MONASH University

MONASH BUSINESS SCHOOL

ETC3550/ETC5550 Applied forecasting

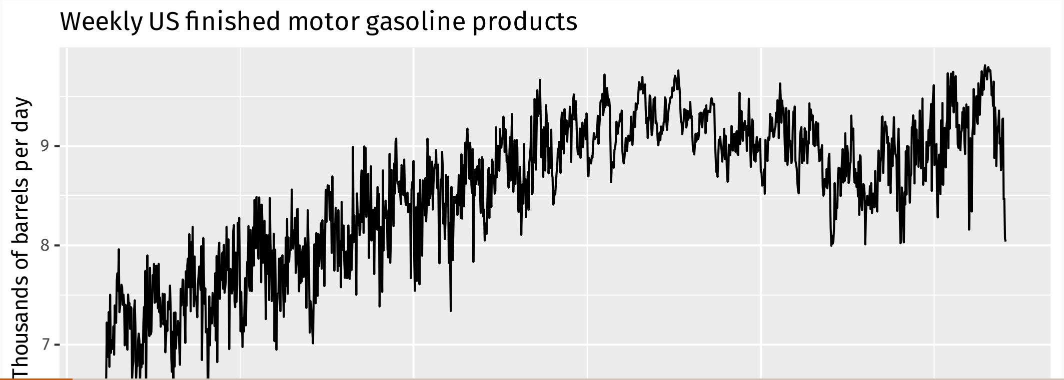
Ch11. Advanced methods

OTexts.org/fpp3/

Outline

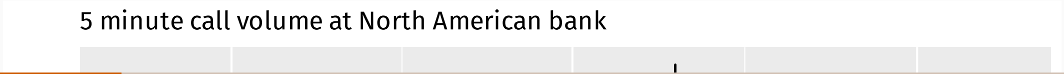
Examples

```
us_gasoline |> autoplot(Barrels) +
  labs(
    x = "Year", y = "Thousands of barrels per day",
    title = "Weekly US finished motor gasoline products"
  )
```



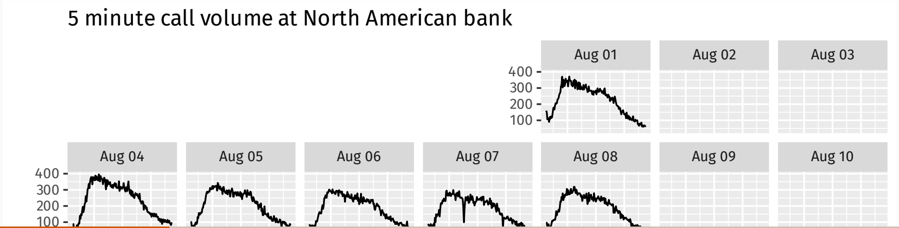
Examples

```r
calls <- read_tsv("http://robjhyndman.com/data/callcenter.txt") |>
  rename(time = `...1`) |>
  pivot_longer(-time, names_to = "date", values_to = "volume") |>
  mutate(
    date = as.Date(date, format = "%d/%m/%Y"),
    datetime = as_datetime(date) + time
  ) |>
  as_tsibble(index = datetime)
calls |>
  fill_gaps() |>
  autoplot(volume) +
  labs(
    x = "Weeks", y = "Call volume",
    title = "5 minute call volume at North American bank"
  )
```



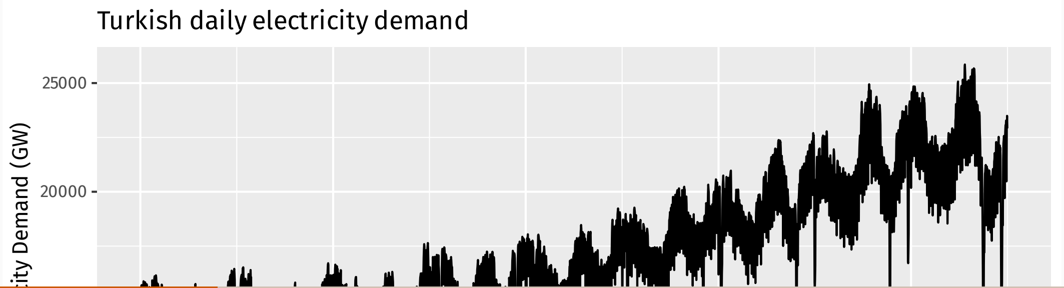
Examples

```r
library(sugrrants)
calls |>
  filter(yearmonth(date) == yearmonth("2003 August")) |>
  ggplot(aes(x = time, y = volume)) +
  geom_line() +
  facet_calendar(date) +
  labs(
    x = "Weeks", y = "Call volume",
    title = "5 minute call volume at North American bank"
  )
```



Examples

```r
turkey_elec <- read_csv("data/turkey_elec.csv", col_names = "Demand") |>
  mutate(Date = seq(ymd("2000-01-01"), ymd("2008-12-31"), by = "day")) |>
  as_tsibble(index = Date)
turkey_elec |> autoplot(Demand) +
  labs(
    title = "Turkish daily electricity demand",
    x = "Year", y = "Electricity Demand (GW)"
  )
```



TBATS model

TBATS

Trigonometric terms for seasonality
Box-Cox transformations for heterogeneity
ARMA errors for short-term dynamics
Trend (possibly damped)
Seasonal (including multiple and non-integer periods)

TBATS model

$$
\begin{aligned}
y_t &= \text{observation at time } t \\
y_t^{(\omega)} &= \begin{cases} (y_t^\omega - 1)/\omega & \text{if } \omega \neq 0; \\ \log y_t & \text{if } \omega = 0. \end{cases} \\
y_t^{(\omega)} &= \ell_{t-1} + \phi b_{t-1} + \sum_{i=1}^{M} s_{t-m_i}^{(i)} + d_t \\
\ell_t &= \ell_{t-1} + \phi b_{t-1} + \alpha d_t \\
b_t &= (1-\phi)b + \phi b_{t-1} + \beta d_t \\
d_t &= \sum_{i=1}^{p} \phi_i d_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t
\end{aligned}
$$

$$
s_t^{(i)} = \sum_{j=1}^{k_i} s_{j,t}^{(i)} \qquad s_{j,t}^{(i)} = s_{j,t-1}^{(i)} \cos\lambda_j^{(i)} + s_{j,t-1}^{*(i)} \sin\lambda_j^{(i)} + \gamma_1^{(i)} d_t
$$

TBATS model

Box-Cox transformation

$$y_t = \text{observation at time } t$$

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$$s_{j,t}^{(i)} = s_{j,t-1}^{(i)} \cos \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \sin \lambda_j^{(i)} + \gamma_1^{(i)} d_t$$

$$s^{(i)} = \sum^{k_i} s^{(i)}$$

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Box-Cox transformation

M seasonal periods

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Box-Cox transformation

$$y_t^{(\omega)} = \ell_{t-1} + \phi b_{t-1} + \sum_{i=1}^{M} s_{t-m_i}^{(i)} + d_t$$

M seasonal periods

$$\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha d_t$$

global and local trend

$$b_t = (1-\phi)b + \phi b_{t-1} + \beta d_t$$

$$d_t = \sum_{i=1}^{p} \phi_i d_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t$$

$$s_t^{(i)} = \sum^{k_i} s^{(i)}$$

$$s_{j,t}^{(i)} = s_{j,t-1}^{(i)} \cos \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \sin \lambda_j^{(i)} + \gamma_1^{(i)} d_t$$

TBATS model

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\end{aligned}
$$

Box-Cox transformation

M seasonal periods

global and local trend

ARMA error

$$s_{j,t}^{(i)} = s_{j,t-1}^{(i)} \cos \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \sin \lambda_j^{(i)} + \gamma_1^{(i)} d_t$$

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TBATS model

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\end{aligned}
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$$
s_t^{(i)} = \sum^{k_i} s^{(i)} \qquad s_{j,t}^{(i)} = s_{j,t-1}^{(i)} \text{ cc}
$$

Box-Cox transformation

M seasonal periods

global and local trend

ARMA error

Fourier-like seasonal terms

TBATS model

y_t = observation at time t

TBATS

Trigonometric

Box-Cox

ARMA

Trend

Seasonal

Box-Cox transformation

M seasonal periods

global and local trend

ARMA error

Fourier-like seasonal terms

Complex seasonality

```
gasoline |>
  tbats() |>
  forecast() |>
  autoplot()
```

Complex seasonality

```r
calls |>
  tbats() |>
  forecast() |>
  autoplot()
```

Complex seasonality

```r
telec |>
  tbats() |>
  forecast() |>
  autoplot()
```

TBATS model

TBATS

Trigonometric terms for seasonality
Box-Cox transformations for heterogeneity
ARMA errors for short-term dynamics
Trend (possibly damped)
Seasonal (including multiple and non-integer periods)

- Handles non-integer seasonality, multiple seasonal periods.
- Entirely automated
- Prediction intervals often too wide
- Very slow on long series

Outline

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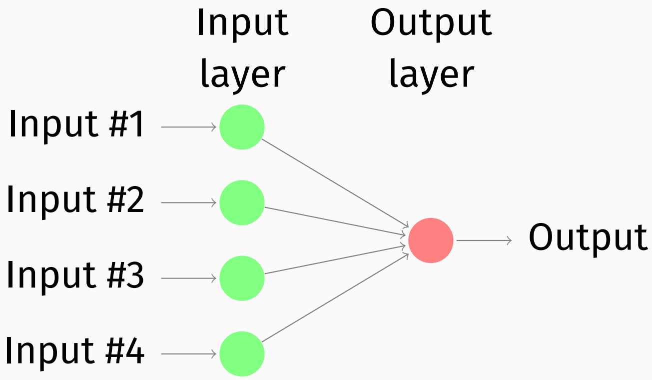
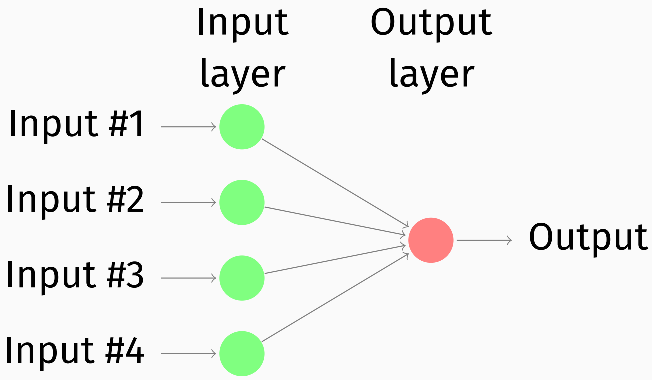
Neural network models

Simplest version: linear regression



Neural network models

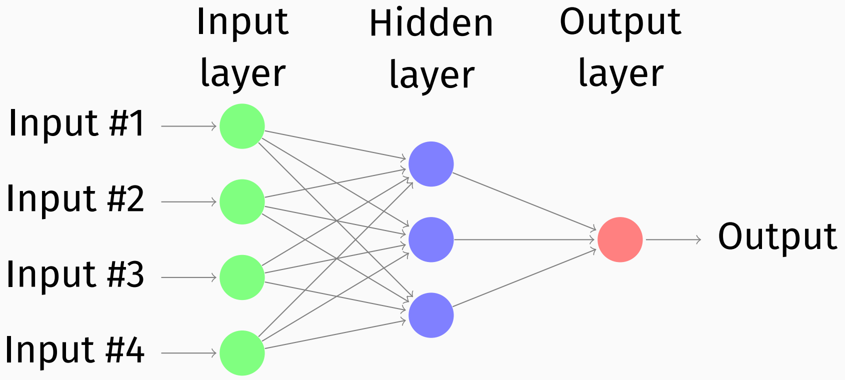
Simplest version: linear regression



- Coefficients attached to predictors are called “weights”.
- Forecasts are obtained by a linear combination of inputs

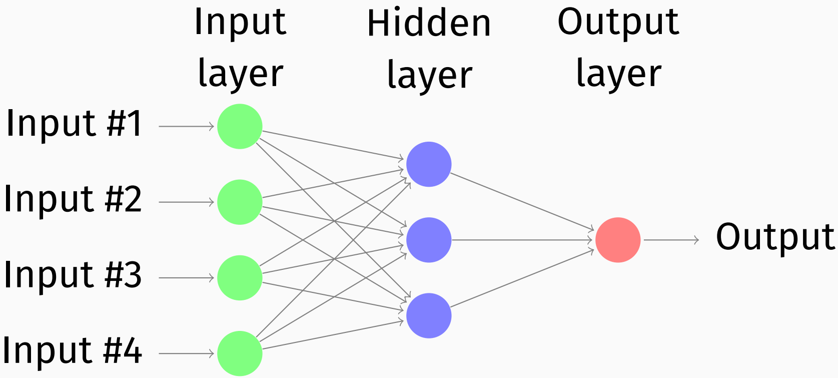
Neural network models

Nonlinear model with one hidden layer



Neural network models

Nonlinear model with one hidden layer



- A **multilayer feed-forward network** where each layer of nodes receives inputs from the previous layers.

Neural network models

Inputs to hidden neuron j linearly combined:

$$z_j = b_j + \sum_{i=1}^{4} w_{i,j} x_i.$$

Modified using nonlinear function such as a sigmoid:

$$s(z) = \frac{1}{1 + e^{-z}},$$

This tends to reduce the effect of extreme input values, thus making the network somewhat robust to outliers.

Neural network models

- Weights take random values to begin with, which are then updated using the observed data.
- There is an element of randomness in the predictions. So the network is usually trained several times using different random starting points, and the results are averaged.
- Number of hidden layers, and the number of nodes in each hidden layer, must be specified in advance.

NNAR models

- Lagged values of the time series can be used as inputs to a neural network.
- NNAR(p, k): p lagged inputs and k nodes in the single hidden layer.
- NNAR($p, 0$) model is equivalent to an ARIMA($p, 0, 0$) model but without stationarity restrictions.
- Seasonal NNAR(p, P, k): inputs $(y_{t-1}, y_{t-2}, \dots, y_{t-p}, y_{t-m}, y_{t-2m}, y_{t-Pm})$ and k neurons in the hidden layer.
- NNAR$(p, P, 0)_m$ model is equivalent to an ARIMA$(p, 0, 0)(P,0,0)_m$ model but without stationarity restrictions.

NNAR models in R

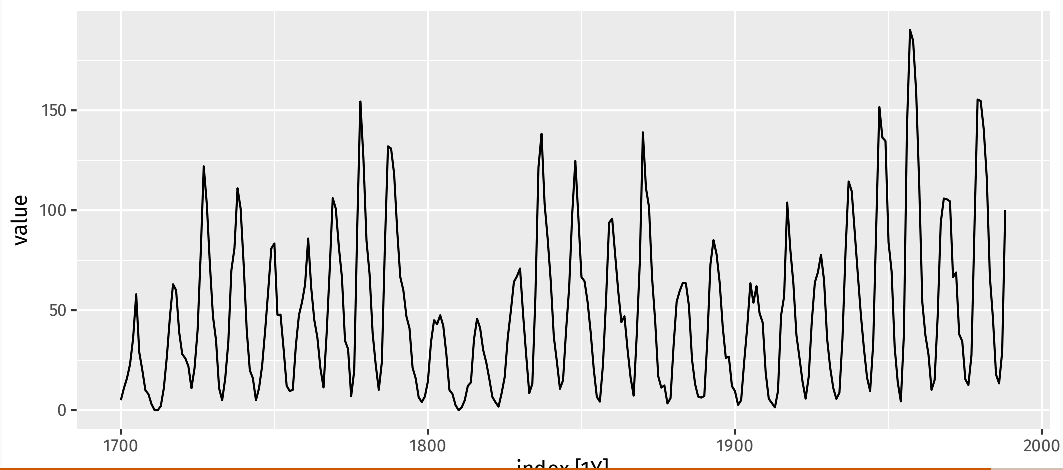
- The `nnetar()` function fits an NNAR$(p, P, k)_m$ model.
- If p and P are not specified, they are automatically selected.
- For non-seasonal time series, default $p =$ optimal number of lags (according to the AIC) for a linear AR(p) model.
- For seasonal time series, defaults are $P = 1$ and p is chosen from the optimal linear model fitted to the seasonally adjusted data.
- Default $k = (p + P + 1)/2$ (rounded to the nearest integer).

Sunspots

- Surface of the sun contains magnetic regions that appear as dark spots.
- These affect the propagation of radio waves and so telecommunication companies like to predict sunspot activity in order to plan for any future difficulties.
- Sunspots follow a cycle of length between 9 and 14 years.

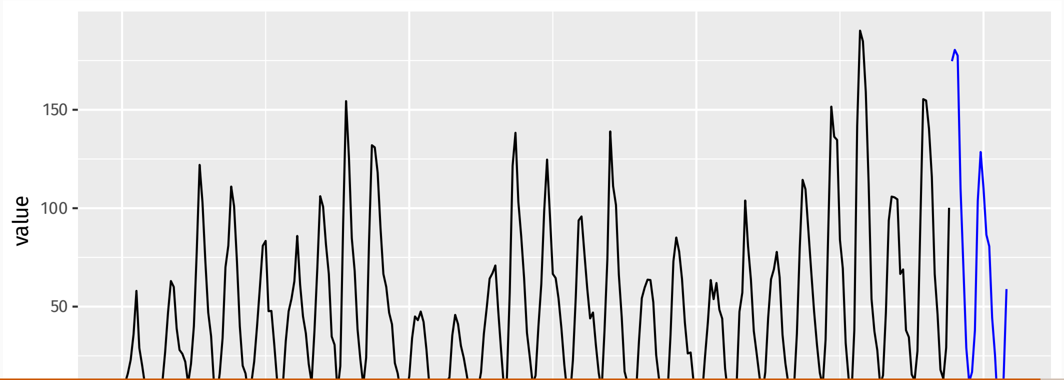
Sunspots

```
sunspots <- sunspot.year |> as_tsibble()
sunspots |> autoplot(value)
```



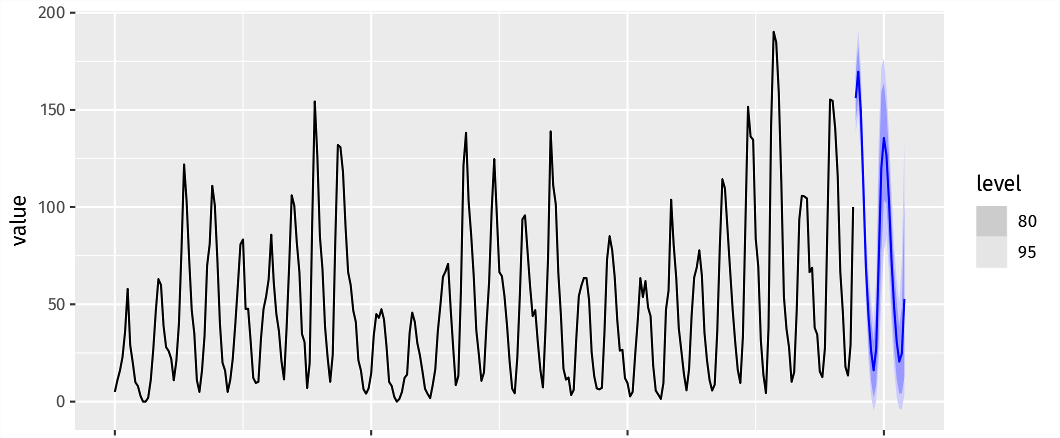
NNAR(9,5) model for sunspots

```
sunspots <- sunspot.year |> as_tsibble()
fit <- sunspots |> model(NNETAR(value))
fit |>
  forecast(h = 20, times = 1) |>
  autoplot(sunspots, level = NULL)
```



Prediction intervals by simulation

```
fit |>
  forecast(h = 20) |>
  autoplot(sunspots)
```



Outline