ETC3550/ETC5550
Applied forecasting

Ch8. Simple Exponential smoothing

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Historical perspective

- Developed in the 1950s and 1960s as methods (algorithms) to produce point forecasts.
- Combine a “level”, “trend” (slope) and “seasonal” component to describe a time series.
- The rate of change of the components are controlled by “smoothing parameters”: α, β and γ respectively.
- Need to choose best values for the smoothing parameters (and initial states).
- Equivalent ETS state space models developed in the 1990s and 2000s.

Big idea: control the rate of change

α controls the flexibility of the **level**

- If $\alpha = 0$, the level never updates (mean)
- If $\alpha = 1$, the level updates completely (naive)

β controls the flexibility of the **trend**

- If $\beta = 0$, the trend is linear
- If $\beta = 1$, the trend changes suddenly every observation

γ controls the flexibility of the **seasonality**

- If $\gamma = 0$, the seasonality is fixed (seasonal means)
- If $\gamma = 1$, the seasonality updates completely (seasonal naive)

Models and methods

Methods

- Algorithms that return point forecasts.

Models

- Generate same point forecasts but can also generate forecast distributions.
- A stochastic (or random) data generating process that can generate an entire forecast distribution.
- Allow for “proper” model selection.

Simple Exponential Smoothing

Iterative form

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Component form

$$\begin{aligned} \text{Forecast equation} \qquad \hat{y}_{t+h|t} &= \ell_t \\ \text{Smoothing equation} \qquad \ell_t &= \alpha y_t + (1-\alpha)\ell_{t-1} \end{aligned}$$

Simple Exponential Smoothing

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Forecast equation	$\hat{y}_{t+h|t} = \ell_t$
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Forecast error: $e_t = y_t - \hat{y}_{t|t-1} = y_t - \ell_{t-1}$.

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Error correction form

$$
\begin{aligned}
y_t &= \ell_{t-1} + e_t \\
\ell_t &= \ell_{t-1} + \alpha(y_t - \ell_{t-1}) \\
&= \ell_{t-1} + \alpha e_t
\end{aligned}
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Specify probability distribution: $e_t = \varepsilon_t \sim \text{NID}(0, \sigma^2)$.

ETS(A,N,N): SES with additive errors

ETS(A,N,N) model

Observation equation	$y_t = \ell_{t-1} + \varepsilon_t$
State equation	$\ell_t = \ell_{t-1} + \alpha\varepsilon_t$

where $\varepsilon_t \sim \text{NID}(0, \sigma^2)$.

- "innovations" or "single source of error" because equations have the same error process, ε_t.
- Observation equation: relationship between observations and states.
- State equation(s): evolution of the state(s) through time.

ETS(M,N,N): SES with multiplicative errors.

- Specify relative errors $\varepsilon_t = \frac{y_t - \hat{y}_{t|t-1}}{\hat{y}_{t|t-1}} \sim \text{NID}(0, \sigma^2)$
- Substituting $\hat{y}_{t|t-1} = \ell_{t-1}$ gives:
  - $y_t = \ell_{t-1} + \ell_{t-1}\varepsilon_t$
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ETS(M,N,N) model

Observation equation	$y_t = \ell_{t-1}(1 + \varepsilon_t)$
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- Models with additive and multiplicative errors with the same parameters generate the same point forecasts but different prediction intervals.