seminar 1

PROGRAMMING LANGUAGE SPECIFICATION

1) BNF (Backus-Naur Form)

Language elements:

- meta linguistic variables (nonterminal) - written between <>
- language primitives (terminals) - written between delimiters
- meta linguistic connectors
  - ::= (equals by definition)
  - | (or)

example 1. all nonempty sequences of letters.

<lettsq> ::= <letter>|<letter><lettsq>

<letter> ::= a|b|...|z|A|B|...|Z

example 2. all signed and unsigned integers with the following constraints: - 0 is unsigned

- numbers of at least 2 digits should not start with 0

<nonzerodigit>::= 1|2|...|9

<digit>::= 0|<nonzerodigit>

<sign>::= -|+

<integer>::=0|<unsignedint>|<signedint>

<digitsq>::= <digit>|<digit><digitsq>

<unsignedint>::=<nonzerodigit>|
<nonzerodigit><digitsq>

<signedint>::= <sign><unsignedint>

2) EBNF (Extended BNF)

Wirth's dialect

- nonterminals without <>
- terminals between " "
- ::= becomes =
- {} - 0 or more
- [] - optionality (0 or 1)
- () - for grouping
- (* *) - comments
- rules end with .

<u>example 3.</u> rewrite example 2 in EBNF.

integer = "0" | ["-" | "+"] nonzerodigit {digit}

nonzerodigit = "1" | "2" | ... | "9"

digit = "0" | nonzero

seminar 2

SCANNING ALGORITHM

E = E"+"T | E"-"T | T

T = T"*"F | T"/"F | F

F = "(" E ")" | ID | NOCONST

example 1. Program.txt

```
program test;
var a,b:integer;
    c: string;
begin
    a:=1;
    if (a>=b) then
        b:=2;
    write("message");
end
```

input: text file containing the program list of tokens from "tokens.txt"

output: PIF + ST + lexical errors (if any)

PIF

token	ST_pos
program	-1
id	0
;	-1
var	-1
id	1
,	-1
id	2
:	-1
integer	-1
;	-1
id	3
:	-1
string	-1
;	-1
begin	-1
id	1
:=	-1
const	4
;	-1
if	-1
(	-1

ST - only id + consts

ST_pos	symbol
0	test
1	a
2	b
3	c
4	1
5	2
6	"message"

seminar 3

GRAMMARS

$G=(N,\Sigma,P,\overset{\in N}{S})$

P - set of production

$P \subseteq (N\cup\Sigma)^* N (N\cup\Sigma) \times (N\cup\Sigma)^*$

$N\cup\Sigma = V$

$(N\cup\Sigma)^* = V^*$

ε - empty sequence
(epsilon, not sigma)

$\alpha \to \beta \Leftrightarrow (\alpha,\beta)$

$\alpha \Rightarrow \beta$ iff $\alpha=\alpha_1\gamma_1\beta_1$ & $\beta=\alpha_1\gamma_2\beta_1$ & $\gamma_1\to\gamma_2 \in P$

$\overset{n}{\Rightarrow} n\in\mathbb{N}^*$

$\overset{*}{\Rightarrow}$

$\overset{+}{\Rightarrow}$

direct derivation

$L(G)=\{w\in\Sigma^* \mid S\overset{*}{\to} w\}$ - language generated by the grammar

ex. 1 $G=(N,\Sigma,P,S)$

$N=\{S,C\},\ \Sigma=\{a,b\}$

$P: S\to ab \mid aCSb$

$C\to S \mid bSb$

$CS\to b$

? $w=ab(ab^2)^2 \in L(G)$

$P(n+1): a^{2n+2}bc = a^2 \cdot a^{2n} \cdot bc$

$S \underset{1}{\Rightarrow} a^2 S \overset{*}{\underset{\text{induction}}{\Rightarrow}} a^2 \cdot a^{2n} bc \Rightarrow P(n+1)$ True

From A+B $\Rightarrow$ $P(n)$ is True, $\forall n \in \mathbb{N}$

2? $L(G) \subseteq L$

$S \underset{2}{\Rightarrow} bc = a^{2\cdot 0} bc \in L$

$\underset{1}{\Rightarrow} a^2 S \underset{2}{\Rightarrow} a^2 bc \in L$

$\underset{1}{\Rightarrow} a^4 S \underset{2}{\Rightarrow} a^4 bc \in L$

$\underset{1}{\Rightarrow} \dots$

! true only if you used all combinations in production

ex. 3 $L = \{0^n 1^n 2^m \mid n, m \in \mathbb{N}^*\}$

? G s.t. $L = L(G)$

$S \underset{1}{\Rightarrow} AB$

$A \rightarrow \underset{2}{0A1} \mid \underset{3}{01}$

$B \rightarrow \underset{4}{2} \mid \underset{5}{2B}$

1. $L(G) \subseteq L$

2. $L \subseteq L(G)$

1. $S \Rightarrow AB$

$A \underset{3}{\Rightarrow} 01$

$\underset{2}{\Rightarrow} 0A1 \underset{3}{\Rightarrow} 0^2 1^2$

$\underset{2}{\Rightarrow} 0^2 A 1^2 \underset{3}{\Rightarrow} 0^3 1^3$

$\underset{2}{\Rightarrow} 0^3 A 1^3 \underset{3}{\Rightarrow} \dots$

$\underset{2}{\Rightarrow} \dots$

(i) A only generates seq. of the shape $0^n 1^n$, $n \in \mathbb{N}^*$

① $S \to ab$

② $S \to aCSb$

③ $C \to S$

④ $C \to bSb$

⑤ $CS \to b$

$(ab)^2 \neq a^2b^2$

abab aabb

$S \underset{2}{\Rightarrow} aCSb \underset{4}{\Rightarrow} abSbSb \overset{2}{\underset{1}{\Rightarrow}} ab(ab^2)^2 \Rightarrow S \overset{4}{\Rightarrow} w \Rightarrow w \in L(G)$

ex.2 $G = (N, \Sigma, P, S)$

$N = \{S\}, \Sigma = \{a, b, c\}$

$P: S \to a^2S \mid bc$

? $L(G)$

$L = \{a^{2n}bc \mid n \in \mathbb{N}\}$

? $L = L(G)$

1 ? $L \subseteq L(G)$

2 ? $L(G) \subseteq L$

1 ? $\forall w \in L, w \in L(G)$

$\forall n \in \mathbb{N}, a^{2n}bc \in L(G)$ (induction) $P(n): a^{2n}bc \in L(G), n \in \mathbb{N}$ to prove that $P(n)$ true, $\forall n \in \mathbb{N}$

A. $n = 0 \Rightarrow bc \in L(G)$ True

$S \underset{2}{\Rightarrow} bc$

B. Assume $P(n): a^{2n}bc \in L(G)$ is True $\Rightarrow S \overset{n}{\Rightarrow} a^{2n}bc$

Prove $P(n+1)$ is True

$B \underset{4}{\Rightarrow} 2$

$\Downarrow_5 \; 2B \underset{4}{\Rightarrow} 2^2$

$\Downarrow_5 \; 2^2B \underset{4}{\Rightarrow} 2^3$

$\Downarrow_5 \; 2^3B \underset{4}{\Rightarrow} \dots$

$\Downarrow_5 \; \dots$

(ii) B only generates seq. of terms 5

$2^m, m \in \mathbb{N}^*$

From (i), (ii), 1 $\Rightarrow$ S only generates seq. of terms of the shape $0^n 1^n 2^m$, $n, m \in \mathbb{N}^*$

2. $\forall n, m \in \mathbb{N}^* \Rightarrow 0^n 1^n 2^m \in L(G)$

Let n, m be fixed ($\in \mathbb{N}^*$)

$S \underset{1}{\Rightarrow} AB \underset{(i)}{\overset{n}{\Rightarrow}} 0^n 1^n B \underset{(ii)}{\overset{m}{\Rightarrow}} 0^n 1^n 2^m \Rightarrow S \overset{1+n+m}{\Longrightarrow} 0^n 1^n 2^m \Rightarrow 0^n 1^n 2^m \in L(G)$

(i) $A \overset{n}{\Rightarrow} 0^n 1^n, \forall n \in \mathbb{N}^*$

(ii) $B \overset{m}{\Rightarrow} 2^m, \forall m \in \mathbb{N}^*$

(i): $P(k): A \overset{k}{\Rightarrow} 0^k 1^k$, $k \in \mathbb{N}^*$ and prove $P(k)$ true $\forall k \in \mathbb{N}^*$ by math ind.

A. $k = 1 \Rightarrow A \Rightarrow 01$ (true based on 3)

B. $P(k) \rightarrow P(k+1)$

[$P(k) \Rightarrow A \overset{k}{\Rightarrow} 0^k 1^k$] - ind. hyp.

$P(k+1): A \overset{k+1}{\Rightarrow} 0^{k+1} 1^{k+1}$

$A \underset{2}{\Rightarrow} 0A1 \underset{hyp.}{\overset{k}{\Rightarrow}} 00^k 1^k 1 = 0^{k+1} 1^{k+1} \Rightarrow A \overset{k+1}{\Rightarrow} 0^{k+1} 1^{k+1} \Rightarrow P(k+1)$ true

$\Rightarrow$ proof step true

seminar 4

FINITE AUTOMATA

FA: $M = \{Q, \Sigma, \delta, q_0, F\}$ $\subseteq Q$

$Q = \{q_0, q_1, q_2, q_3, q_7\}$

$\Sigma = \{1, 2, 3\}$

$F = \{q_7\}$

exercise 1.

δ	1	2	3	
q_0	$\{q_0, q_1\}$	$\{q_0, q_2\}$	$\{q_0, q_3\}$	0
q_1	$\{q_1, q_7\}$	$\{q_1\}$	$\{q_1\}$	0
q_2	$\{q_2\}$	$\{q_2, q_7\}$	$\{q_2\}$	0
q_3	$\{q_3\}$	$\{q_3\}$	$\{q_3, q_7\}$	0
q_7	$\emptyset$	$\emptyset$	$\emptyset$	1

$\delta : Q \times \Sigma \to P(Q)$

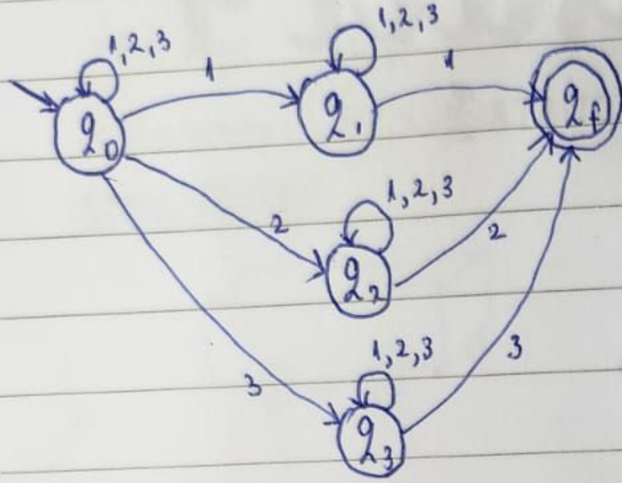
$(q, \underset{\Sigma^*}{\underset{\cap}{x}})$ config. $\qquad (q_0, w)$ - initial config.

(q_7, Σ) - final config.

$(p, ax) \vdash (q, x)$ iff $q \in \delta(p, a)$

$L(M) = \{w \in \Sigma^* \mid (q_0, w) \vdash^* (q_1, \Sigma), q \in \delta\}$

? $w = 12321 \in L(M)$

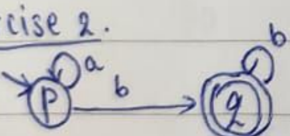


$(q_0, 12321) \vdash (q_1, 2321) \vdash (q_1, 321) \vdash (q_1, 21) \vdash (q_1, 1) \vdash$
$\vdash (q_f, \Sigma)$

$\Rightarrow (q_0, w) \overset{5}{\vdash} (q_f, \Sigma)$

$w \in L(M)$

<u>exercise 2.</u>

M:


? $L(M)$

$L = \{a^n b^m, n \in \mathbb{N}, m \in \mathbb{N}^*\}$

? 1) $L \subseteq L(M)$

? 2) $L(M) \subseteq L$

1) if $n \in \mathbb{N}, m \in \mathbb{N}^*$, $a^n b^m \in L(M)$

Let $n \in \mathbb{N}, m \in \mathbb{N}^*$ be fixed.

$(p, a^n b^m) \underset{a)}{\overset{n}{\vdash}} (p, b^m) \vdash (q, b^{m-1}) \underset{b)}{\overset{m-1}{\vdash}} (q, \Sigma)$

a) $(p, a^n) \overset{n}{\vdash} (p, \Sigma)$, $n \in \mathbb{N}$

b) $(q, b^m) \overset{m}{\vdash} (q, \Sigma)$, $m \in \mathbb{N}$

a) $P(n): (p, a^n) \overset{n}{\vdash} (p, \Sigma)$, $n \in \mathbb{N}$

A. verification

$n=0 \quad (p, \Sigma) \overset{0}{\vdash} (p, \Sigma)$ True

B. proof step

$P(K) \to P(K+1)$, $K \in \mathbb{N}$

$P(K): (p, a^K) \overset{K}{\vdash} (p, \Sigma)$, $K \in \mathbb{N}$

$(p, a^{K+1}) \vdash (p, a^K) \overset{K}{\underset{\text{(ind. hyp.)}}{\vdash}} (p, \Sigma)$

$\Rightarrow P(K+1)$ true

$,1) \vdash$

$(p, a^n b^m) \overset{n+m}{\vdash} (q, \Sigma) \Rightarrow a^n b^m \in L(M)$

2) $a^n \cdot b \cdot b^K$, $n, K \in \mathbb{N}$

$a^n \cdot b^m$, $n \in \mathbb{N}$, $m \in \mathbb{N}^*$

ALL possible paths from initial state to the $\overset{ALL}{\vdash}$ final states.

exercise 3. ? FA

a) $L = \{0^n 1^m 2^q \mid n, m \in \mathbb{N}, q \in \mathbb{N}\}$

b) $L = \{0(01)^n \mid n \in \mathbb{N}\}$

c) $L = \{e^{3n} \mid n \in \mathbb{N}^*\}$

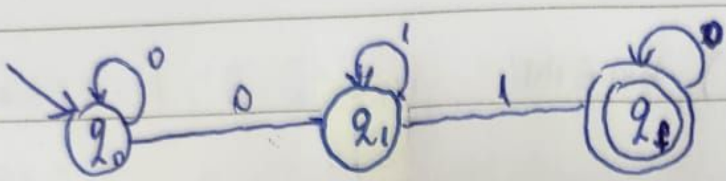
$L = \{e^{3n} \mid n \in \mathbb{N}\}$

d) $\Sigma = \{0, 1\}$ + all sequences have at least two consequtive 0

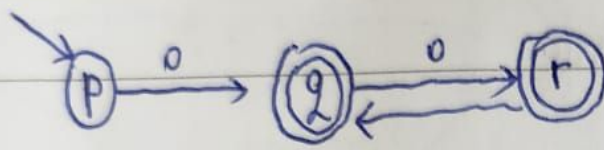
e) integers

f)

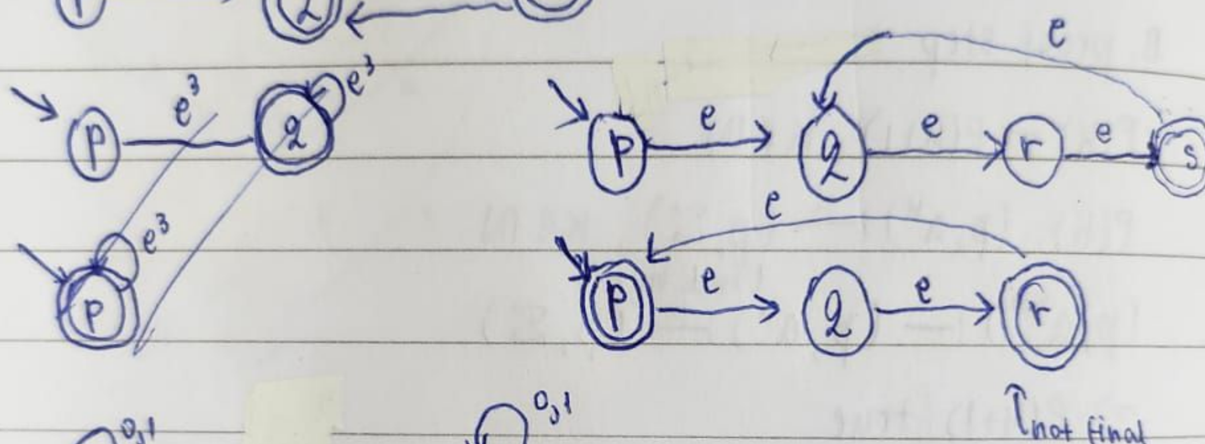
a)



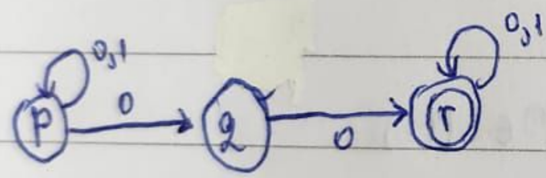
b)



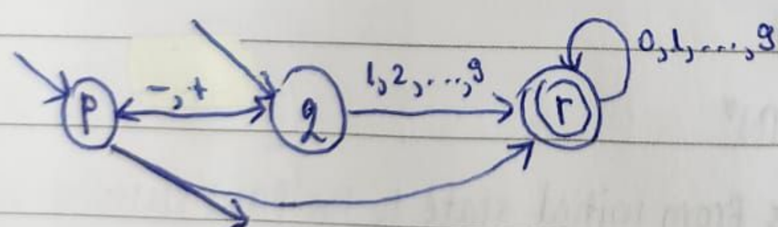
c)



d)



e)



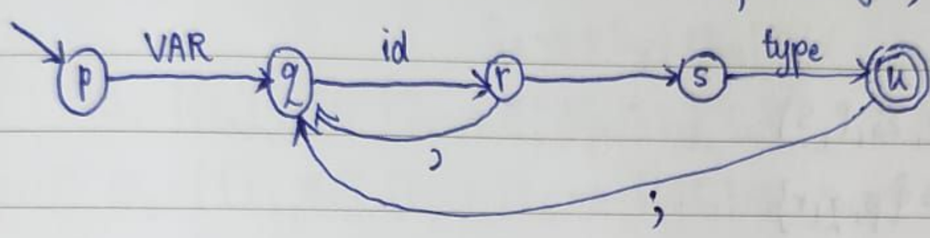
seminar 5

example 1.

$\Sigma = \{$"VAR", type, int, ",", ":", " ", ";"$\}$

VAR

b, a: integer; c, d: char;



FA ⟺ RG ⟺ RE

1) FA ⟺ RG

example 2. $G = \{\{S, A\}, \{a, b\}, P, S\}$

$P: S \to aA$

$A \to aA \mid bA \mid a \mid b$

? FA $M = (Q, \Sigma, \delta, q_0, F)$

$Q = \{S, A, K\}$

$q_0 = S$

$\Sigma = \{a, b\}$

$F = \{K\}$

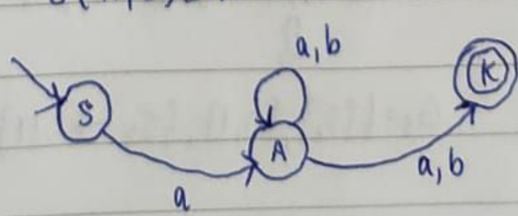
$\delta(S, a) \ni A$

$\delta(A, a) \ni A$

$\delta(A, b) \ni A$

$\delta(A, a) \ni K$

$\delta(A, b) \ni K$



example 3. $M=(Q,\Sigma,\delta,q_0,F)$

$S=\{p,q,r\}$

$q_0=p$

$F=\{r\}$

$\Sigma=\{0,1\}$

? RLG

δ	0	1
p	q	p
q	r	p
r	r	r

$G=(Q,\Sigma,P,S)$

$N=Q=\{p,q,r\}$

$S=q_0=p$

$P: p \to 0q \mid 1p \mid \varepsilon$

$q \to 0r \mid 0 \mid 1p$

$r \to 0r \mid 0 \mid 1r \mid 1$

2) RG $\Leftrightarrow$ RE

example 4. $0(0+1)^*1$

? $\Leftrightarrow$ RG

01, 001, 011, 0001, 0101, ...

$0: G_1=(\{S_1\},\{0,1\},\{S_1\to 0\},S_1)$

$1: G_2=(\{S_2\},\{0,1\},\{S_2\to 1\},S_2)$

$0+1: G_3=(\{S_1,S_2,S_3\},\{0,1\},\{S_1\to 0,\ S_2\to 1,\ S_3\to 0\mid 1\})$

$\Updownarrow$

$G_3=(\{S_3\},\{0,1\},\{S_3\to 0\mid 1\},S_3)$

seminar 6

example 4. (continuation)

$(0+1)^*$: $G_4 = (\{S_3, S_4\}, \{0,1\}, \{S_4 \to \varepsilon|0|1,$
$S_3 \to 0S_4 | 1S_4 | 0 | 1$

$G_4' = \{\{S_4\}, \{0,1\}, \{S_4 \to \varepsilon | 0S_4 | 1S_4\}, S_4\}$

$0(0+1)^*$: $G_5 = (\{S_3, S_1\}, \{0,1\}, \{S_3 \to \varepsilon | 0S_1 | 1S_1, S_1 \to 0S_3\}, S_1)$! not reg

$0(0+1)^*1$: $G_6 = (\{S_1, S_2, S_3\}, \{0,1\}, \{S_2 \to 1, S_3 \to 0S_3 | 1S_3, S_2 \to 0S_3,$
$S_3 \to S_2\}, S_1)$! not reg

$G_6' = (\{S_1, S_3\}, \{0,1\}, \{S_1 \to 0S_3, S_3 \to 0S_3 | 1S_3 | 1\}, S_1)$

example 1. $G = (\{S, A, B\}, \{0,1\}, P, S)$

$P: S \to aA$

$A \to aA | bB | b$

$B \to bB | b$

$X = aX + b$

$X = a^*b$

$aa^*b + b$
$= a^+b + b$
$= (a^+ + \varepsilon)b$
$= a^*b$

? (=) RE

$$\begin{cases} S = aA \\ A = aA + bB + b \\ B = bB + b \end{cases} \Leftrightarrow \begin{cases} B = b^*b = b^+ \\ A = aA + B \\ S = aA \end{cases}$$

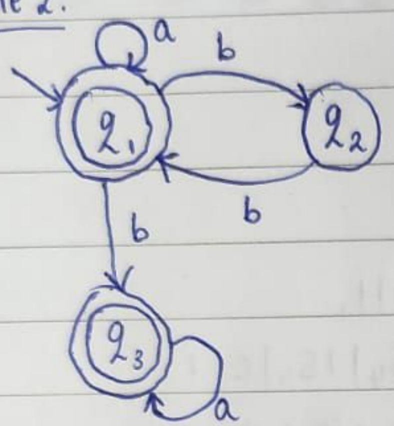
$A = aA + B = aA + b^+ = a^*b^+$

$S = aa^*b^+$

$S = a^+b^+$

3) FA ⇔ RE

example 2.



$X = Xa + b$

$X = ba^*$

? ⇔ RE

$$\begin{cases} q_1 = \varepsilon + q_1 a + q_2 b \\ q_2 = q_1 b \\ q_3 = q_1 b + q_3 a \end{cases} \Leftrightarrow \begin{cases} q_2 = q_1 b \\ q_1 = \varepsilon + q_1 a + q_1 bb \\ q_3 = q_3 a + q_1 b \end{cases}$$

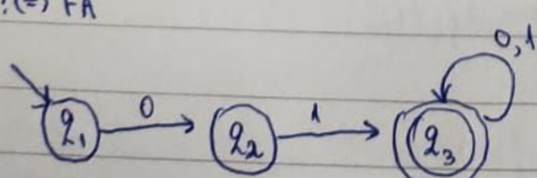
$$q_1 = \varepsilon + q_1(a + bb)$$
$$= (a+bb)^*$$

$$q_3 = (a+bb)^* b + q_3 a = (a+bb)^* b a^*$$

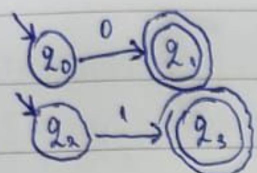
$$q_1 + q_3 = (a+bb)^* + (a+bb)^* b a^*$$
$$= (a+bb)^* (\varepsilon + ba^*)$$

example 3. $01(0+1)^* 1^*$ (RE)

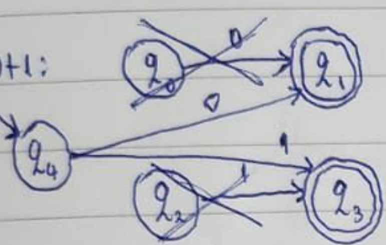
? ⇔ FA



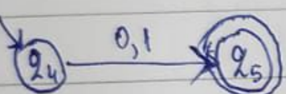
0 :
1 ;



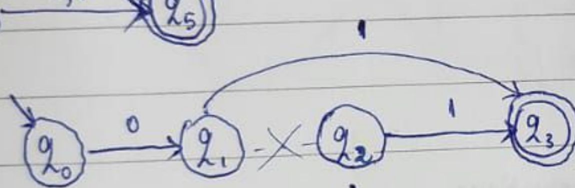
0+1:



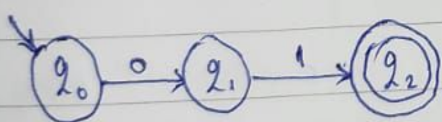
q_0, q_2 - inaccessible



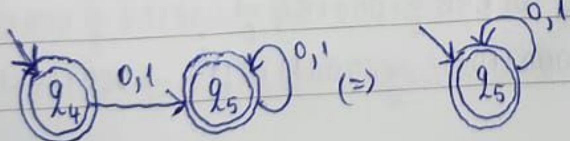
01:



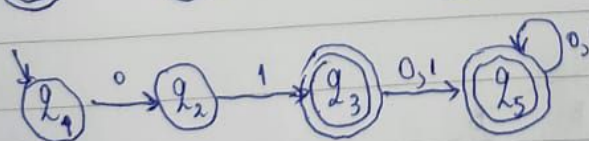
inaccessible



$(0+1)^*$:



$0(0+1)^*$:



seminar 7

CONTEXT FREE GRAMMARS

ex.1. $G = (\{S, A, B\}, \{0, 1\}, P, S)$

$P: S \to \overset{1}{0B} \mid \overset{2}{1A}$

$A \to \overset{3}{0} \mid \overset{4}{0S} \mid \overset{5}{1AA}$

$B \to \overset{6}{1} \mid \overset{7}{1S} \mid \overset{8}{0BB}$

$w = 0001101110$

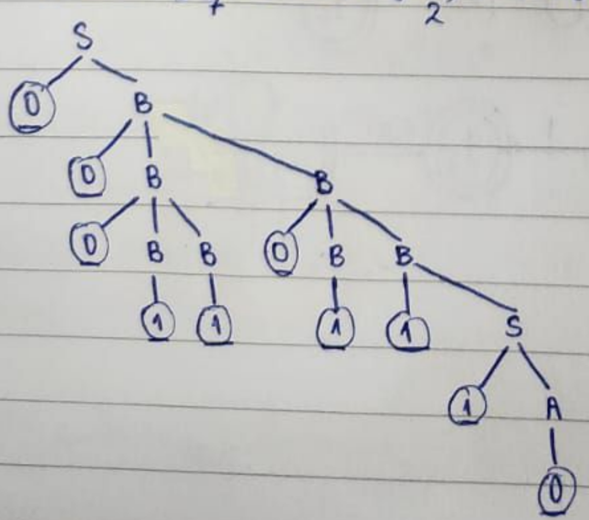
? leftmost / rightmost derivations

for w + corresp. parse tree

1) leftmost

$S \xRightarrow[1]{} 0B \xRightarrow[8]{} 00BB \xRightarrow[8]{} 000BBB \xRightarrow[6]{} 0001BB \xRightarrow[6]{} 00011B \xRightarrow[8]{} 000110BB \xRightarrow[6]{}$

$0001101B \xRightarrow[7]{} 00011011S \xRightarrow[2]{} 000110111A \xRightarrow[3]{} 0001101110$



2) rightmost

$S \xRightarrow[1]{} 0B \xRightarrow[8]{}$ (

00B0

ex.2 prove G

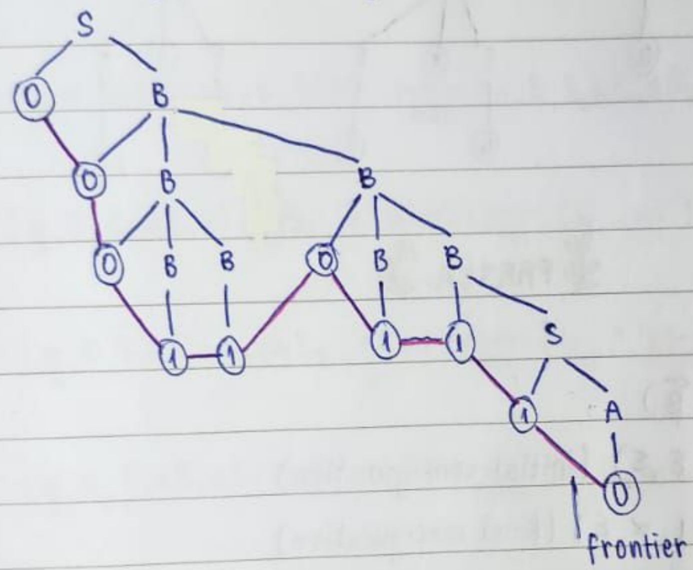
a) G=({S,B

b) G=({E}

a) w = abc

2) rightmost

$S \underset{1}{\Rightarrow} 0B \underset{8}{\Rightarrow} 00BB \underset{8}{\Rightarrow} 00B0BB \underset{7}{\Rightarrow} 00B0B1S \underset{2}{\Rightarrow} 00B0B11A \underset{3}{\Rightarrow} 00B0B110 \underset{6}{\Rightarrow}$
$00B01110 \underset{8}{\Rightarrow} 000BB01110 \overset{2}{\underset{6}{\Rightarrow}} 0001101110$

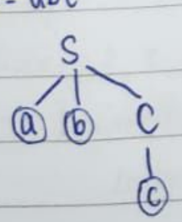


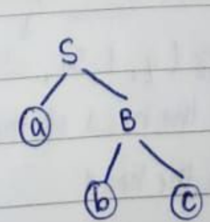
ex.2 prove G is ambiguous

a) $G = (\{S, B, C\}, \{a, b, c\}, \{S \to abC \mid aB, B \to 1C, C \to c\}, S\}$

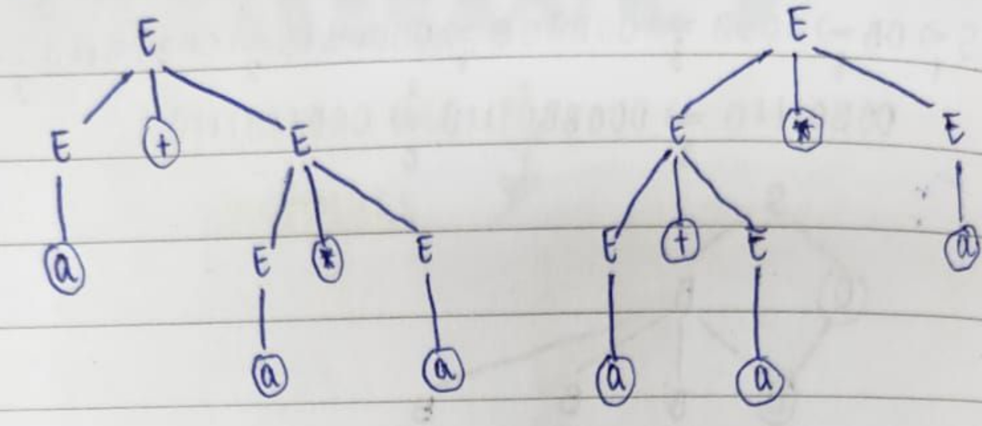
b) $G = (\{E\}, \{a, +, *, (,)\} \{E \to E+E \mid E*E \mid (E) \mid a\}, E)$

a) $w = abc$





b) $w = a+a*a$



PARSER

$(s, i, \vec{\alpha}, \overleftarrow{\beta})$

i.c. $(q, 1, \varepsilon, S)$ (initial configuration)

f.c. $(t, n+1, \alpha, \varepsilon)$ (final configuration)

RECURSIVE DESCENDENT PARSER (RDP)

$G = (\{S\}, \{a, b, c\}, P, S)$

$P: S \rightarrow aSbS \mid aS \mid c$

$w = aacbc$

? $w \in L(G)$ by RDP

$(q, 1, \varepsilon, S) \vdash_{expand} (q, 1, S_1, \overset{\text{top/head}}{\downarrow}aSbS) \vdash_{advance} (q, 2, S_1 a, \overset{\text{top/head}}{\downarrow}SbS)$

! EXPAND only when the head of the input is a nonterminal

! ADVANCE only when the head terminal

$\vdash_{expand} (q, 2, S_1 a S_1, aSbSbS) \vdash_{advance} (q, 3, S_1 a S_1 a, SbSbS)$

$\vdash_{\text{expand}} (q, 3, S_1aS_1aS_1, \underbrace{aS}bSbSbS) \vdash_{\text{momentary insuccess}} (b, 3, S_1aS_1aS_1, \dots)$

$\vdash_{\text{another try 1}} (q, 3, S_1aS_1aS_2, aSbSbS) \vdash_{Mi} (b, \dots) \vdash_{AT1} (q, 3, S_1aS_1aS_3, abSbS)$

$\vdash_{adv.} (q, 4, S_1aS_1aS_3c, bSbS) \vdash_{adv.} (q, 5, S_1aS_1aS_3cb, SbS)$

$\vdash_{exp.} (q, 5, S_1aS_1aS_3cbS_1, aSbSbS) \vdash_{Mi} (b, \dots) \vdash_{AT1}$

$\vdash_{AT1} (q, 5, S_1aS_1aS_3cbS_2, \underset{X}{aSbS}) \vdash_{Mi} (b, \dots) \vdash_{AT1}$

$\vdash_{AT1} (q, 5, S_1aS_1aS_3cbS_3, cbS) \vdash_{adv.} (q, 6, S_1aS_1aS_3cbS_3c, bS)$

$\vdash_{Mi} (b, \dots) \vdash_{back} (b, 5, S_1aS_1aS_3cbS_3, cbS) \vdash_{AT2}$

$\vdash_{AT2} (b, 5, S_1aS_1aS_3cb, SbS) \vdash^{2}_{back} (b, 3, S_1aS_1aS_3, cbSbS)$

$\vdash_{AT2} (b, 3, S_1aS_1a, SbSbS) \vdash_{back} (b, 2, S_1aS_1, aSbSbS) \vdash_{AT1}$

$\vdash_{AT1} (q, 2, S_1aS_2, aSbS) \vdash_{\dots}$

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$(q,1,\varepsilon,S) \vdash_{exp.} \dots \vdash_{AT1} (q,2,S,S_1aS_2, aSbS) \vdash_{adv.} (q,3,S_1aS_2a, SbS)$

$\vdash_{exp.} (q,3,S_1aS_2aS_1, aSbSS) \vdash_{Mi} (b,\dots) \vdash_{AT1}$

$\vdash_{AT1} (q,3,S_1aS_2aS_2, aSbS) \vdash_{Mi} (b,\dots) \vdash_{AT1} (q,3,S_1aS_2aS_3,$ cb

$\vdash_{adv.} (q,4,S_1aS_2aS_3c, bS) \vdash_{adv.} (q,5,S_1aS_2aS_3cb, S) \vdash_{exp.}$

$\vdash_{exp.} (q,5,S_1aS_2aS_3cbS_1, aSbS) \vdash^{4}_{Mi,AT1,Mi,AT1} (q,5,S_1aS_2aS_3cbS$

$\vdash_{adv.} (q,6,S_1aS_2aS_3cbS_3c, \varepsilon) \vdash_{succ.} (f,6,S_1aS_2aS_3cbS_3c,\varepsilon$

LL(1)

$G = (\{S,A,B,C,D\}, \{a,+,*,(,)\}, P, S)$

P:
(1) $S \to BA$
(2) $A \to +BA$
(3) $A \to \varepsilon$
(4) $B \to DC$
(5) $C \to *DC$
(6) $C \to \varepsilon$
(7) $D \to (S)$
(8) $D \to a$

$w = a*(a+a)$

I FIRST +

FIRST(X

DC

$\{(,a\}$

	F_0
S	$\emptyset$
A	$+,\varepsilon$
B	$\emptyset$
C	$*,\varepsilon$
D	(,a

	L_0
S	ε
A	$\emptyset$
B	$\emptyset$
C	$\emptyset$
D	$\emptyset$

I FIRST + FOLLOW

$$FIRST(\underbrace{X_1X_2\dots X_n}_{\alpha\in V^*}) = FIRST(X_1)\oplus FIRST(X_2)\oplus\dots\oplus FIRST(X_n)$$

DC

{(,a} {*,ε}

(*

(

a *

a

{*,ε} {(,a}

*, (, a

	FIRST	FOLLOW
S	(, a	ε,)
A	+, ε	ε,)
B	(, a	+, ε,)
C	*, ε	+, ε,)
D	(, a	*, +, ε,)

	F_0	F_1	F_2	$F_3 = F_2 =$ FIRST
S	∅	∅	(, a	(, a
A	+, ε	+, ε	+, ε	+, ε
B	∅	(, a	(, a	(, a
C	*, ε	*, ε	*, ε	*, ε
D	(, a	(, a	(, a	(, a

	L_0	L_1	L_2	L_3	$L_4 = L_3 =$ FOLLOW
S	ε	ε,)	ε,)	ε,)	ε,)
A	∅	ε	ε,)	ε,)	ε,)
B	∅	+, ε	+, ε,)	+, ε,)	+, ε,)
C	∅	∅	+, ε	+, ε,)	+, ε,)
D	∅	*	*, +, ε	*, +, ε	*, +, ε,)

II LL(1) Parse Table

	a	+	*	(	)	$
S	BA, 1			BA, 1		
A		+BA, 2			ε, 3	ε, 3
B	DC, 4			DC, 4		
C		ε, 6	*DC, 5		ε, 6	ε, 6
D	a, 8			(S), 7		
a	pop					
+		pop				
*			pop			
(				pop		
)					pop	
$						accept

III ANALYSIS

$(a*(a+a)\$, S\$, \varepsilon) \vdash (a*(a+a)\$, BA\$, 1) \vdash (a*(a+a)\$, DCA\$,$

$\vdash (a*(a+a), aCA\$, 148) \vdash_{pop} (*(a+a), CA\$, 148)$

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LL(1) iii ANALYSIS (continuation)

$(*(a+a)\$,\ CA\$,\ 148) \vdash (*(a+a)\$,\ *DCA\$,\ 1485) \vdash_{pop} ((a+a)\$,\ DCA\$,\ 1485)$

$\vdash ((a+a)\$,\ (S)CA,\ 14857) \vdash_{pop} (a+a)\$,\ S)CA\$,\ 14857) \vdash$

$\vdash (a+a)\$,\ BA)CA\$,\ 148571) \vdash (a+a)\$,\ DCA)CA\$,\ 1485714)$

$\vdash (a+a)\$,\ aCA)CA\$,\ 14857148) \vdash_{pop} (+a)\$,\ CA)CA\$,\ 14357148)$

$\vdash (+a)\$,\ A)CA\$,\ 148571486) \vdash (+a)\$,\ +BA)CA\$,\ 1485714862)$

$\vdash_{pop} (a)\$,\ BA)CA\$,\ 1485714862) \vdash (a)\$,\ DCA)CA\$,\ 14857148624)$

$\vdash (a)\$,\ aCA)CA\$,\ 148571486248) \vdash_{pop} ()\$,\ CA)CA\$,\ 14857148624)$

$\vdash^{2} ()\$,\ )CA\$,\ 14857148624 63) \vdash_{pop} (\$,\ CA\$,\ 1485714862463)$

$\vdash^{2} (\$,\ \$,\ 148571486246363) \vdash_{accept}$

how to transform if not in LL(1)?

(1) $A \rightarrow \alpha\beta$
(2) $A \rightarrow \alpha\gamma$

$\Rightarrow$ $A \rightarrow \alpha B$, $B \rightarrow \beta \mid \gamma$

ifstmt → if cond then stmt

  | if cond then stmt else stmt

ifstmt → if cond then stmt B

B → ε | else stmt

LR(0)

example 1. $G = (\{S, A\}, \{a, b, c\}, P, S)$

$S' \rightarrow S$

P: (1) $S \rightarrow aA$

(2) $A \rightarrow bA$

(3) $A \rightarrow c$

$w = abbc \in L(G)$

I Compute the canonical collection of states

$s_0 = closure(\{[S' \rightarrow .S]\}) = \{[S' \rightarrow .S], [S \rightarrow .aA]\}$

$[A \rightarrow \alpha.\beta]$ - LR(0) item

Kernel

$s_1 = goto(s_0, S) = closure(\{[S' \rightarrow S.]\}) = \{[S' \rightarrow S.]\}$

$goto(s_0, A) = closure(\emptyset) = \emptyset$

$s_2 = goto(s_0, a) = closure(\{[S \rightarrow a.A]\}) = \{[S \rightarrow a.A], [A \rightarrow .bA], [A \rightarrow .c]\}$

$goto(s_0, b) = closure(\emptyset) = \emptyset$

$goto(s_0, c) = \emptyset$

$s_3 = goto(s_2, A) = closure(\{[S \rightarrow aA.]\}) = \{[S \rightarrow aA.]\}$

$s_4 = goto(s_2, b) = closure(\{[A \rightarrow b.A]\}) = \{[A \rightarrow b.A], [A \rightarrow .bA], [A \rightarrow .c]\}$

$s_5 = goto(s_2, c) = closure(\{[A \rightarrow c.]\}) = \{[A \rightarrow c.]\}$

$s_6 = goto(s_4, A) = closure(\{[A \rightarrow bA.]\}) = \{[A \rightarrow bA.]\}$

$goto(s_4, b) = closure(\{[A \rightarrow b.A]\}) = s_4$

$goto(s_4, c) = closure(\{[A \rightarrow c.]\}) = s_5$

$\mathcal{C} = \{s_0, s_1, \ldots, s_6\}$

II LR(0) parsi

	A
0	SHIFT
1	ACCEPT
2	SHIFT
3	REDUCE
4	SHIFT
5	REDUCE
6	REDUCE

III Analysis

WORK STA

$0

$0a2

$0a2b4

$0a2b4b

$0a2b4b4

$0a2b4b4

$0a2b4A6

$0a2A3

$0S1

accepted.

II LR(0) parsing table

	ACTION	GOTO				
		S	A	a	b	c
0	SHIFT	1		2		
1	ACCEPT					
2	SHIFT		3		4	5
3	REDUCE (1)					
4	SHIFT		6		4	5
5	REDUCE (3)					
6	REDUCE (2)					

III Analysis

WORK STACK	INPUT STACK	OUTPUT BAND
\$0	abbc\$	ε
\$0a2	bbc\$	ε
\$0a2b4	bc\$	ε
\$0a2b4b4	c\$	ε
\$0a2b4b4<u>c5</u>	\$	ε
\$0a2b4b4<u>A6</u>	\$	3
\$0a2<u>b4A6</u>	\$	23
\$<u>0a2A3</u>	\$	223
\$0S1	\$	1223

accepted.

seminar 10

SLR

$G = (\{E,T\}, \{id, const, +, (,)\}, P, S)$

$P:$ $S' \rightarrow E$

(1) $E \rightarrow T$

(2) $E \rightarrow E+T$

(3) $T \rightarrow (E)$

(4) $T \rightarrow id$

(5) $T \rightarrow const$

$w = id + const \in L(G)$

I Canonical collection

$s_0 = closure(\{[S' \rightarrow .E]\}) = \{[S' \rightarrow .E], [E \rightarrow .T], [E \rightarrow .E+T], [T \rightarrow .(E)],$
$[T \rightarrow .id], [T \rightarrow .const]\}$

$s_1 = goto(s_0, E) = closure(\{[S' \rightarrow E.], [E \rightarrow E.+T]\}) = \{[S' \rightarrow E.], [E \rightarrow E.+T]\}$

$s_2 = goto(s_0, T) = closure(\{[E \rightarrow T.]\}) = \{[E \rightarrow T.]\}$

$s_3 = goto(s_0, () = closure(\{[T \rightarrow (.E)]\}) = \{[T \rightarrow (.E)], [E \rightarrow .E+T], [E \rightarrow .T],$
$[T \rightarrow .(E)], [T \rightarrow .id], [T \rightarrow .const]\}$

$s_4 = goto(s_0, id) = closure(\{[T \rightarrow id.]\}) = \{[T \rightarrow id.]\}$

$s_5 = goto(s_0, const) = closure(\{[T \rightarrow const.]\}) = \{[T \rightarrow const.]\}$

$s_6 = goto(s_1, +) = closure(\{[E \rightarrow E+.T]\}) = \{[E \rightarrow E+.T], [T \rightarrow .(E)], [T \rightarrow .id]$
$[T \rightarrow .const]\}$

$s_7 = goto(s_3, E) = closure(\{[T \rightarrow (E.)], [E \rightarrow E.+T]\}) = \{[T \rightarrow (E.)], [E \rightarrow E.+T$

goto(s_3, T

goto(s_3, (

goto(s_3,

goto(s_3, id

s_8 = goto(s_6, T

goto(s_6, (

goto(s_6, i

goto(s_6, c

s_9 = goto(s_7,)

goto(s_7, +

II SLR Parsing

FOLLOW(E) =

FOLLOW(T) =

	id
0	SHIFT 4
1	
2	
3	SHIFT 4
4	
5	
6	SHIFT 5
7	
8	
9	

$goto(s_3, T) = closure(\{[E \to T.]\}) = s_2$

$goto(s_3, () = closure(\{[T \to (.E)]\}) = s_3$

$goto(s_3, const) = s_5$

$goto(s_3, id) = s_4$

$s_8 = goto(s_6, T) = closure(\{[E \to E+T.]\}) = \{[E \to E+T.]\}$

$goto(s_6, () = s_3$

$goto(s_6, id) = s_4$

$goto(s_6, const) = s_5$

$s_9 = goto(s_7,)) = \{[E \to (T).]\}$

$goto(s_7, +) = s_6$

II SLR Parsing Table

$FOLLOW(E) = \{\varepsilon, +,)\}$

$FOLLOW(T) = \{\varepsilon, +,)\}$

	ACTION						GOTO	
	id	const	+	(	)	$	E	T
0	SHIFT 4	SHIFT 5		SHIFT 3			1	2
1			SHIFT 6			ACC.		
2			RED. 1	RED. 1		RED. 1		
3	SHIFT 4	SHIFT 5		SHIFT 3			7	2
4			RED. 4		RED. 4	RED. 4.		
5			RED. 5		RED. 5	RED. 5		
6-	SHIFT 5	SHIFT 4		SHIFT 3				8
7			SHIFT 6		SHIFT 9			
8			RED. 2		RED. 2	RED. 2		
9			RED. 3		RED. 3	RED. 3		

III Analysis

WORK STACK	INPUT STACK	OUTPUT BAND
\$0	id+const \$	ε
\$0 id4	+ const \$	ε
\$0T2	+ const \$	4
\$0E1	+ const \$	14
\$0E1+6	const \$	14
\$0E1+6 const5	\$	14
\$0E1+6T8	\$	514
\$0E1	\$	2514

accept

seminar 11

LR(1) parser

$G = (\{S, A\}, \{a, b\}, P, S)$

$S' \rightarrow S.$

$P:$ (1) $S \rightarrow AA$

(2) $A \rightarrow aA$

(3) $A \rightarrow b$

$w = abab \overset{?}{\in} L(G)$

LR(1) item: $[A \rightarrow \alpha . \beta, \underset{\underset{\Sigma \cup \{\$\}}{\shortparallel}}{a}]$

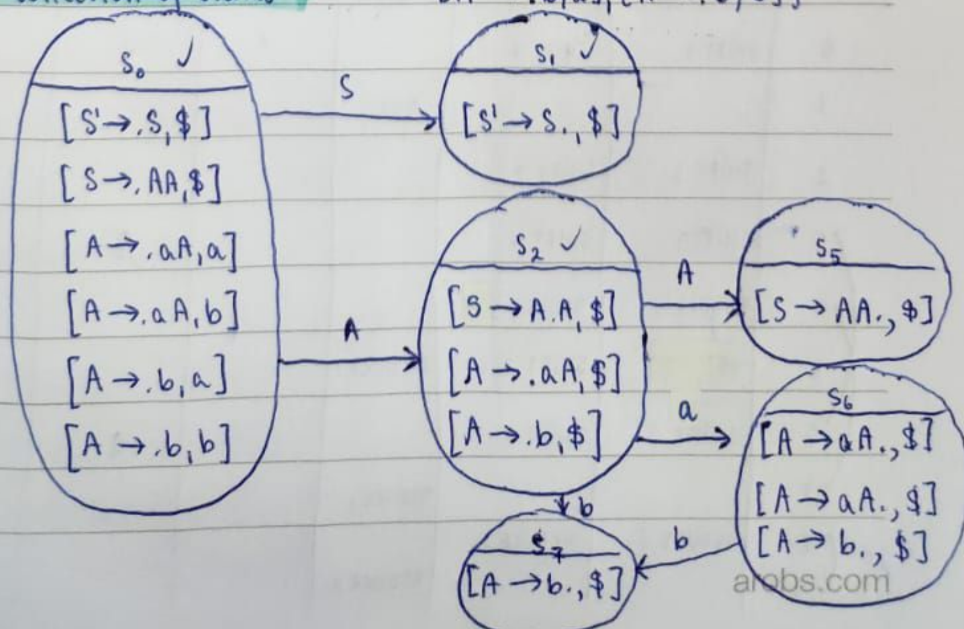
first of ternal - a set of

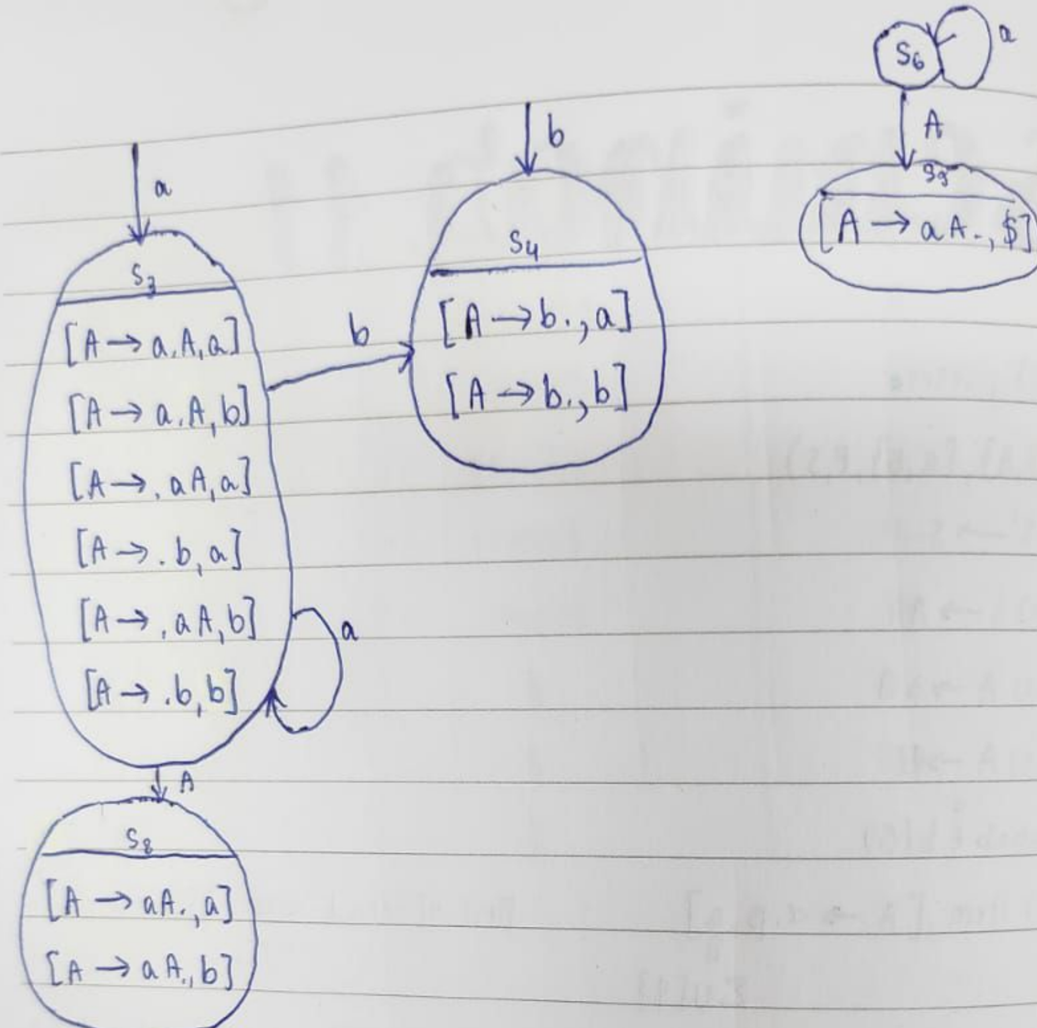
$FIRST(A) = \{a, b\}$

$FIRST(S) = \{a, b\}$

$s_0 = closure(\{[S' \rightarrow .S, \$]\}) = \{[S' \rightarrow .S, \$], [S \rightarrow .AA, \$], [A \rightarrow .aA, a], [A \rightarrow aA, b], [A \rightarrow .b, a], [A \rightarrow .b, b]\}$

I Canonical collection of states





II Parsing table

	ACTION			GOTO	
	a	b	$	S	A
0	SHIFT 3	SHIFT 4		1	2
1			ACC.		
2	SHIFT 6	SHIFT 7			5
3	SHIFT 3	SHIFT 4			8
4	REDUCE 3	REDUCE 3			
5			REDUCE 1		
6	SHIFT 6	SHIFT 7			
7					9
8	REDUCE 2	REDUCE 2	REDUCE 3		
9			REDUCE 2		

III Analysis

WORK

$0

$0a3

$0a3b4

$0a3A

$0A2

$0A2a6

$0A2a6

$0A2a

$0A2A

$0S1

acc.

LALR

I Canonica

S_8+S_9

$S_{89}=\{$

$S_4+S_7=$

$S_{47}=\{$

S_3+S_6

$S_{36}=$

$\mathcal{C}=\{S_0$

III Analysis

WORK STACK	INPUT STACK	OUTPUT BAND
$0	abab$	ε
$0a3	bab$	ε
$0a3b4	ab$	ε
$0a3A8	ab$	3
$0A2	ab$	23
$0A2a6	b$	23
$0A2a6b7	$	23
$0A2a6A9	$	323
$0A2A5	$	2323
$0S1	$	12323
acc.		

LALR

I Canonical collection of states

$s_8 + s_9 \rightarrow s_{89}$

$s_{89} = \{[A \rightarrow aA., a|b|\$]\}$

$s_4 + s_7 = s_{47}$

$s_{47} = \{[A \rightarrow b., a|b|\$]\}$

$s_3 + s_6 = s_{36}$

$s_{36} = \{[A \rightarrow a.A, a|b|\$],$
$[A \rightarrow .aA, a|b|\$],$
$[A \rightarrow .b, a|b|\$]\}$

$\mathcal{C} = \{s_0, s_1, s_2, s_{36}, s_{47}, s_{89}, s_5\}$

II Parsing table

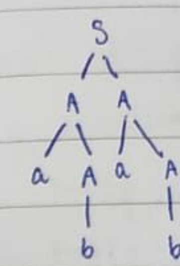
	ACTION			GOTO	
	a	b	$	S	A
0	SHIFT 36	SHIFT 47		1	2
1			ACC.		
2	SHIFT 36	SHIFT 47			5
36	SHIFT 36	SHIFT 47			89
47	REDUCE 3	REDUCE 3	REDUCE 3		
5			REDUCE 1		
89	REDUCE 2	REDUCE 2			

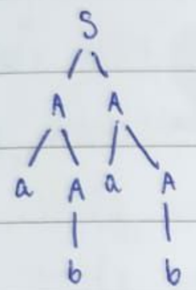
III Analysis

WORK STACK	INPUT STACK	OUTPUT BAND
$0	abab$	ε
$0a36	bab$	ε
$0a36b47	ab$	ε
$0a36A89	ab$	3
$0A2	ab$	23
$0A2a36	b$	23
$0A2a36b47	$	23
$0A2a36A89	$	23
$0A2A5	$	323
$0S1	$	2323
acc.	$	12323

R_1: 12323

R_2: $S \underset{1}{\Rightarrow} AA \underset{2}{\Rightarrow} AaA \underset{3}{\Rightarrow} Aab \underset{2}{\Rightarrow} aAab \underset{3}{\Rightarrow} abab$





R_3

index	symbol	F	S
0	S	-1	-1
1	A	0	2
2	A	0	-1
3	a	1	4
4	A	1	-1

seminar 12

PDA

$M=(Q,\Sigma,\Gamma,\delta,q_0,Z_0,F)$

Γ → stack memory alphabet

Z_0 → initial stack symbol

$FA: \delta: Q\times\Sigma \to \mathcal{P}(Q)$

$PDA: \delta: Q\times\Sigma\cup\{\varepsilon\}\times\Gamma \to \mathcal{P}(Q\times\Gamma^*)$

$L_f(M)=\{w\in\Sigma^* \mid (q_0,w,Z_0)\vdash^* (q_f,\Sigma,\gamma),\ \gamma\in\Gamma,\ q_f\in F\}$ → by the <u>final state</u> criterion

$L_\varepsilon(M)=\{w\in\Sigma^* \mid (q_0,w,Z_0)\vdash^* (q,\varepsilon,\varepsilon),\ q\in Q\}$ → by the <u>empty stack</u> criterion

<u>example 1.</u> ?PDA $L_1=\{ww^R \mid w\in\{a,b\}^+\}$ (R → reverse), $a\overset{R}{a}$, $b\overset{R}{b}$, $ab\overbrace{ba}^{R}$, $abb\overbrace{bba}^{R}$, ...

$M=(\{q_0,q_1\},\{a,b\},\{Z_0,A,B\},\delta,q_0,Z_0,\varepsilon)$

$\delta(q_0,a,Z_0)\ni(q_0,AZ_0)$

$\delta(q_0,b,Z_0)=(q_0,BZ_0)$

$\delta(q_0,a,A)=(q_0,AA)$

$\delta(q_0,a,B)=(q_0,AB)$

$\delta(q_0,b,B)=(q_0,BB)$

$\delta(q_0,b,A)=(q_0,BA)$

$\delta(q_0,a,A)=(q_1,\varepsilon)$

$\delta(q_0,b,B)=(q_1,\varepsilon)$

$\delta(q_1,a,A)=(q_1,\varepsilon)$

$\delta(q_1,b,B)=(q_1,\varepsilon)$

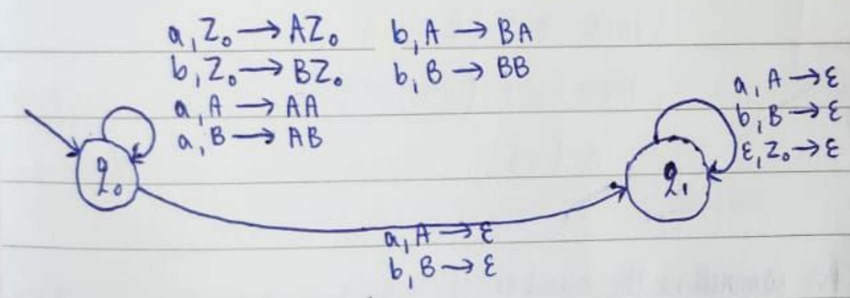
$\delta(q_1,\varepsilon,Z_0)=(q_1,\varepsilon)$

$ww^R =$ abbaabba

$(q_0, abbaabba, Z_0) \vdash (q_0, bbaabba, AZ_0) \vdash (q_0, baabba, BAZ_0) \vdash$
$\vdash (q_0, aabba, BBAZ_0) \vdash (q_0, abba, ABBAZ_0) \vdash (q_1, bba, BBAZ_0) \vdash$
$\vdash (q_1, ba, BAZ_0) \vdash (q_1, a, AZ_0) \vdash (q_1, \varepsilon, Z_0) \vdash (q_1, \varepsilon, \varepsilon)$

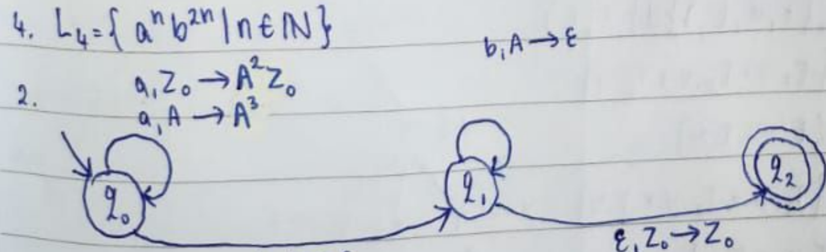
$ww^R =$ abbabaab

$(q_0, abbabaab, Z_0) \vdash \ldots (q_0, baab, ABBAZ_0) \vdash (q_0, aab, BABBAZ_0) \vdash$
$\vdash (q_0, ab, ABABBAZ_0) \vdash (q_0, b, AABABBAZ_0) \vdash (q_0, \varepsilon, BAABABBAZ_0)$
$\vdash (q_1, b, BABBAZ_0) \vdash (q_1, \varepsilon, A \ldots)$

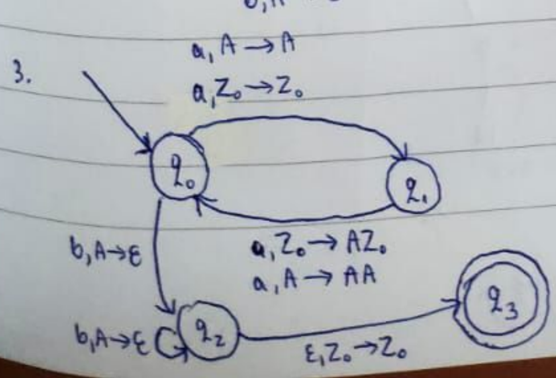


2. $L_2 = \{ a^n b^{2n} \mid n \in \mathbb{N}^* \}$
3. $L_3 = \{ a^{2n} b^n \mid n \in \mathbb{N}^* \}$
4. $L_4 = \{ a^n b^{2n} \mid n \in \mathbb{N} \}$

2.



3.



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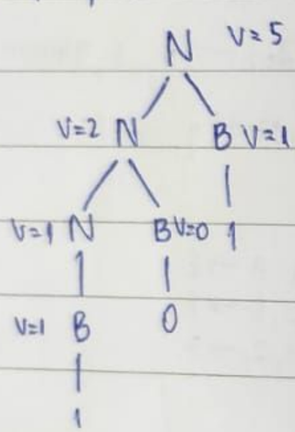
ATTRIBUTE GRAMMARS

cfg

(G, A, R)



(1) $N \to NB$ $\{N_1.v = 2 * N_2.v + B.v\}$

(2) $N \to B$ $\{N.v = B.v\}$

(3) $B \to 0$ $\{B.v = 0\}$

(4) $B \to 1$ $\{B.v = 1\}$

$A = \{v\}$

exercise 1. AG for computing the number of vowels in a non-empty string

exercise 2. AG for computing the value of an arithmetic expression with +, *, (,).

$G = (\{E, T, F\}, \{+, *, (,), id\}, P, E)$

$E \to E + T$ $\{E_1.v = E_2.v + T.v\}$

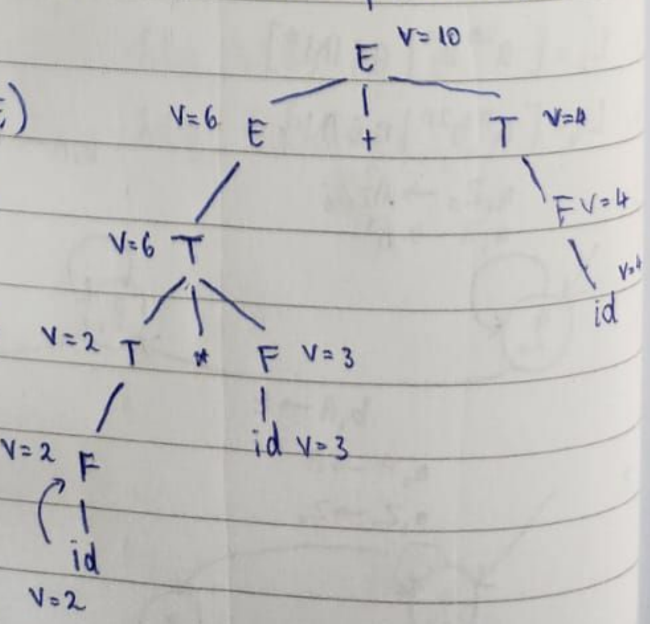
$E \to T$ $\{E.v = T.v\}$

$T \to T * F$ $\{T_1.v = T_2.v * F.v\}$

$T \to F$ $\{T.v = F.v\}$

$F \to (E)$ $\{F.v = E.v\}$

$F \to id$ $\{F.v = id.v\}$



exercise 1. $G = (\{S, L, C, V\}, \{a, b, c, \dots, z\}, P, S)$

$S \to L$ $\{S.n = L.n\}$

$S \to SL$ $\{S_1.n = S_2.n + L.n\}$

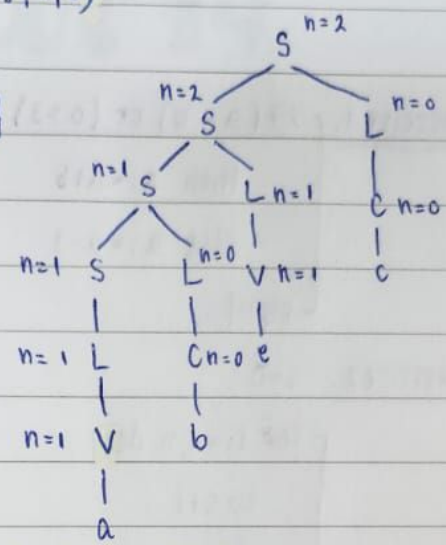
$L \to V$ $\{L.n = V.n\}$

$L \to C$ $\{L.n = C.n\}$

$V = a|e|i|o|u$ $\{C.n = 1\}$

$C = b|c|\dots|z$ $\{C.n = 0\}$

abec



exercise 3. AG for verifying a natural number is divisible by 3.

$N \to D$ $\{N.s = D.s;\ N.d = (D.s \% 3 == 0)\}$

$N \to AS$ $\{N.s = A.s + S.s;\ N.d = (N.s \% 3 == 0)\}$

$A \to 1$ $\{A.s = 1\}$

$\vdots$

$A \to 9$ $\{A.s = 9\}$

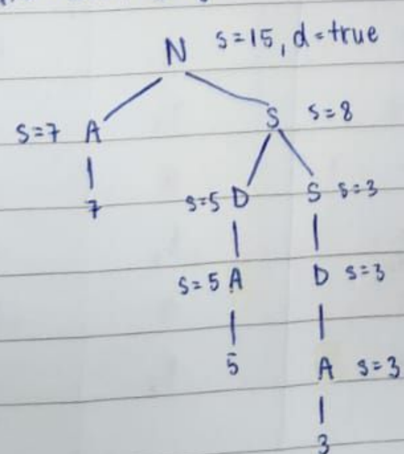
$D \to 0$ $\{D.s = 0\}$

$D \to A$ $\{D.s = A.s\}$

$S \to D$ $\{S.s = D.s\}$

$S \to DS$ $\{S_1.s = D.s + S_2.s\}$

753



3-ADDRESS CODE

<u>exercise 1.</u>

```
if (a<b) or (a>3) and (a<10)
    then a:=a+b
    else a:=a-3
endif
```

<u>exercise 2.</u> s=0

```
for i:=1,n do
    s:=s+i
endfor
```

exercise 1.

label	operator	arg 1	arg 2	result
1	<	a	b	t_1
2	>	a	3	t_2
3	<	a	10	t_3
4	and	t_2	t_3	t_4
5	or	t_1	t_4	t_5
6	goto	t_5		(9)
7	-	a	3	t_7
8 9	:=	t_7		a
10	goto			(12)
11	+	a	b	t_8
12	:=	t_8		a

exercise 2. s

label
1
2
3
4
5
6
7
8
9
10
11
12

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exercise 2. s:=0

```
for i:=1,n do
    b:=-i;
    s:=s+b;
endfor
```

label	operator	arg 1	arg 2	result
1	:=	0		s
2	:=	1		i
3	>	i	n	t_1
4	goto	t_1		(12)
5	@ (-unar)	i		t_2
6	:=	t_2		b
7	+	s	b	t_3
8	:=	t_3		s
9	+	i	1	t_4
10	:=	t_4		i
11	goto			3
12				