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A New Perspective on Eratosthenes' Measurement of the Earth

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Around 240 B.C., Eratosthenes made what is considered to be the most famous and accurate of the ancient measurements of the circumference of the Earth.[1] It was accomplished by making *presumably simultaneous* measurements of the angles of the shadows cast by a vertical stick at Syene (today known as Aswan) and another at Alexandria, at noon on the day of the summer solstice (about June 21 every year). From these measurements, and knowing the distance from Syene to Alexandria along the assumed same meridian of longitude, Eratosthenes was able to provide a remarkably accurate estimate of the radius of the Earth.

The assumption that Syene and Alexandria lie along the same meridian of longitude has been carefully addressed by Fred Hoyle:[2] "It has also been objected that Alexandria does not lie due north of Syene, the difference in longitude being about three degrees. But the error that would arise in this way is only the amount of the difference of the cosine of three degrees from unity, and this is only a little more than 0.1 per cent." This cogent geometric analysis shows that the *consequence* of the difference in longitude of Syene and Alexandria is a small error (about 0.14%) in our analysis.

In practically every description of Eratosthenes' method that has been reported in the literature, the two measurements at Syene and at Alexandria were assumed, either explicitly or implicitly, to be *simultaneous*, in spite of the obvious difficulty inherent in such a process between two points that are about 500 miles apart. However, in the February 2010 issue of *The Physics Teacher*, in a letter to the editor entitled "Eratosthenes' Measurement," Silvia Pugliese Jona[3] argues that Eratosthenes could not possibly have made both measurements simultaneously unless he was able to travel at light speed between the two locations. Pugliese's criticism was referring to an earlier article by Dhevan Gangadharan[4] that mentioned Eratosthenes' measurement of the Earth's radius as being carried out using simultaneous measurements at Alexandria and Syene. This leaves open the question of *how* simultaneous measurements at Syene and at Alexandria might have been made.

Following a suggestion by Kitty Ferguson,[5] the present authors suggest a minimization procedure that could have allowed Eratosthenes to make the two simultaneous measurements between points 5000 stadia apart, at Syene and at Alexandria, while staying at one point, using only information that was known at the time. All that was required was for Eratosthenes to have set up a stick at Alexandria on the day of the summer solstice and to have measured the angle of its shadow *when that shadow was at its shortest (minimum)*.

In order to clarify this minimization procedure, it is necessary to review the standard treatment of Eratosthenes' measurement of the Earth's radius. We assume that Syene was on the Tropic of Cancer when Eratosthenes made his measurement. According to Hoyle[2] and Robert R. Newton,[6] the Sun during the time of Eratosthenes was not strictly overhead at Syene at the time of the summer solstice, but made an angle of 22′ with the vertical. However, by chance, Eratosthenes made a compensating error in measuring the latitude of Alexandria. Thus, at the time of Eratosthenes, the actual difference in latitudes between Syene and Alexandria was 7°5′, as compared to the value of 7°12′ as measured by Eratosthenes. Therefore, the error in Eratosthenes' value of the difference in latitude between Alexandria and Syene is only about 1.6% compared to the actual value of the difference at that time. Accordingly, taking into account errors of both latitude and longitude in Eratosthenes' original measurement, our conclusion is that we expect an error of slightly less than 2% in his overall calculations of the radius of the Earth, a remarkably small number indeed in view of its antiquity.

Accordingly, within the limits of error of the approximations mentioned above, we may reasonably consider that Syene was on the Tropic of Cancer and that Syene and Alexandria lay along the same meridian of longitude at the time of Eratosthenes. What this means first is that, at noon on the day of the summer solstice at Syene, the Sun was directly overhead, reaching its northernmost latitude, so that a stick set up at Syene at this time would cast no shadow and water at the bottom of a well would be illuminated. Second, our assumption that Alexandria has the same longitude as Syene means that a stick set up at Alexandria at noon on the day of the solstice would have a shadow that can allow us to calculate the radius of the Earth, as shown in Fig. 1.

Eratosthenes also assumed that the Sun was far enough away that its rays were essentially parallel. By simple geometry, it can be seen that the angle θ of the stick's shadow at Alexandria, under the given conditions, is precisely the angle subtended at the Earth's center by the arc of the meridian between Syene and Alexandria. Eratosthenes measured this angle to be 7.2°, or one-50th part of a circle. Since the distance from Syene to Alexandria was known to be approximately 5000 stadia, the circumference of the Earth could then be calculated to be $50 \times 5000 = 250{,}000$ stadia.

Figure 2 shows the situation occurring at Syene on the day of the summer solstice, but only *before or after* noon, so that the Sun is at some angle α ($\neq 0$) to the stick at Syene as drawn. This *must* be the case as shown because, as stated earlier, the northernmost latitude (90° from the horizon to the zenith) is reached only at noon, being less than 90° at any other time on this day. Accordingly, the length of the shadow of the stick at Alexandria must be a minimum when the angle $\alpha = 0$ or, equivalently, when it is exactly noon at Syene. Thus, a mea-

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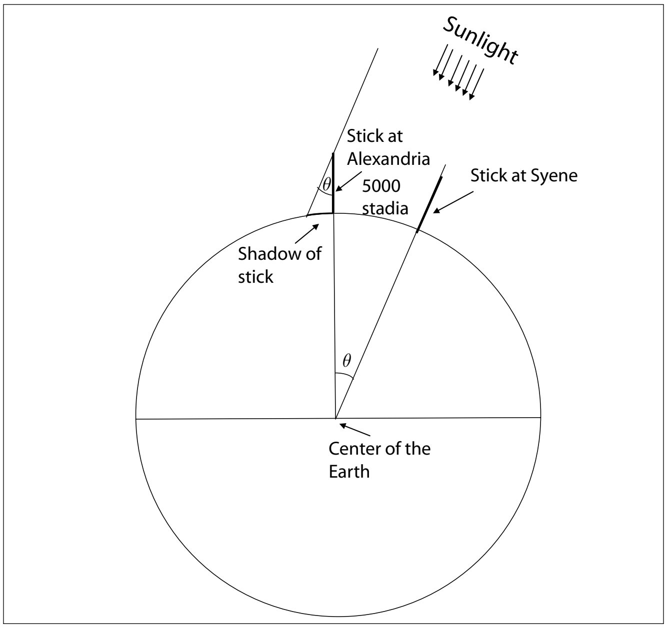

Fig. 1. Eratosthenes' measurement of the circumference of the Earth as viewed when the shadow of the stick at Alexandria is at a minimum, at noon on the day of the summer solstice at Syene.

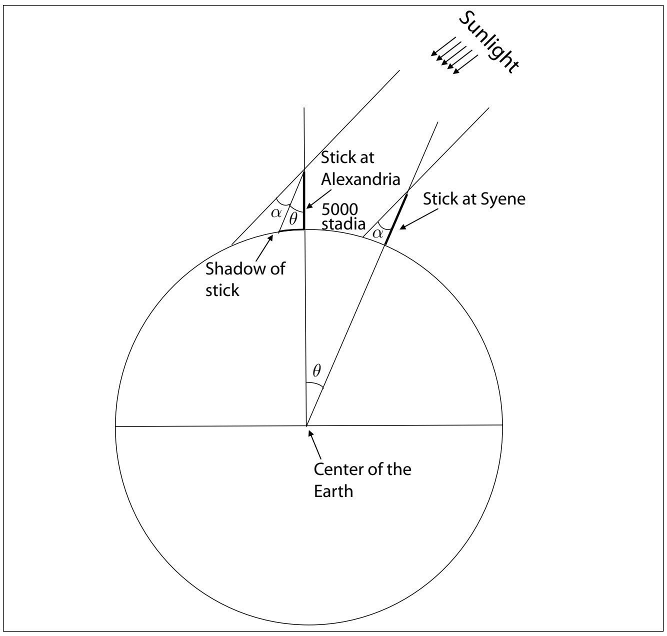

Fig. 2. Eratosthenes' measurement of the circumference of the Earth as viewed when the shadow of the stick at Alexandria is off-minimum, either before or after noon on the day of the summer solstice at Syene.

surement of the stick's shadow at Alexandria when it is a minimum occurs *exactly* when the Sun is directly overhead and the stick casts no shadow at Syene, which completely justifies the standard explanation of Eratosthenes' measurement of the Earth's circumference.

It should be mentioned that Eratosthenes presumably decided upon his method when he learned about a most unusual occurrence that happens yearly at a well on the Island of Elephantine at Syene on the Nile, in Upper Egypt.[7] He learned that at noon on the day of the summer solstice, the

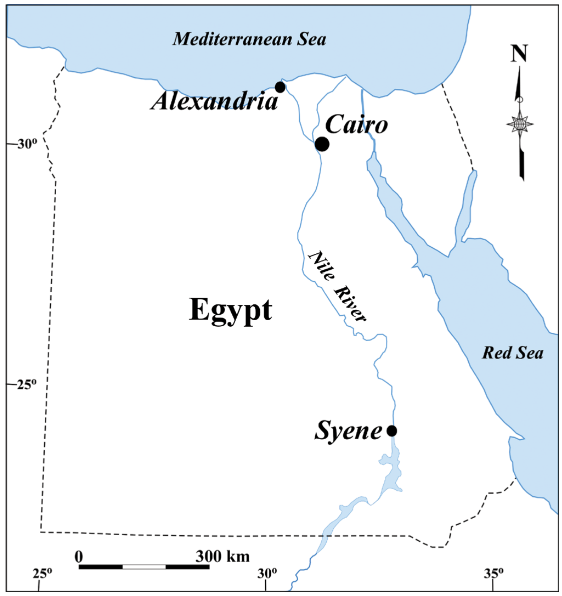

Fig. 3. A map of Egypt. The ancient city of Syene is today known as Aswan. Taken from www.cia.gov/library/publications/the-worldfactbook/maps/maptemplate_eg.html.

Sun was said "to light up the well right down to the water and cast no shadow on the side." This, in turn, must have caused Eratosthenes to realize that the Sun at the place of the well was directly overhead and that the well was located exactly, or very close to, the Tropic of Cancer. This would have indicated to him the procedure for measuring the circumference of the Earth, as shown in Fig. 1.

The definition and value of the stade as a unit of measurement in ancient times is discussed by Aubrey Diller,[8] Fred Hoyle,[2] and others. In particular, Hoyle provides a reasonable argument that Eratosthenes' stade was about 157 meters, which gives remarkably close agreement with the actual circumference of the Earth (through the poles and around the equator).

Syene was an important town of the Egyptian Empire and for administrative reasons its distance from Alexandria must certainly have been well known. Bematists, or professional surveyors trained to walk in equal steps, were employed during the ancient period to measure large distances. The path of the bematists in Eratosthenes' time would have followed the general contour of the Nile, which is irregular in shape rather than in a straight line, so as to overestimate the distance. It is possible that this error is of the order of a few percent, but that would be difficult to prove (Fig. 3).

The exact details of Eratosthenes' measurement of the circumference of the Earth, in particular his manuscript "On the Measurement of the Earth," have been lost except for fragments.[1, 6, 9-10] Fortunately, these latter isolated portions were quoted or rephrased by later, although still ancient, writers and historians such as Cleomedes, Pliny, Strabo, and Ptolemy.[10] Relying on these accounts to be correct, the minimization technique suggested in this article is perhaps the simplest procedure Eratosthenes could have used.

The authors believe that the most readable account of the errors involved in Eratosthenes' measurement is that of Hoyle,[2] who argues persuasively that the total error in Eratosthenes' angle determinations is of the order of 1%, which is quite remarkable. According to Ferguson,[5] Eratosthenes found the diameter of the Earth to be 7850 miles, accurate to within about 1% compared to today's mean value of 7920 miles, and consistent with the conclusions of Hoyle. We agree with Hoyle that there is sufficient independent evidence of the accuracy and sophistication of Eratosthenes as an observer to accept the validity of his extraordinary measurement of the size of the Earth!

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