Chemical Engineering Science 62 (2007) 294–300

Chemical Engineering Science

www.elsevier.com/locate/ces

On the apparent viscosity of a fluidized bed

L.G. Gibilaro[a,*], K. Gallucci[a], R. Di Felice[b], P. Pagliai[b]

[a]*Dipartimento di Chimica, Ingegneria Chimica e Materiali, Università di L'Aquila, Monteluco di Roio, 67040 L'Aquila, Italy*
[b]*Dipartimento di Ingegneria Chimica e di Processo, Università di Genova, Via Opera Pia 15, 16145 Genova, Italy*

Available online 25 August 2006

Abstract

The fluid-dynamic analogy of a typical spherical particle in a fluidized bed as the same particle suspended under terminal conditions in a pseudo-fluid (composed of the fluid and all the other suspended particles) is shown to yield predictive estimates for the apparent viscosity of fluidized beds, in good quantitative agreement with reported experimental measurements for particle concentrations of up to ∼ 40%.

For higher concentrations the predictions fall progressively below measured values, leading to order-of-magnitude underestimates for dense beds at close to minimum fluidization conditions. This latter phenomenon might be ascribable to the dominance, under these conditions, of particle–particle interactions. At high particle concentrations, these would give rise to the observed phenomenon of liquid- and gas-fluidized bed apparent viscosities approaching very similar values, and gas pressure having no influence at all.

Keywords: Fluidization; Apparent viscosity; Pseudo-fluid

1. Introduction

Fluidization is a process whereby a bed of solid particles is transformed into something resembling a fluid: an apparent, or *pseudo*, fluid, possessing apparent fluid properties. Of these, the apparent density presents little conceptual difficulty and no problem at all in expressing directly in terms of the properties of the particle and fluidizing fluid: it becomes simply the mean density of the bed, i.e. its total mass divided by its total volume:

$$\rho_{\mathrm{app}} = \rho_f \varepsilon_f + \rho_p \varepsilon_p. \tag{1}$$

A solid object placed in a fluidized bed will tend to sink if its density is greater than ρ_{app} and to rise to the surface if its density is less than ρ_{app}. The pressure gradient in the bed is given by

$$\frac{\mathrm{d}p}{\mathrm{d}z} = -\rho_{\mathrm{app}} g \tag{2}$$

and this imparts a force on a submerged object, analogous to what would be the Archimedean upthrust in a true fluid. So far so good. However, when it comes to the other basic fluid-dynamic property of a fluidized suspension, its apparent viscosity, the situation becomes less clearly defined. The bed certainly displays a property analogous to fluid viscosity, which influences the velocity with which the submerged objects considered above either fall or rise. But although empirical studies of this phenomenon have been reported, and the measured fall and rise velocities used to determine values of apparent viscosity for the particular experimental conditions employed (see, for example, Prudhoe and Whitmore, 1964; King et al., 1981; Martin et al., 1981; Poletto and Joseph, 1995; Rees et al., 2005), no means have yet been proposed for estimating this important property of a fluidized suspension as a function of the basic system parameters—as becomes immediately possible for the apparent density, Eq. (1). The main purpose of this paper is to present a simple analysis which addresses this deficiency, and to compare its predictions with measured values of apparent viscosity reported in the literature.

Theoretical treatments of the general problem have been limited to cases in which the suspension is maintained without a fluid velocity relative to that of the particles, in particular, for the situation of 'zero buoyancy' particles, having the same density as the liquid in which they are held ($\rho_{\mathrm{app}} = \rho_f = \rho_p$). For this case, the classic Einstein relation, derived rigorously for infinitely dilute suspensions, matches well the reported measurements for very low particle

* Corresponding author. Tel.: +39 0862 434 214; fax: +39 0862 434 203.
E-mail address: gibilaro@ing.univaq.it (L.G. Gibilaro).

0009-2509/$ - see front matter
doi:10.1016/j.ces.2006.08.030

concentrations:

$$\mu_{app} = \mu_f(1 + 2.5\varepsilon_p), \quad \varepsilon_p < 0.01. \tag{3}$$

Also, the cell model of Happel (1957) may be applied over a considerably extended concentration range. Probably the most effective correlation for the purpose is the semi-empirical relation due to Thomas (1965), which matches well the extensive experimental data for apparent viscosity of both zero buoyancy and high-viscosity-fluid suspensions—in which, in contrast to the situation of the fluidized or sedimenting systems considered here, the relative fluid–particle velocity is zero or negligibly small. It should perhaps be mentioned at this stage that particle suspensions in Newtonian fluids will undoubtedly exhibit non-Newtonian behaviour.

2. The pseudo-fluid analogy for a fluidized bed

Implicit in the notion of an apparent viscosity for a fluidized suspension is that of a pseudo-fluid, visualized as a continuum, whose properties depend on those of its component parts: the fluid itself, together with those of the suspended particles. The problem of how these separate properties are to be effectively combined to yield an apparent viscosity is by no means as immediately obvious as is the case for the apparent density, but may be approached by means of the following argument.

Consider a typical particle in a fluidized bed, shown shaded in Fig. 1(a); the forces to which this particle is subjected may be quantified in terms of the primary equilibrium forces for a fluidized suspension, for which various formulations are readily available. Alternatively, the same typical particle may be considered to be suspended alone, under terminal conditions, in a pseudo-fluid consisting of the fluid itself and all the remaining particles—the situation depicted in Fig. 1(b).

The parameters of the pseudo-fluid are its apparent density ρ_{app}, given by Eq. (1), and its, unknown, apparent viscosity μ_{app}. By suitably equating the two alternate views, a value for apparent viscosity may be deduced. It remains only to specify the criterion whereby the equivalence of these separate descriptions of particle suspension is to be defined.

2.1. The bulk mobility of the particles, B_p

A key concept, adopted by Batchelor (1988) in his analysis of fluidization dynamics, is that of particle mobility. This relates to the small change in velocity which the particles in a fluidized suspension would undergo as a result of the application of a small force. Specifically, the fluidized particles, initially under equilibrium conditions ($f = 0$), are subjected to a small force; the particles thus undergo a small change in velocity to return the system to equilibrium; the bulk mobility is simply the ratio of these small changes.

Particle mobility thus represents a model-independent property of the fluidized state, which relates intuitively to the notion of an apparent viscosity—a relationship which we now see to be fully quantifiable under ideal limiting conditions. It thus appears particularly well suited to the purpose in hand.

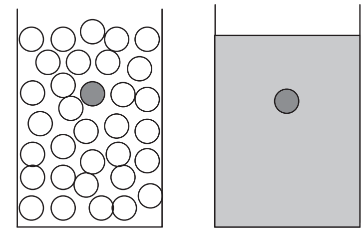

Fig. 1. Alternative views of particle suspension in a fluidized bed: (a) typical particle, one of many, subjected to the equilibrium forces for fluidization; (b) the same typical particle suspended in a pseudo-fluid of density ρ_{app} and viscosity μ_{app}.

As it is the relative fluid–particle velocity, $u_f - u_p$, which responds to an applied force, we may define the *bulk mobility* B_p of the particles with reference to either the fluid or the particle velocity, on the understanding that the other one remains constant:

$$B_p = \left.\frac{\partial u_f}{\partial f}\right|_{f=0,u_p\text{const.}} = -\left.\frac{\partial u_p}{\partial f}\right|_{f=0,u_f\text{const.}}. \tag{4}$$

On applying this definition of B_p to the case of a single sphere swept by a Newtonian fluid under low Reynolds number conditions, for which

$$f = 3\pi d_p \mu_f(u_f - u_p) - \frac{\pi d_p^3}{6}(\rho_p - \rho_f)g, \tag{5}$$

we find the fluid viscosity to be simply inversely proportional to B_p:

$$\frac{1}{B_p} = 3\pi d_p \mu_f. \tag{6}$$

We now apply this concept to the problem of quantifying the equivalence of the two views of particle suspension.

2.2. B_p evaluated from the equilibrium force balance for a fluidized particle

Given an expression for the force on a single particle in a fluidized bed as a function of the relative fluid–particle velocity, the bulk mobility B_p becomes readily available. Consider a typical particle in a fluidized bed; the particle bed model (Foscolo and Gibilaro, 1987; Gibilaro, 2001) employs the following expression for the net force it experiences under equilibrium conditions:

$$f = \frac{\pi d_p^3}{6}(\rho_p - \rho_f)g \times \left[\left(\frac{(u_f - u_p)\varepsilon_f}{u_t}\right)^{4.8/n} \varepsilon_f^{-3.8} - \varepsilon_f\right] = 0; \tag{7}$$

in this relation, the fluid flux term U_o of the original expression (equal to $u_f\varepsilon_f$ for the equilibrium condition of $u_p = 0$) has been expressed in terms of the fluid velocity relative to that of the particles: $U_o = (u_f - u_p)\varepsilon_f$. Eq. (7) thus provides the required expression for the force on a fluidized particle as a function of fluid velocity u_f, from which the bulk mobility may be evaluated for perturbations about the equilibrium state: $u_p=0$, $U_o=u_f\varepsilon_f=u_t\varepsilon_f^n$:

$$\left.\frac{\partial f}{\partial u_f}\right|_{f=0,u_p\text{const.}}=\frac{1}{B_p}=\frac{\pi d_p^3}{6}\cdot\frac{g(\rho_p-\rho_f)}{u_t}\cdot\frac{4.8}{n}\cdot\varepsilon_f^{2-n}, \tag{8}$$

where the single particle terminal velocity u_t and the Richardson–Zaki parameter n may be evaluated as explicit functions of the Archimedes number Ar, rendering B_p immediately available for any fluidized bed, regardless of the flow regime, in terms of the basic fluid and particle properties and the bed void fraction: μ_f, ρ_f, ρ_p, d_p and ε_f (Gibilaro, 2001).

$$Ar=\frac{gd_p^3\rho_f(\rho_p-\rho_f)}{\mu_f^2}, \tag{9}$$

$$u_t=\frac{\mu_f}{d_p\rho_f}[-3.809+(3.809^2+1.832Ar^{0.5})^{0.5}]^2, \tag{10}$$

$$n=\frac{4.8+0.1032Ar^{0.57}}{1+0.043Ar^{0.57}}. \tag{11}$$

2.3. B_p evaluated from the equilibrium force balance for a particle suspended in a pseudo-fluid

We now turn to the alternative view of the typical particle, this time considered suspended alone, in a pseudo-fluid of apparent density ρ_{app} and apparent viscosity μ_{app}. Under low Reynolds number conditions, B_p may be related to μ_{app} by means of Eq. (6), with μ_{app} taking the place of μ_f. More generally, it may be obtained from the single particle limit of Eq. (7) ($\varepsilon_f=1$), expressed in terms of the properties of the pseudo-fluid ($\rho_{\text{app}}=\rho_f$, $\mu_{\text{app}}=\mu_f$); this leads to the required alternative expression for particle mobility:

$$\frac{1}{B_p}=\frac{\pi d_p^3}{6}\cdot\frac{g(\rho_p-\rho_{\text{app}})}{u_{\text{tap}}}\cdot\frac{4.8}{n_{\text{app}}}. \tag{12}$$

The terminal velocity u_{tap} and Richardson–Zaki parameter n_{app} appearing in Eq. (12) are to be evaluated from Eqs. (9)–(11) with: $u_t=u_{\text{tap}}$, $n=n_{\text{app}}$, $\rho_f=\rho_{\text{app}}$, $\mu_f=\mu_{\text{app}}$.

3. The apparent viscosity

We are now in a position to evaluate the apparent viscosity by imposing the criterion that the mobility of a fluidized particle takes on identical values for the two suspension models considered above. Thus, on equating the right hand sides of Eqs. (8) and (12) we obtain

$$u_{\text{tap}}n_{\text{app}}=\frac{\rho_p-\rho_{\text{app}}}{\rho_p-\rho_f}\cdot u_t n\varepsilon_f^{n-2}=u_t n\varepsilon_f^{n-1}. \tag{13}$$

The left hand side of Eq. (13) is a known function of the apparent viscosity μ_{app} , all other terms in the relation being directly available: μ_{app} may therefore be obtained by iteration from Eq. (13) for any defined fluidized bed, regardless of the flow regime.

3.1. Apparent viscosity under low Reynolds number conditions

For the commonly encountered case of low particle Reynolds numbers,

$$Re_t<0.2,\quad Re_{\text{tap}}<0.2,\quad n=n_{\text{app}}=4.8, \tag{14}$$

where

$$Re_t=\frac{\rho_f d_p u_t}{\mu_f},\quad Re_{\text{tap}}=\frac{\rho_{\text{app}}d_p u_{\text{tap}}}{\mu_{\text{app}}}, \tag{15}$$

the relation for μ_{app} reduces to a very simple explicit form. Under these conditions, the bulk mobility relation for fluidized particles, Eq. (8), becomes

$$\frac{1}{B_p}=\frac{\pi_p^3}{6}\cdot\frac{g(\rho_p-\rho_f)}{u_t}\cdot\varepsilon_f^{-2.8}, \tag{16}$$

where the terminal particle settling velocity u_t may now be evaluated using Stokes law:

$$u_t=\frac{g\,d_p^2(\rho_p-\rho_f)}{18\mu_f}, \tag{17}$$

yielding

$$\frac{1}{B_p}=3\pi d_p\mu_f\varepsilon_f^{-2.8}. \tag{18}$$

On equating the right hand side of Eq. (18) with the low Reynolds number relation for the bulk mobility of the single particle in the pseudo-fluid, Eq. (6) with μ_{app} in place of μ_f, we obtain

$$\mu_{\text{app}}=\mu_f\varepsilon_f^{-2.8}=\mu_f(1-\varepsilon_p)^{-2.8}. \tag{19}$$

Note that the density difference term, $\rho_p-\rho_f$, in the fluidized particle bulk mobility expression, Eq. (16), has cancelled out, so that Eq. (19) remains valid for the limiting situation of zero buoyancy suspensions. For low particle concentration, this relation approximates to

$$\mu_{\text{app}}=\mu_f\,(1+2.8\varepsilon_p), \tag{20}$$

in very reasonable accord with the Einstein result, Eq. (3).

4. Comparison of model predictions with experimental measurements

In principle, the predictive expressions derived above for apparent viscosity should be applicable to both gas- and liquid-fluidization. In practice, for reasons referred to in the following section, only liquid beds, with very few exceptions, can give rise to homogeneous dispersions with particle concentrations lower than $\sim 40\%$. For this reason, the results presented in this section relate solely to particle suspensions in liquids.

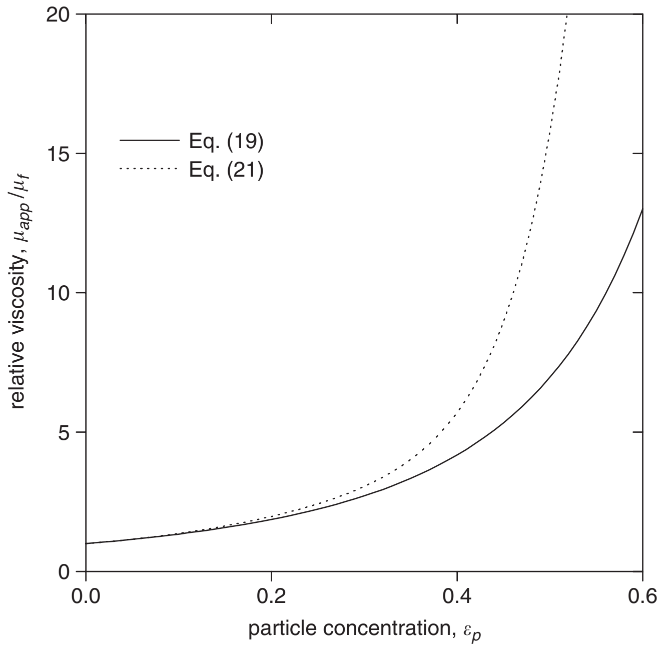

Fig. 2. Apparent viscosity of zero buoyancy and other zero-flux particle suspensions. Solid line: predictions of Eq. (19). Broken line: empirical correlation of Thomas (1965), Eq. (21).

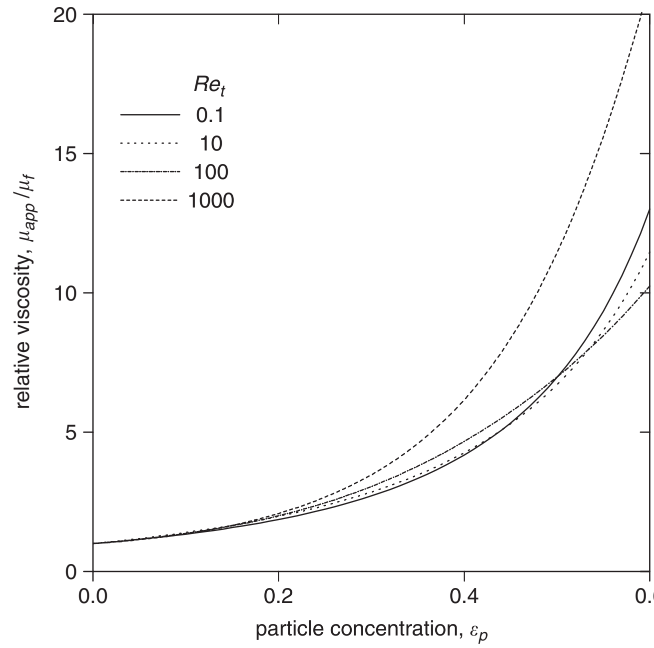

Fig. 3. Apparent viscosity predictions for water fluidization of glass spheres ($\rho_p = 2500\,\text{kg/m}^3$, $\rho_f = 1000\,\text{kg/m}^3$, $\mu_f = 0.001\,\text{Ns/m}^2$): solutions of Eq. (13) for full range of attainable flow regimes.

4.1. Zero buoyancy suspensions

The semi-empirical correlation of Thomas (1965), already referred to, is

$$\frac{\mu_{\text{app}}}{\mu_f} = 1 + 2.5\,\varepsilon_p + 10.05\varepsilon_p^2 + 0.00273\,\exp(16.6\,\varepsilon_p). \qquad (21)$$

This was obtained by fitting the extensive experimental measurements reported in the literature for zero buoyancy particle systems, as well as a number of other effectively stable suspensions (fine powders in high viscosity fluids) requiring no relative fluid–particle motion for their maintenance. Eq. (21) thus represents a limiting case for the fluidized bed relations considered in this work and falls unambiguously in the low Reynolds number regime, for which the analytical relation of Eq. (19) may be applied. These two relations are compared in Fig. 2.

It is clear that the simple predictive expression, Eq. (19), derived above matches well the empirical data up to particle concentrations approaching 0.4. Beyond this point, the predictions fall progressively below measured values, a phenomenon discussed in the following section.

4.2. Fluidized and sedimenting particle suspensions

Apparent viscosity predictions, resulting from solutions of Eq. (13), are shown in Fig. 3 for water fluidization of glass spheres of various diameters, covering the viscous ($Re_t < 0.2$), intermediate ($0.2 < Re_t < 500$) and inertial ($Re_t > 500$) flow regimes. The viscous regime results follow exactly the simple explicit form, Eq. (19). Somewhat surprisingly, deviations from this form remain, for the most part, negligible over the full particle concentration range for the other regimes as well, only the inertial regime showing marked deviations for high concentrations; this suggests the applicability of Eq. (19) as a reasonable approximation for most practical purposes.

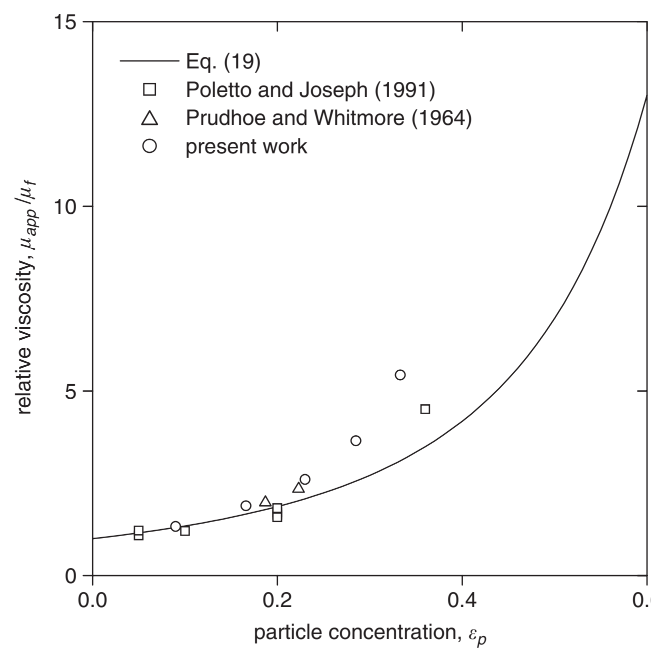

Fig. 4. Apparent viscosities of fluidized and sedimenting particle suspensions in high viscosity fluids—low Reynolds number regime ($Re_t < 0.04$). Comparison of experimental results (points) with model predictions (continuous curve).

Figs. 4 and 5 show the experimental results of various workers for the viscous and intermediate regimes, respectively,

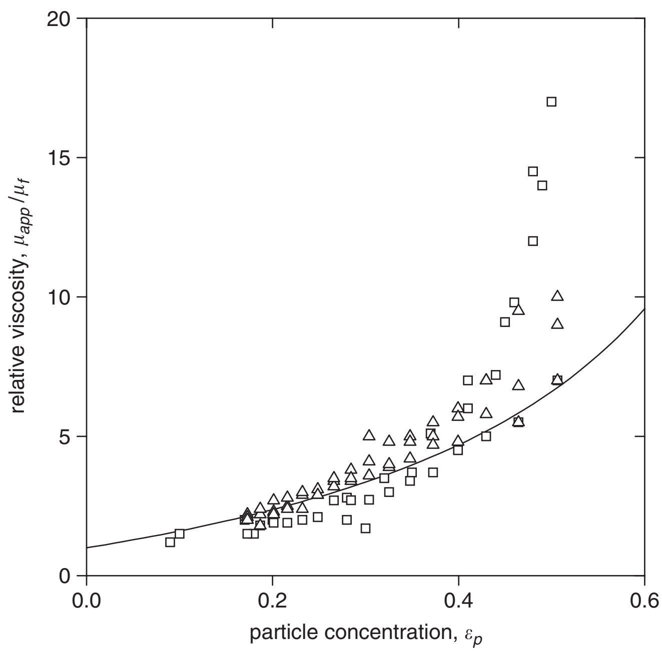

Fig. 5. Apparent viscosities of water-fluidized beds in the intermediate flow regime ($103 > Re_t > 28$). Comparison of experimental results (points) with model predictions (continuous curve). (——) Eq. (13); □ Martin et al. (1981); △ Van der Wielen et al. (1996).

all evaluated from the measured velocity of a sphere falling through either a fluidized bed or a sedimenting suspension. It is clear that the predictions of Eqs. (13) and (19) are in good correspondence with measured apparent viscosities in all cases, up to particle concentrations of around 0.4.

The 'present work' results shown in Fig. 5 were obtained using glass particle suspensions in silicon oil (ε_p up to =0.33, $d_p = 53\,\mu m$, $\rho_p = 2570\,kg/m^3$, $\rho_f = 970\,kg/m^3$, $\mu_f = 1.2\,Pa\,s$). Measurements were made using a lead test sphere of diameter 3.35 mm and density of 11400 kg/m^3, which was allowed to settle through the homogeneous suspension in a 107 mm diameter, 500 mm high cylindrical column. The settling velocity was obtained from the average of 15 tests for each particle concentration. Stoke's equation was then used to obtain the apparent viscosity. Full details of the experimental method are given by Di Felice and Pagliai (2003).

5. High suspended particle concentrations

Apparent viscosities, predicted by means of Eq. (13) or (19), represent intrinsic values, evaluated on the basis of the invisible, equilibrium force interactions within a homogeneously fluidized suspension. The *measurement* of apparent viscosity, however, requires some physical intervention, which can result in extraneous effects, capable of influencing, in many cases dominating, the values obtained. This is particularly so for the most common experimental procedure adopted, which is to allow a dense sphere, having a diameter very much larger than that of the fluidized particles, to fall through the bed, thereby tending to force the fluidized particles into direct contact with one another. Such phenomena are to be expected in highly concentrated beds, which can assume concentrations approaching the close packed limit, for which particle–particle separations are vanishingly small. What is measured in these cases is therefore likely to be more representative of transient granular interaction brought about by the passage of the test sphere, giving rise to measured apparent viscosity values orders of magnitude higher than those of fluid dynamic origin that pertain to the equilibrium fluidized state. Such phenomena could well account for the growing discrepancy between predicted and measured values reported above for particle concentrations in excess of around 0.4. In fact, such conditions represent the norm for gas-fluidization, for which bubble free, effectively homogeneous behaviour, is only possible close to the fixed bed limit ($\varepsilon_p \approx 0.6$, the minimum fluidization point), or, for fine powder systems, down to concentrations typically not far below 0.5. Homogeneous liquid fluidization, on the other hand, is usually stable over the entire realisable concentration range: $0.6 > \varepsilon_p > 0$.

A quantitative analysis of the problems encountered in the measurement of apparent viscosity in high-particle-concentration suspensions is beyond the scope of the present work: the truly enormous variation in reported values, by various authors, using various methods, has been pointed out by Newton et al. (1999) and leaves a question mark over the likelihood of success in such a task. One expected outcome of the above explanation, however, is that the properties of the suspending fluid should cease to play a significant role in whatever it is that occurs when attempts are made at measuring apparent viscosity in high concentration suspensions: there is some evidence to support this expectation which we now briefly examine.

5.1. The effect of gas pressure: the experiments of King et al. (1981)

The apparent viscosity was measured by King et al. (1981) for three gas-fluidized beds (consisting of 475, 101 and 64 μm glass spheres), at pressures ranging from ambient to 20 bar. The measurements were made at 'just below the minimum bubbling velocity' in each case, which for the 475 μm particles would have corresponded to very close to the unchanging minimum fluidization condition: $\varepsilon_p \approx 0.6$, at all pressures. The very high value of apparent viscosity, 2.2 Ns/m^2, found for this bed, obtained from the terminal velocity of a falling stainless steel sphere, remained unchanged over the entire pressure range, in accord with the above hypothesis.

The smallest diameter particle bed exhibited a significant progressive reduction in apparent viscosity with increasing gas pressure. For this system, however, it is known that a region of homogeneous expansion is to be expected before the minimum bubbling point is reached, and that this expansion range *increases with increasing gas pressure*. The particle bed model (Foscolo and Gibilaro, 1987; Gibilaro, 2001) enables this

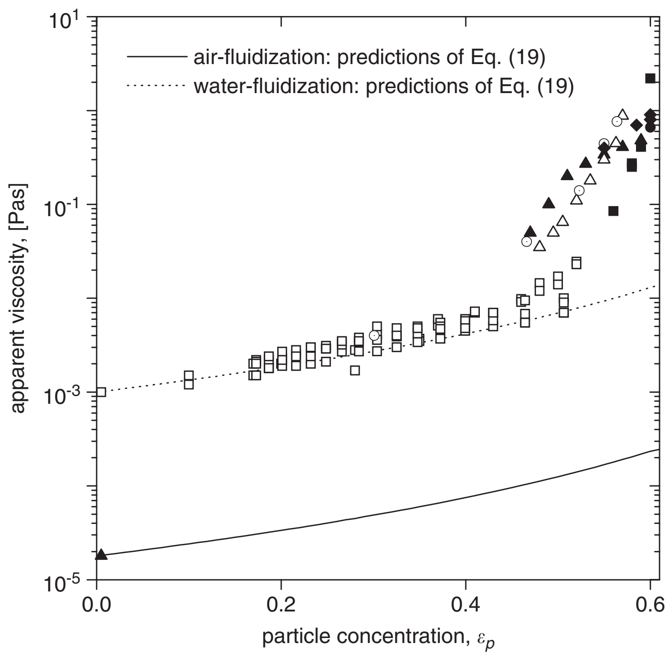

Fig. 6. Apparent viscosities of water- and gas-fluidized beds. Comparison of experimental results for spherical particles (points) with model predictions. Experimental particle concentrations in the gas beds, where not specifically reported, were assumed to be either the minimum fluidization value, $\varepsilon_p = 0.6$, or, for expanded beds of fine powders, the minimum bubbling value, estimated using the Foscolo–Gibilaro (1987) stability criterion. Gas-fluidization results: reported by Grace (1970): solid diamonds; reported by King et al. (1981): solid squares; reported by Rees et al. (2005): solid circles; reported by Reiling (1992): solid triangles. Water-fluidization results: previously reported in Fig. 5: open squares; reported by Tsuchiya et al. (1997): open triangles; reported by Tsuchiya and Furumoto (1995): open circles.

phenomenon to be quantified: for the 64 μm article bed, a particle concentration decrease from 0.60 to 0.58 is predicted over the applied pressure range. Such a reduction is significant close to the fixed bed condition, where all the tests were performed, and could well provide the explanation for the observed reductions in measured apparent viscosity with increasing gas pressure. These results are included in Fig. 6, shown along with other reported measurements of apparent viscosity in gas-fluidized and water-fluidized beds.

5.2. *Gas- and water-fluidization at close to minimum fluidization conditions* ($\varepsilon_p \approx 0.6$)

The problems referred to above with regard to the measurement of apparent viscosity at high particle concentrations are well known and attempts to overcome them, or to compensate for them in one way or another, have been the subject of a number of research studies. A particularly imaginative one is due to Grace (1970), who likened the shape of rising gas bubbles in viscous liquids to their shape in heterogeneously gas-fluidized beds—where they occur naturally, as a result of the intrinsic instability of the homogeneously fluidized state. In spite of obvious mechanistic differences between the two systems, his results for apparent viscosity of the densely fluidized phase turn out to be in reasonable agreement with those obtained by other methods. Problems of local defluidization, which can greatly influence results obtained using falling or rising sphere techniques, are avoided, as the rising bubble is swept by an upflow of gas which maintains the fluidized state immediately above and below it. Severe defluidization problems were encountered, and compensated for, in a recent study, which employed a rising buoyant sphere (Rees et al., 2005), and which led to results in good accord with those found by Grace. These results, together with those of King et al. (1981) and Reiling (1992), for high concentration gas-fluidization are shown in Fig. 6, together with corresponding results for water fluidization: Tsuchiya et al. (1997), Tsuchiya and Furumoto (1995). Although, as is to be expected, considerable scatter is evident, the closeness of the apparent viscosity results for these high particle concentration gas- and liquid-fluidized systems is noteworthy, particularly on consideration of the order of magnitude differences in the viscosities of the fluids involved: this finding supports the hypothesis that granular flow, rather than fluid dynamic, phenomena dominate the measured suspension properties at high particle concentrations.

6. Conclusions

The primary force formulations of the particle bed model for fluidization have been shown to lead to fully predictive estimates for the intrinsic apparent viscosity of a fluidized suspension for particle concentrations of up to ∼40%. Beyond this value, the predictions fall progressively below measured values, almost certainly due to the growing dominance of particle–particle interaction phenomena, brought about by the experimental test conditions adopted. At particle concentrations close to the minimum fluidization condition, this dominance is evidenced by the fact that the measurements cease to be influenced by the properties of the suspending fluid. This offers an explanation for why pressure has no effect on the measured apparent viscosity in gas-fluidized systems, and why both liquid- and gas-fluidization give rise to very similar values for the measured apparent viscosity as the minimum fluidization limit is approached.

The concept of a pseudo-fluid, consisting of a fluid together with a dispersed particle phase, is an extremely simple and useful one, which has found application in the analysis of numerous multiphase systems (see, for example, Di Felice et al., 1991; Poletto and Joseph, 1995; Di Felice, 1998; Wang and Li, 2002). The expressions derived here for its intrinsic apparent viscosity, based on no more than the primary force interactions for homogeneous fluidization, speak loudly for its suitability in this respect, encouraging its adoption as a valid analytical tool in further research. The remarkably compact form for low Reynolds numbers, $\mu_{\text{app}} = \mu_f \varepsilon_f^{-2.8}$ (which, it has been shown, can provide a remarkably close approximation of the model predictions over the entire Reynolds range of practical interest), could be particularly appealing in this respect.

Notation

Ar	Archimedes number, dimensionless
B_p	bulk mobility of the particles, m/Ns
d_p	particle diameter, m
f	net force, N
g	gravitational field strength, N/kg
p	fluid pressure, N/m^2
Re_t	Reynolds number, dimensionless
Re_{tap}	Reynolds number for pseudo-fluid, dimensionless
n	Richardson–Zaki parameter, dimensionless
n_{app}	Richardson–Zaki parameter for pseudo-fluid, dimensionless
u_f	fluid velocity, m/s
u_p	particle velocity, m/s
u_t	terminal particle velocity, m/s
u_{tap}	terminal particle velocity in pseudo-fluid, m/s
U_o	fluid flux, m/s
z	distance variable, m

Greek letters

ε_f	fluid volumetric concentration, dimensionless
ε_p	particle volumetric concentration, dimensionless
μ_{app}	apparent viscosity, Pa s
μ_f	fluid viscosity, Pa s
ρ_{app}	apparent density, kg/m^3
ρ_f	fluid density, kg/m^3
ρ_p	particle density, kg/m^3

Acknowledgement

The financial support of the Ministero dell'Istruzione, dell'Universita' e della Ricerca (MIUR) under their PRIN 2003 and 2005 programmes is gratefully acknowledged.

References

Batchelor, G.K., 1988. A new theory for the instability of a uniform fluidized bed. Journal of Fluid Mechanics 183, 75–110.

Di Felice, R., 1998. The applicability of the pseudo-fluid model to the settling velocity of a foreign particle in a suspension. Chemical Engineering Science 53, 371–375.

Di Felice, R., Pagliai, P., 2003. Interaction of a sphere with a suspension of other spheres. A.I.Ch.E. Journal 49, 3270–3274.

Di Felice, R., Foscolo, P.U., Gibilaro, L.G., Rapagnà, S., 1991. The interaction of particles with a fluid-particle pseudo-fluid. Chemical Engineering Science 44, 1873–1877.

Foscolo, P.U., Gibilaro, L.G., 1987. Fluid dynamic stability of fluidized suspensions: the particle bed model. Chemical Engineering Science 42, 1489–1500.

Gibilaro, L.G., 2001. Fluidization-Dynamics. Butterworth Heinemann, Oxford.

Grace, J.R., 1970. The viscosity of fluidized beds. The Canadian Journal of Chemical Engineering 48, 30–33.

Happel, J., 1957. Viscosity of suspensions of uniform spheres. Journal of Applied Physics 28, 1288–1292.

King, D.F., Mitchell, F.R.G., Harrison, D., 1981. Dense phase viscosities of fluidized beds at elevated pressures. Powder Technology 28, 55–58.

Martin, B.L.A., Kolar, Z., Wesselingh, J.A., 1981. The falling velocity of a sphere in a swarm of different spheres. Transactions of the Institution of Chemical Engineers 59, 100–104.

Newton, D., Smith, G., Hird, N., 1999. Comments and experimental data on the suitability of assigning a viscosity to the fluidized bed dense phase. Fluid–Particle Interactions V: Engineering Foundation Conference, 9–14 May, Santa Fè USA (unpublished presentation).

Poletto, M., Joseph, D.D., 1995. Effective density and viscosity of a suspension. Journal of Rheology 39, 323–343.

Prudhoe, J., Whitmore, R.L., 1964. Terminal velocity of spheres in fluidized beds. British Chemical Engineering 9, 371–375.

Rees, A.C., Davidson, J.F., Dennis, J.S., Hayhurst, A.N., 2005. The rise of a buoyant sphere in a gas-fluidized bed. Chemical Engineering Science 60, 1143–1153.

Reiling, V.G., 1992. Effect of type C particles on cohesion and viscosity of Type A powders. In: Potter, O.E., Nicklin, D.J. (Eds.), Fluidization VII. Engineering Foundation, New York.

Thomas, D.G., 1965. Transport characteristics of suspension: VIII. A note on the viscosity of Newtonian suspensions of uniform spherical particles. Journal of Colloid Science 20, 267–277.

Tsuchiya, K., Furumoto, A., 1995. Tortuosity of bubble rise path in a liquid–solid fluidized bed: effect of particle shape. A.I.Ch.E. Journal 41, 1368–1374.

Tsuchiya, K., Furumoto, A., Fan, L.-S., Zhang, J., 1997. Suspension viscosity and bubble rise velocity in liquid–solid fluidized beds. Chemical Engineering Science 52, 3053–3066.

Van der Wielen, L.A.M., van Dam, M.H.H., Luyben, K.Ch.A.M., 1996. On the relative motion of a particle in a swarm of different particles. Chemical Engineering Science 51, 995–1008.

Wang, W., Li, Y.C., 2002. Pseudo-fluid simulation of transient behaviour in a CFB riser. Chinese Journal of Chemical Engineering 10, 77–83.