We take the diffusion coefficient to be constant and take it and any constant multiples of other defining terms to be 1.

In[106]:=
```
DiffusionHeatEqn = D[u[x, t], t] == D[u[x, t], {x, 2}];
(* Defines the diffusion eqn
 (for constant diffusion constant the diffusion eqn is just the heat eqn) *)
```

Below we consider various cases of distributions.

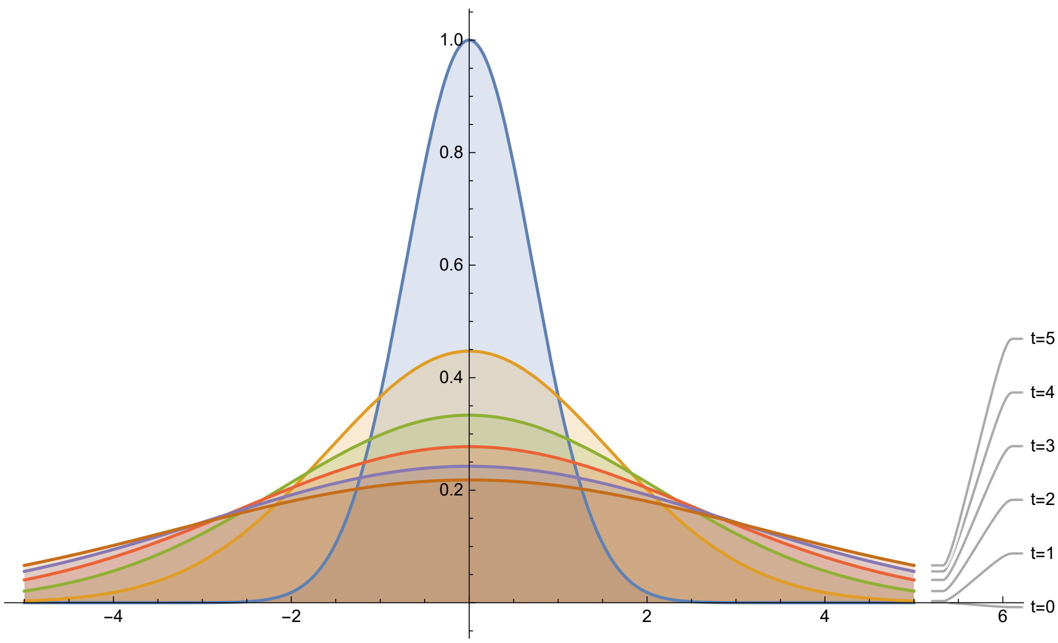
The 2D plots are the solutions at various time steps. The plots are labelled according to their time steps.

Gaussian Initial Distribution

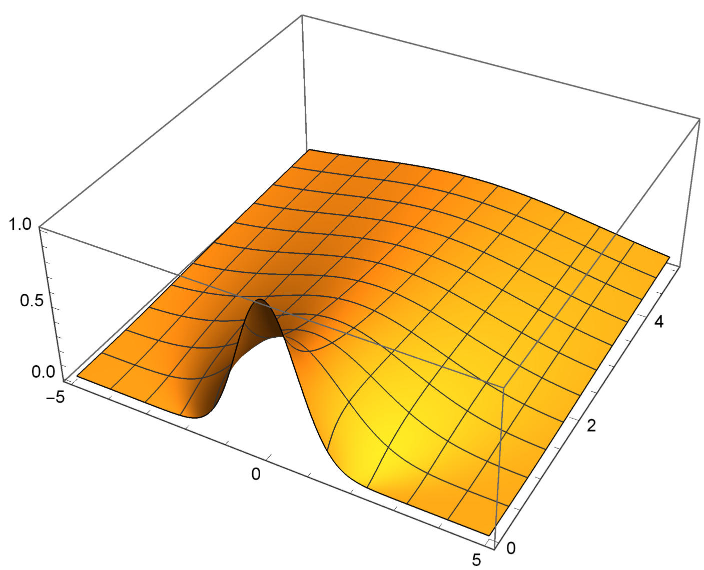
In[107]:=
```
distr = u[x, 0] == E^(-x^2);

(* Sets initial distribution to Gaussian distribution *)
solution = DSolveValue[{DiffusionHeatEqn, distr}, u[x, t], {x, t}]
Plot[Evaluate[Table[solution, {t, 0, 5}]], {x, -5, 5}, PlotRange → All,
 Filling → Axis, PlotLabels → {"t=0", "t=1", "t=2", "t=3", "t=4", "t=5"}]
Plot3D[solution, {x, -5, 5}, {t, 0, 5}, PlotRange → All, PlotPoints → 250, Mesh → 10]
```

Out[108]= $$\frac{e^{-\frac{x^2}{1+4t}}}{\sqrt{1+4t}}$$

Out[109]=



Out[110]=

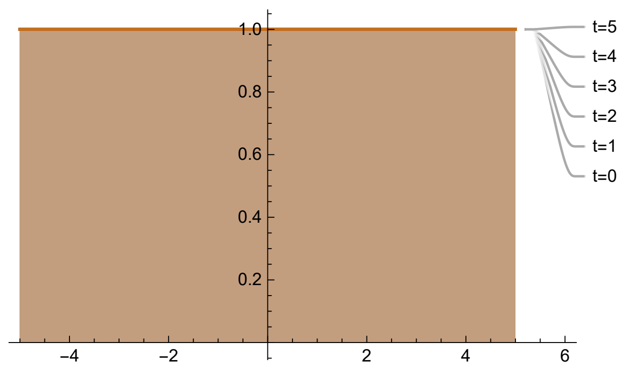


Uniform Initial Distribution

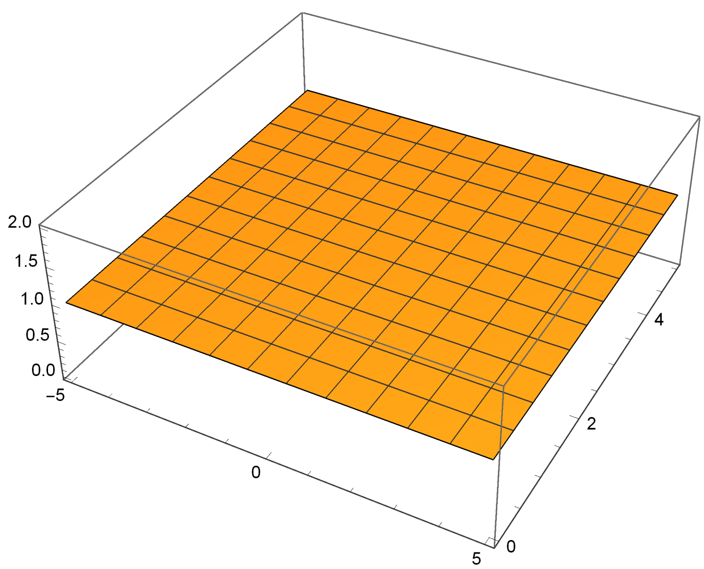
In[111]:=
```
distr = u[x, 0] == 1;

(* Sets initial distribution to Gaussian distribution *)
solution = DSolveValue[{DiffusionHeatEqn, distr}, u[x, t], {x, t}]
Plot[Evaluate[Table[solution, {t, 0, 5}]], {x, -5, 5}, PlotRange → All,
 Filling → Axis, PlotLabels → {"t=0", "t=1", "t=2", "t=3", "t=4", "t=5"}]
Plot3D[solution, {x, -5, 5}, {t, 0, 5}, PlotRange → All, PlotPoints → 250, Mesh → 10]
```

Out[112]= 1

Out[113]=



Out[114]=



Piecewise Box Initial Distribution (u[x,0] = 1 for |x| ≤ 1/2 , 0 outside the box)

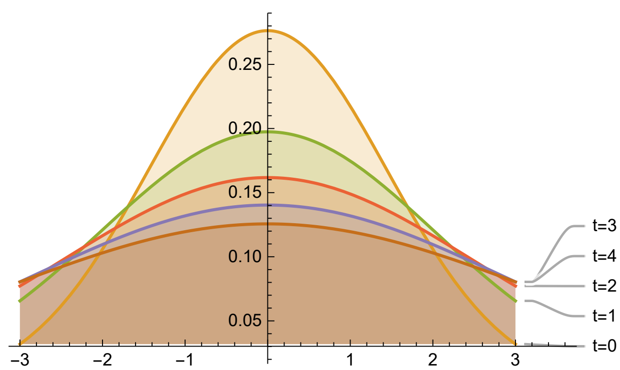
In[115]:=
```
distr = u[x, 0] == UnitBox[x];

solution = DSolveValue[{DiffusionHeatEqn, distr}, u[x, t], {x, t}]
Plot[Evaluate[Table[solution, {t, 0, 5}]], {x, -3, 3}, PlotRange → All,
 Filling → Axis, PlotLabels → {"t=0", "t=1", "t=2", "t=3", "t=4", "t=5"}]
Plot3D[solution, {x, -2, 2}, {t, 0, 5}, PlotRange → All, PlotPoints → 250, Mesh → 10]
```

Out[116]= $\frac{1}{2}\left(\mathrm{Erf}\left[\frac{1-2x}{4\sqrt{t}}\right]+\mathrm{Erf}\left[\frac{1+2x}{4\sqrt{t}}\right]\right)$

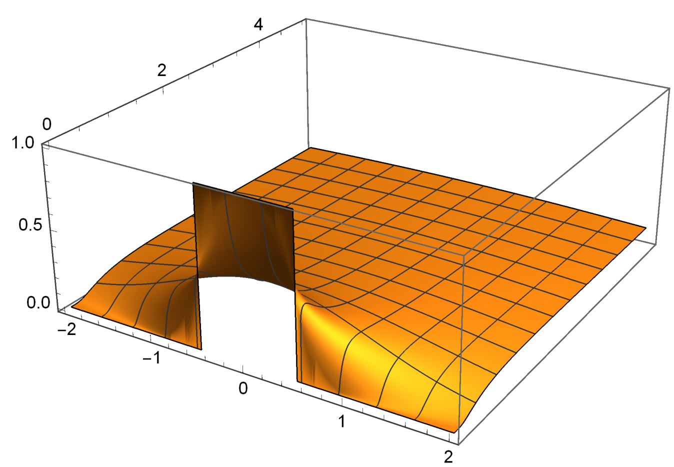
Power: Infinite expression $\frac{1}{\sqrt{0}}$ encountered.

Power: Infinite expression $\frac{1}{\sqrt{0}}$ encountered.

Out[117]=



Out[118]=



Piecewise Box Initial Distribution (u[x,0] = 1 for |x| ≥ 1/2 , 0 inside the box)

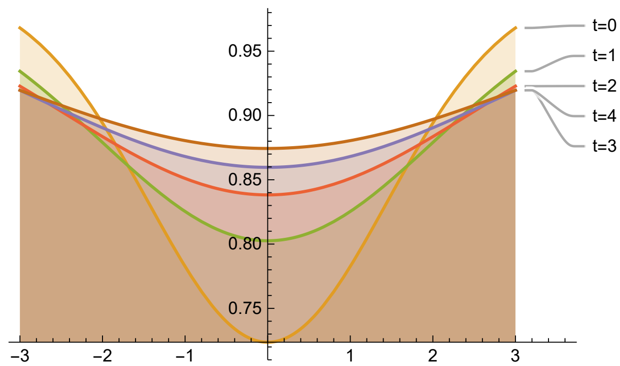
In[119]:=
```
distr = u[x, 0] == 1 - UnitBox[x];

solution = DSolveValue[{DiffusionHeatEqn, distr}, u[x, t], {x, t}]
Plot[Evaluate[Table[solution, {t, 0, 5}]], {x, -3, 3}, PlotRange → All,
 Filling → Axis, PlotLabels → {"t=0", "t=1", "t=2", "t=3", "t=4", "t=5"}]
Plot3D[solution, {x, -2, 2}, {t, 0, 5}, PlotRange → All, PlotPoints → 250, Mesh → 10]
```

Out[120]= $$\frac{1}{2}\left(\text{Erfc}\left[\frac{1-2x}{4\sqrt{t}}\right]+\text{Erfc}\left[\frac{1+2x}{4\sqrt{t}}\right]\right)$$

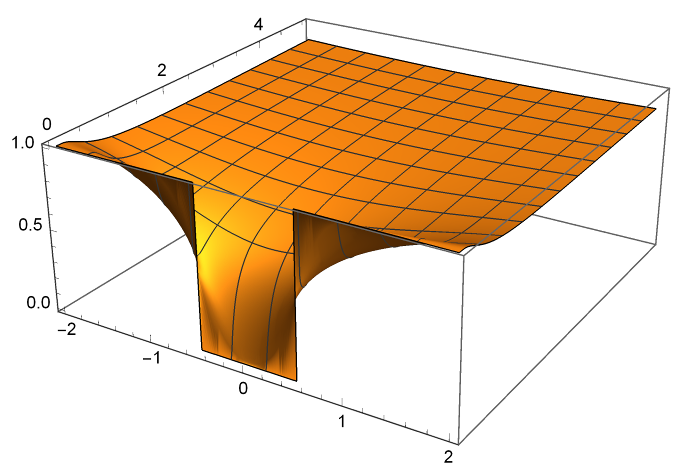
... Power: Infinite expression $\frac{1}{\sqrt{0}}$ encountered.

... Power: Infinite expression $\frac{1}{\sqrt{0}}$ encountered.

Out[121]=



Out[122]=



Dirac Delta Initial Distribution

In[123]:=
```
distr = u[x, 0] == DiracDelta[x];

solution = DSolveValue[{DiffusionHeatEqn, distr}, u[x, t], {x, t}]
Plot[Evaluate[Table[solution, {t, 0, 5}]], {x, -5, 5}, PlotRange → All,
 Filling → Axis, PlotLabels → {"t=0", "t=1", "t=2", "t=3", "t=4", "t=5"}]
Plot3D[solution, {x, -1, 1}, {t, 0, 5}, PlotRange → All, PlotPoints → 250, Mesh → 10]
```

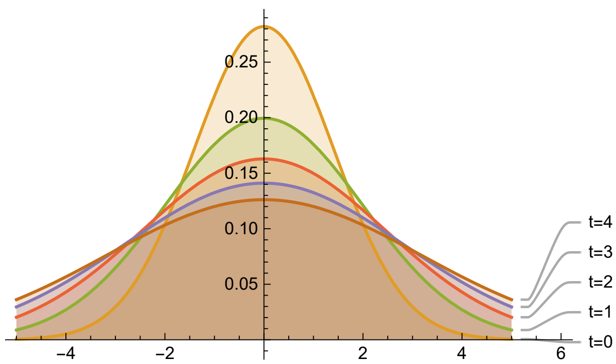
Out[124]= $$\frac{e^{-\frac{x^2}{4t}}}{2\sqrt{\pi}\sqrt{t}}$$

Power: Infinite expression $\frac{1}{0}$ encountered.

Infinity: Indeterminate expression $e^{\text{ComplexInfinity}}$ encountered.

Power: Infinite expression $\frac{1}{\sqrt{0}}$ encountered.

Out[125]=



General: Exp[-12437.4] is too small to represent as a normalized machine number; precision may be lost.

Out[126]=