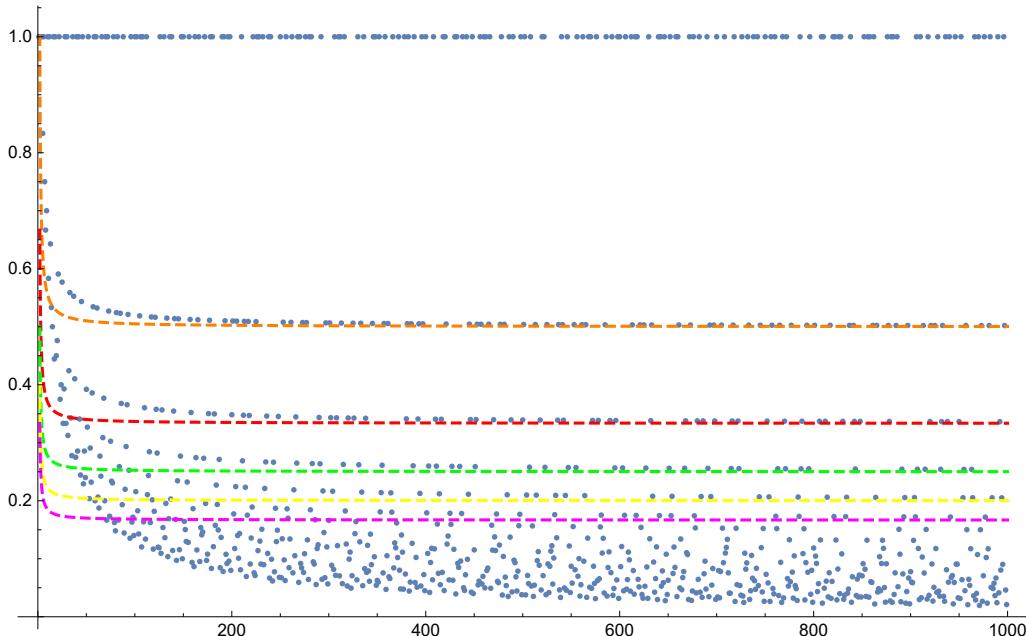


```

Show[ListPlot[Table[ $\frac{\text{PrimeFactorSum}[k]}{k}$ , {k, 2, 1000}]] ,
Plot[{ $\frac{\frac{1}{2}}{\left(1 - \frac{1}{k}\right)}$ ,  $\frac{\frac{1}{3}}{\left(1 - \frac{1}{k}\right)}$ ,  $\frac{\frac{1}{4}}{\left(1 - \frac{1}{k}\right)}$ ,  $\frac{\frac{1}{5}}{\left(1 - \frac{1}{k}\right)}$ ,  $\frac{\frac{1}{6}}{\left(1 - \frac{1}{k}\right)}$ },
{k, 2, 1000}, PlotStyle -> {{Orange, Dashed}, {Red, Dashed},
{Green, Dashed}, {Yellow, Dashed}, {Magenta, Dashed}}}]
(*Tiny bit better of a fit of the fns the pts actually tend towards,
but still not perfect *)

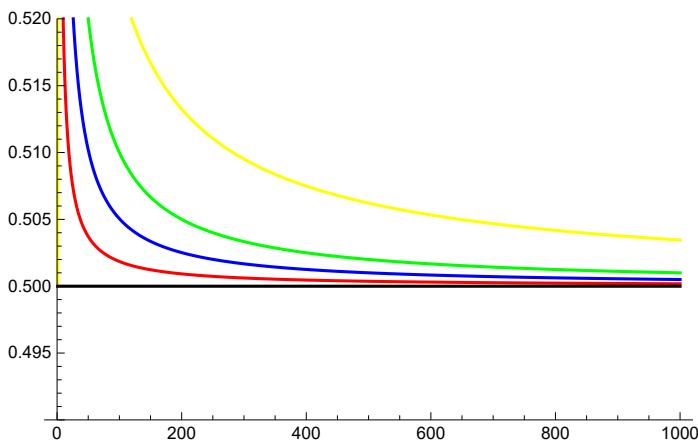
```



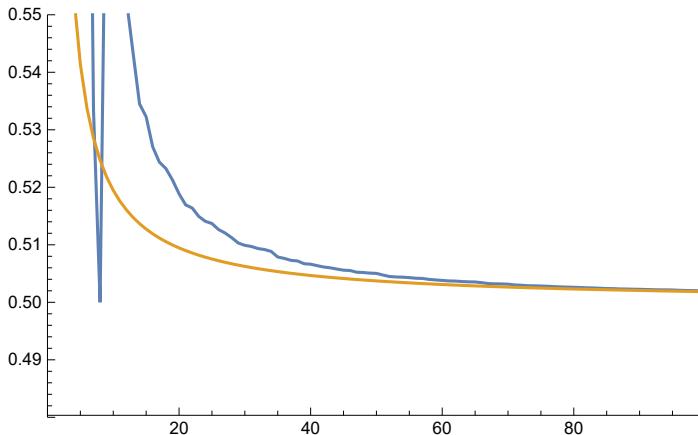
```

Plot[{ $\frac{1/2}{\left(1 - \frac{1}{k}\right)}$ ,  $1/2 + \frac{\log[k]}{2k}$ ,  $1/2 + \frac{1}{k}$ ,  $\left(2 \log[\epsilon - \frac{1}{k}]^(-1), 1/2\right)$ , {k, 1, 1000},
PlotStyle -> {Blue, Yellow, Green, Red, Black}, PlotRange -> {Automatic, {0.49, 0.52}}]

```



```
ListLinePlot[{PFSnTermsList[2, kmax], Table[(2 Log[E - 1/k])^(-1), {k, 1, 99}]},  
PlotRange -> {{0, 99}, {0.48, 0.55}}]
```



So $(2 \log\left(E - \frac{1}{k}\right))^{-1}$ seems to bound PFSnTermsList[2, kmax] from below,
but doesn't meet it as well near $k \rightarrow 1$,
which would be ideal, so might still need some adjustment
Although this very property might be proper in order for

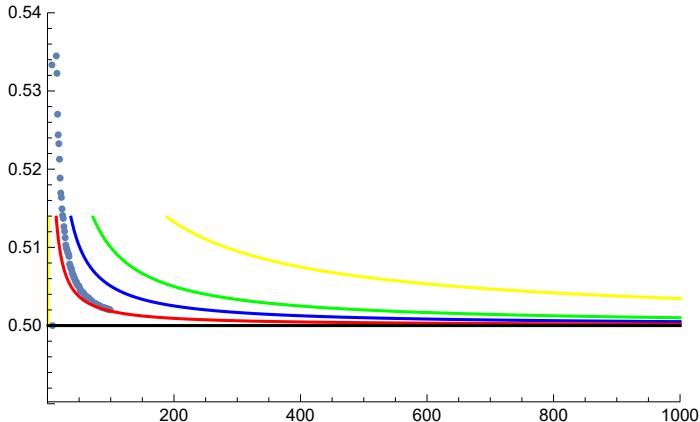
$(n \log\left(E - \frac{1}{k}\right))^{-1}$ to bound PFSnTermsList[n, kmax] from below for $n > 2$

```
maxiterboundn2 = Length[PFSnTermsList[2, 100]];  
Table[(2 Log[E - 1/k])^(-1) - PFSnTermsList[2, 100][[k]], {k, 1, maxiterboundn2}] // N  
{0.0903265, -0.122437, -0.0914102, -0.14661, -0.0419599, -0.109083, -0.00480018,  
0.0247007, -0.0691331, -0.0574524, -0.0412168, -0.0365631, -0.0287008,  
-0.0208043, -0.0195264, -0.0151196, -0.0132068, -0.0127134, -0.0113057}
```

```

kmax = 1000;
Show[
  ListPlot[PFSnTermsList[2, kmax], PlotRange -> {{0, 1000}, {.49, 0.54}}],
  Plot[{1/2, 1/2 + Log[k]/(2 k), 1/2 + 1/k, (2 Log[E - 1/k])^(-1), 1/2},
    {k, 1, kmax}, PlotStyle -> {Blue, Yellow, Green, Red, Black}]
]

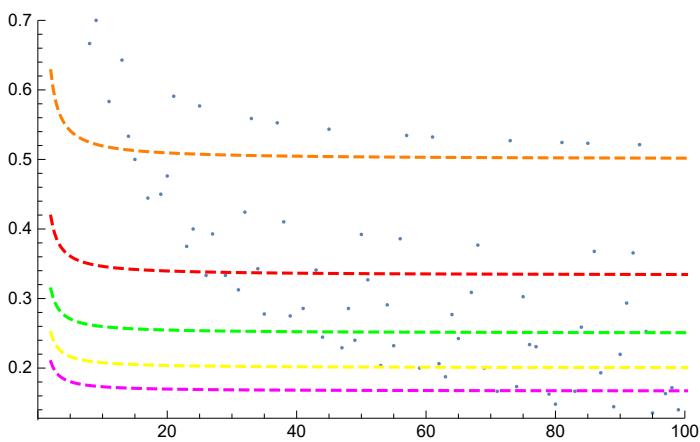
```



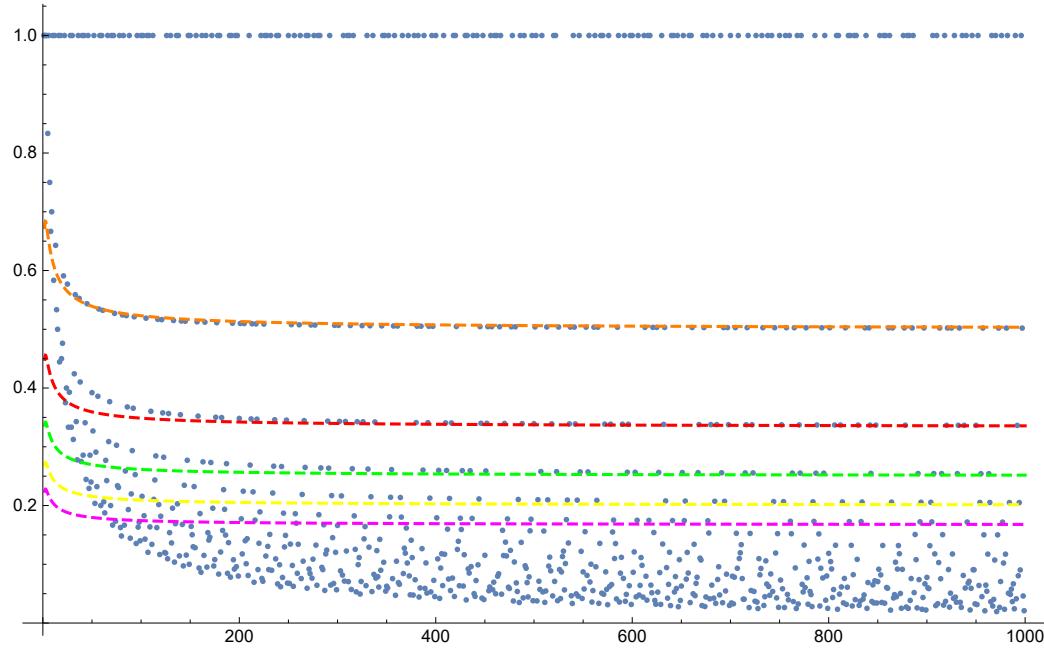
```

Show[ListPlot[Table[PrimeFactorSum[k],
  {k, 2, 1000}], PlotRange -> {{0, 100}, {1/8, 0.7}}],
  Plot[{(2 Log[E - 1/k])^(-1), (3 Log[E - 1/k])^(-1), (4 Log[E - 1/k])^(-1),
    (5 Log[E - 1/k])^(-1), (6 Log[E - 1/k])^(-1)}, {k, 2, 1000}, PlotStyle ->
    {Orange, Dashed}, {Red, Dashed}, {Green, Dashed}, {Yellow, Dashed}, {Magenta, Dashed}]]

```

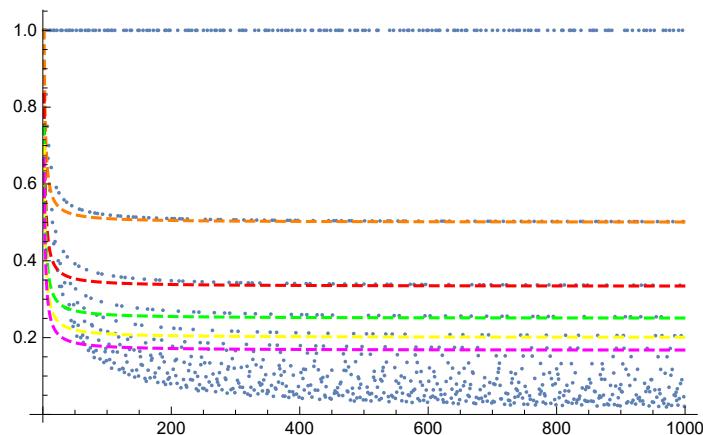


```
Show[ListPlot[Table[PrimeFactorSum[k], {k, 2, 1000}]], Plot[
{1/2 + Log[k]/(2 k), 1/3 + Log[k]/(3 k), 1/4 + Log[k]/(4 k), 1/5 + Log[k]/(5 k), 1/6 + Log[k]/(6 k)}, {k, 2, 1000}, PlotStyle -> {{Orange, Dashed}, {Red, Dashed}, {Green, Dashed}, {Yellow, Dashed}, {Magenta, Dashed}}]]
```



```
Show[ListPlot[Table[PrimeFactorSum[k], {k, 2, 1000}]], Plot[
{1/2 + 1/k, 1/3 + 1/k, 1/4 + 1/k, 1/5 + 1/k, 1/6 + 1/k}, {k, 2, 1000}, PlotStyle -> {{Orange, Dashed}, {Red, Dashed}, {Green, Dashed}, {Yellow, Dashed}, {Magenta, Dashed}}]]
```

(*Probably the best one so far *)



Comparison of Terms

```

kmax = 1000;
ListPNS2 = PFSnTermsList[2, kmax];
LenListPNS2 = Length[PFSnTermsList[2, kmax]]
99

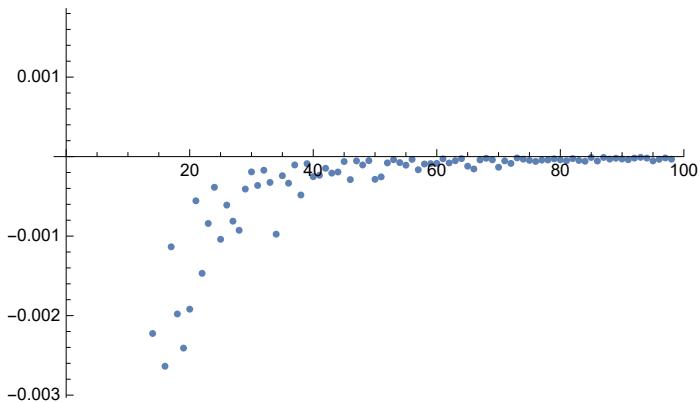
ListPNS2[[2]] - ListPNS2[[1]]

$$-\frac{1}{12}$$


Table[ListPNS2[[j]] - ListPNS2[[j - 1]], {j, 2, 99}]
{
$$\begin{aligned} & -\frac{1}{12}, -\frac{1}{12}, \frac{1}{30}, -\frac{7}{60}, \frac{5}{84}, -\frac{23}{210}, -\frac{1}{30}, \frac{1}{11}, -\frac{2}{143}, -\frac{4}{221}, -\frac{2}{323}, -\frac{4}{437}, -\frac{6}{667}, -\frac{2}{899}, -\frac{6}{1147}, \\ & -\frac{4}{1517}, -\frac{2}{1763}, -\frac{4}{2021}, -\frac{6}{2491}, -\frac{6}{3127}, -\frac{2}{3599}, -\frac{6}{4087}, -\frac{4}{4757}, -\frac{2}{5183}, -\frac{6}{5767}, -\frac{4}{6557}, \\ & -\frac{6}{7387}, -\frac{8}{8633}, -\frac{4}{9797}, -\frac{2}{10403}, -\frac{4}{11021}, -\frac{2}{11663}, -\frac{4}{12317}, -\frac{14}{14351}, -\frac{4}{16637}, -\frac{6}{17947}, \\ & -\frac{2}{19043}, -\frac{10}{20711}, -\frac{2}{22499}, -\frac{6}{23707}, -\frac{6}{25591}, -\frac{4}{27221}, -\frac{6}{28891}, -\frac{6}{30967}, -\frac{2}{32399}, -\frac{10}{34571}, \\ & -\frac{2}{36863}, -\frac{4}{38021}, -\frac{2}{39203}, -\frac{12}{41989}, -\frac{12}{47053}, -\frac{4}{50621}, -\frac{2}{51983}, -\frac{4}{53357}, -\frac{6}{55687}, \\ & -\frac{2}{57599}, -\frac{10}{60491}, -\frac{6}{64507}, -\frac{6}{67591}, -\frac{2}{70747}, -\frac{72899}{72899}, -\frac{75067}{75067}, -\frac{77837}{77837}, -\frac{79523}{79523}, \\ & -\frac{10}{82919}, -\frac{14}{89951}, -\frac{4}{95477}, -\frac{2}{97343}, -\frac{4}{99221}, -\frac{14}{104927}, -\frac{6}{111547}, -\frac{10}{116939}, -\frac{2}{121103}, \\ & -\frac{4}{123197}, -\frac{6}{126727}, -\frac{8}{131753}, -\frac{6}{136891}, -\frac{6}{141367}, -\frac{4}{145157}, -\frac{6}{148987}, -\frac{8}{154433}, -\frac{4}{159197}, \\ & -\frac{8}{164009}, -\frac{10}{171371}, -\frac{2}{176399}, -\frac{10}{181451}, -\frac{2}{186623}, -\frac{6}{190087}, -\frac{4}{194477}, -\frac{6}{198907}, \\ & -\frac{8}{205193}, -\frac{4}{210677}, -\frac{2}{213443}, -\frac{4}{216221}, -\frac{12}{223693}, -\frac{8}{233273}, -\frac{4}{239117}, -\frac{8}{245009} \end{aligned}\}$$


```

```
ListPlot[Table[ListPNS2[[j]] - ListPNS2[[j - 1]], {j, 2, 99}]]
```

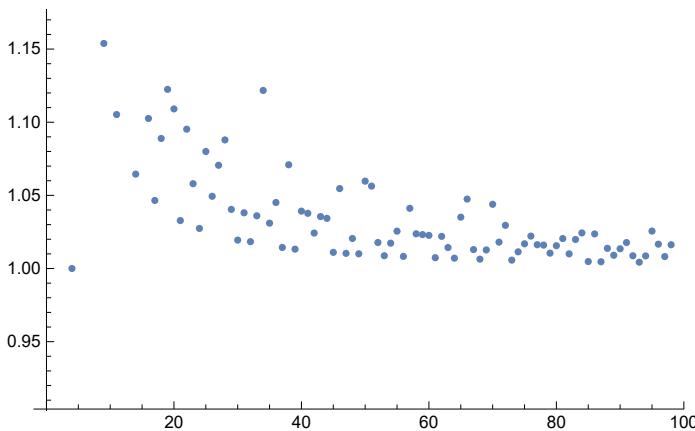


```

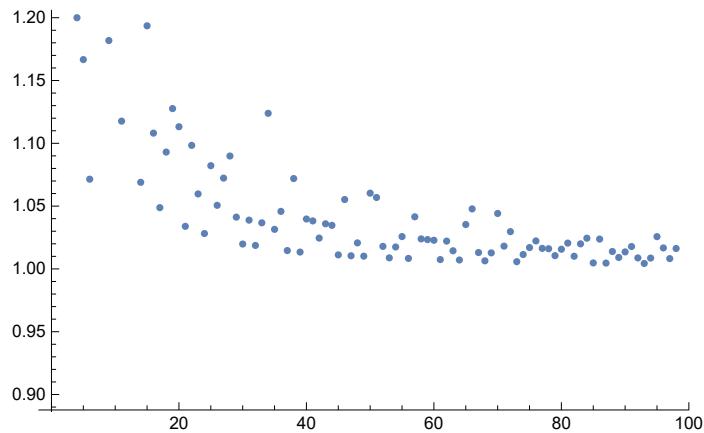
Table[Numerator[ListPNS2[[j]]] - Numerator[ListPNS2[[j - 1]]], {j, 2, 99}]
Table[Denominator[ListPNS2[[j]]] - Denominator[ListPNS2[[j - 1]]], {j, 2, 99}]
{-2, -1, 5, 0, 2, -1, -7, 12, 2, 4, 2, 4, 6, 2, 6, 4, 2, 4, 6, 6, 2, 6,
 4, 2, 6, 4, 6, 8, 4, 2, 4, 2, 4, 14, 4, 6, 2, 10, 2, 6, 6, 4, 6, 6, 2, 10, 2, 4,
 2, 12, 12, 4, 2, 4, 6, 2, 10, 6, 6, 6, 2, 6, 4, 2, 10, 14, 4, 2, 4, 14, 6, 10,
 2, 4, 6, 8, 6, 6, 4, 6, 8, 4, 8, 10, 2, 10, 2, 6, 4, 6, 8, 4, 2, 4, 12, 8, 4, 8}
{-2, -1, 7, 2, 2, 1, -13, 20, 4, 8, 4, 8, 12, 4, 12, 8, 4, 8, 12, 12, 4, 12, 8, 4, 12,
 8, 12, 16, 8, 4, 8, 4, 8, 28, 8, 12, 4, 20, 4, 12, 12, 8, 12, 12, 4, 20, 4, 8, 4, 24,
 24, 8, 4, 8, 12, 4, 20, 12, 12, 12, 4, 12, 8, 4, 20, 28, 8, 4, 8, 28, 12, 20, 4, 8,
 12, 16, 12, 12, 8, 12, 16, 8, 16, 20, 4, 20, 4, 12, 8, 12, 16, 8, 4, 8, 24, 16, 8, 16}

Table[ListPNS2[[j]], {j, 1, 99}];
Table[ $\frac{\text{Numerator}[\text{ListPNS2}[[\text{j}]]]}{\text{Numerator}[\text{ListPNS2}[[\text{j}-1]]]}$ , {j, 2, 99}]
ListPlot[Table[ $\frac{\text{Numerator}[\text{ListPNS2}[[\text{j}]]]}{\text{Numerator}[\text{ListPNS2}[[\text{j}-1]]]}$ , {j, 2, 99}]]
Table[ $\frac{\text{Denominator}[\text{ListPNS2}[[\text{j}]]]}{\text{Denominator}[\text{ListPNS2}[[\text{j}-1]]]}$ , {j, 2, 99}]
ListPlot[Table[ $\frac{\text{Denominator}[\text{ListPNS2}[[\text{j}]]]}{\text{Denominator}[\text{ListPNS2}[[\text{j}-1]]]}$ , {j, 2, 99}]]
{ $\frac{3}{5}, \frac{2}{3}, \frac{7}{2}, 1, \frac{9}{7}, \frac{8}{9}, \frac{1}{8}, 13, \frac{15}{13}, \frac{19}{15}, \frac{21}{19}, \frac{25}{21}, \frac{31}{25}, \frac{33}{31}, \frac{13}{11}, \frac{43}{39}, \frac{45}{43}, \frac{49}{45}, \frac{55}{49}, \frac{61}{55}, \frac{63}{61},$ 
  $\frac{23}{21}, \frac{73}{69}, \frac{75}{73}, \frac{27}{25}, \frac{85}{81}, \frac{91}{85}, \frac{99}{91}, \frac{103}{99}, \frac{105}{103}, \frac{109}{105}, \frac{111}{109}, \frac{115}{111}, \frac{129}{115}, \frac{133}{129}, \frac{139}{133}, \frac{141}{139},$ 
  $\frac{151}{141}, \frac{153}{151}, \frac{53}{51}, \frac{55}{53}, \frac{169}{165}, \frac{175}{169}, \frac{181}{175}, \frac{183}{181}, \frac{193}{183}, \frac{195}{193}, \frac{199}{195}, \frac{201}{199}, \frac{71}{67}, \frac{75}{71}, \frac{229}{225}, \frac{231}{229},$ 
  $\frac{235}{231}, \frac{241}{235}, \frac{243}{241}, \frac{253}{243}, \frac{259}{253}, \frac{265}{259}, \frac{271}{265}, \frac{273}{271}, \frac{93}{91}, \frac{283}{279}, \frac{285}{283}, \frac{59}{57}, \frac{309}{295}, \frac{313}{309}, \frac{315}{313},$ 
  $\frac{319}{315}, \frac{333}{319}, \frac{113}{111}, \frac{349}{339}, \frac{351}{349}, \frac{355}{351}, \frac{361}{355}, \frac{369}{361}, \frac{125}{123}, \frac{127}{125}, \frac{385}{381}, \frac{391}{385}, \frac{399}{391}, \frac{403}{399}, \frac{411}{403},$ 
  $\frac{421}{411}, \frac{423}{421}, \frac{433}{423}, \frac{435}{433}, \frac{147}{145}, \frac{445}{441}, \frac{451}{445}, \frac{459}{451}, \frac{463}{459}, \frac{465}{463}, \frac{469}{465}, \frac{481}{469}, \frac{489}{481}, \frac{493}{489}, \frac{501}{493}$ 

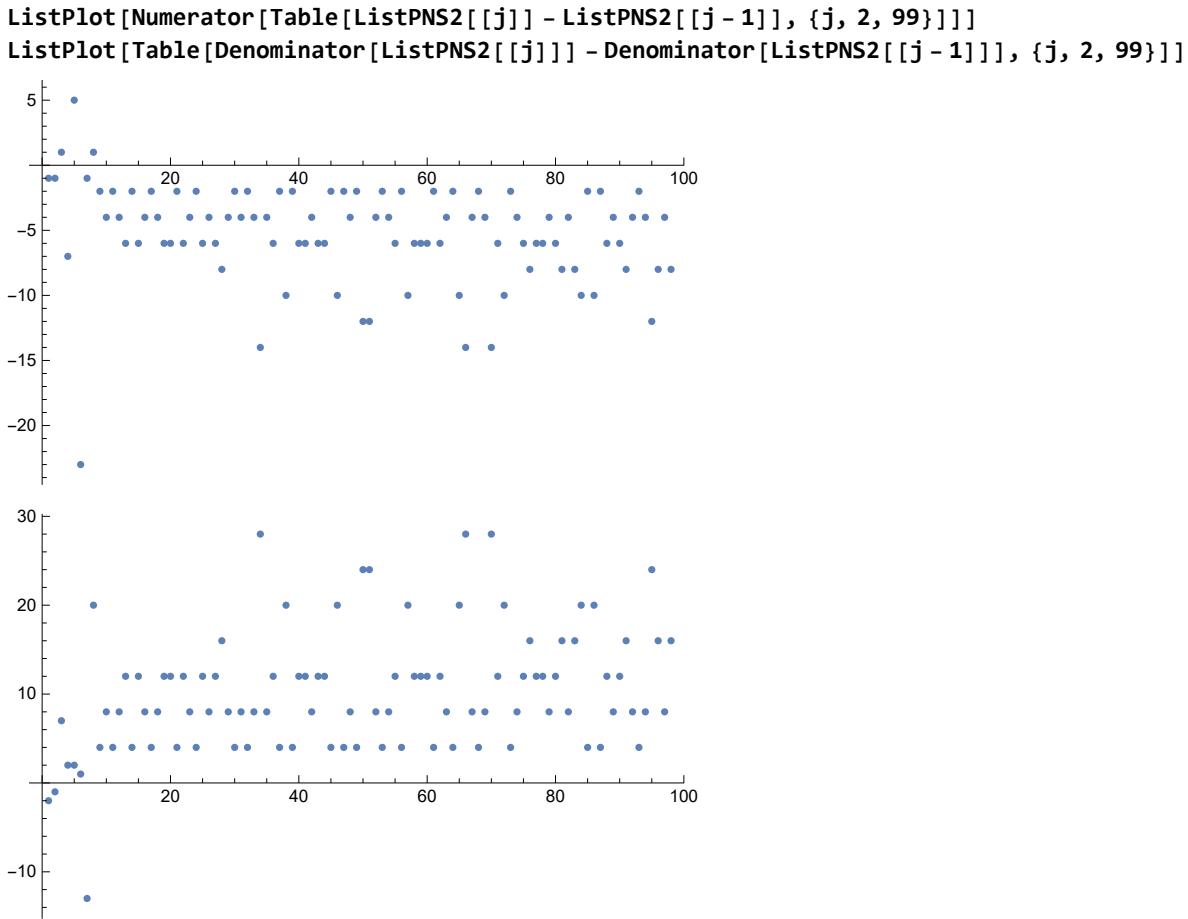
```



$$\left\{ \frac{2}{3}, \frac{3}{4}, \frac{10}{3}, \frac{6}{5}, \frac{7}{6}, \frac{15}{14}, \frac{2}{15}, 11, \frac{13}{11}, \frac{17}{13}, \frac{19}{17}, \frac{23}{19}, \frac{29}{23}, \frac{31}{29}, \frac{37}{31}, \frac{41}{37}, \frac{43}{41}, \frac{47}{43}, \frac{53}{47}, \frac{59}{53}, \frac{61}{59}, \right. \\ \left. \frac{67}{61}, \frac{71}{67}, \frac{73}{71}, \frac{79}{73}, \frac{83}{79}, \frac{89}{83}, \frac{97}{89}, \frac{101}{97}, \frac{103}{101}, \frac{107}{103}, \frac{109}{107}, \frac{113}{109}, \frac{127}{113}, \frac{131}{127}, \frac{137}{131}, \frac{139}{137}, \frac{149}{139}, \right. \\ \left. \frac{151}{149}, \frac{157}{151}, \frac{163}{157}, \frac{167}{163}, \frac{173}{167}, \frac{179}{173}, \frac{181}{179}, \frac{191}{181}, \frac{193}{191}, \frac{197}{193}, \frac{199}{197}, \frac{211}{199}, \frac{223}{211}, \frac{227}{223}, \frac{229}{227}, \right. \\ \left. \frac{233}{229}, \frac{239}{233}, \frac{241}{239}, \frac{251}{241}, \frac{257}{251}, \frac{263}{257}, \frac{269}{263}, \frac{271}{269}, \frac{277}{271}, \frac{281}{277}, \frac{283}{281}, \frac{293}{283}, \frac{307}{293}, \frac{311}{307}, \frac{313}{311}, \right. \\ \left. \frac{317}{313}, \frac{331}{317}, \frac{337}{331}, \frac{347}{337}, \frac{349}{347}, \frac{353}{349}, \frac{359}{353}, \frac{367}{359}, \frac{373}{367}, \frac{379}{373}, \frac{383}{379}, \frac{389}{383}, \frac{397}{389}, \frac{401}{397}, \frac{409}{401}, \right. \\ \left. \frac{419}{409}, \frac{421}{419}, \frac{431}{421}, \frac{433}{431}, \frac{439}{433}, \frac{443}{439}, \frac{449}{443}, \frac{457}{449}, \frac{461}{457}, \frac{463}{461}, \frac{467}{463}, \frac{479}{467}, \frac{487}{479}, \frac{491}{487}, \frac{499}{491} \right\}$$



```
Table[ListPNS2[[j]], {j, 1, 99}];
Table[Numerator[ListPNS2[[j]]], {j, 1, 99}];
Table[Denominator[ListPNS2[[j]]], {j, 1, 99}];
```



Setup and check of PFSnTerms fn

```
PFSnTerms[4, 1000]
PFSnTermsList[4, 1000] // N
{32, 36, 40, 42, 49, 52, 55, 65, 68, 76, 85, 92, 95, 116, 124, 148, 164, 172, 188, 212, 236,
244, 268, 284, 292, 316, 332, 356, 388, 404, 412, 428, 436, 452, 508, 524, 548, 556, 596,
604, 628, 652, 668, 692, 716, 724, 764, 772, 788, 796, 844, 892, 908, 916, 932, 956, 964}

{0.3125, 0.277778, 0.275, 0.285714, 0.285714, 0.326923, 0.290909, 0.276923, 0.308824,
0.302632, 0.258824, 0.293478, 0.252632, 0.284483, 0.282258, 0.277027, 0.27439,
0.273256, 0.271277, 0.268868, 0.266949, 0.266393, 0.264925, 0.264085, 0.263699,
0.262658, 0.262048, 0.261236, 0.260309, 0.259901, 0.259709, 0.259346, 0.259174,
0.25885, 0.257874, 0.257634, 0.257299, 0.257194, 0.256711, 0.256623, 0.256369,
0.256135, 0.255988, 0.25578, 0.255587, 0.255525, 0.255236, 0.255181, 0.255076,
0.255025, 0.254739, 0.254484, 0.254405, 0.254367, 0.254292, 0.254184, 0.254149}

Table[{k, Floor[2 (PrimeFactorSum[k]) / k]}, {k, 2, 1000}]
```

```
{{{2, 2}, {3, 2}, {4, 2}, {5, 2}, {6, 1}, {7, 2}, {8, 1}, {9, 1}, {10, 1}, {11, 2}, {12, 1},
{13, 2}, {14, 1}, {15, 1}, {16, 1}, {17, 2}, {18, 0}, {19, 2}, {20, 0}, {21, 0}, {22, 1},
{23, 2}, {24, 0}, {25, 0}, {26, 1}, {27, 0}, {28, 0}, {29, 2}, {30, 0}, {31, 2}, {32, 0},
{33, 0}, {34, 1}, {35, 0}, {36, 0}, {37, 2}, {38, 1}, {39, 0}, {40, 0}, {41, 2}, {42, 0}},
```

```

{43, 2}, {44, 0}, {45, 0}, {46, 1}, {47, 2}, {48, 0}, {49, 0}, {50, 0}, {51, 0}, {52, 0},
{53, 2}, {54, 0}, {55, 0}, {56, 0}, {57, 0}, {58, 1}, {59, 2}, {60, 0}, {61, 2}, {62, 1},
{63, 0}, {64, 0}, {65, 0}, {66, 0}, {67, 2}, {68, 0}, {69, 0}, {70, 0}, {71, 2}, {72, 0},
{73, 2}, {74, 1}, {75, 0}, {76, 0}, {77, 0}, {78, 0}, {79, 2}, {80, 0}, {81, 0}, {82, 1},
{83, 2}, {84, 0}, {85, 0}, {86, 1}, {87, 0}, {88, 0}, {89, 2}, {90, 0}, {91, 0},
{92, 0}, {93, 0}, {94, 1}, {95, 0}, {96, 0}, {97, 2}, {98, 0}, {99, 0}, {100, 0},
{101, 2}, {102, 0}, {103, 2}, {104, 0}, {105, 0}, {106, 1}, {107, 2}, {108, 0},
{109, 2}, {110, 0}, {111, 0}, {112, 0}, {113, 2}, {114, 0}, {115, 0}, {116, 0},
{117, 0}, {118, 1}, {119, 0}, {120, 0}, {121, 0}, {122, 1}, {123, 0}, {124, 0},
{125, 0}, {126, 0}, {127, 2}, {128, 0}, {129, 0}, {130, 0}, {131, 2}, {132, 0}, {133, 0},
{134, 1}, {135, 0}, {136, 0}, {137, 2}, {138, 0}, {139, 2}, {140, 0}, {141, 0},
{142, 1}, {143, 0}, {144, 0}, {145, 0}, {146, 1}, {147, 0}, {148, 0}, {149, 2}, {150, 0},
{151, 2}, {152, 0}, {153, 0}, {154, 0}, {155, 0}, {156, 0}, {157, 2}, {158, 1}, {159, 0},
{160, 0}, {161, 0}, {162, 0}, {163, 2}, {164, 0}, {165, 0}, {166, 1}, {167, 2}, {168, 0},
{169, 0}, {170, 0}, {171, 0}, {172, 0}, {173, 2}, {174, 0}, {175, 0}, {176, 0}, {177, 0},
{178, 1}, {179, 2}, {180, 0}, {181, 2}, {182, 0}, {183, 0}, {184, 0}, {185, 0}, {186, 0},
{187, 0}, {188, 0}, {189, 0}, {190, 0}, {191, 2}, {192, 0}, {193, 2}, {194, 1}, {195, 0},
{196, 0}, {197, 2}, {198, 0}, {199, 2}, {200, 0}, {201, 0}, {202, 1}, {203, 0}, {204, 0},
{205, 0}, {206, 1}, {207, 0}, {208, 0}, {209, 0}, {210, 0}, {211, 2}, {212, 0}, {213, 0},
{214, 1}, {215, 0}, {216, 0}, {217, 0}, {218, 1}, {219, 0}, {220, 0}, {221, 0}, {222, 0},
{223, 2}, {224, 0}, {225, 0}, {226, 1}, {227, 2}, {228, 0}, {229, 2}, {230, 0}, {231, 0},
{232, 0}, {233, 2}, {234, 0}, {235, 0}, {236, 0}, {237, 0}, {238, 0}, {239, 2}, {240, 0},
{241, 2}, {242, 0}, {243, 0}, {244, 0}, {245, 0}, {246, 0}, {247, 0}, {248, 0}, {249, 0},
{250, 0}, {251, 2}, {252, 0}, {253, 0}, {254, 1}, {255, 0}, {256, 0}, {257, 2}, {258, 0},
{259, 0}, {260, 0}, {261, 0}, {262, 1}, {263, 2}, {264, 0}, {265, 0}, {266, 0}, {267, 0},
{268, 0}, {269, 2}, {270, 0}, {271, 2}, {272, 0}, {273, 0}, {274, 1}, {275, 0}, {276, 0},
{277, 2}, {278, 1}, {279, 0}, {280, 0}, {281, 2}, {282, 0}, {283, 2}, {284, 0}, {285, 0},
{286, 0}, {287, 0}, {288, 0}, {289, 0}, {290, 0}, {291, 0}, {292, 0}, {293, 2}, {294, 0},
{295, 0}, {296, 0}, {297, 0}, {298, 1}, {299, 0}, {300, 0}, {301, 0}, {302, 1}, {303, 0},
{304, 0}, {305, 0}, {306, 0}, {307, 2}, {308, 0}, {309, 0}, {310, 0}, {311, 2}, {312, 0},
{313, 2}, {314, 1}, {315, 0}, {316, 0}, {317, 2}, {318, 0}, {319, 0}, {320, 0}, {321, 0},
{322, 0}, {323, 0}, {324, 0}, {325, 0}, {326, 1}, {327, 0}, {328, 0}, {329, 0}, {330, 0},
{331, 2}, {332, 0}, {333, 0}, {334, 1}, {335, 0}, {336, 0}, {337, 2}, {338, 0}, {339, 0},
{340, 0}, {341, 0}, {342, 0}, {343, 0}, {344, 0}, {345, 0}, {346, 1}, {347, 2}, {348, 0},
{349, 2}, {350, 0}, {351, 0}, {352, 0}, {353, 2}, {354, 0}, {355, 0}, {356, 0}, {357, 0},
{358, 1}, {359, 2}, {360, 0}, {361, 0}, {362, 1}, {363, 0}, {364, 0}, {365, 0}, {366, 0},
{367, 2}, {368, 0}, {369, 0}, {370, 0}, {371, 0}, {372, 0}, {373, 2}, {374, 0}, {375, 0},
{376, 0}, {377, 0}, {378, 0}, {379, 2}, {380, 0}, {381, 0}, {382, 1}, {383, 2}, {384, 0},
{385, 0}, {386, 1}, {387, 0}, {388, 0}, {389, 2}, {390, 0}, {391, 0}, {392, 0}, {393, 0},
{394, 1}, {395, 0}, {396, 0}, {397, 2}, {398, 1}, {399, 0}, {400, 0}, {401, 2}, {402, 0},
{403, 0}, {404, 0}, {405, 0}, {406, 0}, {407, 0}, {408, 0}, {409, 2}, {410, 0}, {411, 0},
{412, 0}, {413, 0}, {414, 0}, {415, 0}, {416, 0}, {417, 0}, {418, 0}, {419, 2}, {420, 0},
{421, 2}, {422, 1}, {423, 0}, {424, 0}, {425, 0}, {426, 0}, {427, 0}, {428, 0}, {429, 0},
{430, 0}, {431, 2}, {432, 0}, {433, 2}, {434, 0}, {435, 0}, {436, 0}, {437, 0}, {438, 0},
{439, 2}, {440, 0}, {441, 0}, {442, 0}, {443, 2}, {444, 0}, {445, 0}, {446, 1}, {447, 0},
{448, 0}, {449, 2}, {450, 0}, {451, 0}, {452, 0}, {453, 0}, {454, 1}, {455, 0}, {456, 0},
{457, 2}, {458, 1}, {459, 0}, {460, 0}, {461, 2}, {462, 0}, {463, 2}, {464, 0}, {465, 0},
{466, 1}, {467, 2}, {468, 0}, {469, 0}, {470, 0}, {471, 0}, {472, 0}, {473, 0}, {474, 0},
{475, 0}, {476, 0}, {477, 0}, {478, 1}, {479, 2}, {480, 0}, {481, 0}, {482, 1}, {483, 0},
{484, 0}, {485, 0}, {486, 0}, {487, 2}, {488, 0}, {489, 0}, {490, 0}, {491, 2}, {492, 0},
{493, 0}, {494, 0}, {495, 0}, {496, 0}, {497, 0}, {498, 0}, {499, 2}, {500, 0}, {501, 0},

```

```

{502, 1}, {503, 2}, {504, 0}, {505, 0}, {506, 0}, {507, 0}, {508, 0}, {509, 2}, {510, 0},
{511, 0}, {512, 0}, {513, 0}, {514, 1}, {515, 0}, {516, 0}, {517, 0}, {518, 0}, {519, 0},
{520, 0}, {521, 2}, {522, 0}, {523, 2}, {524, 0}, {525, 0}, {526, 1}, {527, 0}, {528, 0},
{529, 0}, {530, 0}, {531, 0}, {532, 0}, {533, 0}, {534, 0}, {535, 0}, {536, 0}, {537, 0},
{538, 1}, {539, 0}, {540, 0}, {541, 2}, {542, 1}, {543, 0}, {544, 0}, {545, 0}, {546, 0},
{547, 2}, {548, 0}, {549, 0}, {550, 0}, {551, 0}, {552, 0}, {553, 0}, {554, 1}, {555, 0},
{556, 0}, {557, 2}, {558, 0}, {559, 0}, {560, 0}, {561, 0}, {562, 1}, {563, 2}, {564, 0},
{565, 0}, {566, 1}, {567, 0}, {568, 0}, {569, 2}, {570, 0}, {571, 2}, {572, 0}, {573, 0},
{574, 0}, {575, 0}, {576, 0}, {577, 2}, {578, 0}, {579, 0}, {580, 0}, {581, 0}, {582, 0},
{583, 0}, {584, 0}, {585, 0}, {586, 1}, {587, 2}, {588, 0}, {589, 0}, {590, 0}, {591, 0},
{592, 0}, {593, 2}, {594, 0}, {595, 0}, {596, 0}, {597, 0}, {598, 0}, {599, 2}, {600, 0},
{601, 2}, {602, 0}, {603, 0}, {604, 0}, {605, 0}, {606, 0}, {607, 2}, {608, 0}, {609, 0},
{610, 0}, {611, 0}, {612, 0}, {613, 2}, {614, 1}, {615, 0}, {616, 0}, {617, 2}, {618, 0},
{619, 2}, {620, 0}, {621, 0}, {622, 1}, {623, 0}, {624, 0}, {625, 0}, {626, 1}, {627, 0},
{628, 0}, {629, 0}, {630, 0}, {631, 2}, {632, 0}, {633, 0}, {634, 1}, {635, 0}, {636, 0},
{637, 0}, {638, 0}, {639, 0}, {640, 0}, {641, 2}, {642, 0}, {643, 2}, {644, 0}, {645, 0},
{646, 0}, {647, 2}, {648, 0}, {649, 0}, {650, 0}, {651, 0}, {652, 0}, {653, 2}, {654, 0},
{655, 0}, {656, 0}, {657, 0}, {658, 0}, {659, 2}, {660, 0}, {661, 2}, {662, 1}, {663, 0},
{664, 0}, {665, 0}, {666, 0}, {667, 0}, {668, 0}, {669, 0}, {670, 0}, {671, 0}, {672, 0},
{673, 2}, {674, 1}, {675, 0}, {676, 0}, {677, 2}, {678, 0}, {679, 0}, {680, 0}, {681, 0},
{682, 0}, {683, 2}, {684, 0}, {685, 0}, {686, 0}, {687, 0}, {688, 0}, {689, 0}, {690, 0},
{691, 2}, {692, 0}, {693, 0}, {694, 1}, {695, 0}, {696, 0}, {697, 0}, {698, 1}, {699, 0},
{700, 0}, {701, 2}, {702, 0}, {703, 0}, {704, 0}, {705, 0}, {706, 1}, {707, 0}, {708, 0},
{709, 2}, {710, 0}, {711, 0}, {712, 0}, {713, 0}, {714, 0}, {715, 0}, {716, 0}, {717, 0},
{718, 1}, {719, 2}, {720, 0}, {721, 0}, {722, 0}, {723, 0}, {724, 0}, {725, 0}, {726, 0},
{727, 2}, {728, 0}, {729, 0}, {730, 0}, {731, 0}, {732, 0}, {733, 2}, {734, 1}, {735, 0},
{736, 0}, {737, 0}, {738, 0}, {739, 2}, {740, 0}, {741, 0}, {742, 0}, {743, 2}, {744, 0},
{745, 0}, {746, 1}, {747, 0}, {748, 0}, {749, 0}, {750, 0}, {751, 2}, {752, 0}, {753, 0},
{754, 0}, {755, 0}, {756, 0}, {757, 2}, {758, 1}, {759, 0}, {760, 0}, {761, 2}, {762, 0},
{763, 0}, {764, 0}, {765, 0}, {766, 1}, {767, 0}, {768, 0}, {769, 2}, {770, 0}, {771, 0},
{772, 0}, {773, 2}, {774, 0}, {775, 0}, {776, 0}, {777, 0}, {778, 1}, {779, 0}, {780, 0},
{781, 0}, {782, 0}, {783, 0}, {784, 0}, {785, 0}, {786, 0}, {787, 2}, {788, 0}, {789, 0},
{790, 0}, {791, 0}, {792, 0}, {793, 0}, {794, 1}, {795, 0}, {796, 0}, {797, 2}, {798, 0},
{799, 0}, {800, 0}, {801, 0}, {802, 1}, {803, 0}, {804, 0}, {805, 0}, {806, 0}, {807, 0},
{808, 0}, {809, 2}, {810, 0}, {811, 2}, {812, 0}, {813, 0}, {814, 0}, {815, 0}, {816, 0},
{817, 0}, {818, 1}, {819, 0}, {820, 0}, {821, 2}, {822, 0}, {823, 2}, {824, 0}, {825, 0},
{826, 0}, {827, 2}, {828, 0}, {829, 2}, {830, 0}, {831, 0}, {832, 0}, {833, 0}, {834, 0},
{835, 0}, {836, 0}, {837, 0}, {838, 1}, {839, 2}, {840, 0}, {841, 0}, {842, 1}, {843, 0},
{844, 0}, {845, 0}, {846, 0}, {847, 0}, {848, 0}, {849, 0}, {850, 0}, {851, 0}, {852, 0},
{853, 2}, {854, 0}, {855, 0}, {856, 0}, {857, 2}, {858, 0}, {859, 2}, {860, 0}, {861, 0},
{862, 1}, {863, 2}, {864, 0}, {865, 0}, {866, 1}, {867, 0}, {868, 0}, {869, 0}, {870, 0},
{871, 0}, {872, 0}, {873, 0}, {874, 0}, {875, 0}, {876, 0}, {877, 2}, {878, 1}, {879, 0},
{880, 0}, {881, 2}, {882, 0}, {883, 2}, {884, 0}, {885, 0}, {886, 1}, {887, 2}, {888, 0},
{889, 0}, {890, 0}, {891, 0}, {892, 0}, {893, 0}, {894, 0}, {895, 0}, {896, 0}, {897, 0},
{898, 1}, {899, 0}, {900, 0}, {901, 0}, {902, 0}, {903, 0}, {904, 0}, {905, 0}, {906, 0},
{907, 2}, {908, 0}, {909, 0}, {910, 0}, {911, 2}, {912, 0}, {913, 0}, {914, 1}, {915, 0},
{916, 0}, {917, 0}, {918, 0}, {919, 2}, {920, 0}, {921, 0}, {922, 1}, {923, 0}, {924, 0},
{925, 0}, {926, 1}, {927, 0}, {928, 0}, {929, 2}, {930, 0}, {931, 0}, {932, 0}, {933, 0},
{934, 1}, {935, 0}, {936, 0}, {937, 2}, {938, 0}, {939, 0}, {940, 0}, {941, 2}, {942, 0},
{943, 0}, {944, 0}, {945, 0}, {946, 0}, {947, 2}, {948, 0}, {949, 0}, {950, 0}, {951, 0},
{952, 0}, {953, 2}, {954, 0}, {955, 0}, {956, 0}, {957, 0}, {958, 1}, {959, 0}, {960, 0}

```


(Should give all such values)

```

Table[ $\frac{\text{PrimeFactorSum}[\text{PFSnTerms}[3, 1000][[j]]]}{\text{PFSnTerms}[3, 1000][[j]]}$ , {j, 1, Length[PFSnTerms[3, 1000]]}] // N
(* The function
Union[Floor[Table[ $\frac{\text{PrimeFactorSum}[\text{PFSnTerms}[3, 1000][[j]]}{\text{PFSnTerms}[3, 1000][[j]]}$ , {j, 1, Length[PFSnTerms[3, 1000]]}] -  $\frac{1}{3}$ ] // N]
should return only 0 if the function works correctly *)
{0.444444, 0.45, 0.47619, 0.375, 0.4, 0.333333, 0.392857, 0.333333, 0.424242,
0.342857, 0.410256, 0.340909, 0.392157, 0.385965, 0.376812, 0.367816, 0.365591,
0.36036, 0.357724, 0.356589, 0.35461, 0.352201, 0.350282, 0.349727, 0.348259,
0.347418, 0.347032, 0.345992, 0.345382, 0.344569, 0.343643, 0.343234, 0.343042,
0.342679, 0.342508, 0.342183, 0.341207, 0.340967, 0.340633, 0.340528, 0.340045,
0.339956, 0.339703, 0.339468, 0.339321, 0.339114, 0.33892, 0.338858, 0.338569,
0.338515, 0.338409, 0.338358, 0.338073, 0.337818, 0.337739, 0.3377, 0.337625,
0.337517, 0.337483, 0.337317, 0.337224, 0.337136, 0.337051, 0.337023, 0.336943,
0.336892, 0.336867, 0.336746, 0.336591, 0.336549, 0.336528, 0.336488, 0.336354}

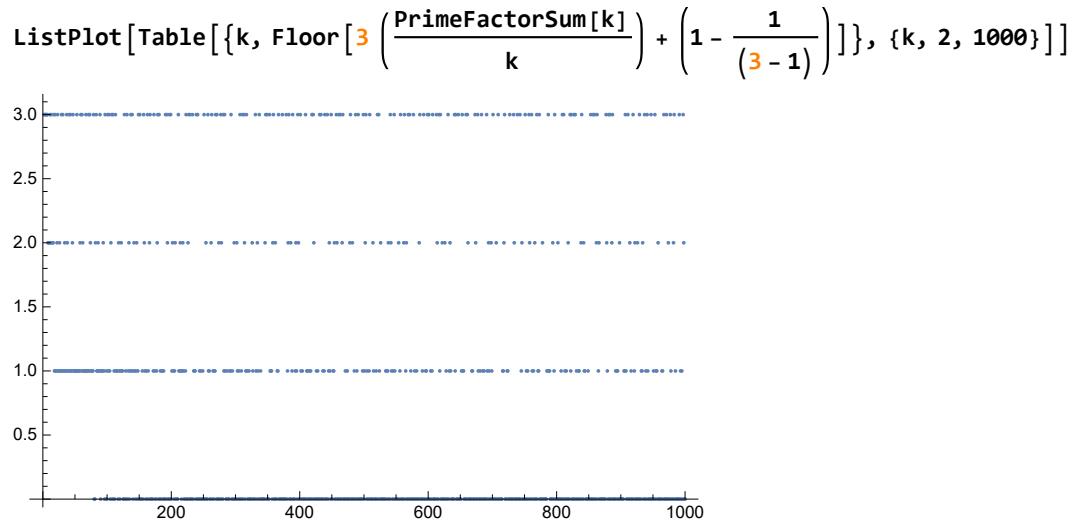
(* Union[
Floor[Table[ $\frac{\text{PrimeFactorSum}[\text{PFSnTerms}[3, 1000][[j]]}{\text{PFSnTerms}[3, 1000][[j]]}$ , {j, 1, Length[PFSnTerms[3, 1000]]}] -  $\frac{1}{3}$ ] // N] *)
(*FN1ntest[n_, testvalue_] := ((n-1) n(testvalue) + (2-n)) (*Seems to work *)
FN2ntest[n_, testvalue_] := Floor[n(testvalue) + (1 -  $\frac{1}{(n-1)}$ )]*)

(* ((n-1) n(testvalue) + (2-n)),
(n-1) (n(testvalue) + (1 -  $\frac{n}{(n-1)}$ )) *)

(* {{FN1ntest[2, 0.52], FN1ntest[2, 0.334], FN1ntest[2, 0.26], FN1ntest[2, 0.21]}, 
{FN1ntest[3, 0.52], FN1ntest[3, 0.334], FN1ntest[3, 0.26], FN1ntest[3, 0.21]}, 
{FN1ntest[4, 0.52], FN1ntest[4, 0.334], FN1ntest[4, 0.26], FN1ntest[4, 0.21]}, 
{FN1ntest[5, 0.52], FN1ntest[5, 0.334], FN1ntest[5, 0.26], FN1ntest[5, 0.21]}] // MatrixForm;
Floor[{{FN1ntest[2, 0.52], FN1ntest[2, 0.334], FN1ntest[2, 0.26], FN1ntest[2, 0.21]}, 
{FN1ntest[3, 0.52], FN1ntest[3, 0.334], FN1ntest[3, 0.26], FN1ntest[3, 0.21]}, 
{FN1ntest[4, 0.52], FN1ntest[4, 0.334], FN1ntest[4, 0.26], FN1ntest[4, 0.21]}, 
{FN1ntest[5, 0.52], FN1ntest[5, 0.334], FN1ntest[5, 0.26], FN1ntest[5, 0.21]}]] // 
MatrixForm *)
(*If the fn works only the diagnol should have any 1's *)

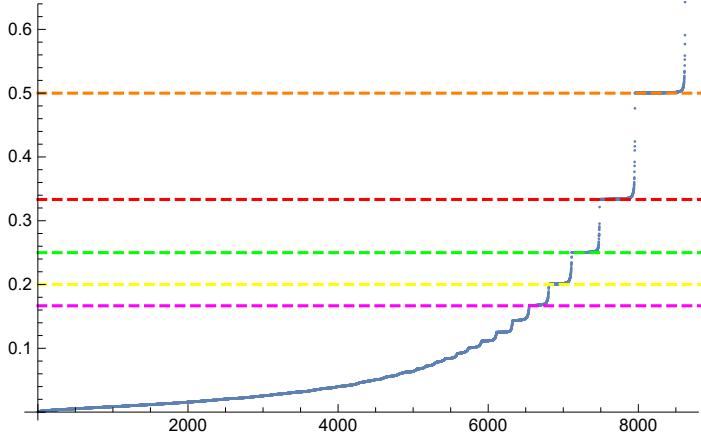
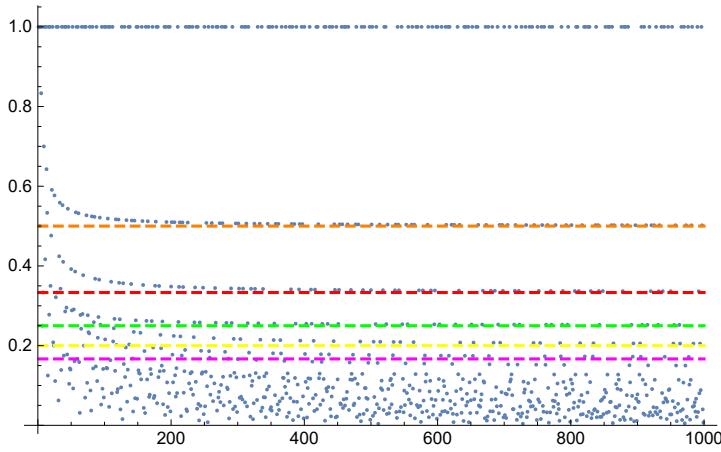
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 4 & 2 & 1 & 0 \\ 7 & 3 & 2 & 1 \end{pmatrix}$$


```



Analogous stuff for SumOfEachPrimeFactorOnce function

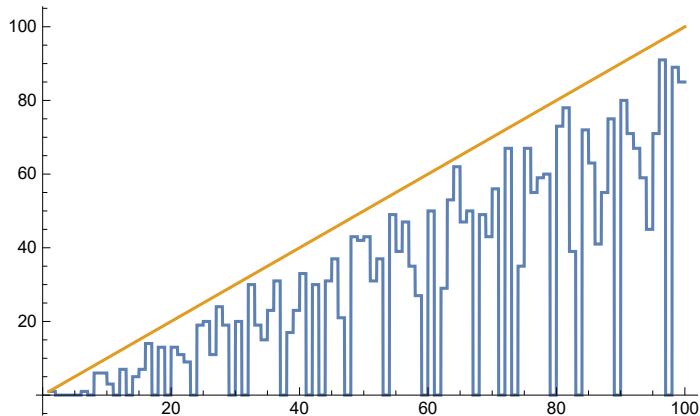
```
Show[ListPlot[Table[\frac{SumOfEachPrimeFactorOnce[k]}{k}, {k, 2, 1000}]]],  
Plot[\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}\}, {k, 2, 1000}, PlotStyle -> {{Orange, Dashed},  
{Red, Dashed}, {Green, Dashed}, {Yellow, Dashed}, {Magenta, Dashed}}]]  
Show[ListPlot[Union[Table[\frac{SumOfEachPrimeFactorOnce[k]}{k}, {k, 2, 1000}]]]],  
Plot[\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}\}, {k, 2, 1000}, PlotStyle ->  
{Orange, Dashed}, {Red, Dashed}, {Green, Dashed}, {Yellow, Dashed}, {Magenta, Dashed}}]]
```



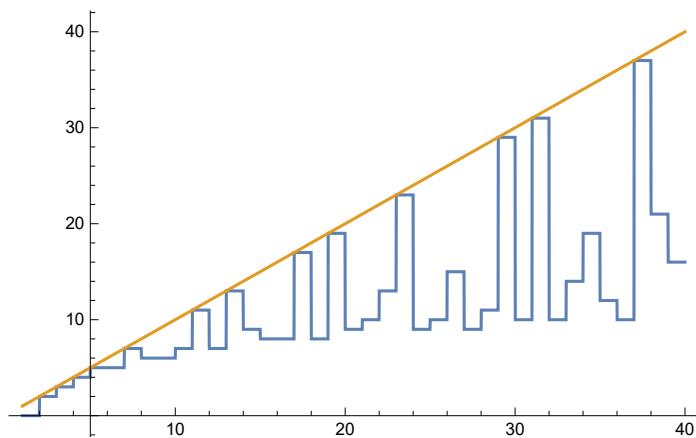
Relation to EulerPhi

```
(* Plot[{SumOfEachPrimeFactorOnce[Floor[x]], {x, 1, 100}] *)
```

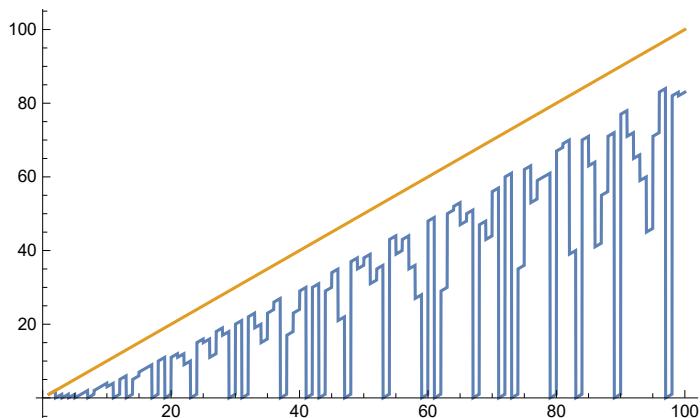
```
Plot[{Floor[x] - SumOfEachPrimeFactorOnce[Floor[x]], x}, {x, 1, 100}]
```



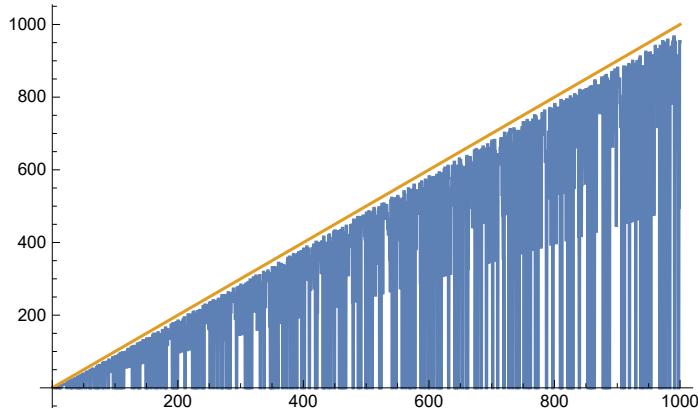
```
Plot[{PrimeFactorSum[Floor[x]], x}, {x, 1, 40}]
```



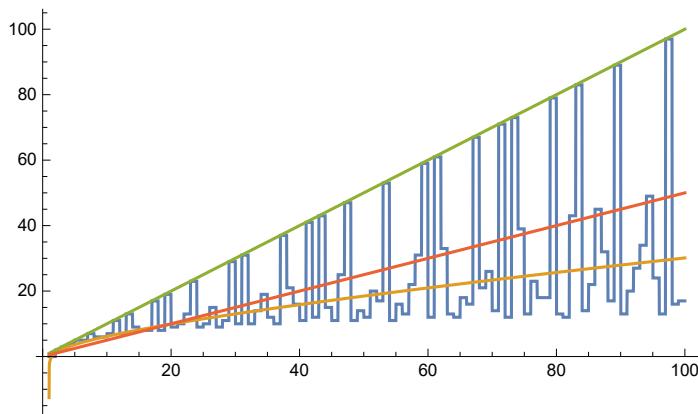
```
Plot[{(x - PrimeFactorSum[Floor[x]]), x}, {x, 1, 100}]
```



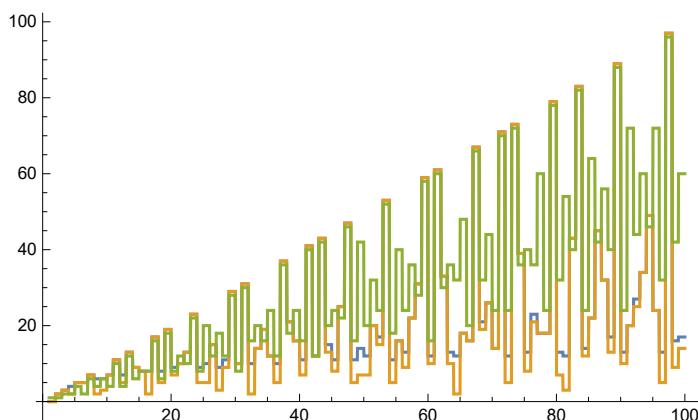
```
Plot[{Floor[x] - PrimeFactorSum[Floor[x]], x}, {x, 1, 1000}]
```

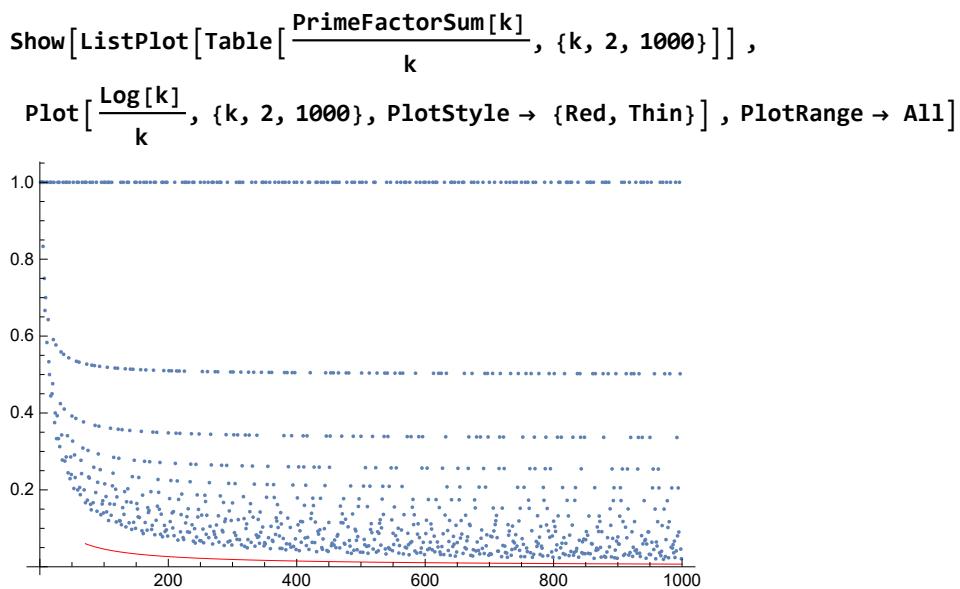
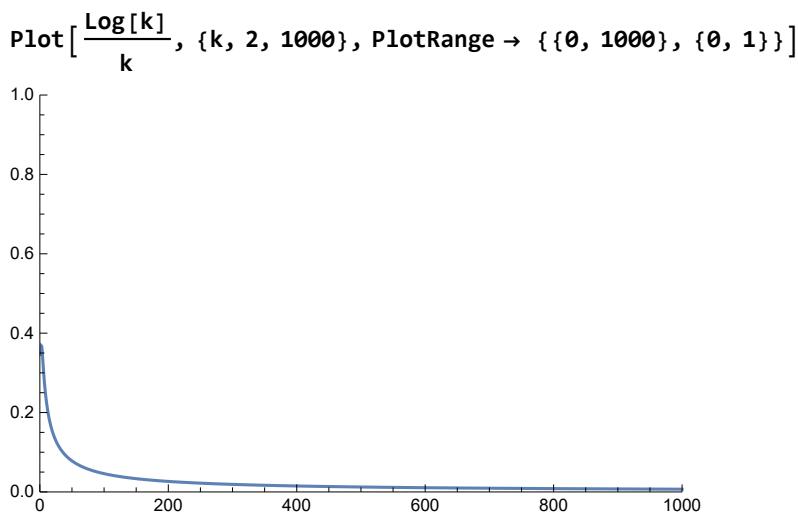
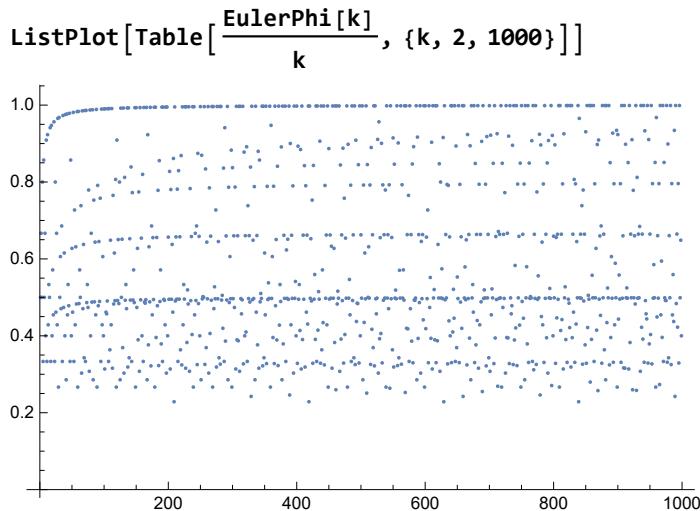


```
Plot[{PrimeFactorSum[Floor[x]], LogIntegral[x], x, 1/2 x}, {x, 1, 100}]
```

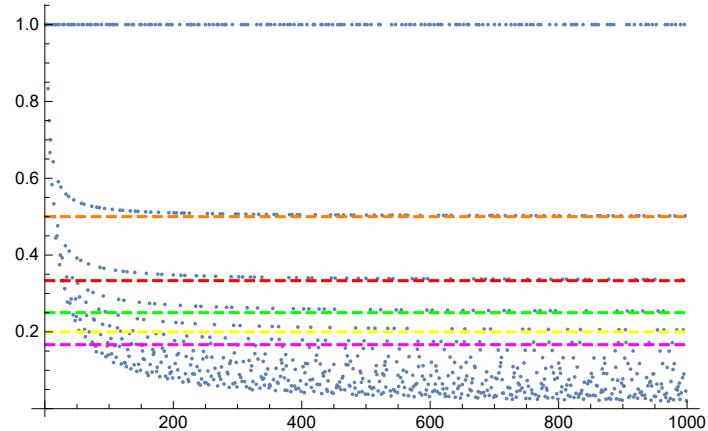


```
Plot[{PrimeFactorSum[Floor[x]], SumOfEachPrimeFactorOnce[Floor[x]], EulerPhi[Floor[x]]}, {x, 1, 100}]
```

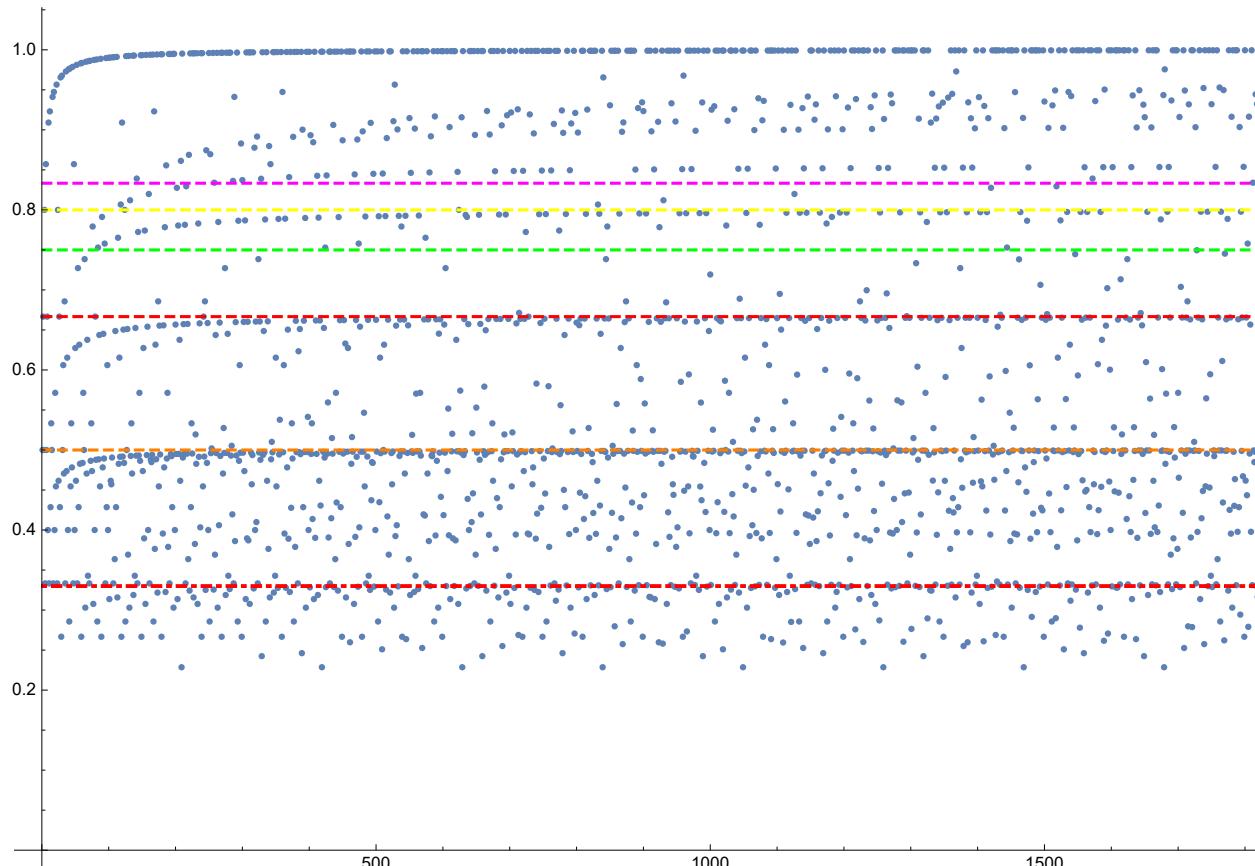




```
Show[ListPlot[Table[PrimeFactorSum[k], k, 2, 1000]],  
 Plot[{1/2, 1/3, 1/4, 1/5, 1/6}, {k, 2, 1000}, PlotStyle ->  
 {{Orange, Dashed}, {Red, Dashed}, {Green, Dashed}, {Yellow, Dashed}, {Magenta, Dashed}}]]
```



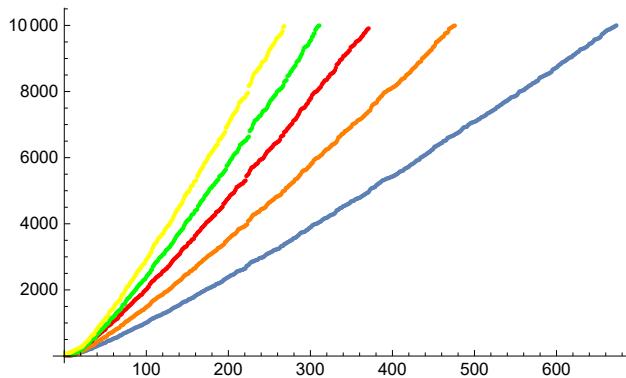
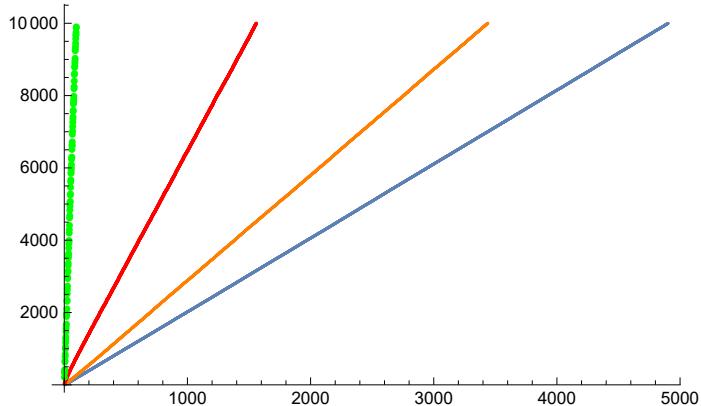
```
KMAXplot = 2000;
Show[ListPlot[Table[ $\frac{\text{EulerPhi}[k]}{k}$ , {k, 2, KMAXplot}]] ,
Plot[{ $\frac{1}{2}$ ,  $1 - \frac{1}{3}$ ,  $1 - \frac{1}{4}$ ,  $1 - \frac{1}{5}$ ,  $1 - \frac{1}{6}$ }, {k, 2, KMAXplot}, PlotStyle ->
{{Orange, Dashed}, {Red, Dashed}, {Green, Dashed}, {Yellow, Dashed}, {Magenta, Dashed}}],
Plot[{0.33}, {k, 2, KMAXplot}, PlotStyle -> {{Red, DotDashed, Thick}}]]
```

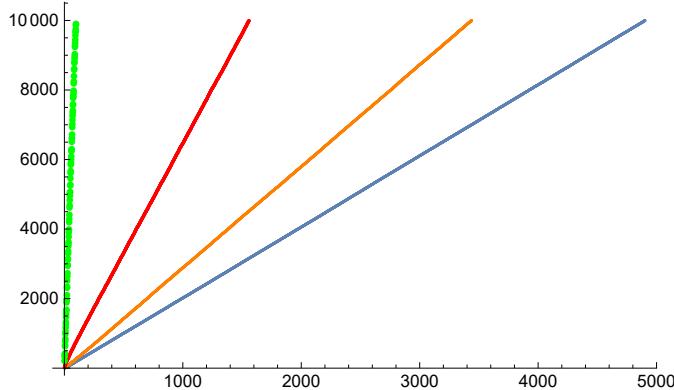


```

EulerPhinTerms[n_, kmax_] :=
  Select[Table[{k, Floor[(n - 1) n (EulerPhi[k]) / k] + (2 - n)}, {k, 2, kmax}], #[[2]] == 1 &] [[
  All, 1]]
kmax = 10000;
Show[
  ListPlot[EulerPhinTerms[2, kmax]],
  ListPlot[EulerPhinTerms[3, kmax], PlotStyle -> Orange],
  ListPlot[EulerPhinTerms[4, kmax], PlotStyle -> Red],
  ListPlot[EulerPhinTerms[5, kmax], PlotStyle -> Green],
  ListPlot[EulerPhinTerms[6, kmax], PlotStyle -> Yellow]
]
(* This plot is for comparison b/w with the analogous graph for PrimeFactorSum,
pasted below *)
(* LABEL: EulerPhi n vs. fn graph,
label for the analogous graph for PrimeFactorSum is
LABEL: PrimeFactorSum n vs. fn graph *)

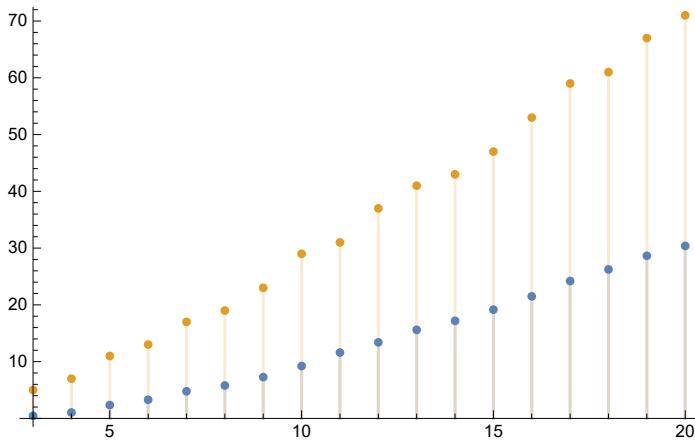
```





Numerical Integration of PrimePi

```
DiscretePlot[
 {NIntegrate[LogIntegral[x], {x, 1, y}] - NIntegrate[PrimePi[x], {x, 1, y}], Prime[y]},
 {y, 3, 20}]
NIntegrate[PrimePi[x], {x, 1, 100}]
```



1440.

```
Table[{NIntegrate[PrimePi[x], {x, 1, y}]}, {y, 2, 20}]
{{0.}, {1.}, {3.}, {5.}, {8.}, {11.}, {15.}, {19.}, {23.},
 {27.}, {32.}, {37.}, {43.}, {49.}, {55.}, {61.}, {68.}, {75.}, {83.}}
```

```
Table[
 {NIntegrate[LogIntegral[x], {x, 1, y}] - NIntegrate[PrimePi[x], {x, 1, y}]}, {y, 2, 20}]
{{-0.18411}, {0.462675}, {1.04377}, {2.35313}, {3.28678}, {4.7801},
 {5.78823}, {7.27785}, {9.223}, {11.6027}, {13.3997}, {15.5993}, {17.1889},
 {19.1576}, {21.4955}, {24.1943}, {26.246}, {28.6438}, {30.3813}}
```

```

yyy = 100;
NIntegrate[LogIntegral[x], {x, 1, yyy}]
NIntegrate[PrimePi[x], {x, 1, yyy}]
NIntegrate[LogIntegral[x], {x, 1, yyy}] - NIntegrate[PrimePi[x], {x, 1, yyy}]
1767.17
1440.
327.17

```

Relation to Divisor Function

```

Divisors[20]
{1, 2, 4, 5, 10, 20}

```

Sum of divisors:

```

DivisorSigma[1, 20]
42

```

Sum of squares of divisors:

```

DivisorSigma[2, 20]
546

Divisors[10]
DivisorSigma[1, 10]
{1, 2, 5, 10}

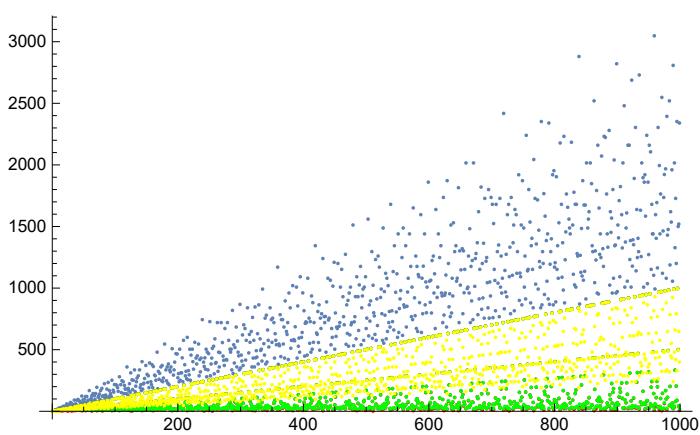
18

Divisors[9]
DivisorSigma[1, 9]
{1, 3, 9}

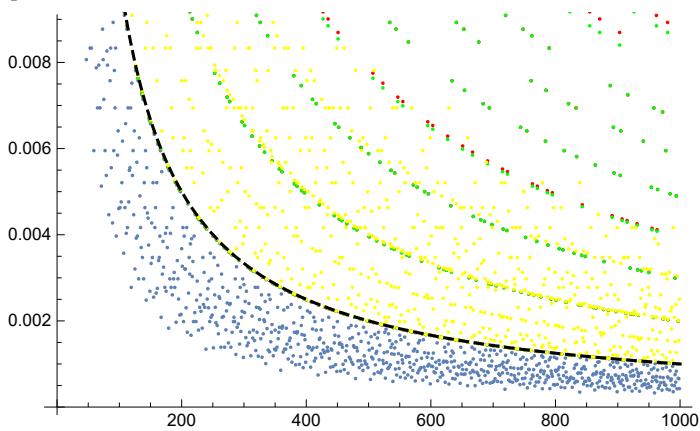
13

```

```
KMAX = 1000;
Show[
  ListPlot[Table[DivisorSigma[1, k], {k, 2, KMAX}]] ,
  ListPlot[Table[SumOfEachPrimeFactorOnce[k], {k, 2, KMAX}], PlotStyle -> Red] ,
  ListPlot[Table[PrimeFactorSum[k], {k, 2, KMAX}], PlotStyle -> Green] ,
  ListPlot[Table[EulerPhi[k], {k, 2, KMAX}], PlotStyle -> Yellow]
]
```



```
KMAX = 1000;
Show[
  ListPlot[Table[1/DivisorSigma[1, k], {k, 2, KMAX}]] ,
  ListPlot[Table[1/SumOfEachPrimeFactorOnce[k], {k, 2, KMAX}], PlotStyle -> Red] ,
  ListPlot[Table[1/PrimeFactorSum[k], {k, 2, KMAX}], PlotStyle -> Green] ,
  ListPlot[Table[1/EulerPhi[k], {k, 2, KMAX}], PlotStyle -> Yellow] ,
  Plot[{1/x}, {x, 2, KMAX}, PlotStyle -> {{Black, Dashed}}]
]
```



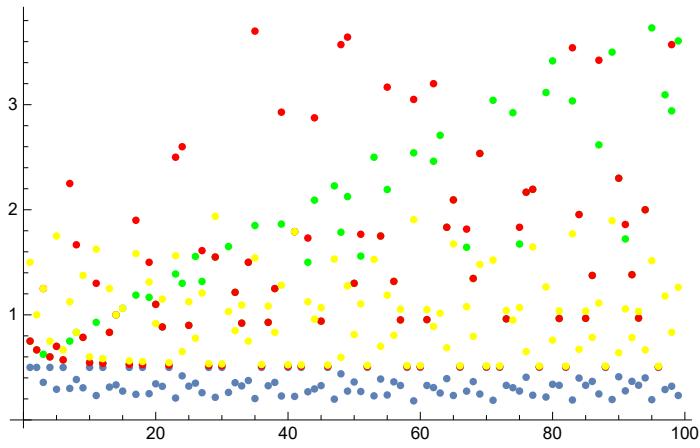
```

Sum[j, {j, 1, k}]
SumFN[Integer_] := Sum[j, {j, 1, Integer}]
SumFNtype1[k_] :=  $\frac{\text{Sum}[j, \{j, 1, k\}]}{\text{PrimeFactorSum}[k]}$ 
SumFNtype2[k_] :=  $\frac{\text{Sum}[j, \{j, 1, k\}]}{\text{DivisorSigma}[1, k]}$ 
SumFNtype3[k_] :=  $\frac{\text{Sum}[j, \{j, 1, k\}]}{\text{SumOfEachPrimeFactorOnce}[k]}$ 
SumFNtype4[k_] :=  $\frac{\text{Sum}[j, \{j, 1, k\}]}{\text{EulerPhi}[k]}$ 
 $\frac{1}{2} k (1 + k)$ 

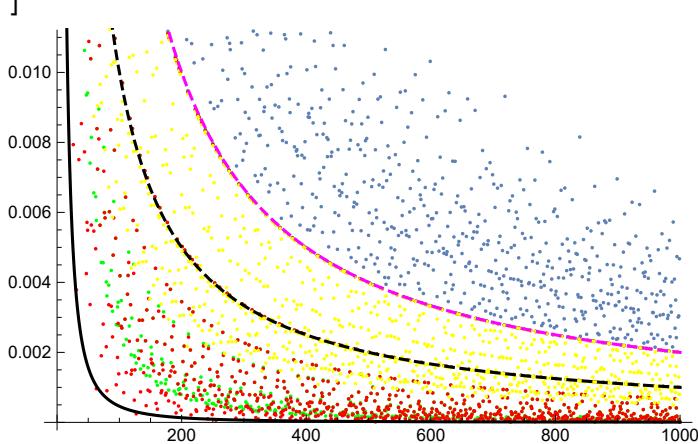
SumFN[5]
SumFNtype1[5]
SumFNtype2[5]
SumFNtype3[5]
SumFNtype4[5]
15
3
 $\frac{5}{2}$ 
3
 $\frac{15}{4}$ 

```

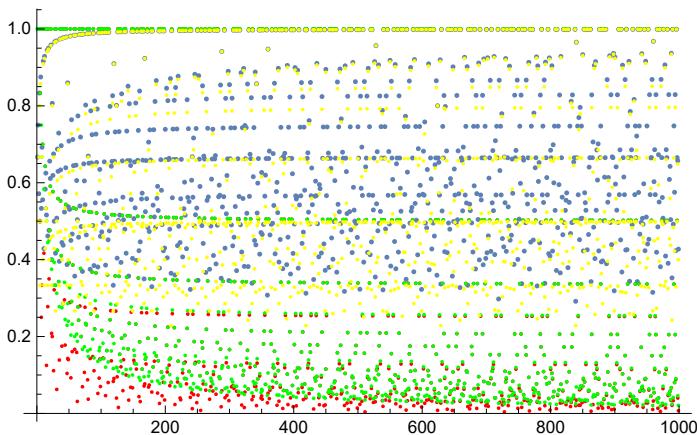
```
kMAX = 100;
Show[
  ListPlot[Table[ $\frac{\text{SumFNtype1}[k]}{k}$ , {k, 2, kMAX}], PlotStyle -> Green],
  ListPlot[Table[ $\frac{\text{SumFNtype2}[k]}{k}$ , {k, 2, kMAX}]],
  ListPlot[Table[ $\frac{\text{SumFNtype3}[k]}{k}$ , {k, 2, kMAX}], PlotStyle -> Red],
  ListPlot[Table[ $\frac{\text{SumFNtype4}[k]}{k}$ , {k, 2, kMAX}], PlotStyle -> Yellow]
]
```



```
kMAX = 1000;
Show[
  ListPlot[Table[1/SumFNtype1[k], {k, 2, kMAX}], PlotStyle -> Green],
  ListPlot[Table[1/SumFNtype2[k], {k, 2, kMAX}]],
  ListPlot[Table[1/SumFNtype3[k], {k, 2, kMAX}], PlotStyle -> Red],
  ListPlot[Table[1/SumFNtype4[k], {k, 2, kMAX}], PlotStyle -> Yellow],
  Plot[{ $\frac{1}{x}$ ,  $\frac{\log[x]}{x^2}$ }, {x, 2, kMAX},
    PlotStyle -> {{Black, Dashed}, {Black}, {Magenta, Dashed}}]
]
```



```
KMAX = 1000;
Show[
  ListPlot[Table[k / DivisorSigma[1, k], {k, 2, KMAX}]] ,
  ListPlot[Table[ $\frac{\text{SumOfEachPrimeFactorOnce}[k]}{k}$ , {k, 2, KMAX}], PlotStyle -> Red] ,
  ListPlot[Table[ $\frac{\text{PrimeFactorSum}[k]}{k}$ , {k, 2, KMAX}], PlotStyle -> Green] ,
  ListPlot[Table[ $\frac{\text{EulerPhi}[k]}{k}$ , {k, 2, KMAX}], PlotStyle -> Yellow]
]
```



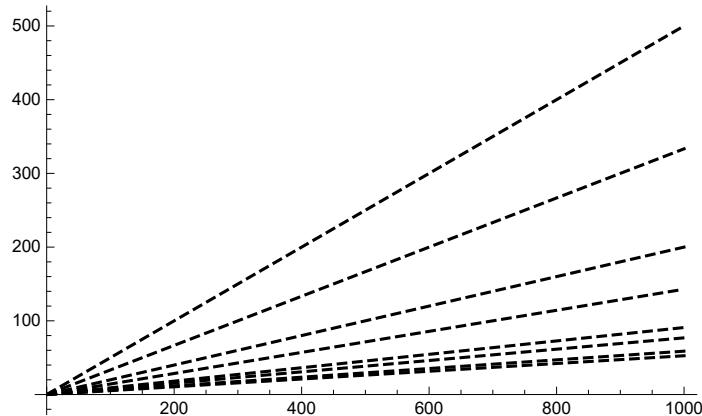
```
PrimeFactorSum[Integer_] :=
  Sum[(FactorInteger[Integer][[i, 1]]) (FactorInteger[Integer][[i, 2]]),
    {i, 1, PrimeNu[Integer]}]
SumOfEachPrimeFactorOnce[Integer_] :=
  Sum[(FactorInteger[Integer][[i, 1]]), {i, 1, PrimeNu[Integer]}]
```

```
PFSnTerms[n_, kmax_] :=
  Select[Table[{k, Floor[ $\left( (n-1) \frac{\text{PrimeFactorSum}[k]}{k} + (2-n) \right)$ ]}, {k, 2, kmax}], #[[2]] == 1 &][[All, 1]] (*Gives the k values for which
the function  $\frac{\text{PrimeFactorSum}[k]}{k}$  lies in  $[\frac{1}{n}, \frac{1}{n-1}]$  and/or tends to  $\frac{1}{n}$ ,
although some values given by this might actually be part of the set
of points tending towards  $\frac{1}{n+1}$  as elaborated upon in the orange
box and subsection "Finding Fns pts actually tend towards" *)
PFSnTermsList[n_, kmax_] :=
  Table[ $\frac{\text{PrimeFactorSum}[\text{PFSnTerms}[n, \text{kmax}][[j]]]}{\text{PFSnTerms}[n, \text{kmax}][[j]]}$ , {j, 1, Length[PFSnTerms[n, kmax]]}]
(*Gives the actual values of the function  $\frac{\text{PrimeFactorSum}[k]}{k}$  for
the k produced in the previous fn PFSnTerms[n,kmax] *)
PFSnPlotList[n_, kmax_] := Table[{PFSnTerms[n, kmax][[j]], PFSnTermsList[n, kmax][[j]]},
  {j, 1, Length[PFSnTerms[n, kmax]]}]
(*Puts the previous two fns in a format easier to get some plots in *)
```

Reciprocal of Primes tending fn

Want to construct or find a similar function but instead want the points to tend towards the reciprocals of the primes

```
Plot[Table[ $\frac{1}{\text{Prime}[i]}$  k, {i, 1, 8}], {k, 2, KMAX}, PlotStyle -> {{Black, Dashed}}]
```

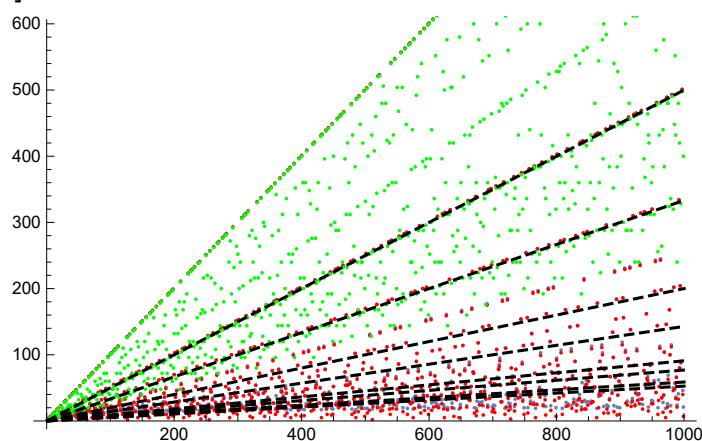


```
KMAX = 1000;
```

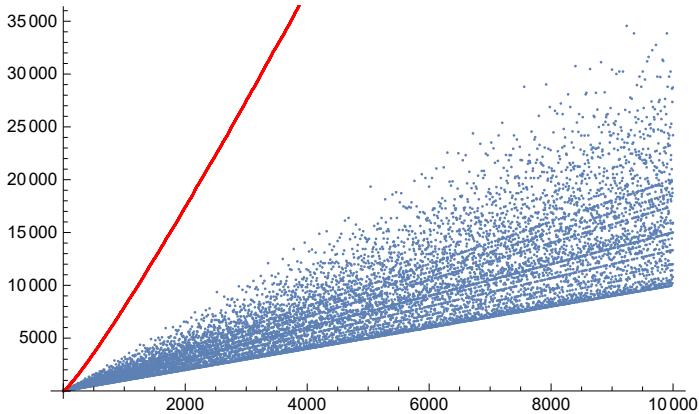
```
Show[
```

```
ListPlot[Table[PrimeFactorSum[k], {k, 2, KMAX}]] ,  
ListPlot[Table[SumOfEachPrimeFactorOnce[k], {k, 2, KMAX}], PlotStyle -> Red] ,  
ListPlot[Table[EulerPhi[k], {k, 2, KMAX}], PlotStyle -> Green] ,  
Plot[Table[ $\frac{1}{\text{Prime}[i]}$  k, {i, 1, 8}], {k, 2, KMAX}, PlotStyle -> {{Black, Dashed}}]
```

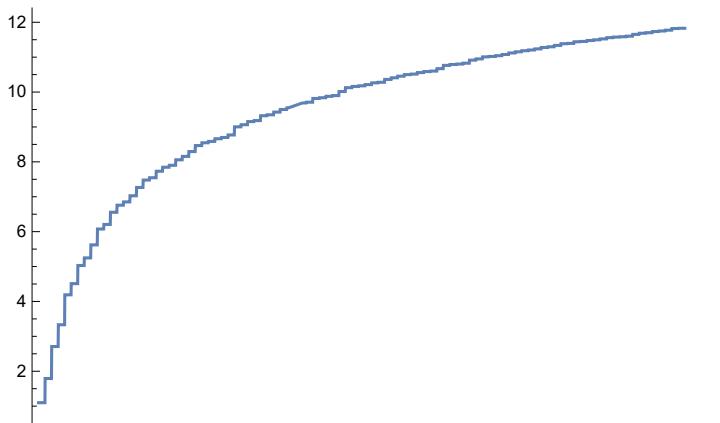
```
]
```



```
KMAX = 10000;
Show[
  ListPlot[Table[DivisorSigma[1, k], {k, 2, KMAX}]] ,
  ListPlot[Table[DivisorSigma[1, Prime[k]], {k, 2, KMAX}], PlotStyle -> Red]
]
```

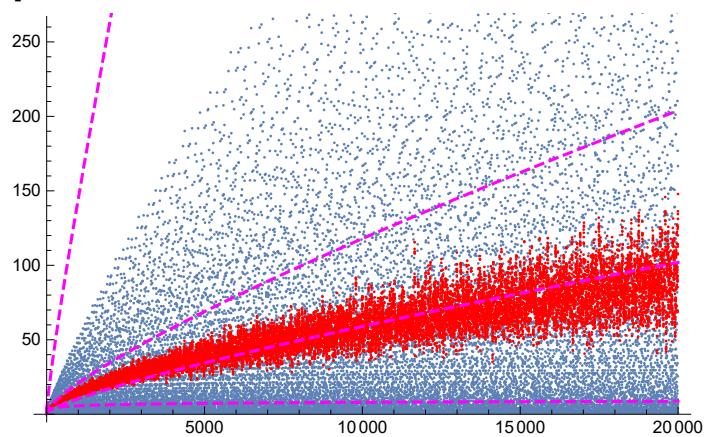


```
KMAX = 1000;
Show[
  ListPlot[Table[PrimeFactorSum[k], {k, 2, KMAX}]] ,
  ListPlot[Table[SumOfEachPrimeFactorOnce[k], {k, 2, KMAX}], PlotStyle -> Red] ,
  ListPlot[Table[EulerPhi[k], {k, 2, KMAX}], PlotStyle -> Green] ,
  Plot[Table[ $\frac{1}{\text{Prime}[i]}$  k, {i, 1, 8}], {k, 2, KMAX}, PlotStyle -> {{Black, Dashed}}]
]
SumNaturals[x_] := Sum[k, {k, 1, x}]
Sum[j, {j, 1, k}]
 $\frac{1}{2} k (1 + k)$ 
Plot[Log[SumNaturals[Prime[Floor[x]]]], {x, 1, 100}]
```

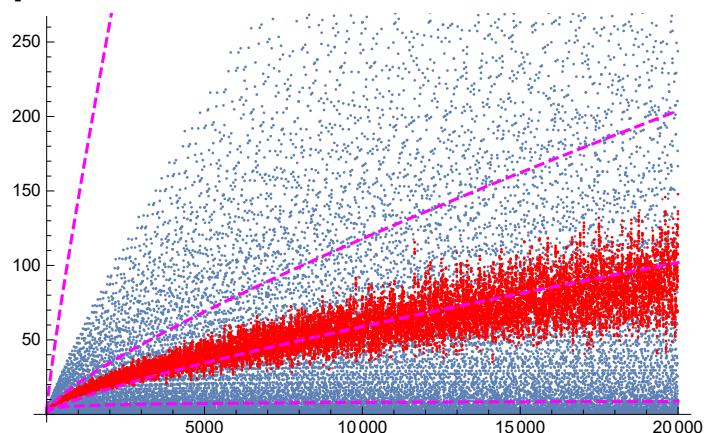


GBC

```
KMAX = 20000;
Show[
  ListPlot[Table[k / PrimeFactorSum[k], {k, 2, KMAX}]] ,
  ListPlot[MeanFilter[Table[k / PrimeFactorSum[k], {k, 2, KMAX}], 10], PlotStyle -> Red] ,
  Plot[{ $\frac{k}{\text{Log}[k]}$ ,  $\frac{k}{(\text{Log}[k])^2}$ ,  $\frac{k}{\text{LogIntegral}[k]}$ ,  $\frac{k}{2((\text{Log}[k])^2)}$ },
    {k, 2, KMAX}, PlotStyle -> {{Magenta, Dashed}}]
]
```



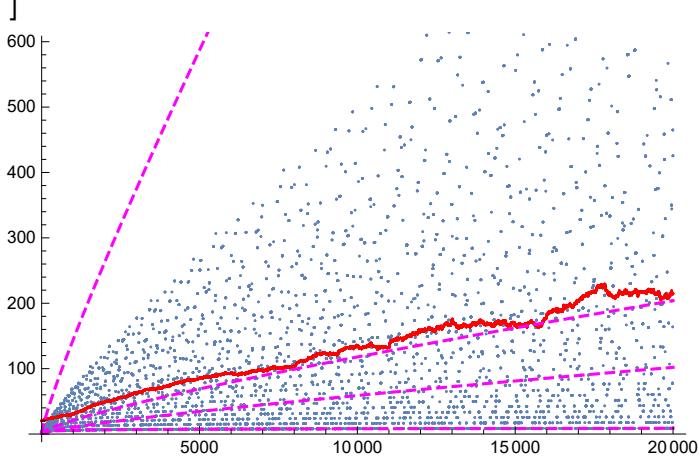
```
KMAX = 20000;
Show[
  ListPlot[Table[k / PrimeFactorSum[k], {k, 2, KMAX}]] ,
  ListPlot[MeanFilter[Table[k / PrimeFactorSum[k], {k, 2, KMAX}], 10], PlotStyle -> Red] ,
  Plot[{ $\frac{k}{\text{Log}[k]}$ ,  $\frac{k}{(\text{Log}[k])^2}$ ,  $\frac{k}{\text{LogIntegral}[k]}$ ,  $\frac{k}{2((\text{Log}[k])^2)}$ },
    {k, 2, KMAX}, PlotStyle -> {{Magenta, Dashed}}]
]
```



The mean of this function, $\frac{k}{\text{PrimeFactorSum}[k]}$, is approx $\frac{k}{2((\text{Log}[k])^2)}$, i.e. similar to the value tended

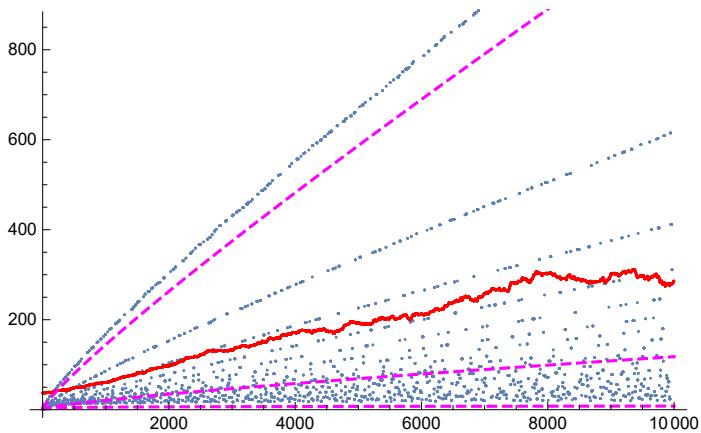
towards in the *extended Goldbach conjecture*

```
KMAX = 20000;
Show[
  ListPlot[Table[k / PrimeFactorSum[PrimePi[k]], {k, 3, KMAX}]] ,
  ListPlot[
    MeanFilter[Table[k / PrimeFactorSum[PrimePi[k]], {k, 3, KMAX}], 1000], PlotStyle -> Red] ,
  Plot[{k / Log[k], k / ((Log[k])^2), k / LogIntegral[k], k / (2 ((Log[k])^2))},
    {k, 2, KMAX}, PlotStyle -> {{Magenta, Dashed}}]
]
```



The mean of this function, $\frac{k}{\text{PrimeFactorSum}[\text{PrimePi}[k]]}$, tends towards $\frac{k}{((\log[k])^2)}$, i.e. similar to the value tended towards in the *extended Goldbach conjecture*

```
KMAX = 10000;
Show[
  ListPlot[Table[PrimeFactorSum[PrimePi[k]], {k, 2, KMAX}]] ,
  ListPlot[
    MeanFilter[Table[PrimeFactorSum[PrimePi[k]], {k, 2, KMAX}], 1000], PlotStyle -> Red] ,
  Plot[{ $\frac{k}{\log[k]}$ ,  $\frac{k}{(\log[k])^2}$ ,  $\frac{k}{\text{LogIntegral}[k]}$ ,  $\frac{k}{2(\log[k])^2}$ },
    {k, 2, KMAX}, PlotStyle -> {{Magenta, Dashed}}]
]
```



```
KMAX = 10000;
Show[
  ListPlot[Table[PrimeFactorSum[k], {k, 2, KMAX}]] ,
  Plot[Table[ $\frac{1}{i}k$ , {i, 1, 10}], {k, 2, KMAX}, PlotStyle -> {{Black, Dashed, Thin}}]
]
```

```
Table[PrimeFactorSum[2 k], {k, 2, KMAX}]
```

```
{4, 5, 6, 7, 7, 9, 8, 8, 9, 13, 9, 15, 11, 10, 10, 19, 10, 21, 11, 12, 15, 25, 11, 12, 17, 11, 13, 31, 12, 33, 12, 16, 21, 14, 12, 39, 23, 18, 13, 43, 14, 45, 17, 13, 27, 49, 13, 16, 14, 22, 19, 55, 13, 18, 15, 24, 33, 61, 14, 63, 35, 15, 14, 20, 18, 69, 23, 28, 16, 73, 14, 75, 41, 15, 25, 20, 20, 81, 15, 14, 45, 85, 16, 24, 47, 34, 19, 91, 15, 22, 29, 36, 51, 26, 15, 99, 18, 19, 16, 103, 24, 105, 21, 17, 57, 109, 15, 111, 20, 42, 17, 115, 26, 30, 35, 21, 63, 26, 16, 24, 65, 46, 37, 17, 17, 129, 16, 48, 22, 133, 20, 28, 71, 16, 25, 139, 30, 141, 18, 52, 75, 26, 16, 36, 77, 19, 43, 151, 17, 153, 27, 25, 22, 38, 22, 159, 83, 58, 17, 32, 16, 165, 47, 21, 87, 169, 18, 28, 26, 27, 49, 175, 36, 19, 21, 64, 93, 181, 17, 183, 24, 66, 31, 44, 38, 30, 53, 18, 28, 193, 17, 195, 101, 23, 20, 199, 21, 201, 18, 72, 105, 38, 26, 48, 107, 31, 23, 32, 19, 213, 59, 76, 111, 50, 17, 40, 113, 78, 22, 32, 44, 225, 19, 18, 117, 229, 28, 231, 32, 23, 37, 235, 23, 54, 65, 84, 28, 241, 18, 243, 26, 17, 67, 21, 48, 34, 39, 88, 19, 253, 19, 36, 131, 27, 18, 259, 50, 46, 24, 37, 135, 265, 22, 60, 30, 94, 73, 271, 18, 273, 27, 25, 141, 23, 32, 279, 143, 39, 20, 283, 54, 285, 77, 29, 28, 50, 18, 36, 38, 102, 79, 295, 21, 66, 45, 22, 153, 38, 19, 52, 155, 106, 29, 68, 27, 309, 24, 108, 40, 313, 24, 315, 161, 20, 85, 319, 60, 42, 19, 112, 34, 38, 18, 25, 167, 114, 49, 56, 23, 333, 89, 45, 171, 74, 20, 339, 30, 118, 28, 44, 29, 23, 51, 33, 177, 349, 38, 351, 21, 24, 23, 355, 66, 78, 95, 29, 183, 361, 19, 40, 185, 27, 26, 80, 68, 369, 33, 49, 46, 62, 40, 375, 32, 20, 55, 44, 20, 381, 30, 132, 195, 385, 19, 25, 197, 51, 103, 391, 25, 42, 22, 136, 201, 86, 23, 399, 203, 31, 20, 403, 74, 46, 107, 19, 40, 50, 28, 411, 50, 142, 109, 68, 33, 90, 25, 144, 34, 421, 21, 423, 215, 55, 61, 29, 78, 70, 113, 29, 52, 433, 19, 435, 42, 39, 115, 44, 80, 441, 24, 22, 34, 445, 46, 96, 227, 154, 21, 451, 20, 54, 119, 156, 231, 27, 30, 459, 233, 28, 34, 463, 25, 465, 39, 41, 237, 469, 25, 76, 56, 162, 67, 56, 86, 31, 30, 61, 243, 481, 20, 52, 245, 35, 28, 104, 19, 489, 69, 168, 23, 493, 50, 48, 36, 24, 41, 80, 90, 501, 21, 172, 255, 505, 21, 108, 38, 31, 133, 511, 29, 82, 20, 30, 261, 110, 52, 60, 48, 178, 26, 523, 39, 525, 137, 22, 267, 50, 24, 48, 62, 67, 32, 56, 96, 114, 75, 184, 273, 27, 20, 543, 275, 186, 29, 116, 27, 549, 143, 69, 25, 50, 34, 88, 281, 47, 145, 559, 41, 58, 22, 33, 285, 565, 56, 120, 287, 21, 79, 571, 31, 573, 30, 196, 52, 35, 20, 579, 38, 198, 40, 92, 104, 66, 81, 26, 297, 589, 23, 52, 68, 202, 47, 595, 24, 31, 155, 204, 40, 601, 21, 603, 54, 75, 157, 29, 108, 609, 31, 41, 70, 62, 29, 615, 311, 51, 26, 619, 110, 621, 42, 34, 315, 98, 26, 22, 317, 35, 163, 56, 22, 633, 87, 216, 321, 134, 62, 29, 44, 79, 21, 643, 114, 645, 36, 53, 40, 649, 20, 72, 27, 43, 169, 655, 116, 138, 51, 81, 58, 661, 25, 663, 335, 35, 91, 33, 47, 54, 173, 228, 76, 74, 22, 675, 341, 21, 32, 679, 120, 106, 30, 232, 46, 685, 31, 144, 25, 234, 53, 68, 35, 693, 179, 26, 351, 146, 40, 60, 353, 238, 23, 703, 26, 58, 25, 57, 357, 110, 68, 711, 80, 87, 97, 56, 31, 31, 185, 244, 363, 721, 21, 112, 42, 246, 187, 41, 29, 729, 28, 20, 82, 62, 70, 735, 371, 24, 35, 80, 51, 741, 48, 37, 64, 745, 42, 156, 377, 91, 34, 116, 22, 753, 57, 256, 46, 158, 22, 759, 383, 39, 32, 763, 134, 118, 197, 30, 387, 74, 21, 771, 27, 262, 199, 775, 53, 43, 105, 49, 393, 62, 27, 84, 44, 40, 24, 164, 138, 789, 203, 268, 88, 122, 25, 76, 401, 63, 205, 799, 33, 66, 22, 97, 405, 86, 76, 37, 48, 274, 109, 811, 21, 813, 42, 276, 52, 170, 30, 64, 413, 28, 52, 823, 144, 825, 111, 26, 70, 829, 35, 831, 92, 282, 27, 33, 146, 174, 36, 42, 423, 841, 23, 60, 425, 286, 217, 33, 57, 31, 63, 288, 31, 62, 80, 855, 72, 32, 115, 859, 31, 861, 54, 53, 435, 865, 21, 180, 437, 39, 44, 92, 41, 82, 117, 105, 46, 24, 82, 879, 443, 298, 26, 883, 24, 885, 36, 69, 447, 889, 48, 136, 98, 25, 229, 68, 156, 186, 23, 41, 453, 62, 22, 72, 56, 55, 121, 188, 158, 909, 233, 109, 29, 913, 32, 96, 461, 71, 235, 140, 30, 921, 36, 312, 465, 86, 27, 49, 467, 111, 41, 931, 43, 35, 239, 316, 471, 35, 27, 939, 78, 318, 58, 943, 164, 66, 69, 23, 58, 949, 88, 88, 33, 322, 32, 955, 63, 198, 245, 45, 483, 146, 22, 64, 54, 115, 247, 200, 37, 969, 30, 41, 106, 973, 21, 148, 491, 28, 71, 979, 170, 102, 25, 117, 495, 985, 52, 204, 50, 59, 38, 68, 26, 993, 43, 336, 82, 206, 92, 999, 503, 48, 23}
```