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COMP 476

Advanced Game Development

Session 3

Pathfinding AI, World Representations

Pathfinding AI (Reading: AI for G, Millington § 4.1-4.4)

Lecture Overview

- Pathfinding
- Dijkstra’s Algorithm
- A* Algorithm
- Pathfinding Lists
- World Representation

Pathfinding

If we need to design an AI…

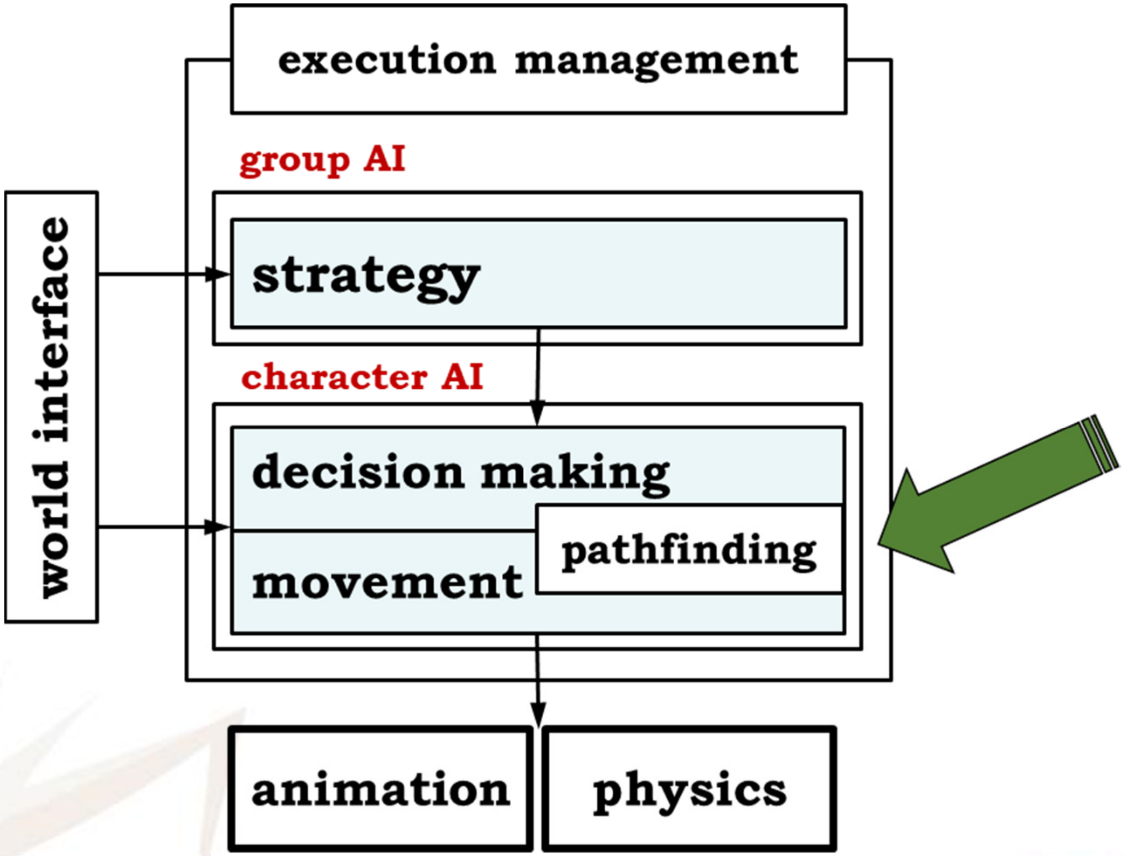
- That is able to calculate a ***suitable route*** through the game level to get from where it currently is to its goal position …
- … and that route is to be ***as short or rapid as possible***, or at least, **looks smart enough** (!) …
- … we need to do … **pathfinding**

Pathfinding

- Sometimes called path planning (but pathfinding is still the usual term used)
- Simple task: *Work out where to move to reach a goal*
- In Millington and Funge's AI model – sits on the border between decision-making and movement.
  - The goal is decided by decision-making AI
  - Pathfinding simply works out how to get there
  - Movement AI gets the character there

Pathfinding AI

Millington and Funge's Model



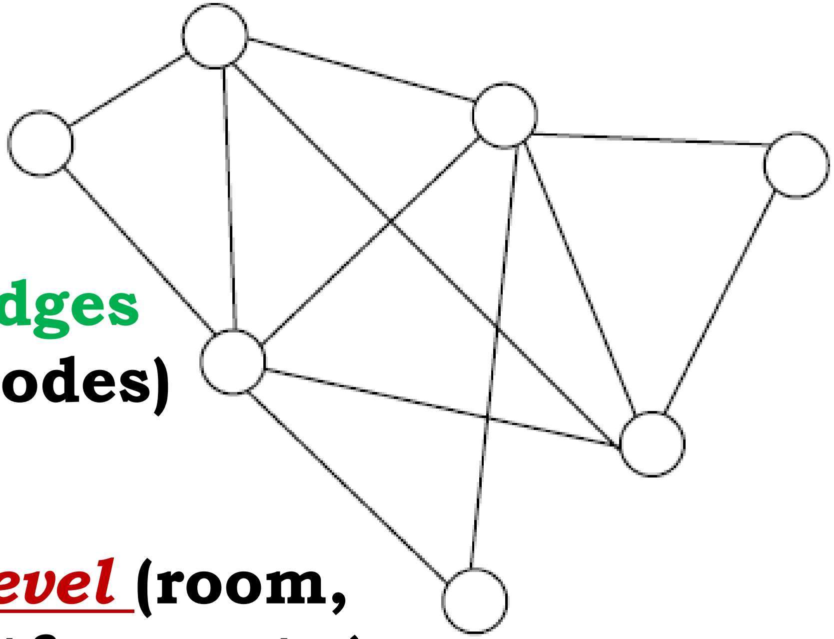
Using Pathfinding

- Vast majority of games use pathfinding solutions that are *efficient and easy to implement* – but pathfinding AI **cannot work directly with game level data**.
- Requires game level *to be represented in a perticular data structure*:

  A directed non-negative weighted graph

- In particular, a graph is a **simplified version of the level**; the **better the simplification, the better the path**

Graphs



- **Formal representation:**
  - **A set of nodes**
  - **A set of connections/edges (an unordered pair of nodes)**
- **Each node represents**
  - **A *region of the game level* (room, section of corridor, platform, *etc.*)**
- **Connections represent**
  - ***Which locations (nodes) are connected***
    - **E.g., if a room adjoins a corridor**
- **A graph splits the whole game level into regions which are connected together**

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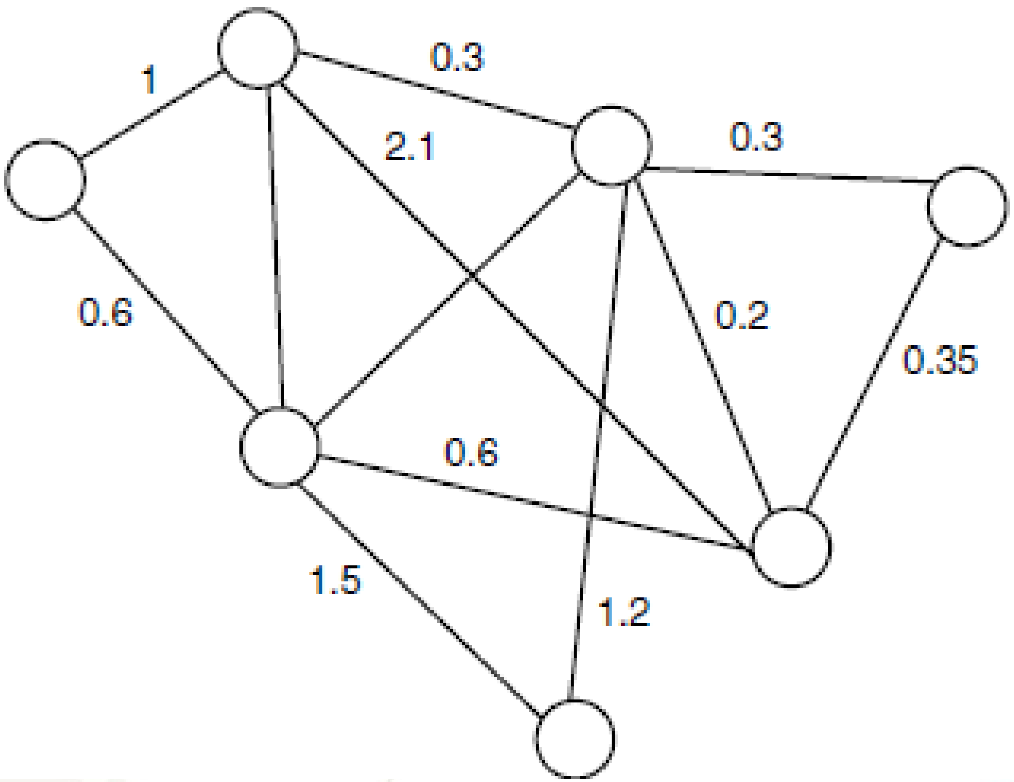
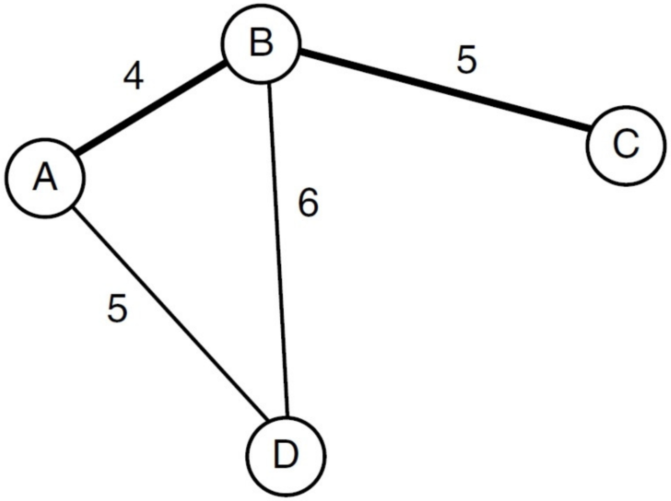
Weighted Graphs

- In addition to having nodes and connections, a numerical value or "**weight**" is used <u>to label each connection with an associated cost value</u>



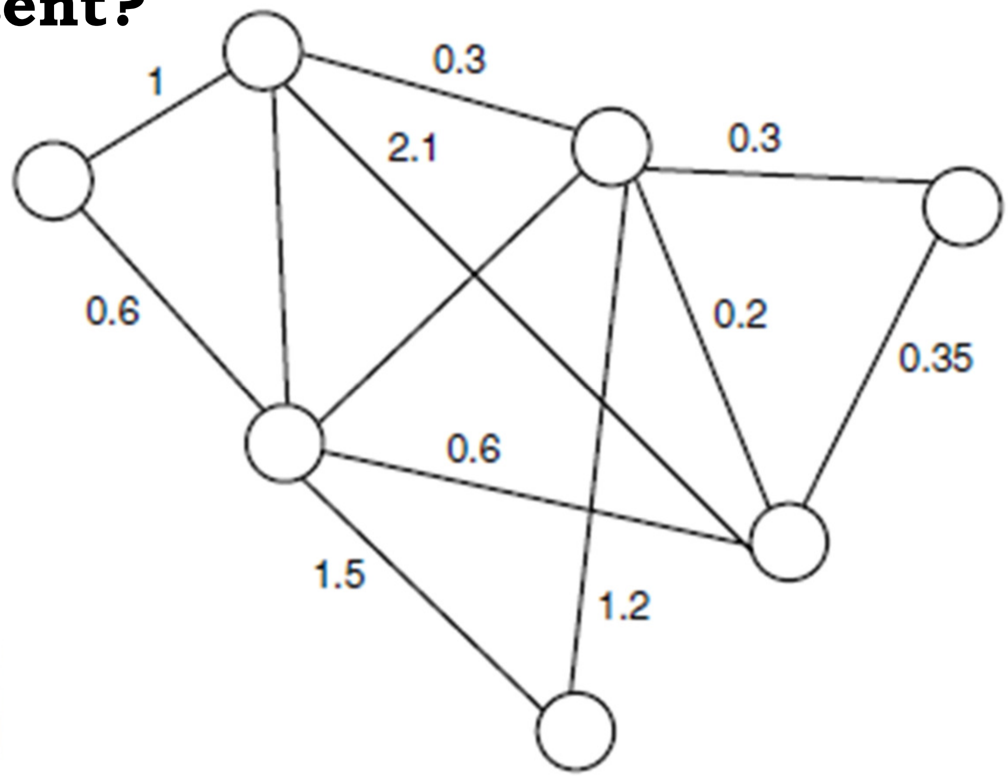
Total path cost



Graphs in Games

Weights?

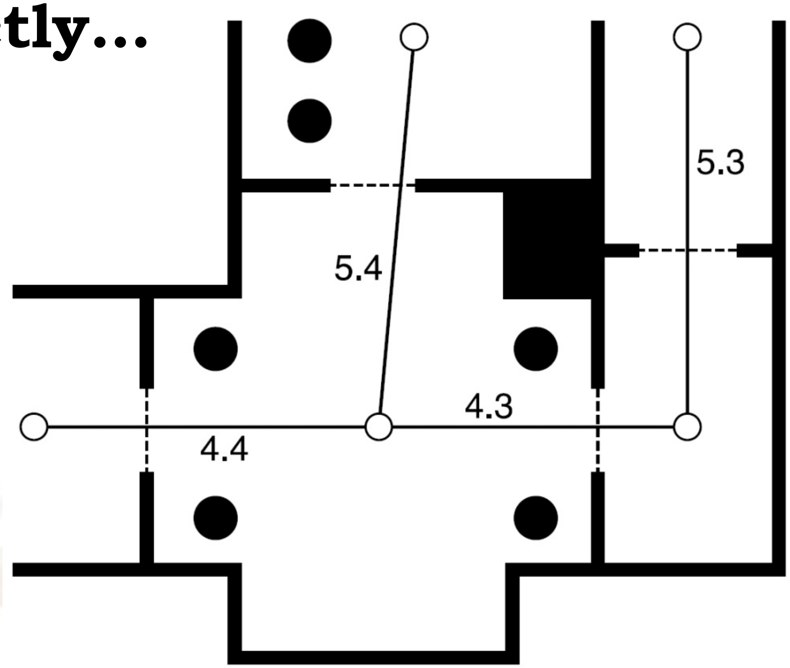
- In games, what can we use the weights (costs) to represent?



Graphs in Games

Nodes/Connections?

- **How shall we overlay** a weighted graph onto game level geometry?
- Suggest some ways to place nodes/connections correctly…

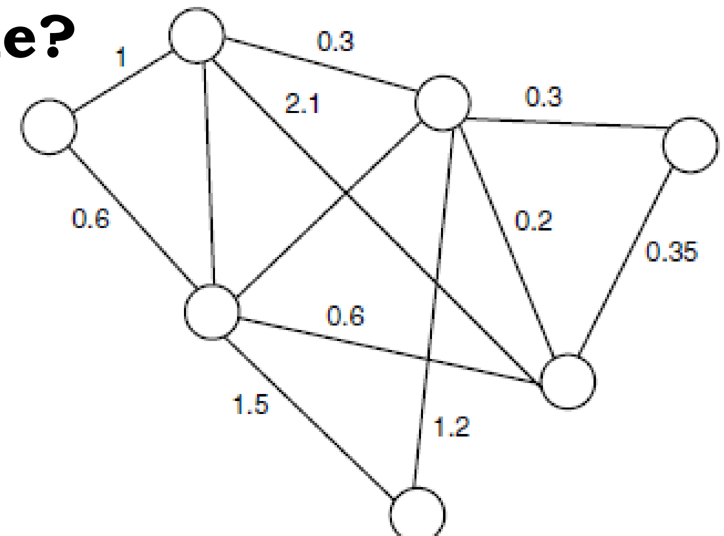


Weighted graph *overlaid* onto level geometry

Graphs in Games

Now, what's the whole idea?

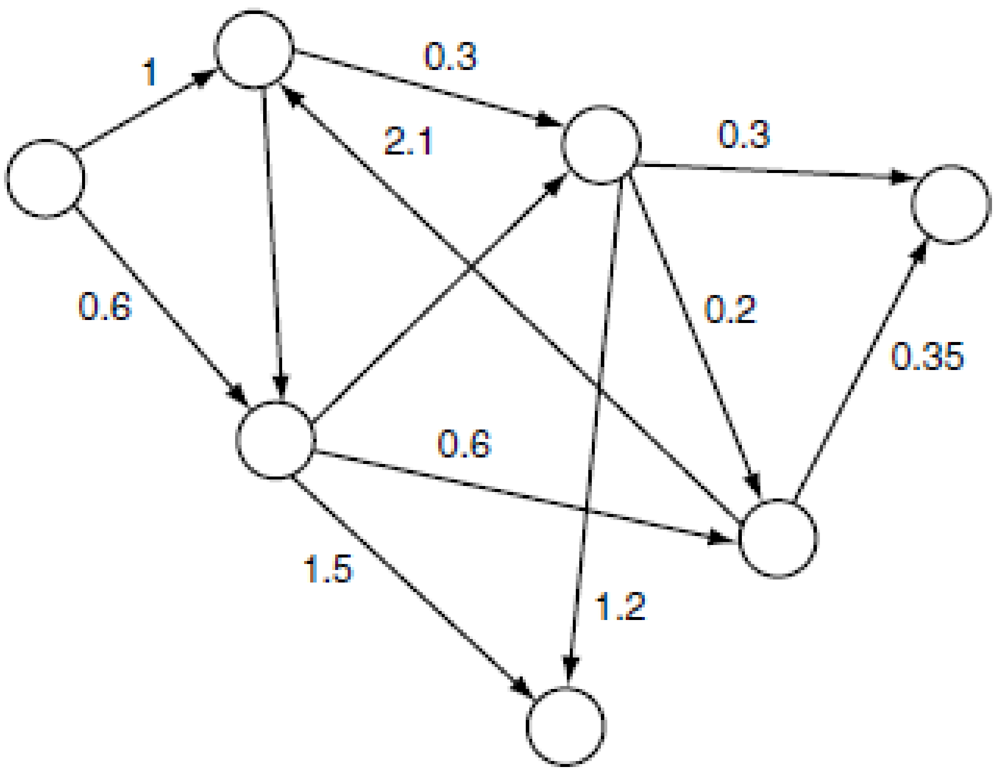
- So, back to the original idea of pathfinding: ***How shall we use a weighted graph*** (like below) to find paths in the game?



- Can we have **negative weights** in a graph?
- Can certain nodes be only connected **one way**(just like a one-way street)?

Directed Weighted Graphs

- Connections of graph are ***directed***, or allows movement between 2 nodes to be from ***one direction only***



Directed Weighted Graphs

- Can be useful in 2 special situations:
  - ***Reachability between two locations*** **(node A can reach node B, but node B cannot reach node A)**
  - **Allow both *connections in opposite directions to have different weights* (the cost to move from node A to node B is different from the cost to move from node B to node A)**



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Graph Representations

Now, what's the whole idea?

- **Graph class** – store an array of connection objects for any node
- **Connection class**– store cost, 'from' node, 'to' node

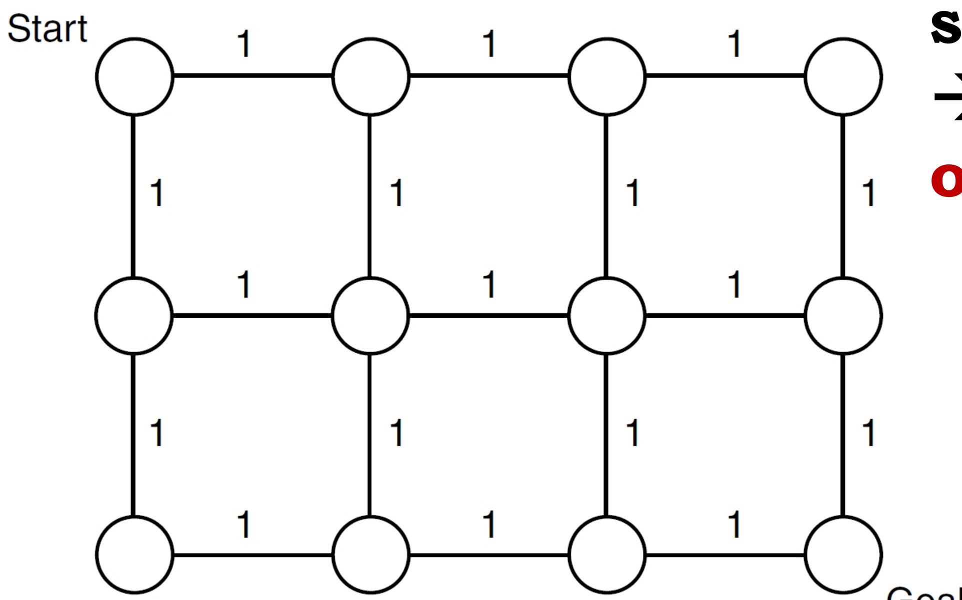
```
class Graph:
  # Returns an array of connections (of class
  # Connection) outgoing from the given node
  def getConnections(fromNode)

class Connection:
  # Returns the non-negative cost of the connection
  def getCost()

  # Returns the node that this connection came from
  def getFromNode()

  # Returns the node that this connection leads to
  def getToNode()
```

Dijkstra's Algorithm

- Named after Edsger Dijkstra, a mathematician
- Originally designed to solve a problem in mathematical graph theory called "shortest path", not for games
- Idea: Finding the *shortest path from one start point to one goal point*
- Dijkstra's Algorithm is designed to find the shortest routes to everywhere from an initial point – Wasteful?
- We will examine Dijkstra's because it is a simpler version of the main pathfinding algorithm A*.

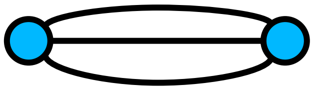
The Problem

- **Aim: Given a graph and two nodes (start and goal), *generate a path such that the total path cost of that path is minimal* among all possible paths from start to goal**
- **There may be any number of paths with the same minimal cost → Just returning any one will do**



Same Cost Paths

- **The path we expect to be returned consists of a *set of connections*, *not nodes*.**
- **Many games do not consider having more than one connection between any pair of nodes (only one possible path?)**



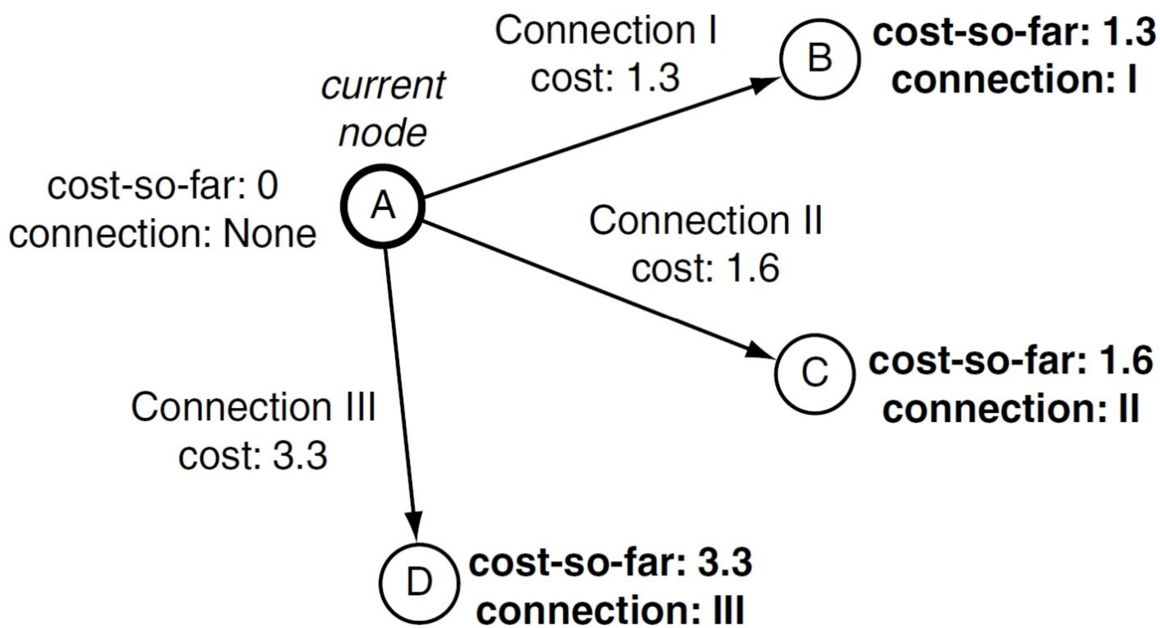
- **With many connections between any pair of nodes, an optimum path is needed to make the AI look smart! So, keep track of multiple connections, use the connection of these with smallest cost.**
- **This is particularly useful if the costs *change over the course of the game* or between different characters**

The Algorithm

- Spread out from the start node along its connections
- Each time, keep a record of the direction it came from with "arrows" (stored as directed edges)
- Eventually, when the goal is reached, *follow the "arrows", in reverse*, back to the start point to generate the complete minimal route
- Works in iterations
- Each iteration: consider one node of a graph and *follow its outgoing connections* (to other nodes)
- Each iteration's node is called the "*current node*"

The Algorithm

- Processing the ***Current Node*** at each iteration
  - Consider outgoing connections from current node
  - For each connection, **finds the end node** and stores the total cost of the path so far as the "cost-so-far" (from the start node)

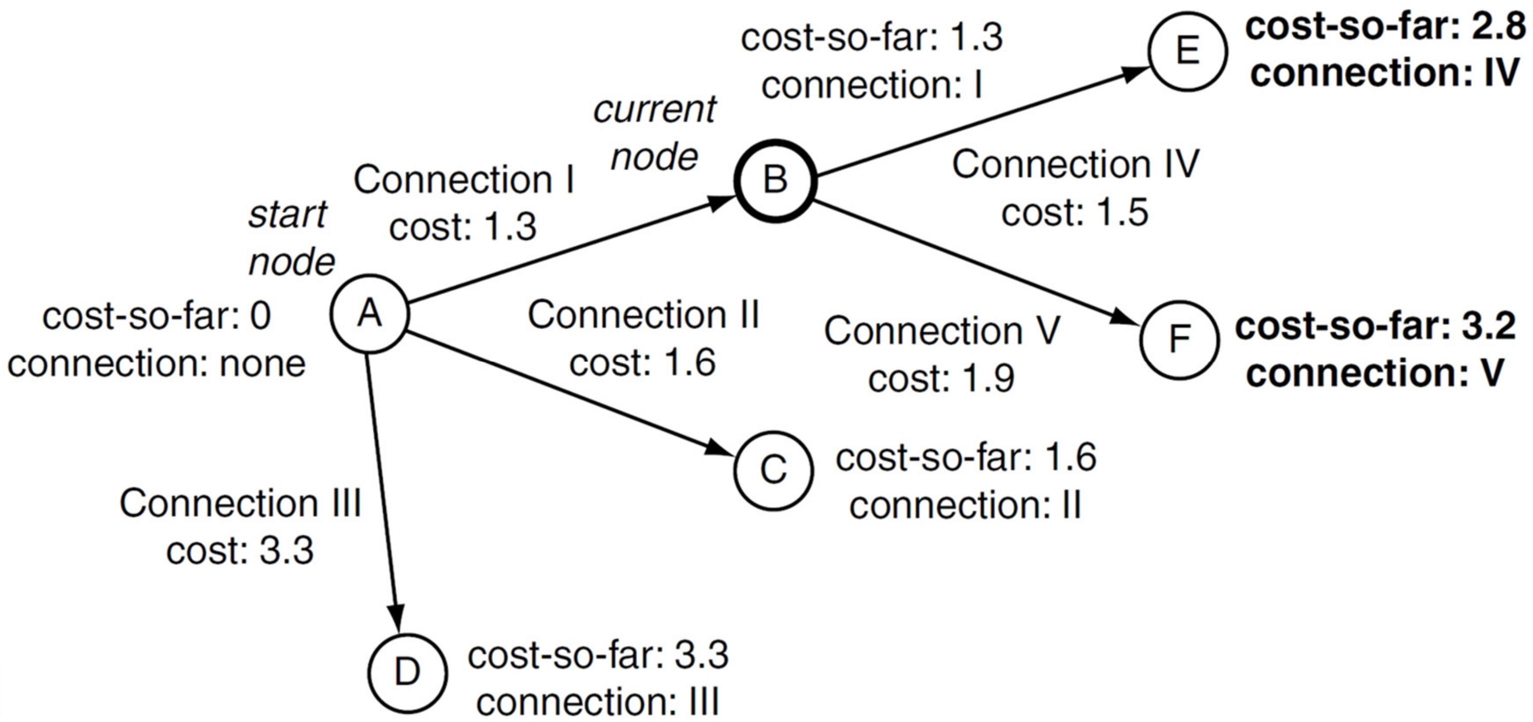


- Each connected node stores a) ***cost-so-far*** and b) which ***connection to the node*** was used to get the cost-so-far

The Algorithm

- After 1st iteration, for neighboring unvisited nodes

  Cost-so-far = Sum of connection cost + cost-so-far of current node



Node Lists

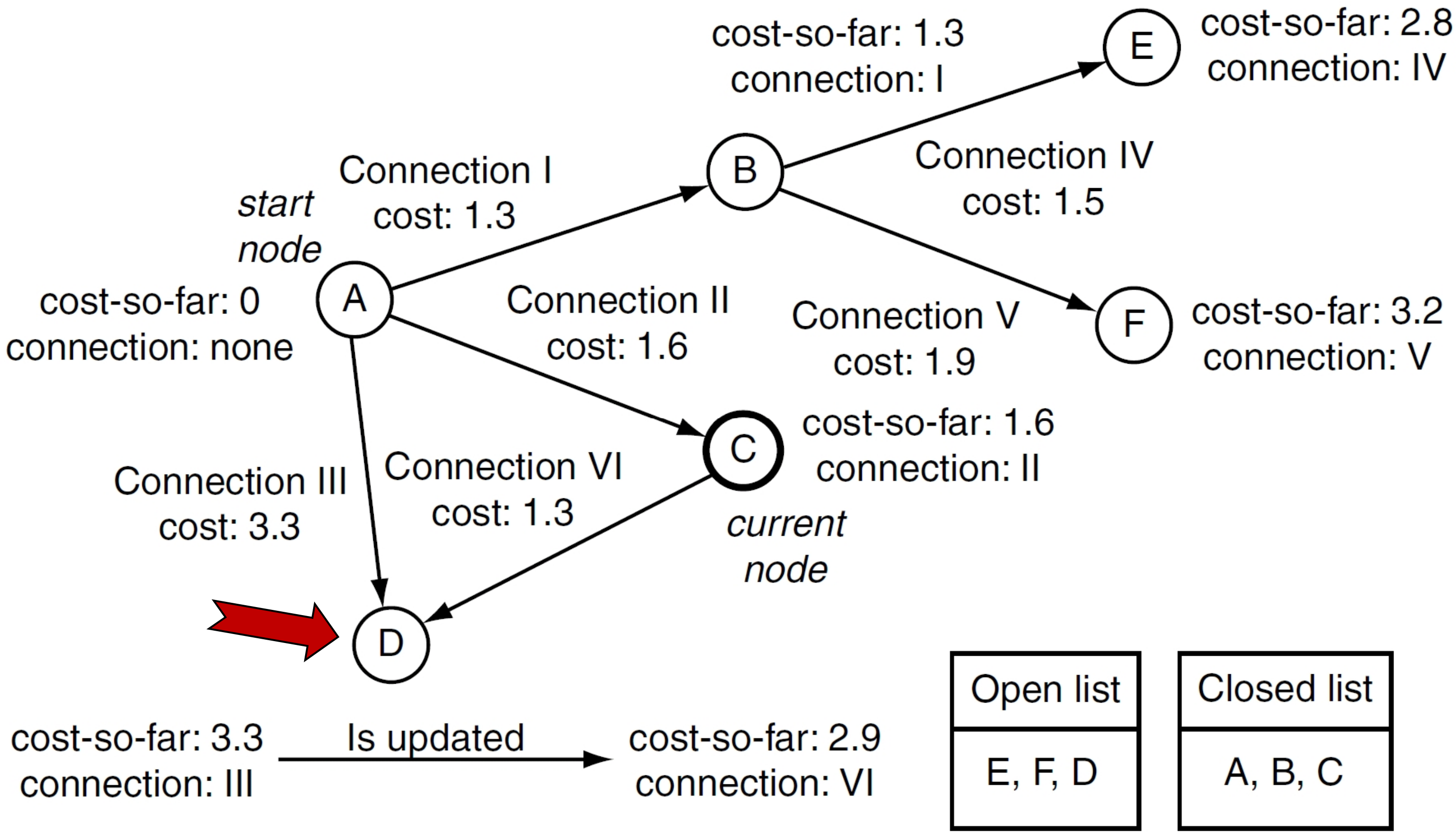
- **Keeps track of all the nodes seen so far in 2 lists:**
  - **Open List**
    - **Records all nodes seen, but haven't been processed in its own iteration yet**
  - **Closed List**
    - **Records all nodes that have been processed in its own iteration (all "current node" s)**
- **Nodes in neither list are "unvisited"**
- **At each iteration, a node from the open list with <u>smallest cost-so-far</u> is chosen (and processed), then removed from the open list and placed in the closed list**

Cost-so-far for Open and Closed Nodes

- What happens when we arrive at an open or closed node (again) during an iteration?
- If there are ***new values*** of the cost-so-far that are lower than the existing value of node, an update of that node is needed, regardless of which list it's in:
  - The open nodes that have their values updated simply stay on the open list
  - For closed nodes — simply force the algorithm to recalculate and re-propagate new values:
    - ✓ Remove node from closed list, place back into open list with its values revised
    - ✓ It will be processed once again and have its connections reconsidered

A few iterations later…

- Notice the updating done on node D



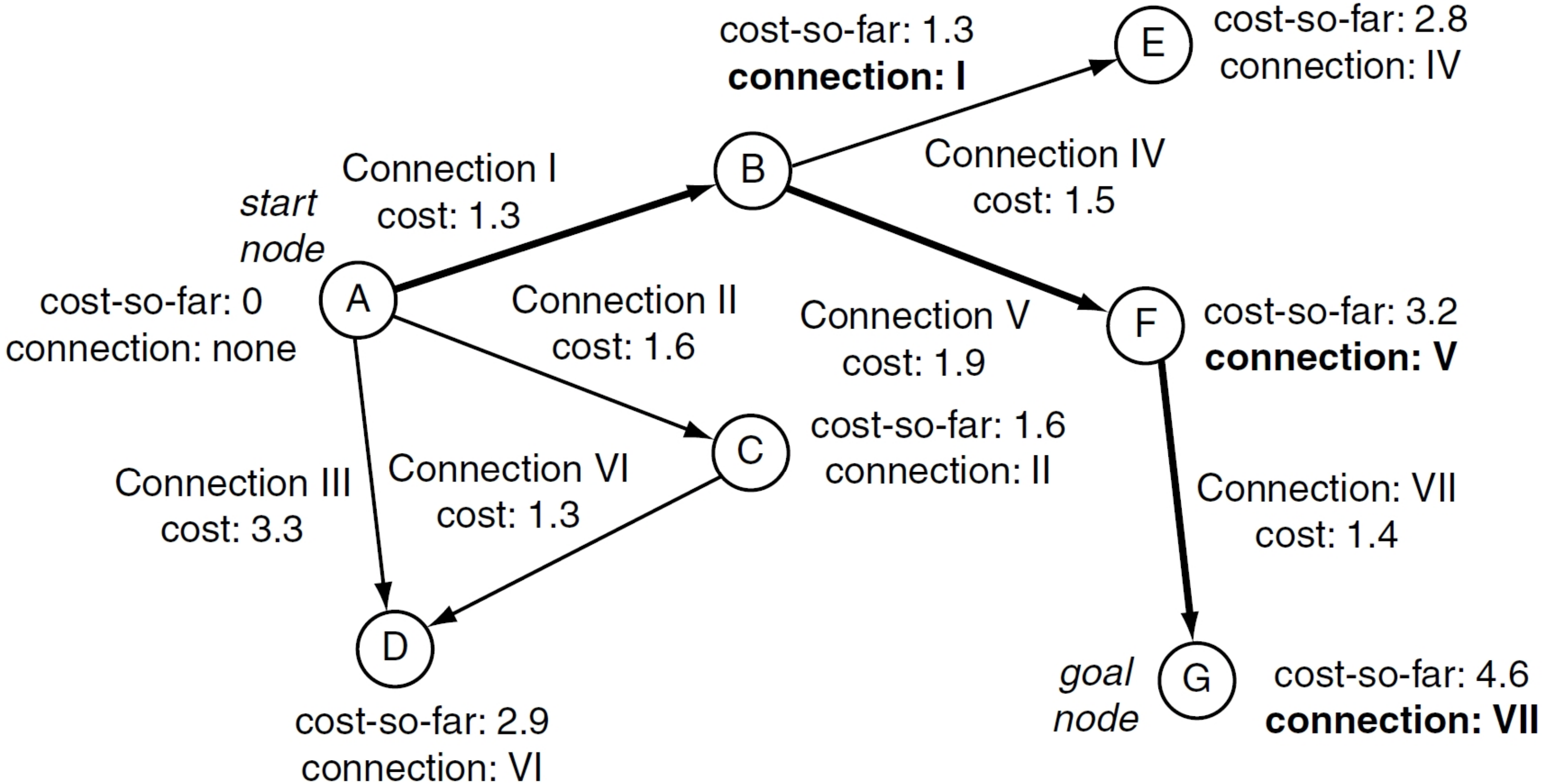
Terminating the Algorithm

- Basic Dijkstra's Algorithm – ***terminates when the open list is empty*** **(no more nodes to process),** ***and all nodes are in the closed list***
- **In practical pathfinding, we can terminate earlier once we have**
  - ***placed the goal node on the open list*****, and**
  - **the goal node is the node with the smallest cost-so-far** ***on the open list*** **(an added heuristic)**
- **Why do we add this heuristic rule?**

Retrieving the Path

- Final step
- Start from goal node, ***look back at the connection*** that was used to arrive to this node; then for the node at the other end of the connection, ***look at the connection used to arrive at that node***; etc. Continue until reaching the start goal
- The list of connections ***need to be reversed to obtain the right path order***

Final Result



Connections working back from goal: **VII, V, I**
Final path: **I, V, VII**

Data Structures & Interfaces

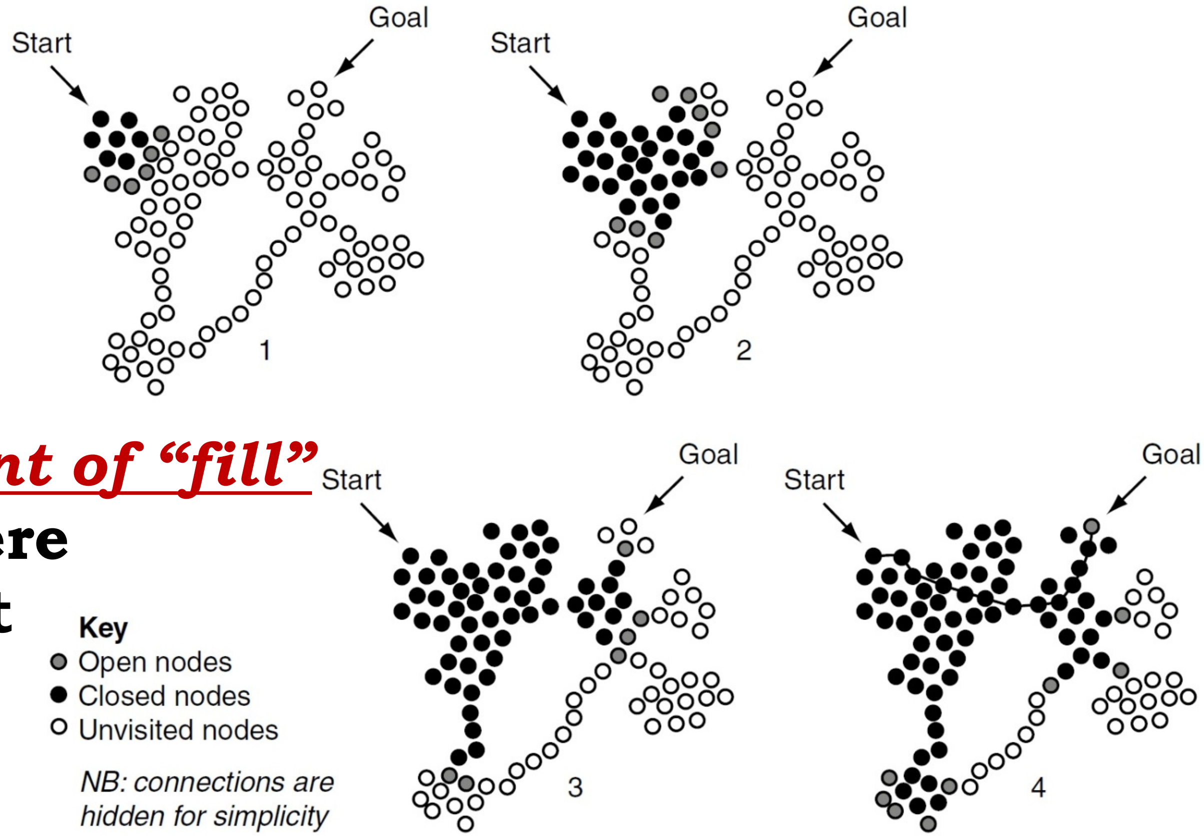
- Graph – Rarely a performance bottleneck (can be created offline)
- Simple List (for reporting connections in Graph)
  - Not performance critical, use a basic linked list (list) or resizable array (vector)
- Pathfinding Lists
  - Large Open and Closed lists can affect performance. ***Need optimization to perform these operations***:
    A. Adding/removing entry,
    B. Finding smallest element,
    C. Finding entry in list corresponding to a particular node

Practical Performance of Dijkstra's

- **Depends mostly on performance of operations in pathfinding list(s) data structure**
- **Theoretically (n nodes, m average number of connections per node):**
  - **Time complexity (execution speed): O(nm)**
  - **Space complexity (memory): O(nm) worst-case**
- **Select implementations:**
  - **Min Priority Queue: O($n^2 + nm$)**
  - **Fibonacci Heap: O($n \log n + nm$)**
- **Note: $1 < m \leq n-1$ for a connected graph with $n > 2$**

Weaknesses

- Searches whole graph indiscriminately for shortest path
- ***Wasteful*** **for point-to-point pathfinding**
- **Suffers from *terrible amount of "fill"* (nodes that were considered but never made part of the final route)**

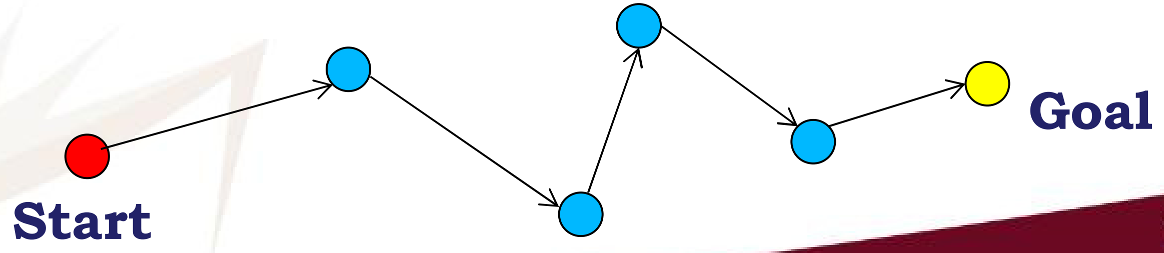


A* Algorithm

- Most widely used for pathfinding in games
  - *Simple to implement*
  - *Efficient*
  - Lots of *scope for optimization*
- Unlike Dijkstra algorithm, A* is designed for *point-to-point pathfinding*, not for solving the *shortest path problem* in graph theory.
  - Always returns a single path from source to goal
  - The "best" possible path

The Problem

- Given a graph (a **directed non-negative weighted** graph) and two nodes in that graph (a start and a goal node)
- Generate a path → the ***total path cost of the path is minimal*** among all paths from start to goal
- If there are more than one possible solution, ***any minimal path will do***
- Path ***should consist of a list of connections from the start node to goal node***



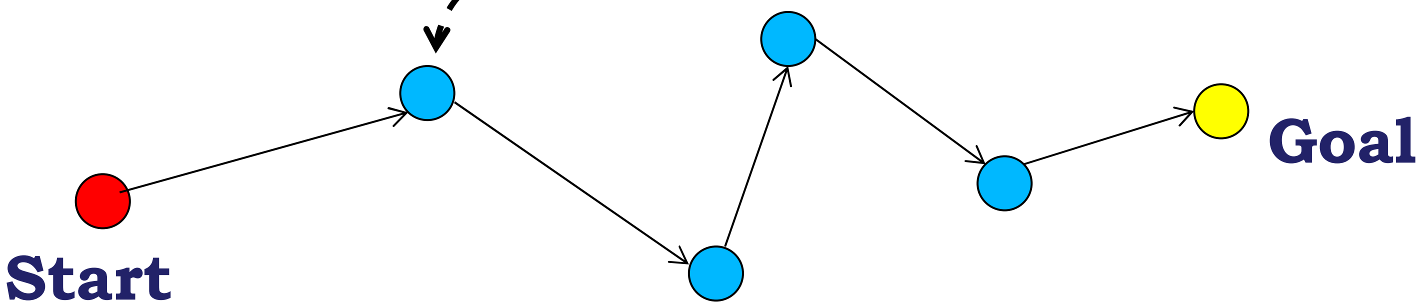
Differences from Dijkstra

- **Dijkstra → *Always considered the open node with lowest cost-so-far* for processing**
- **A* → Consider node that most likely to lead to shortest overall path**
- **“most likely” → controlled by a “heuristic”, an *estimated rule of thumb***
- **If accurate heuristic used: Efficient**
- **If bad heuristic used: May perform worse than Dijkstra!**

“heuristic” for A*

- Think about **how** an “estimated total cost” **should be calculated**?...

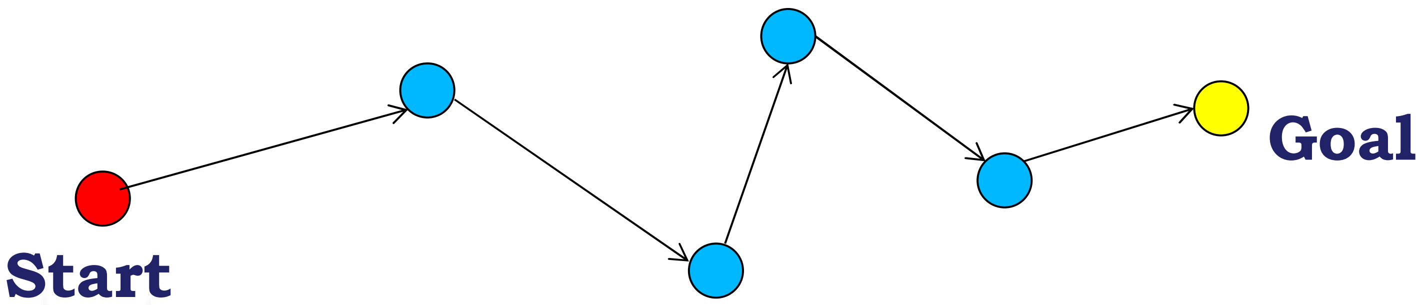
How to estimate total cost from here to goal for this node?



- Think of a good heuristic to use?

“heuristic” for A*

- A node with smallest estimated-total-cost should have:
  - A relatively small cost-so-far (Dijkstra only has this)
  - A relatively small estimated distance to go to reach goal (the heuristic)



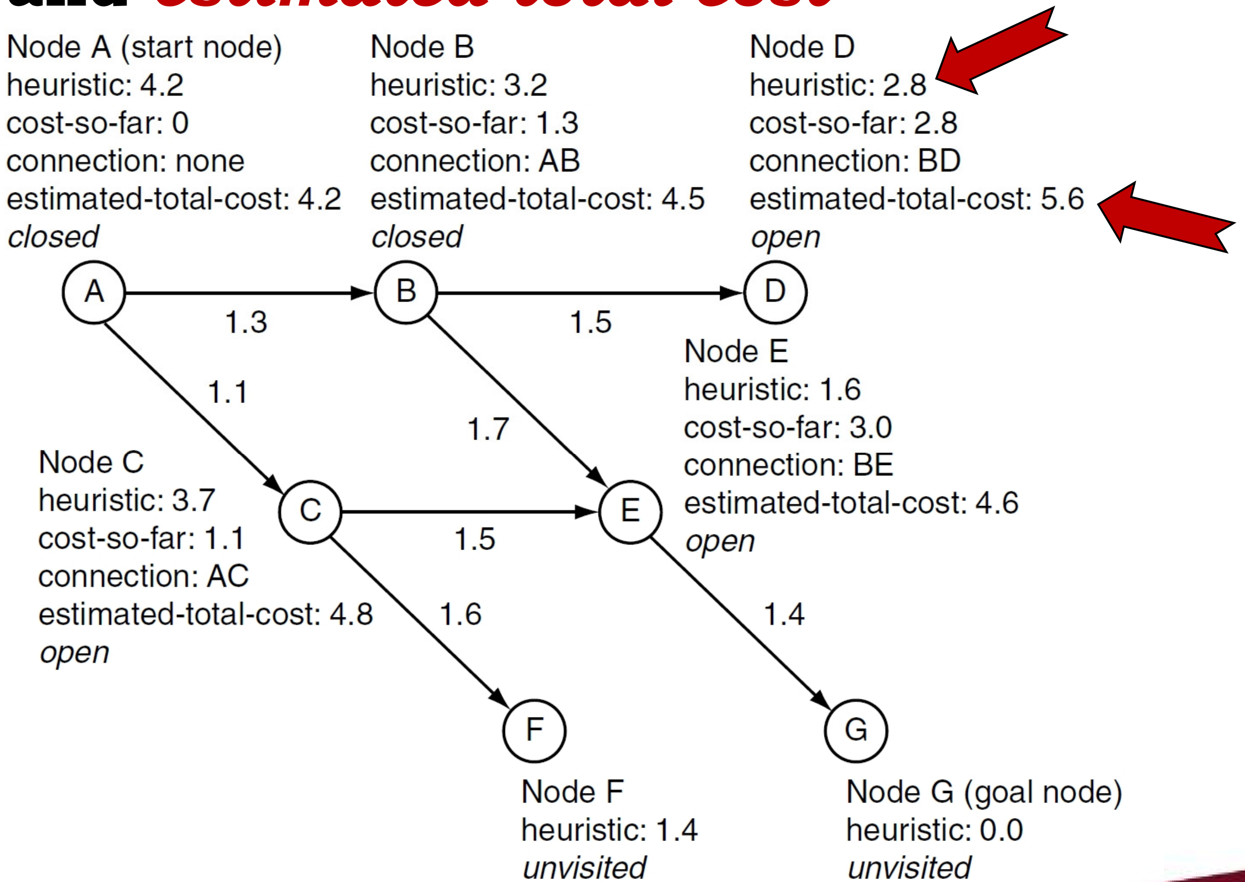
- estimated-total-cost = cost-so-far + heuristic cost

Pathfinding Lists

- Just like Dijkstra, A* also keeps an open list of nodes and a closed list of nodes
  - Nodes are moved to open list as they are found at end of connections (previously "unvisited")
  - Nodes are moved to closed list *as they are processed in their own iteration*
- Unlike Dijkstra, node with smallest estimated-total-cost is <u>selected</u> from the open list, not the node with smallest cost-so-far

Using the "heuristic"

- A* stores 2 additional values – heuristic value and ***estimated total cost***



Calculating Costs

- estimated-total-cost =<u>cost-so-far + Heuristic cost</u>
  - As with Dijkstra's algorithm, the cost-so-far is calculated using the actual ***total cost of the path*** travelled so far from the start to the current node
  - The heuristic cost **cannot be negative** (obviously!)
- To update a <u>previously visited node</u>
  - Compare cost-so-far, not the estimated total cost (with heuristic added)
  - This is the only reliable "real" value. Heuristics involve estimation (and are usually fixed values)

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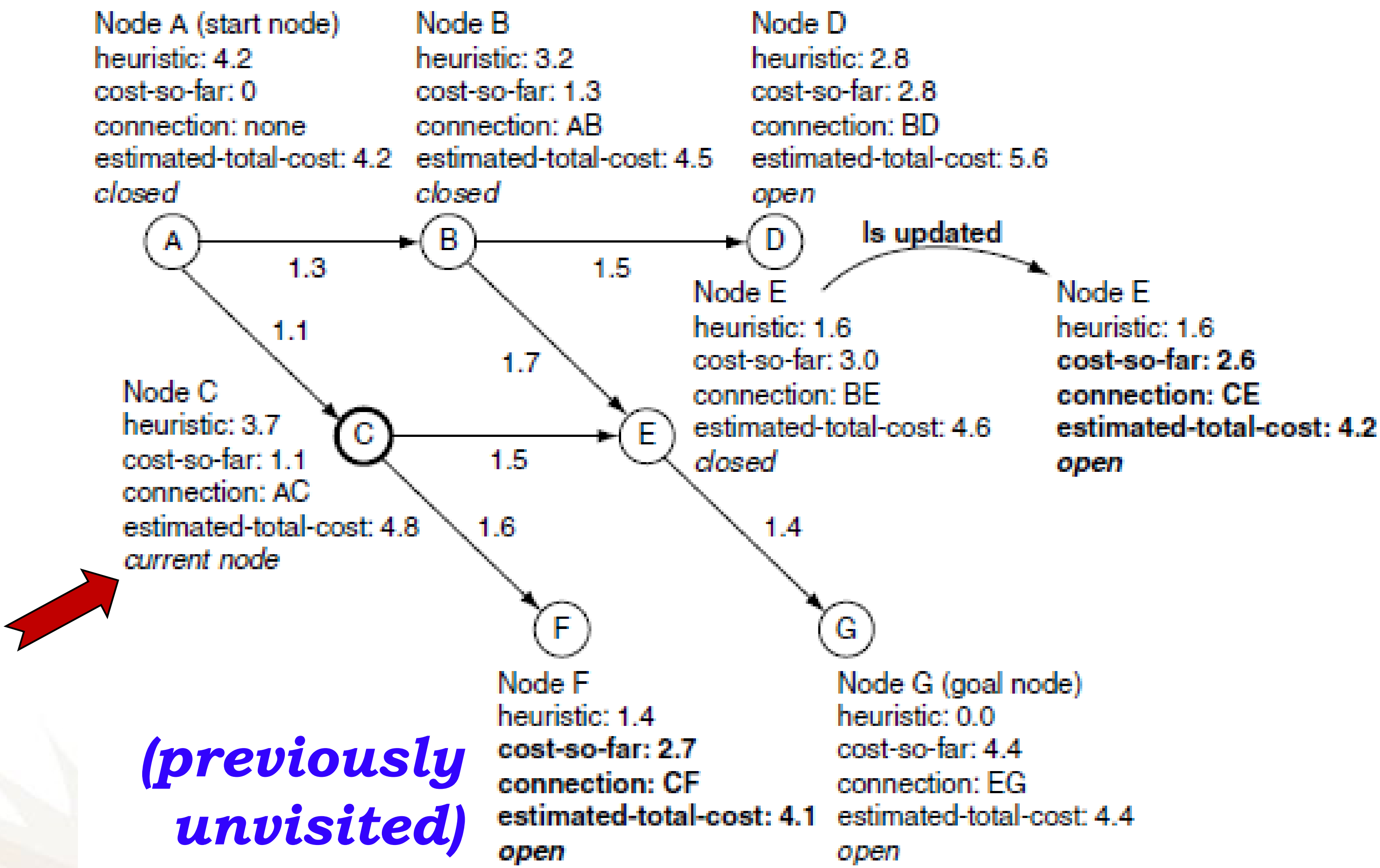
Updating Costs

- If there are *new values* of the cost-so-far that are lower than the existing value of node, an update of that node is needed, as with Dijkstra, regardless of which list it is in:
  - The open nodes that have their values revised simply stay on the open list
  - For closed nodes, remove the node from closed list, place it back into open list with its values revised - to be processed once again and have its connections reconsidered

Updating Costs



(previously unvisited)

Terminating the Algorithm

- A* terminates when the goal node is the smallest node on the open list (with smallest estimated-total-cost)
- This does not guarantee that shortest path is found. Why?
- It is natural for A* to run a bit longer to generate a guaranteed optimal result
- One way: Require that A* terminates when node in open list with smallest cost-so-far has a cost-so-far greater than cost of path found to goal (almost like Dijkstra)
- Drawback: Same fill problem as Dijkstra!!

Algorithm Performance

- Depends on data structure used
  - Time complexity, O(*lm*)
  - Space complexity, O(*lm*)
  - l: number of nodes whose total estimated-path-cost is less than that of goal, < n from Dijkstra
  - m: *average number of outgoing connections from each node*
- Heuristic calculation is usually O(1) execution time and memory, so no effect on overall performance. Can be even calculated offline in some cases.
- Refer to [M&F] for more detail on pseudocode and data structures involved

l = amount of "fill"