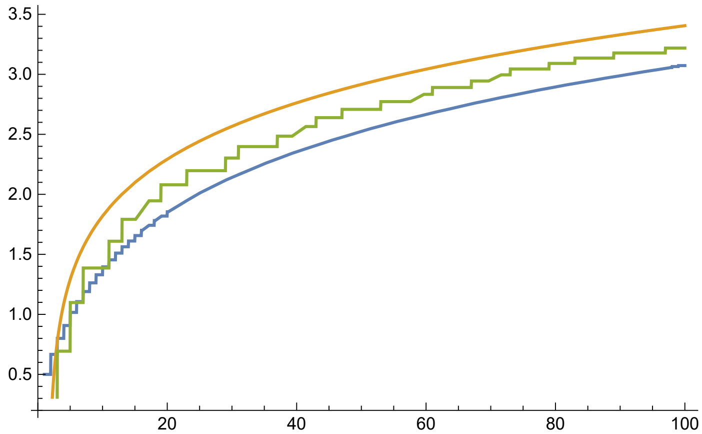
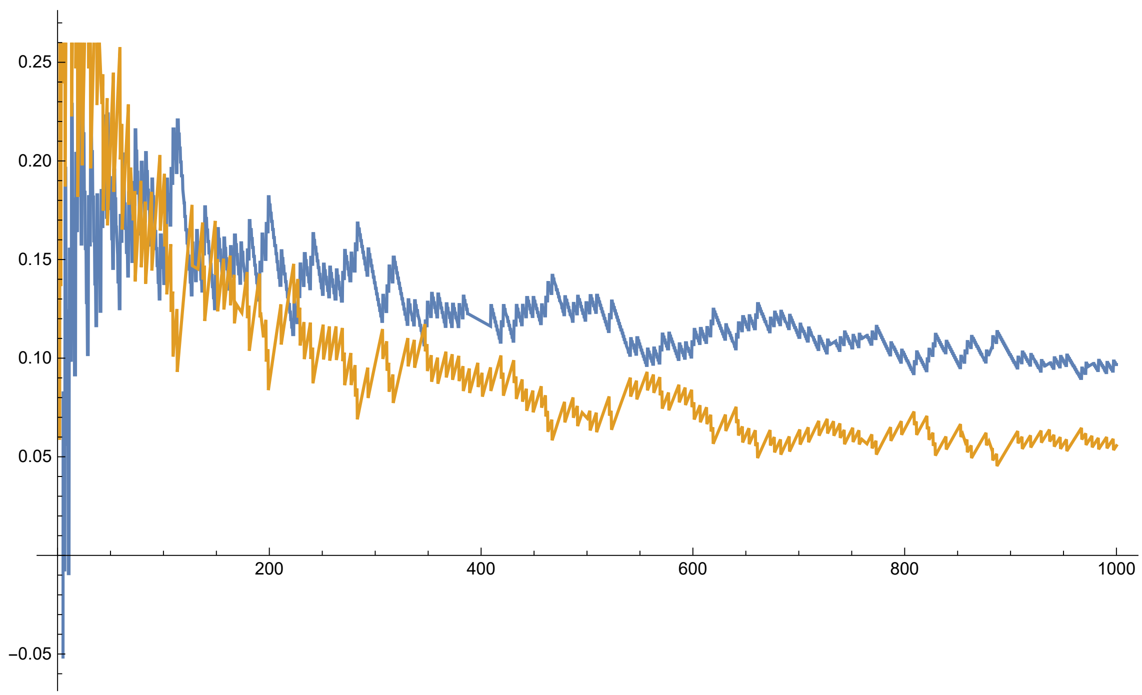
```
Plot[{Sum[1/i - 1/Prime[i], {i, 1, x}], Log[LogIntegral[x]], Log[PrimePi[x]]}, {x, 1, 100}]
```

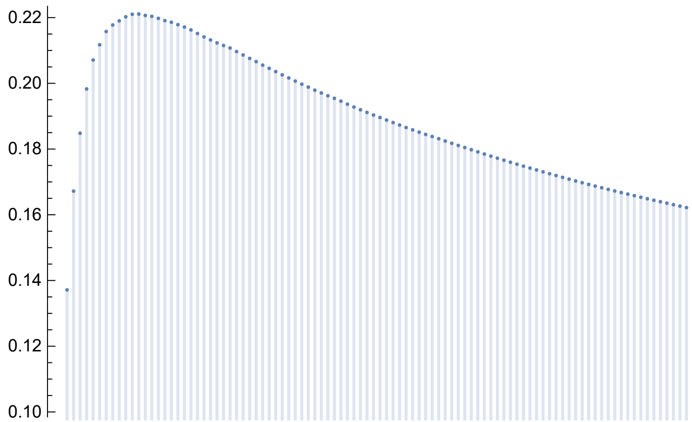



```
Ω[x_] := Sum[1/i - 1/Prime[i], {i, 1, x}]
```

```
Plot[{Log[PrimePi[x]] - Sum[1/i - 1/Prime[i], {i, 1, x}],
  Log[LogIntegral[x]] - Log[PrimePi[x]]}, {x, 1, 1000}]
```

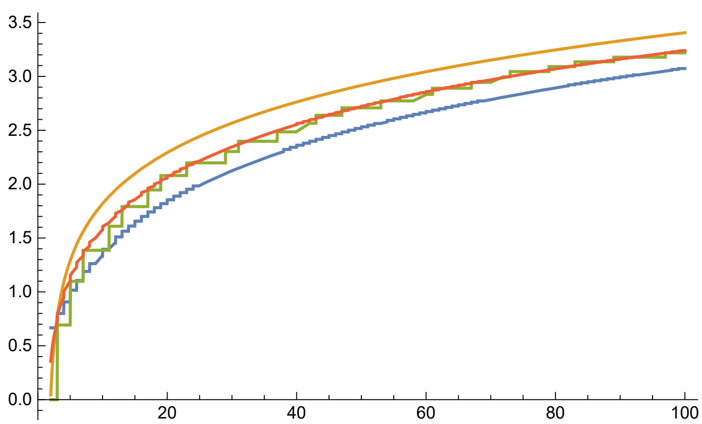


```
(*Plot the Mean of these *)
DiscretePlot[Mean[{Log[PrimePi[x]] - Sum[1/i - 1/Prime[i], {i, 1, x}],
    Log[LogIntegral[x]] - Log[PrimePi[x]]}], {x, 2, 100}]
```

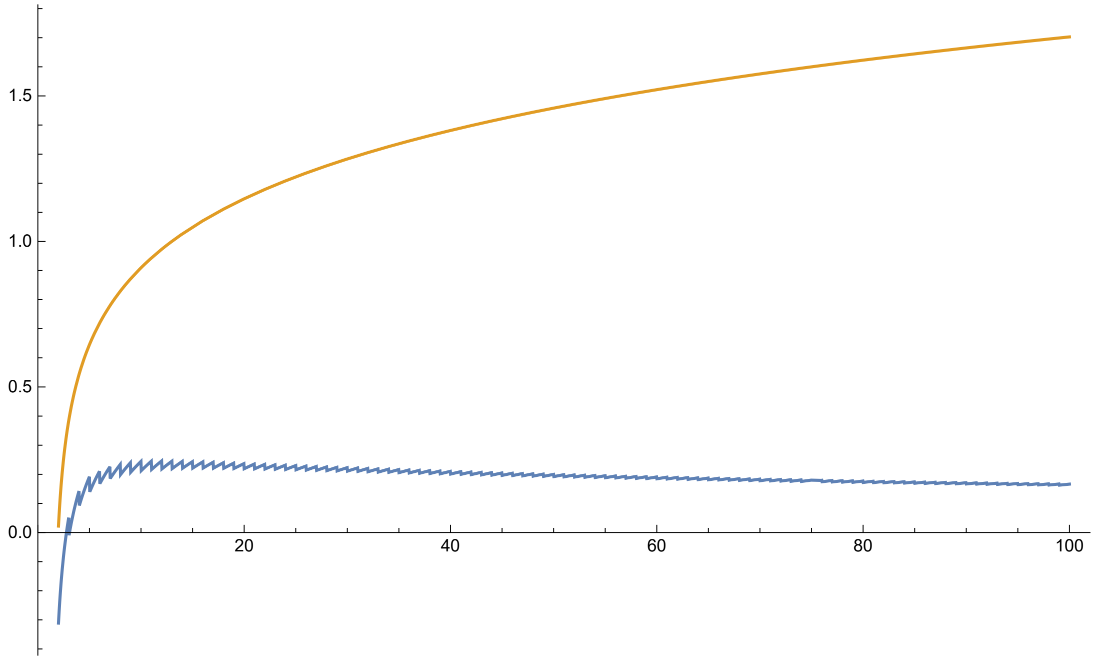



```
A[x_] := 1/2 (Log[LogIntegral[x]] - Ω[x])
B[x_] := (Log[PrimePi[x]]) - (1/2 (Log[LogIntegral[x]] + Ω[x]))
mean[x_] := (1/2 (Log[LogIntegral[x]] + Ω[x]))
```

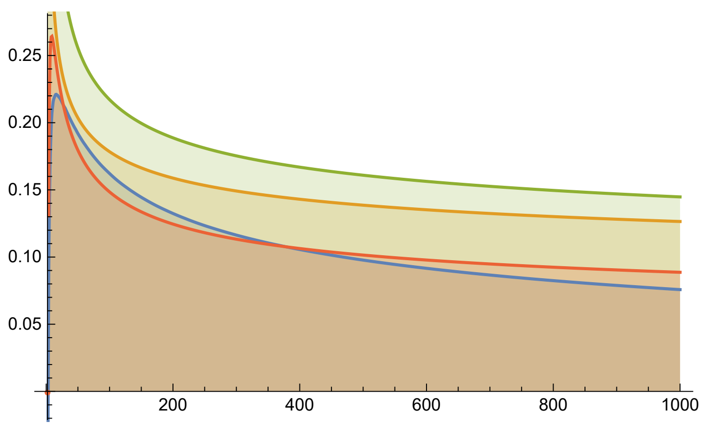
```
Plot[{Sum[1/i - 1/Prime[i], {i, 1, x}],
  Log[LogIntegral[x]], Log[PrimePi[x]], mean[x]}, {x, 2, 100}]
```



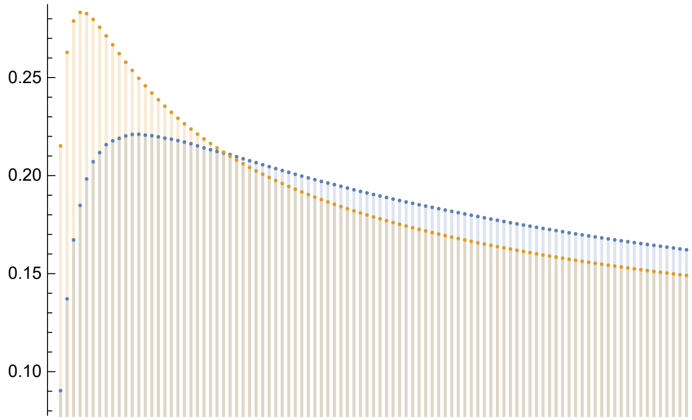
```
Plot[{A[x], 1/2 Log[LogIntegral[x]]}, {x, 2, 100}]
```



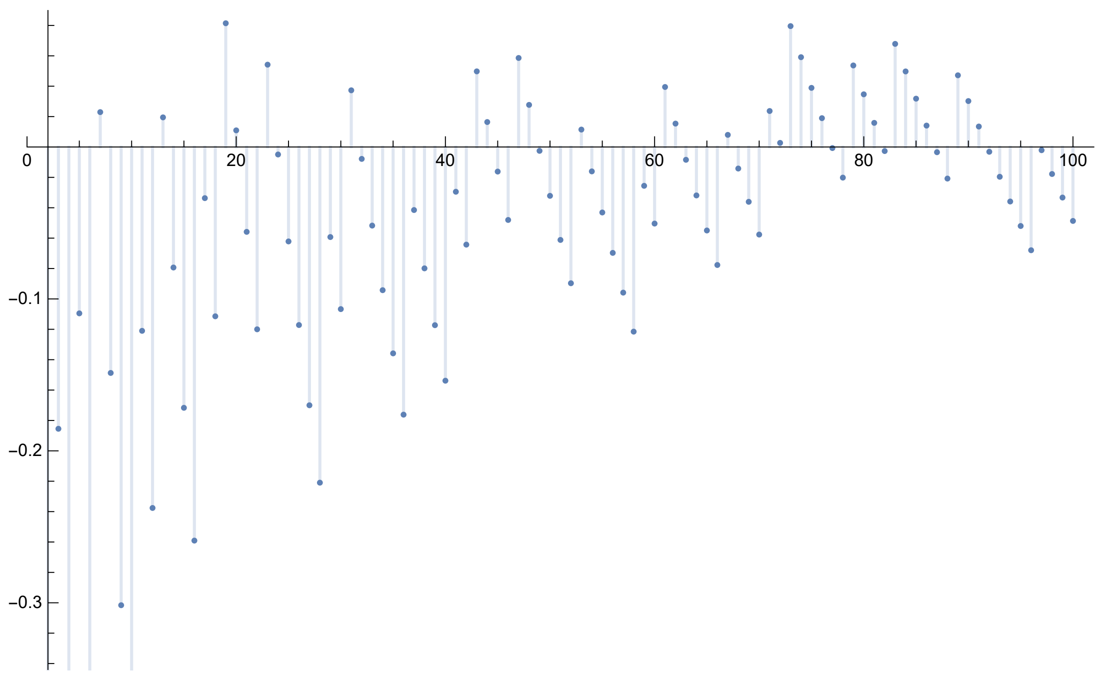

```
DiscretePlot[{Mean[{Log[PrimePi[x]] - Sum[1/i - 1/Prime[i], {i, 1, x}],
    Log[LogIntegral[x]] - Log[PrimePi[x]]}],
  1/(Log[x] + 1), 1/Log[x], 1/2 LogIntegral[x - 2]/x}, {x, 2, 1000}]
```



```
DiscretePlot[{Mean[{Log[PrimePi[x]] - Sum[1/i - 1/Prime[i], {i, 1, x}],
    Log[LogIntegral[x]] - Log[PrimePi[x]]}], 1/2 LogIntegral[x - 1.45]/(x)}, {x, 2, 100}]
```




```
(*Plot the difference between these *)
DiscretePlot[{(Log[PrimePi[x]] - Sum[1/i - 1/Prime[i], {i, 1, x}]) -
   (Log[LogIntegral[x]] - Log[PrimePi[x]])}, {x, 2, 100}]
```



```
Table[Mean[{Log[PrimePi[x]] - Sum[1/i - 1/Prime[i], {i, 1, x}],
    Log[LogIntegral[x]] - Log[PrimePi[x]]}], {x, 2, 100}] // N
max = Max[Table[Mean[{Log[PrimePi[x]] - Sum[1/i - 1/Prime[i], {i, 1, x}],
      Log[LogIntegral[x]] - Log[PrimePi[x]]}], {x, 2, 100}]] // N
```

```
{-0.311247, -0.0141159, 0.0903028, 0.137131, 0.167192, 0.184809, 0.198278, 0.207086,
 0.211728, 0.215763, 0.217751, 0.218997, 0.220235, 0.221014, 0.221084, 0.220657, 0.22038,
 0.219766, 0.219099, 0.218578, 0.217853, 0.217116, 0.216246, 0.215166, 0.214119,
 0.213198, 0.212302, 0.211515, 0.210749, 0.209667, 0.208627, 0.207572, 0.206609,
 0.205541, 0.204558, 0.203572, 0.202584, 0.201634, 0.200685, 0.19974, 0.19886, 0.197926,
 0.197053, 0.19621, 0.195422, 0.194567, 0.193659, 0.192781, 0.191951, 0.191149, 0.190355,
 0.189604, 0.188828, 0.188061, 0.187303, 0.186555, 0.185843, 0.185139, 0.184456,
 0.183806, 0.183139, 0.182436, 0.181753, 0.181099, 0.180463, 0.179798, 0.179142,
 0.178478, 0.17784, 0.177218, 0.176605, 0.175992, 0.175387, 0.174791, 0.174209,
 0.173636, 0.173063, 0.172504, 0.171946, 0.171384, 0.170841, 0.170294, 0.169765,
 0.169242, 0.168732, 0.168227, 0.167725, 0.167233, 0.166757, 0.166291, 0.165817,
 0.165345, 0.164882, 0.164422, 0.16397, 0.163525, 0.163073, 0.162634, 0.162179}
```

```
0.221084
```

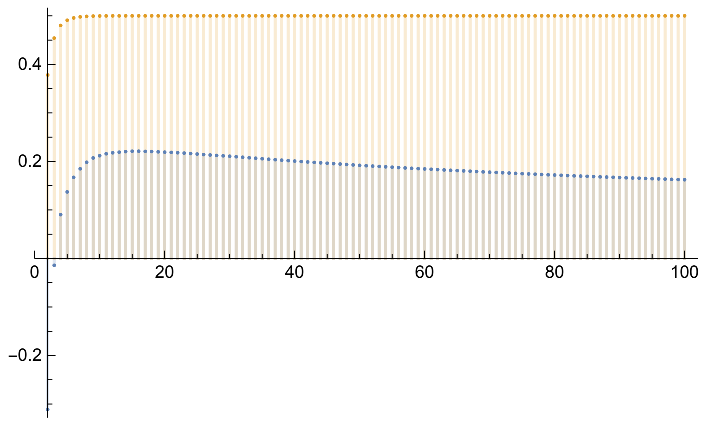
```
x = 16;
Mean[{Log[PrimePi[x]] - Sum[1/i - 1/Prime[i], {i, 1, x}],
   Log[LogIntegral[x]] - Log[PrimePi[x]]}] // N
(*Max occurs at x = 16 *)
```

```
0.221084
```

```
1 / max
Exp[max]
Log[max]
√max
√max Log[max]
```

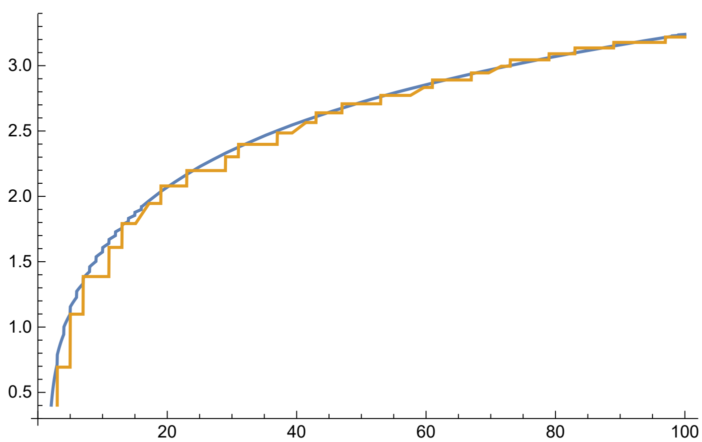
```
4.52316

1.24743

-1.50921

0.470196

-0.709625
```

```
(*Try to compare this mean curve to some fns  *)
DiscretePlot[{Mean[{Log[PrimePi[x]] - Sum[1/i - 1/Prime[i], {i, 1, x}],
    Log[LogIntegral[x]] - Log[PrimePi[x]]}], 1/(1 + Zeta[x])}, {x, 2, 100}]
```

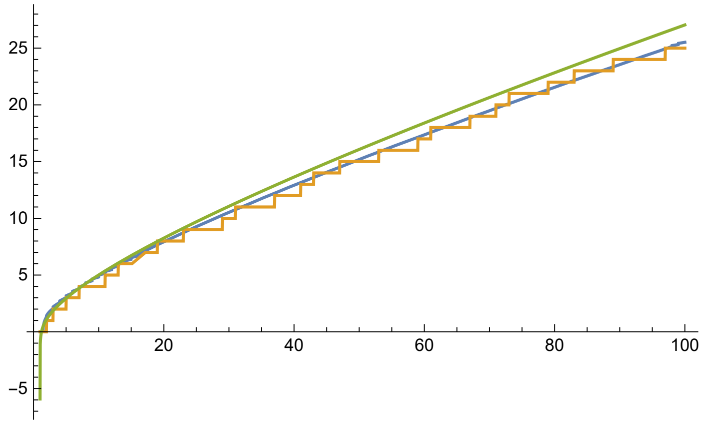


Omega fn cont

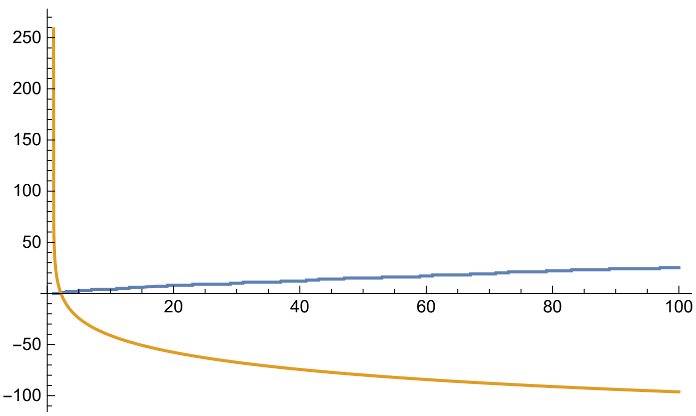
```
Plot[
 {1/2 (Sum[1/i - 1/Prime[i], {i, 1, x}] + Log[LogIntegral[x]]), Log[PrimePi[x]]}, {x, 1, 100}]
```



```
Plot[{Exp[1/2 (Sum[1/i - 1/Prime[i], {i, 1, x}] + Log[LogIntegral[x]])],
  PrimePi[x], LogIntegral[x] - 1/2 LogIntegral[Sqrt[x]]}, {x, 1, 100}]
```




```
Plot[{PrimePi[x], LogIntegral[x] - 1/2 LogIntegral[Sqrt[x]] -
   Sum[LogIntegral[Abs[x^ZetaZero[k]]], {k, 1, 20}]}, {x, 1, 100}]
```




```
Sum[LogIntegral[x^ZetaZero[k]], {k, 1, 20}] // N
```

$\mathrm{LogIntegral}\left[x^{0.5+14.1347\,i}\right] + \mathrm{LogIntegral}\left[x^{0.5+21.022\,i}\right] + \mathrm{LogIntegral}\left[x^{0.5+25.0109\,i}\right] + \mathrm{LogIntegral}\left[x^{0.5+30.4249\,i}\right] + \mathrm{LogIntegral}\left[x^{0.5+32.9351\,i}\right] + \mathrm{LogIntegral}\left[x^{0.5+37.5862\,i}\right] + \mathrm{LogIntegral}\left[x^{0.5+40.9187\,i}\right] + \mathrm{LogIntegral}\left[x^{0.5+43.3271\,i}\right] + \mathrm{LogIntegral}\left[x^{0.5+48.0052\,i}\right] + \mathrm{LogIntegral}\left[x^{0.5+49.7738\,i}\right] + \mathrm{LogIntegral}\left[x^{0.5+52.9703\,i}\right] + \mathrm{LogIntegral}\left[x^{0.5+56.4462\,i}\right] + \mathrm{LogIntegral}\left[x^{0.5+59.347\,i}\right] + \mathrm{LogIntegral}\left[x^{0.5+60.8318\,i}\right] + \mathrm{LogIntegral}\left[x^{0.5+65.1125\,i}\right] + \mathrm{LogIntegral}\left[x^{0.5+67.0798\,i}\right] + \mathrm{LogIntegral}\left[x^{0.5+69.5464\,i}\right] + \mathrm{LogIntegral}\left[x^{0.5+72.0672\,i}\right] + \mathrm{LogIntegral}\left[x^{0.5+75.7047\,i}\right] + \mathrm{LogIntegral}\left[x^{0.5+77.1448\,i}\right]$

Rieman

So we can see that the leading terms of Log[PrimePi[x]] are

$$\mathrm{Log}[\mathrm{PrimePi}[x]] = \frac{1}{2}\left(\mathrm{Sum}\left[\frac{1}{i} - \frac{1}{\mathrm{Prime}[i]}, \{i, 1, x\}\right] + \mathrm{Log}[\mathrm{LogIntegral}[x]]\right) + (\text{Smaller Terms})$$

$\Rightarrow$

$$\text{PrimePi}[x] \simeq \text{Exp}\left[\frac{1}{2}\left(\text{Sum}\left[\frac{1}{i}-\frac{1}{\text{Prime}[i]},\{i,1,x\}\right]+\text{Log}[\text{LogIntegral}[x]]\right)\right]$$

$$\text{PrimePi}[x] \simeq \text{Exp}\left[\frac{1}{2}\text{Sum}\left[\frac{1}{i}-\frac{1}{\text{Prime}[i]},\{i,1,x\}\right]\right]\text{Exp}\left[\frac{1}{2}\text{Log}[\text{LogIntegral}[x]]\right]$$

$$\text{PrimePi}[x] \simeq \text{Exp}\left[\frac{1}{2}\text{Sum}\left[\frac{1}{i}-\frac{1}{\text{Prime}[i]},\{i,1,x\}\right]\right]\text{Exp}\left[\text{Log}\left[\sqrt{\text{LogIntegral}[x]}\right]\right]$$

$$\text{PrimePi}[x] = \text{Exp}\left[\frac{1}{2}\text{Sum}\left[\frac{1}{i}-\frac{1}{\text{Prime}[i]},\{i,1,x\}\right]\right]\sqrt{\text{LogIntegral}[x]}+(\text{Smaller Terms})$$

Note that Riemann ' s formula for the leading terms is

$$\text{PrimePi}[x] = \text{LogIntegral}[x]-\frac{1}{2}\text{LogIntegral}\left[\sqrt{x}\right]-\sum_{\rho}\text{LogIntegral}[x\verb|^|\rho]+(\text{Smaller Terms})$$

$\text{Exp}\left[\frac{1}{2}\left(\text{Sum}\left[\frac{1}{i}-\frac{1}{\text{Prime}[i]},\{i,1,x\}\right]+\text{Log}[\text{LogIntegral}[x]]\right)\right]$ and $\text{LogIntegral}[x]-\frac{1}{2}\text{LogIntegral}\left[\sqrt{x}\right]$ are very good appxs, first is a bit better for lower x but second seems to be better as x goes up (look around 10, 000)

Very likely that after the inclusion of the ρ dependent term they ' re very nearly, if not exactly, the same.

Briefly assume near exactitude, then :

$$\text{Exp}\left[\frac{1}{2}\left(\text{Sum}\left[\frac{1}{i}-\frac{1}{\text{Prime}[i]},\{i,1,x\}\right]+\text{Log}[\text{LogIntegral}[x]]\right)\right] \simeq \text{LogIntegral}[x]-\frac{1}{2}\text{LogIntegral}\left[\sqrt{x}\right]-\sum_{\rho}\text{LogIntegral}[x\verb|^|\rho]$$

$\Rightarrow$

$$\text{Exp}\left[\frac{1}{2}\text{Sum}\left[\frac{1}{i}-\frac{1}{\text{Prime}[i]},\{i,1,x\}\right]\right]\sqrt{\text{LogIntegral}[x]} \simeq \text{LogIntegral}[x]-\frac{1}{2}\text{LogIntegral}\left[\sqrt{x}\right]-\sum_{\rho}\text{LogIntegral}[x\verb|^|\rho]$$

$$\Rightarrow \text{Exp}\left[\frac{1}{2}\text{Sum}\left[\frac{1}{i}-\frac{1}{\text{Prime}[i]},\{i,1,x\}\right]\right] \simeq \left(\text{LogIntegral}[x]-\frac{1}{2}\text{LogIntegral}\left[\sqrt{x}\right]-\sum_{\rho}\text{LogIntegral}[x\verb|^|\rho]\right)\Big/\left(\sqrt{\text{LogIntegral}[x]}\right)$$

$$\Rightarrow \frac{1}{2}\,\mathrm{Sum}\left[\frac{1}{i}-\frac{1}{\mathrm{Prime}[i]},\ \{i,1,x\}\right] \simeq \mathrm{Log}\left[\left(\mathrm{LogIntegral}[x]-\frac{1}{2}\,\mathrm{LogIntegral}\left[\sqrt{x}\right]-\sum_{\rho}\mathrm{LogIntegral}[x^{\wedge}\rho]\right)\Big/\left(\sqrt{\mathrm{LogIntegral}[x]}\right)\right]$$

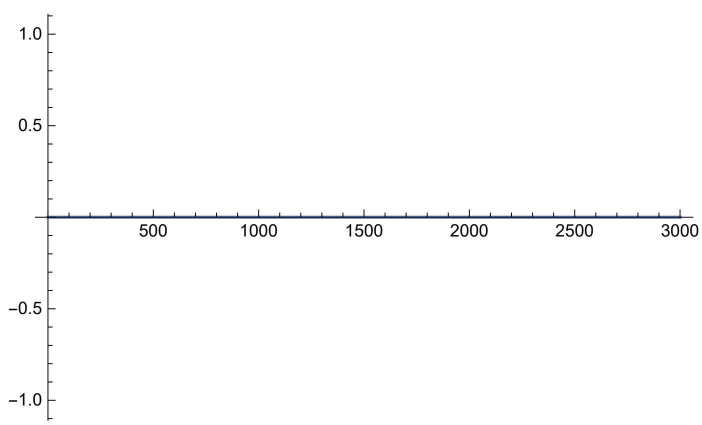
$$\Rightarrow \mathrm{Sum}\left[\frac{1}{i}-\frac{1}{\mathrm{Prime}[i]},\ \{i,1,x\}\right] \simeq 2\,\mathrm{Log}\left[\left(\mathrm{LogIntegral}[x]-\frac{1}{2}\,\mathrm{LogIntegral}\left[\sqrt{x}\right]-\sum_{\rho}\mathrm{LogIntegral}[x^{\wedge}\rho]\right)\Big/\left(\sqrt{\mathrm{LogIntegral}[x]}\right)\right]$$

$$\Rightarrow \mathrm{Sum}\left[\frac{1}{i}-\frac{1}{\mathrm{Prime}[i]},\ \{i,1,x\}\right] \simeq \mathrm{Log}\left[\left(\mathrm{LogIntegral}[x]-\frac{1}{2}\,\mathrm{LogIntegral}\left[\sqrt{x}\right]-\sum_{\rho}\mathrm{LogIntegral}[x^{\wedge}\rho]\right)^{\wedge}2\Big/\mathrm{LogIntegral}[x]\right]$$

$$\Rightarrow \mathrm{Sum}\left[\frac{1}{i},\ \{i,1,x\}\right]-\mathrm{Sum}\left[\frac{1}{\mathrm{Prime}[i]},\ \{i,1,x\}\right] \simeq \mathrm{Log}\left[\left(\mathrm{LogIntegral}[x]-\frac{1}{2}\,\mathrm{LogIntegral}\left[\sqrt{x}\right]-\sum_{\rho}\mathrm{LogIntegral}[x^{\wedge}\rho]\right)^{\wedge}2\Big/\mathrm{LogIntegral}[x]\right]$$

$$\Rightarrow \mathrm{Sum}\left[\frac{1}{\mathrm{Prime}[i]},\ \{i,1,x\}\right] \simeq \mathrm{Sum}\left[\frac{1}{i},\ \{i,1,x\}\right]-\mathrm{Log}\left[\left(\mathrm{LogIntegral}[x]-\frac{1}{2}\,\mathrm{LogIntegral}\left[\sqrt{x}\right]-\sum_{\rho}\mathrm{LogIntegral}[x^{\wedge}\rho]\right)^{\wedge}2\Big/\mathrm{LogIntegral}[x]\right]$$

Plot[{ Sum[1/i - 1/Prime[i], {i, 1, x}], (Sum[1/i, {i, 1, x}] - Sum[1/Prime[i], {i, 1, x}])}, {x, 1, 3000}]



Sum[1/i, {i, 1, x}]

HarmonicNumber[x]

$$\Rightarrow \text{Sum}\left[\frac{1}{\text{Prime}[i]}, \{i, 1, x\}\right] \simeq \text{HarmonicNumber}[x] - \text{Log}\left[\left(\text{LogIntegral}[x] - \frac{1}{2}\text{LogIntegral}\left[\sqrt{x}\right] - \sum_{\rho}\text{LogIntegral}[x\text{^}\rho]\right)\text{^}2 \Big/ \text{LogIntegral}[x]\right]$$

$$\Rightarrow \text{Sum}\left[\frac{1}{\text{Prime}[i]}, \{i, 1, x\}\right] \simeq \text{HarmonicNumber}[x] + \text{Log}\left[\text{LogIntegral}[x] \Big/ \left(\text{LogIntegral}[x] - \frac{1}{2}\text{LogIntegral}\left[\sqrt{x}\right] - \sum_{\rho}\text{LogIntegral}[x\text{^}\rho]\right)\text{^}2\right]$$

$$\text{Sum}\left[\frac{1}{\text{Prime}[i]}, \{i, 1, x\}\right] = \frac{1}{\text{Prime}[x]} + \frac{1}{\text{Prime}[x-1]} + \ldots + \frac{1}{\text{Prime}[3]} + \frac{1}{\text{Prime}[2]} + \frac{1}{\text{Prime}[1]} = \frac{1}{\text{Prime}[x]} + \frac{1}{\text{Prime}[x-1]} + \ldots + \frac{1}{5} + \frac{1}{3} + \frac{1}{2}$$

or more precisely the floor of x

$$\Rightarrow \frac{1}{\text{Prime}[x]} \simeq \left(\text{HarmonicNumber}[x] + \text{Log}\left[\text{LogIntegral}[x] \Big/ \left(\text{LogIntegral}[x] - \frac{1}{2}\text{LogIntegral}\left[\sqrt{x}\right] - \sum_{\rho}\text{LogIntegral}[x\text{^}\rho]\right)\text{^}2\right]\right) - \left(\frac{1}{\text{Prime}[x-1]} + \ldots + \frac{1}{5} + \frac{1}{3} + \frac{1}{2}\right)$$

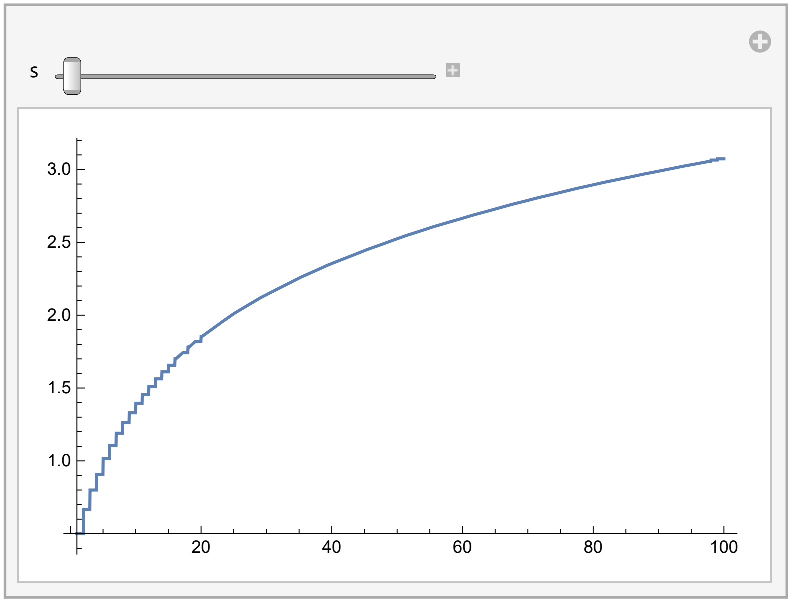
$$\textbf{HarZeta}[x\_, s\_] := \textbf{Sum}\left[\frac{1}{((i)\text{^}s)}, \{i, 1, x\}\right]$$

$$\textbf{HarZetaPrime}[x\_, s\_] := \textbf{Sum}\left[\frac{1}{((\text{Prime}[i])\text{^}s)}, \{i, 1, x\}\right]$$

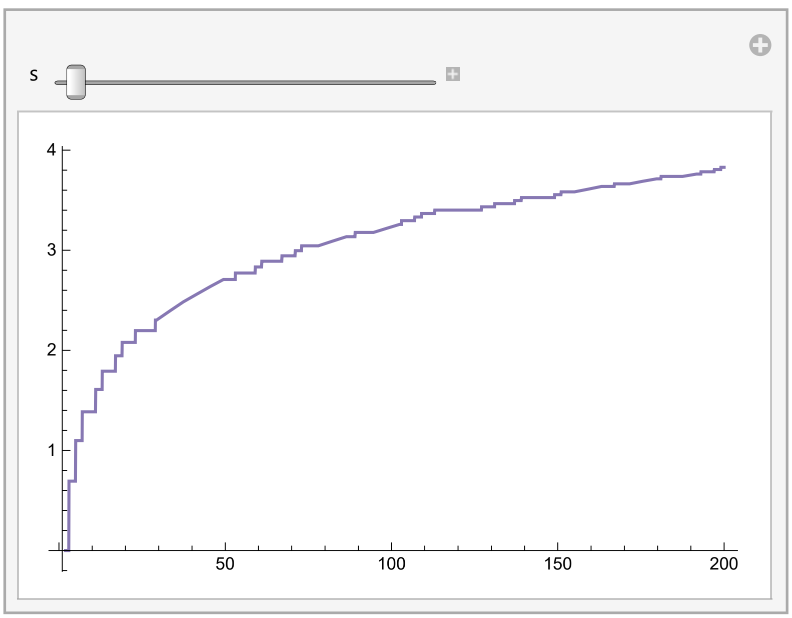
$$\textbf{OmegaZeta}[x\_, s\_] := \textbf{Sum}\left[\left(\frac{1}{i} - \frac{1}{\text{Prime}[i]}\right)\text{^}s, \{i, 1, x\}\right]$$

$$\Omega[x\_] := \textbf{Sum}\left[\frac{1}{i} - \frac{1}{\text{Prime}[i]}, \{i, 1, x\}\right]$$

```
Manipulate[Plot[{Sum[1/i - 1/Prime[i], {i, 1, x}], HarZeta[x, s], HarZetaPrime[x, s],
   OmegaZeta[x, s], (HarZeta[x, s] - HarZetaPrime[x, s])}, {x, 1, 100}], {s, 1, 10}]
```




```
Manipulate[Plot[{HarZeta[x, s], HarZetaPrime[x, s], OmegaZeta[x, s],
   (HarZeta[x, s] - HarZetaPrime[x, s]), Log[PrimePi[x]]}, {x, 1, 200}], {s, 1, 10}]
```



$$\Omega[x\_] := \mathrm{Sum}\left[\frac{1}{i} - \frac{1}{\mathrm{Prime}[i]}, \{i, 1, x\}\right]$$

$$\Omega\mathrm{Series}[x\_, s\_] :=$$

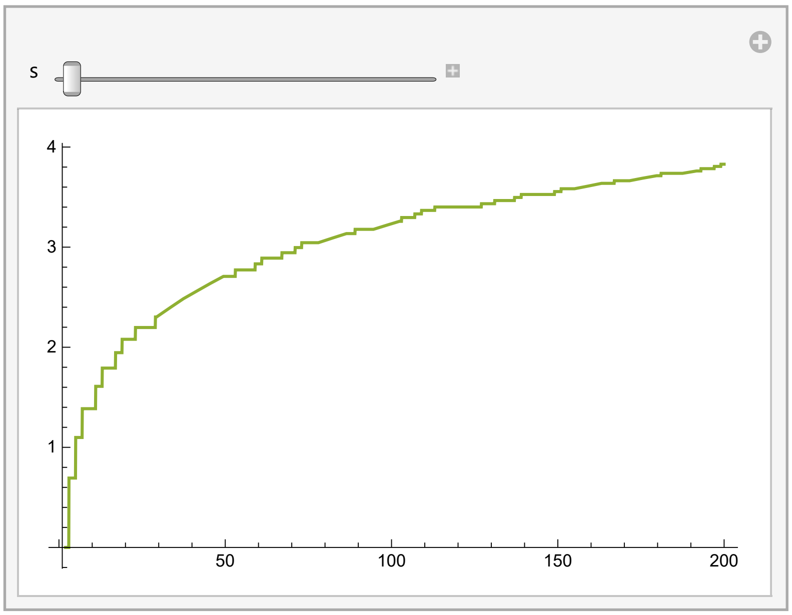
$$\mathrm{Sum}\left[\frac{1}{i} - \frac{1}{\mathrm{Prime}[i]}, \{i, 1, x\}\right] + \mathrm{Sum}\left[\left(\frac{1}{k}\,(\mathrm{HarZeta}[x, k] - \mathrm{HarZetaPrime}[x, k])\right), \{k, 2, s\}\right]$$

$$\mathrm{Alt}\Omega\mathrm{Series}[x\_, s\_] := \mathrm{Sum}\left[\left(\frac{((-1)\,\hat{}\,(k-1))}{k}\,(\mathrm{HarZeta}[x, k] - \mathrm{HarZetaPrime}[x, k])\right), \{k, 1, s\}\right]$$

$$\Omega\mathrm{ZetaSeries}[x\_, s\_] := \mathrm{Sum}\left[\frac{1}{i} - \frac{1}{\mathrm{Prime}[i]}, \{i, 1, x\}\right] + \mathrm{Sum}\left[\left(\frac{1}{k}\,(\mathrm{OmegaZeta}[x, k])\right), \{k, 2, s\}\right]$$

$$\mathrm{Alt}\Omega\mathrm{ZetaSeries}[x\_, s\_] := \mathrm{Sum}\left[\left(\frac{((-1)\,\hat{}\,(k-1))}{k}\,(\mathrm{OmegaZeta}[x, k])\right), \{k, 1, s\}\right]$$

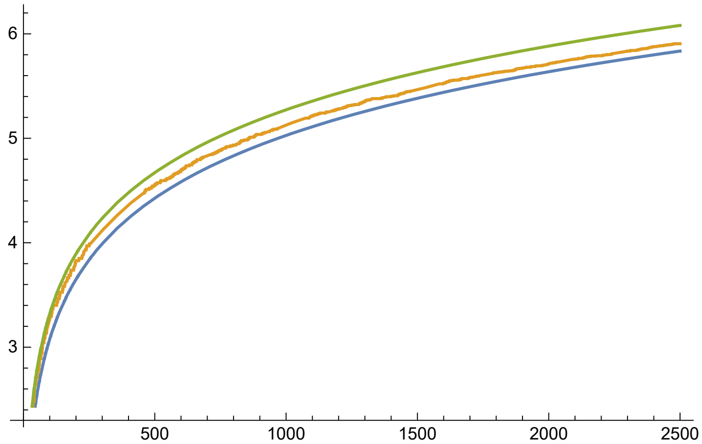
```
Manipulate[Plot[{ΩSeries[x, s], AltΩSeries[x, s], Log[PrimePi[x]],
  ΩZetaSeries[x, s], AltΩZetaSeries[x, s]}, {x, 1, 200}], {s, 1, 10}]
```




```
ΩZetaSeries[x, 2]
```

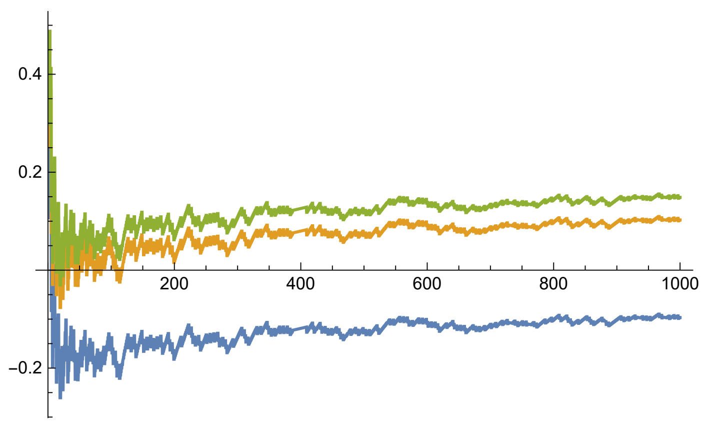
$$\sum_{i=1}^{x}\left(\frac{1}{i} - \frac{1}{\mathrm{Prime}[i]}\right) + \frac{1}{2}\sum_{i=1}^{x}\left(\frac{1}{i} - \frac{1}{\mathrm{Prime}[i]}\right)^2$$

```
Plot[{Ω[x], Log[PrimePi[x]], ΩZetaSeries[x, 3]}, {x, 1, 2500}]
```

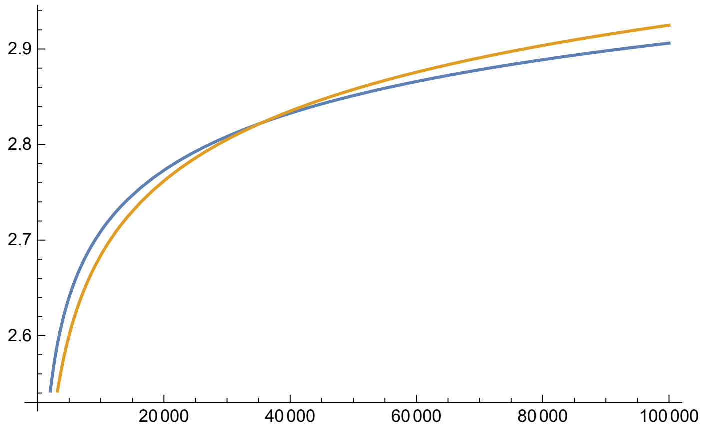


```
Table[(ΩZetaSeries[Floor[x], s] - Log[PrimePi[Floor[x]]]), {s, 1, 4}]
```

```
Plot[{(ΩZetaSeries[Floor[x], 1] - Log[PrimePi[Floor[x]]]),
  (ΩZetaSeries[Floor[x], 2] - Log[PrimePi[Floor[x]]]),
  (ΩZetaSeries[Floor[x], 3] - Log[PrimePi[Floor[x]]])}, {x, 1, 1000}]
```



```
Plot[{Sum[1/Prime[i], {i, 1, x}],
  HarmonicNumber[x] + Log[LogIntegral[x]/(LogIntegral[x] - 1/2 LogIntegral[Sqrt[x]])^2]}, {x, 1, 100000}]
```




```
b = 3;
(HarmonicNumber[b] + Log[LogIntegral[b]/(LogIntegral[b] - 1/2 LogIntegral[Sqrt[b]])^2]) // N
```

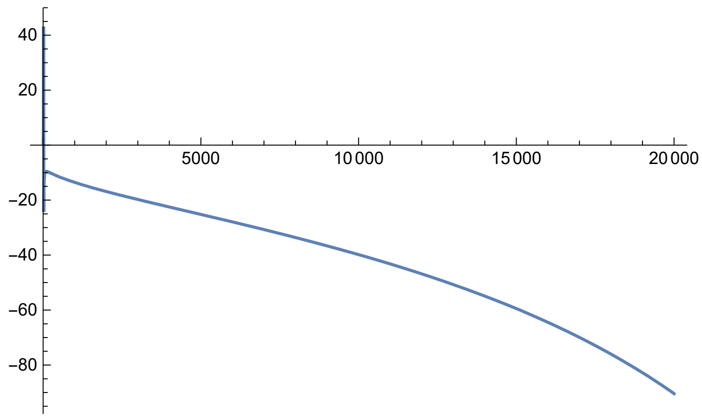
1.36714

```
b = 1010;
((HarmonicNumber[b] + Log[LogIntegral[b]/(LogIntegral[b] - 1/2 LogIntegral[Sqrt[b]])^2]) -
    Sum[1/Prime[i], {i, 1, b - 2}])^(-1) // N
Prime[
 b]
```

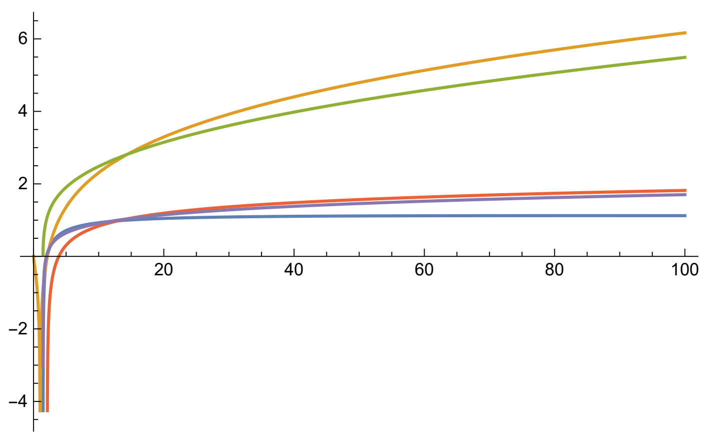
-13.586

8017

```
Plot[((HarmonicNumber[b] + Log[LogIntegral[b] / (LogIntegral[b] - 1/2 LogIntegral[Sqrt[b]])^2]) -
    Sum[1/Prime[i], {i, 1, b - 2}])^(-1), {b, 2, 20000}]
```




```
Plot[{LogIntegral[Sqrt[x]] / Sqrt[LogIntegral[x]], LogIntegral[Sqrt[x]], Sqrt[LogIntegral[x]],
  Log[LogIntegral[Sqrt[x]]], 1/2 Log[LogIntegral[x]]}, {x, 0, 100}]
```



$$\frac{\text{LogIntegral}[\sqrt{x}]}{\sqrt{\text{LogIntegral}[x]}} \approx \frac{1}{2}\text{Log}[\text{LogIntegral}[x]] = \text{Log}\left[\sqrt{\text{LogIntegral}[x]}\right],$$

this $\frac{\text{LogIntegral}[\sqrt{x}]}{\sqrt{\text{LogIntegral}[x]}}$ approximation only holds for smaller x,

want an exact result if possible

Zeta fn zeroes

```
N[ZetaZero[1]]
Im[N[ZetaZero[1]]]
```

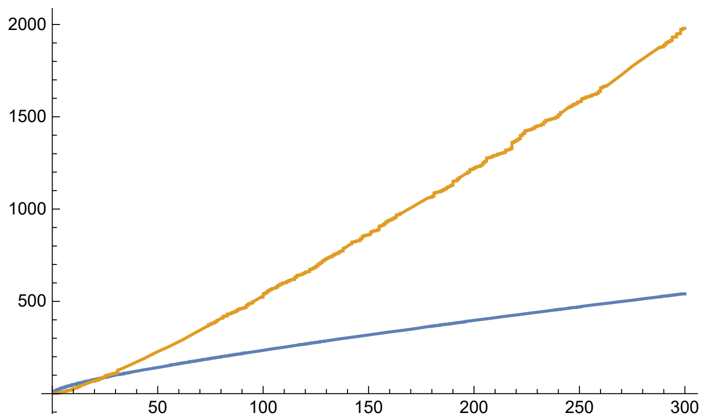
0.5 + 14.1347 i

14.1347

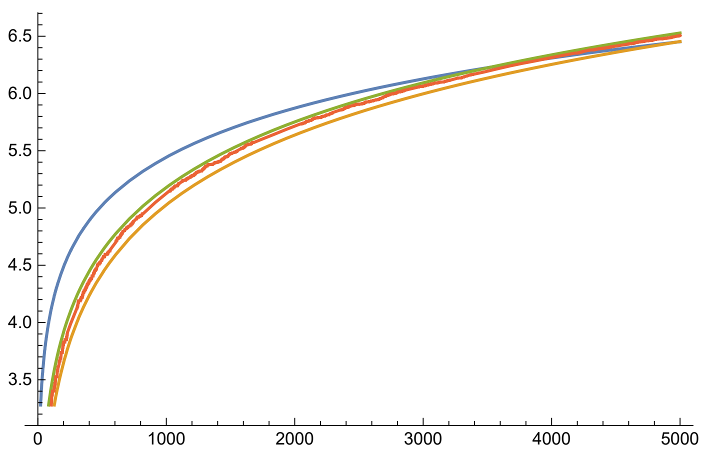
```
Table[{Im[N[ZetaZero[Floor[x]]]]}, {x, 1, 30}]
```

{{14.1347}, {21.022}, {25.0109}, {30.4249}, {32.9351}, {37.5862}, {40.9187}, {43.3271}, {48.0052}, {49.7738}, {52.9703}, {56.4462}, {59.347}, {60.8318}, {65.1125}, {67.0798}, {69.5464}, {72.0672}, {75.7047}, {77.1448}, {79.3374}, {82.9104}, {84.7355}, {87.4253}, {88.8091}, {92.4919}, {94.6513}, {95.8706}, {98.8312}, {101.318}}

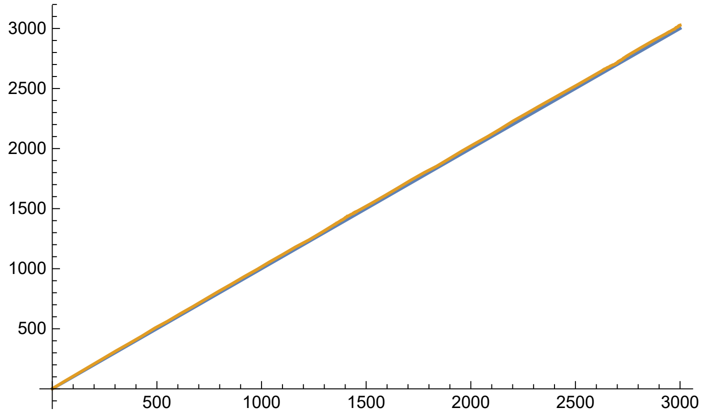
```
Plot[{Im[N[ZetaZero[Floor[x]]]], Prime[Floor[x]]}, {x, 1, 300}]
```

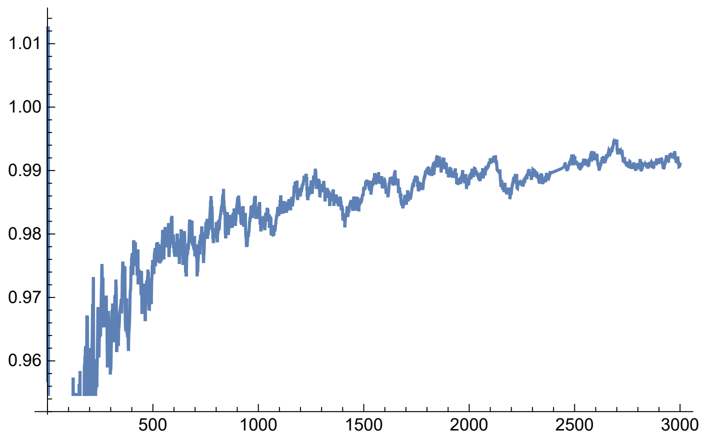


$$\text{Plot}\left[\left\{\text{Log}[\text{Im}[\text{N}[\text{ZetaZero}[\text{Floor}[x]]]]], \text{Sum}\left[\frac{1}{i} - \frac{1}{\text{Prime}[i]}, \{i, 1, x\}\right], \text{Log}[\text{LogIntegral}[x]], \text{Log}[\text{PrimePi}[x]]\right\}, \{x, 1, 5000\}\right]$$

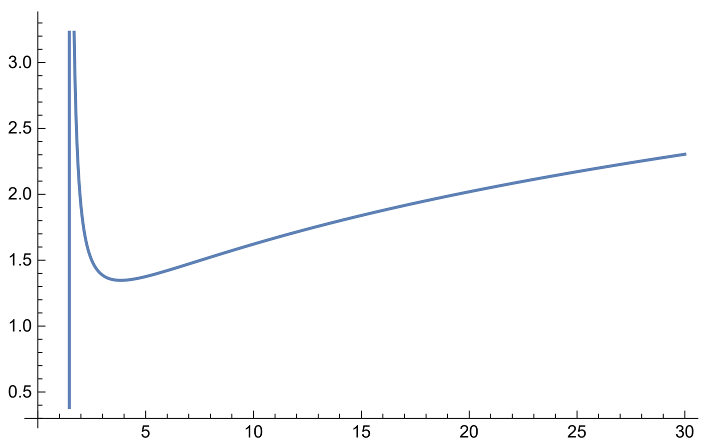


```
Plot[{x, LogIntegral[Prime[Floor[x]]]}, {x, 1, 3000}]
Plot[{x / LogIntegral[Prime[Floor[x]]]}, {x, 1, 3000}]
```

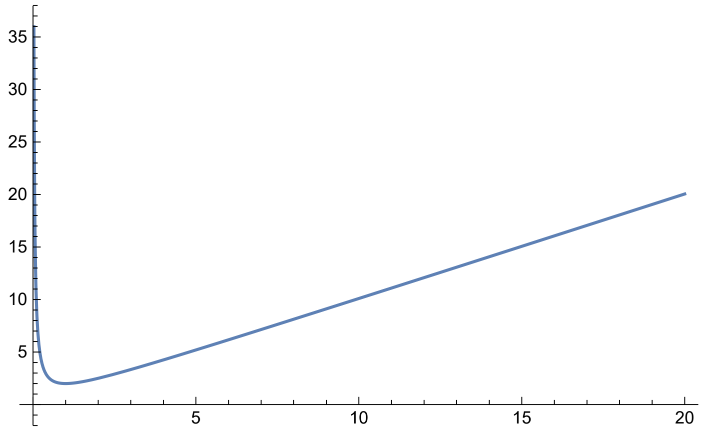





```
Plot[{x / LogIntegral[x]}, {x, 0, 30}]
```



```
Plot[{(1/x + x)}, {x, 0, 20}]
```



```
Table[Sum[LogIntegral[x^ZetaZero[k]], {k, 1, 20}], {x, 1, 20}]
Plot[Sum[LogIntegral[x^ZetaZero[k]], {k, 1, 20}], {x, 1, 20}]
```