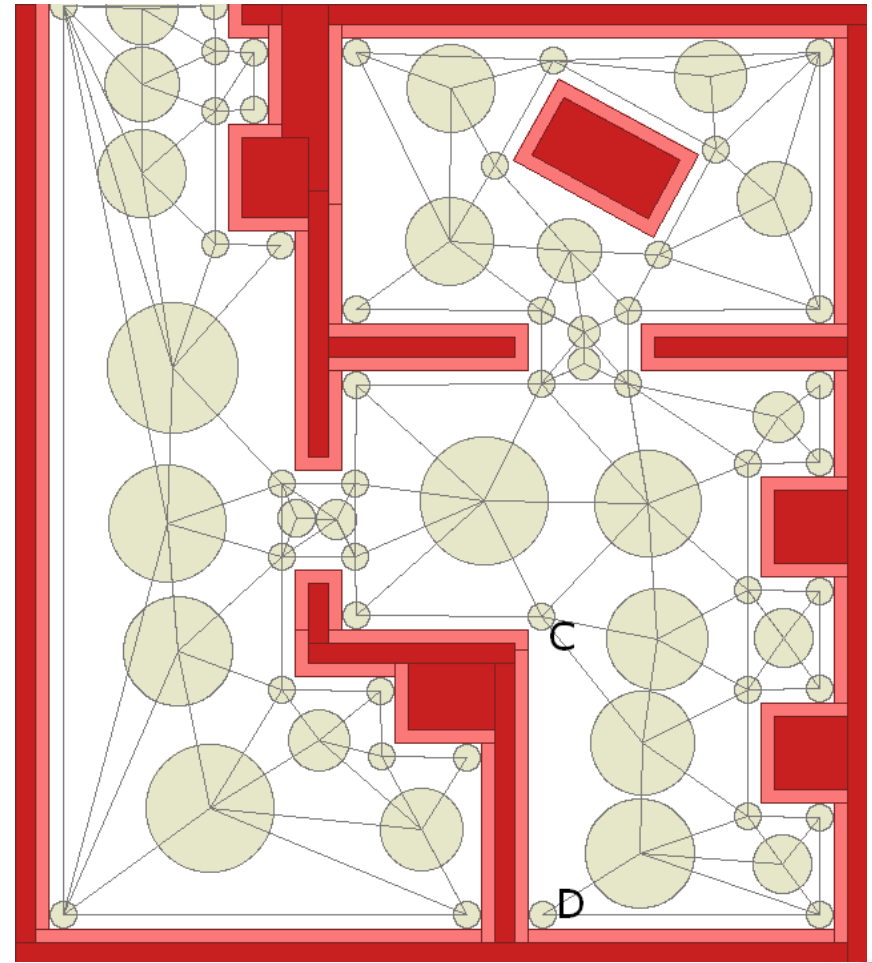
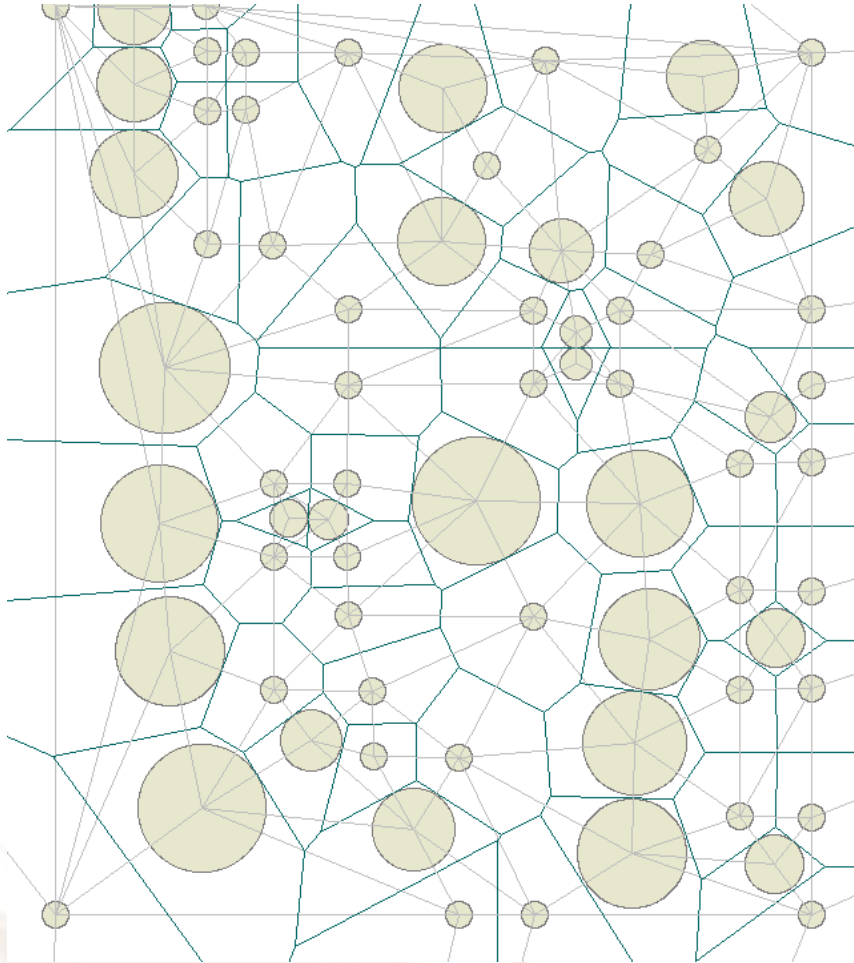


Top view

Dirichlet Domains

- ❑ **Division Scheme:** Connections are placed between bordering domains (this creates a *Delaunay triangulation*). You will need to check that domains that touch can actually be moved between (e.g., *there might be a wall in the way*).
- ❑ Frequently, either
 - make the artist specify connections as part of their level design, or
 - ray cast between points to check for connections (see the points of visibility method below).

Removing Blocked Edges



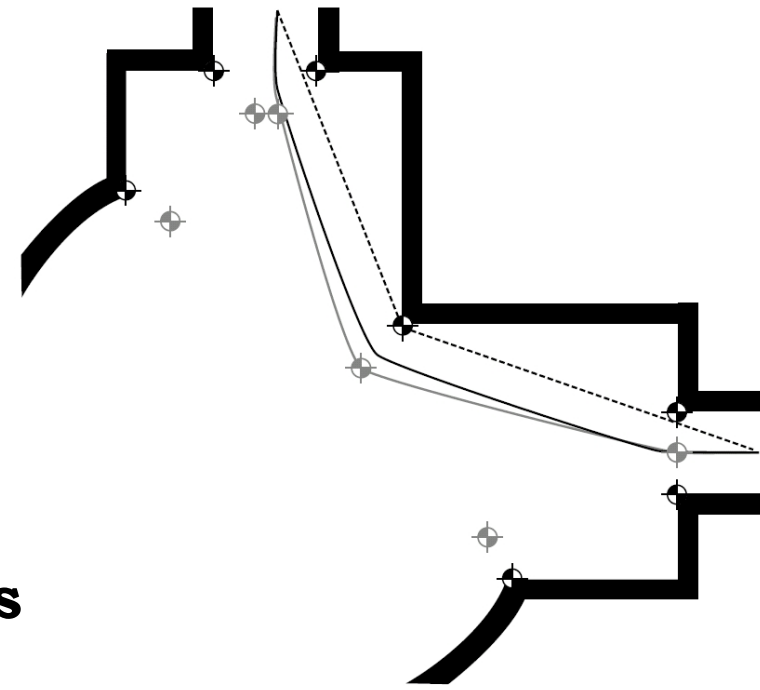
Images from http://www.gamedev.net/page/resources/_/technical/artificial-intelligence/navigation-graph-generation-r2805

Dirichlet Domains

- ❑ **Quantization and Localization:** Positions are quantized by finding the *characteristic point that is closest* (aided by a spatial partition data structure). The localization of a node is given by the *position of the characteristic point that forms the domain* (i.e., the one that is closest).
- ❑ **Validity:** To ensure the graph is valid, the *placement of nodes* is *often based on the structure of obstacles*. Obstacles are not normally given their own domains, and so the invalidity of the graph is rarely exposed.

Points of Visibility (POV)

- The optimal path through any 2D environment *will always have inflection points* (i.e., points on the path where *the direction changes*) at convex vertices in the environment. If the character that is moving has some radius, these *inflection points are replaced by arcs of a circle* at a distance away from the vertex.

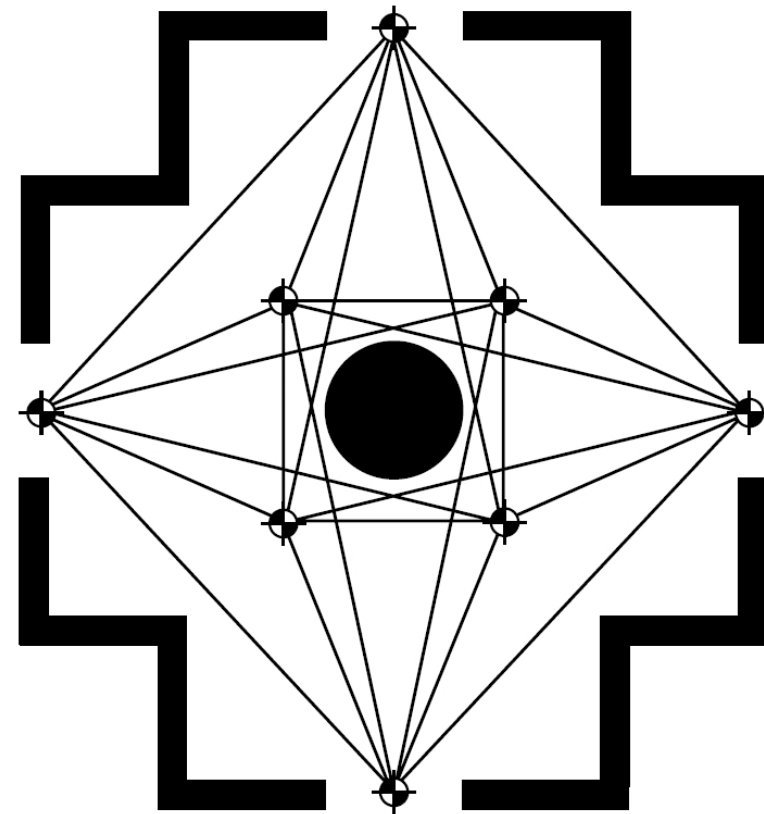


Key

- Optimal path for zero-width character
- Path for character with width
- Path using vertex offsets
- Original characteristic points at vertices
- Offset characteristic points

Points of Visibility (POV)

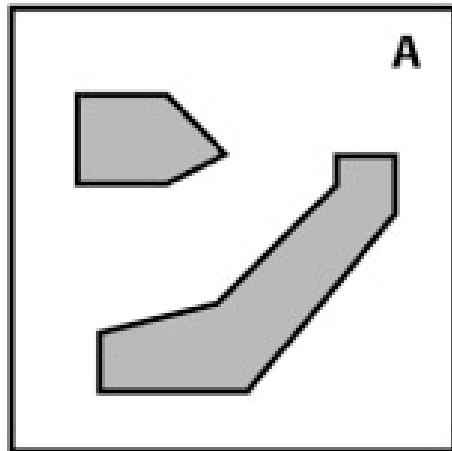
- Since these *inflection points naturally occur in the shortest path*, we can use them as nodes in the pathfinding graph.
- To work out how these points are connected, *rays are cast between them*, and a connection is made *if the ray doesn't collide* with any other geometry.
- The *resulting graph can be large*.



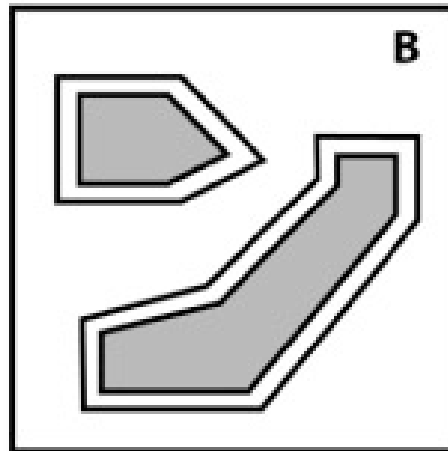
Key

— Connection between nodes

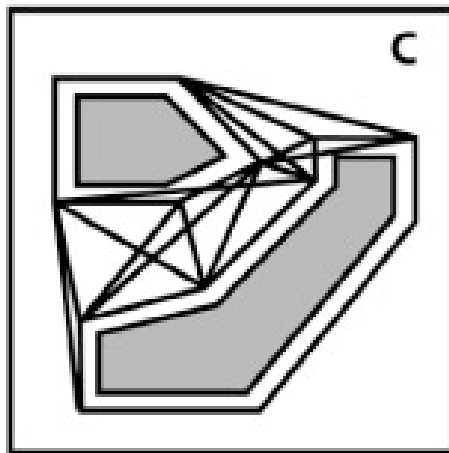
Points of Visibility (POV)



Simple Geometry



Expanded Geometry



The finished POV graph

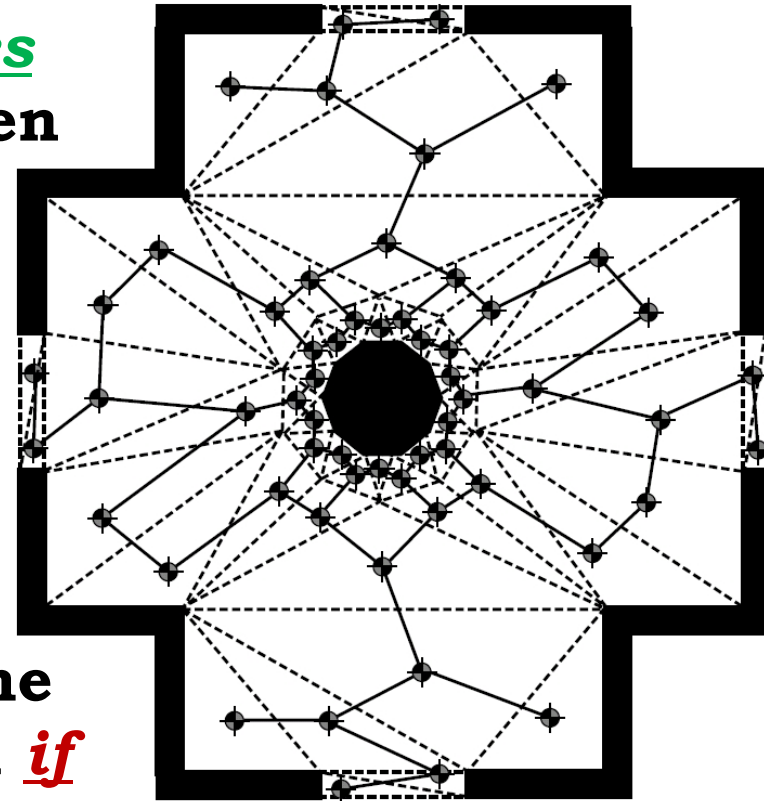
1. **Expand geometry** by an amount *proportional to bounding radius of the NPCs* (this expansion is called the **Minkowski sum** of the two geometries)
2. **Add vertices** to graph
3. *Prune non-line-of-sight points*

Points of Visibility

- ❑ **Quantization and Localization:** *Points of visibility* are usually taken *to represent the centers of Dirichlet domains* for the purpose of quantization.
- ❑ **Validity:** In addition, if Dirichlet domains are used for quantization, points quantized to two connected nodes *may not be able to reach each other*. As with Dirichlet domains above, this means that the graph is strictly invalid.
- ❑ Although a relatively popular method for automatic graph generation, [AI for G, M] don't think the results are worth the effort and suggest *Navigation Meshes instead...*

Navigation Meshes

- A majority of modern games use navigation meshes (often abbreviated to “navmesh”) for pathfinding. Since levels are usually made up of polygons, these are used as the basis of the graph.
- **Division Scheme:** Each polygon acts as a node in the graph. Nodes are connected if their corresponding polygons share an edge.



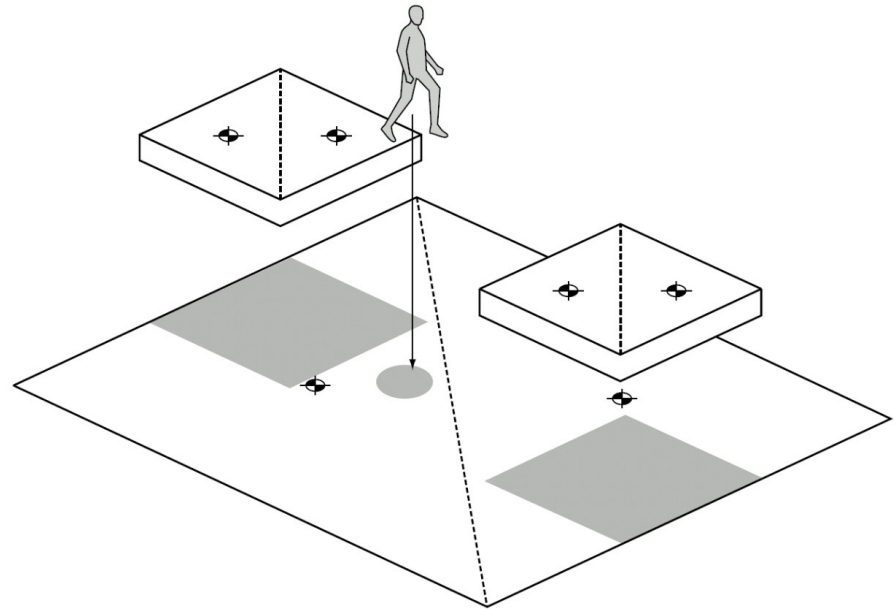
Key

- Edge of a floor polygon
- Connection between nodes

Navigation Meshes

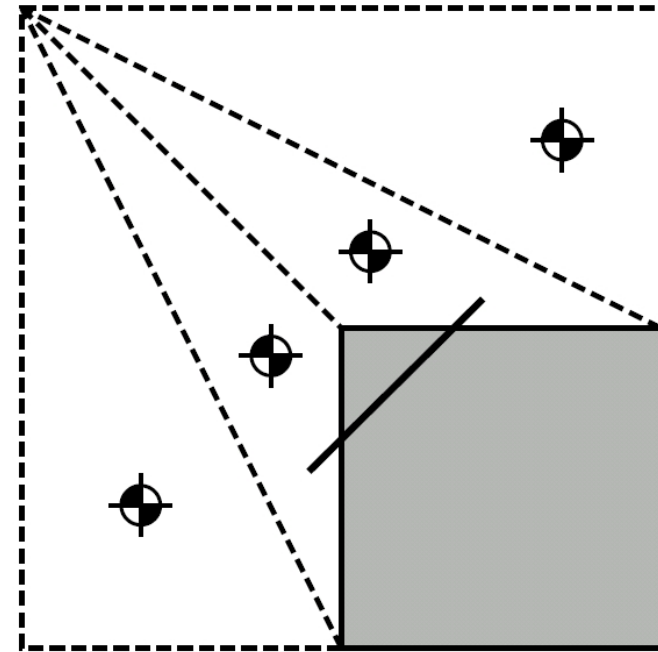
- **Quantization and Localization:** A *position* is localized into the floor polygon that contains it. The only wrinkle occurs *when a character is not touching the floor*. We can simply *find the first polygon below it and quantize it to that*.

Unfortunately, it is possible for the character to be placed in a completely *inappropriate ode as it falls or jumps*

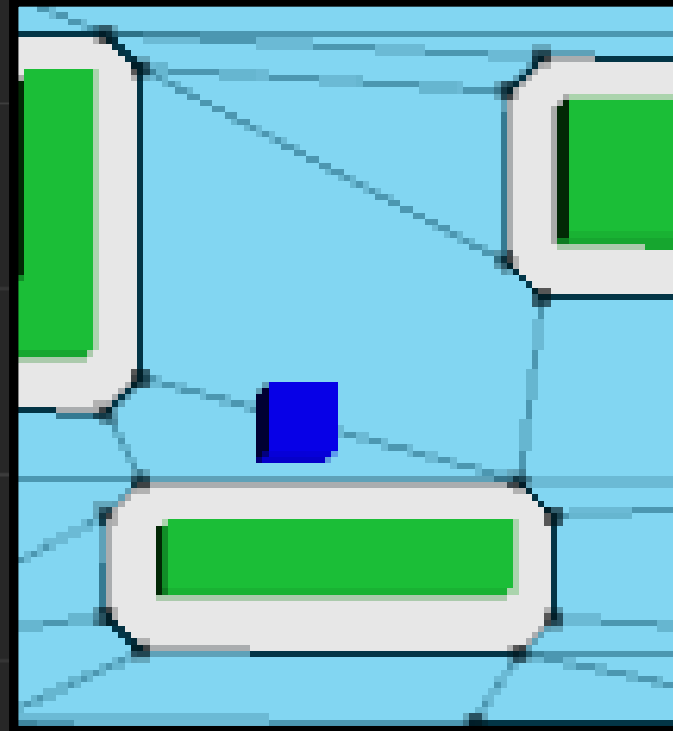
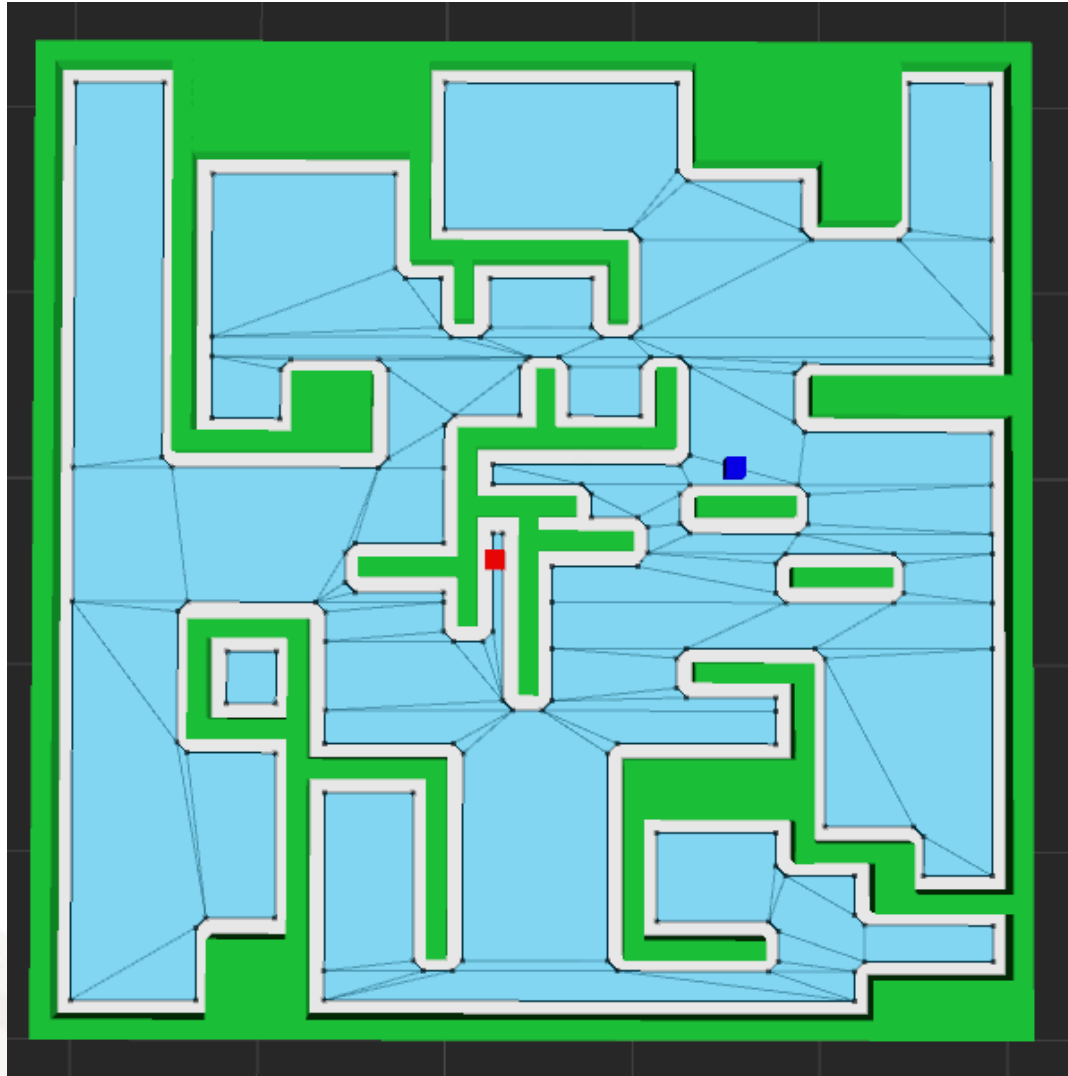


Navigation Meshes

- ❑ **Quantization and Localization:** Localization can choose any point in the polygon, but normally uses the geometric center (the average position of its vertices).
- ❑ **Validity:** The regions generated by navigation meshes *can be problematic*. We have assumed that any point in one region can move directly to any point in a connected region, but *this may not be the case*. A level designer can *create geometry that avoid this*



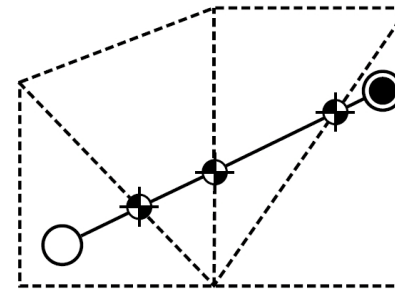
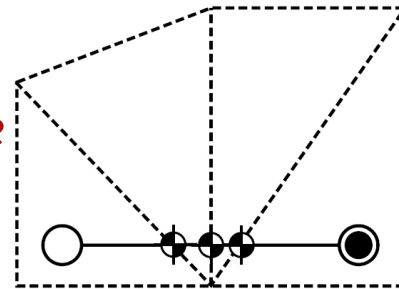
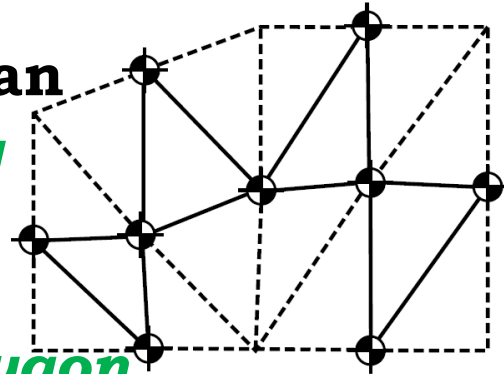
Unity's Navmesh



<http://www.binpress.com/tutorial/unity3d-ai-navmesh-navigation/119>

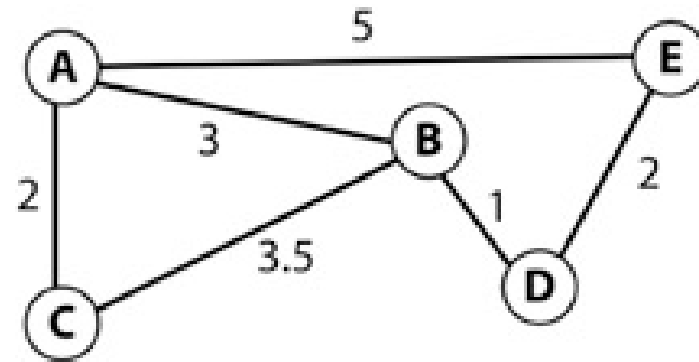
Navigation Meshes

- ❑ **Edges as Nodes:** Floor polygons can also be *converted into a pathfinding graph* by *assigning nodes to edges between polygons* and *using connections across face of each polygon*
- ❑ This approach is also commonly used in association with portal-based rendering, where nodes are assigned to portals and where *connections link all portals within the line of sight of one another*.
- ❑ The nodes on the edges of floor polygons *can be placed dynamically in the best position* for the path-finder.



Reducing CPU Overhead

□ Time/space trade-off



	A	B	C	D	E
A	A	B	C	B	E
B	A	B	C	D	D
C	A	B	C	B	B
D	B	B	B	D	E
E	A	D	D	D	E

shortest path table

	A	B	C	D	E
A	0	3	2	4	5
B	3	0	3.5	1	3
C	2	3.5	0	4.5	6.5
D	4	1	4.5	0	2
E	5	3	6.5	2	0

path cost table