

$$\frac{1}{\pi} \text{Sum}\left[\frac{\text{Sin}[2 \pi k x]}{k}, \{k, 1, \infty\}\right]$$

$$\frac{i \left( \text{Log}\left[1 - e^{2 i \pi x}\right] - \text{Log}\left[e^{-2 i \pi x} \left(-1 + e^{2 i \pi x}\right)\right] \right)}{2 \pi}$$

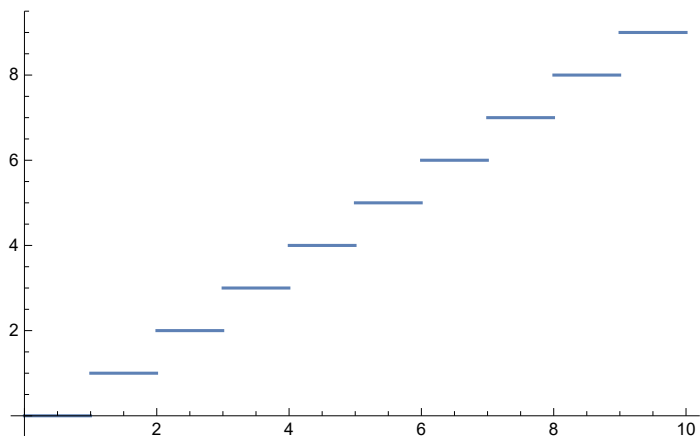
Note that  $\text{Floor}[x] = \left(x - \frac{1}{2}\right) + \frac{1}{\pi} \text{Sum}\left[\frac{\text{Sin}[2 \pi k x]}{k}, \{k, 1, \infty\}\right]$

from defn of sawtooth fn

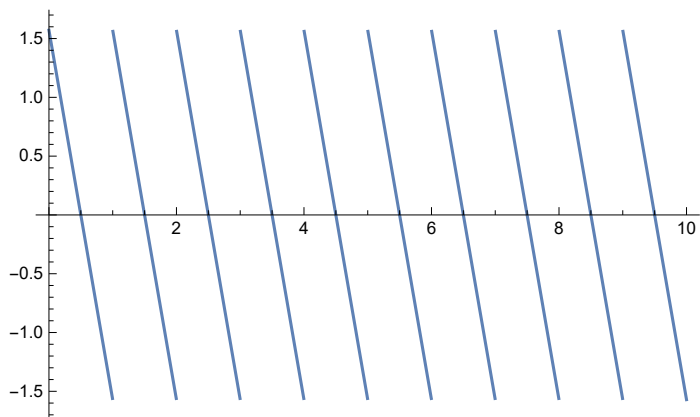
$$\left(x - \frac{1}{2}\right) + \frac{1}{\pi} \text{Sum}\left[\frac{\text{Sin}[2 \pi k x]}{k}, \{k, 1, \infty\}\right] // \text{FullSimplify}$$

$$-\frac{\pi - 2 \pi x + i \text{Log}\left[1 - e^{-2 i \pi x}\right] - i \text{Log}\left[1 - e^{2 i \pi x}\right]}{2 \pi}$$

$$\text{Plot}\left[\left(x - \frac{1}{2}\right) + \frac{1}{\pi} \text{Sum}\left[\frac{\text{Sin}[2 \pi k x]}{k}, \{k, 1, \infty\}\right], \{x, 0, 10\}\right]$$



$$\text{Plot}\left[\text{Sum}\left[\frac{\text{Sin}[2 \pi k x]}{k}, \{k, 1, \infty\}\right], \{x, 0, 10\}\right]$$



$$\frac{i \left( \text{Log}\left[1 - e^{2 i \pi x}\right] - \text{Log}\left[e^{-2 i \pi x} \left(-1 + e^{2 i \pi x}\right)\right] \right)}{2 \pi} = \frac{i}{2 \pi} \left( \text{Log}\left[1 - e^{2 i \pi x}\right] - \text{Log}\left[\left(-e^{-2 i \pi x} + 1\right)\right] \right)$$

$$= \frac{i}{2\pi} \left( \text{Log} \left[ 1 - e^{2i\pi x} \right] - \text{Log} \left[ 1 - e^{-2i\pi x} \right] \right) = \frac{i}{2\pi} \text{Log} \left[ \frac{1 - e^{2i\pi x}}{1 - e^{-2i\pi x}} \right]$$

$$\frac{1}{\pi} \text{Sum} \left[ \frac{\text{Cos}[2\pi k x]}{k}, \{k, 1, \infty\} \right] \\ \frac{-\text{Log} \left[ 1 - e^{2i\pi x} \right] - \text{Log} \left[ e^{-2i\pi x} (-1 + e^{2i\pi x}) \right]}{2\pi}$$

$$\frac{1}{\pi} \text{Sum} \left[ \frac{\text{Exp}[2\pi k y] \text{Cos}[2\pi k x]}{k}, \{k, 1, \infty\} \right] \\ \frac{-\text{Log} \left[ 1 - e^{2\pi (-ix+y)} \right] - \text{Log} \left[ 1 - e^{2\pi (ix+y)} \right]}{2\pi}$$