```
Integrate[((x^1) (E^x))/(E^x - 1)^1, {x, 0, Infinity}]
```

Integrate::idiv : Integral of $\frac{e^x x}{-1 + e^x}$ does not converge on {0, ∞}. >>

$$\int_0^\infty \frac{e^x x}{-1 + e^x} \, dx$$

```
n0 = 1;
Integrate[((x^n0) (E^(-x)))/(E^(-x) - 1)^n0, x] // FullSimplify
```

$$\frac{1}{2} x \left(x - 2 \text{Log}\left[1 - e^x\right]\right) - \text{PolyLog}\left[2, e^x\right]$$

```
Integrate[((x^2) (E^x))/(E^x - 1)^2, x]
```

$$x \left(\frac{e^x x}{1 - e^x} + 2 \text{Log}\left[1 - e^x\right]\right) + 2 \text{PolyLog}\left[2, e^x\right]$$

```
Integrate[(e^x x^1)/(-1 + e^x)^1, x] // FullSimplify
```

$$x \text{Log}\left[1 - e^x\right] + \text{PolyLog}\left[2, e^x\right]$$

Gauss-Riemann Zeta

Bottom term of integral raised to a power. Reverse sign of bottom term, and no gamma fns in these. Left E^(-x) on top, so differes from zeta integral except at n=1

```
n = 2;
Integrate[((x^n) (E^(-x)))/(1 - E^(-x))^n, {x, 0, Infinity}] // FullSimplify(*increases as n → ∞*)
Integrate[(x^(n))/(E^(x) - 1)^(n), {x, 0, Infinity}] // FullSimplify(*decreases as n → ∞*)
Integrate[(x^(n + 1))/(E^(x) - 1)^(n), {x, 0, Infinity}] // FullSimplify(*decreases as n → ∞*)
```

$$\frac{\pi^2}{3}$$

$$\frac{1}{3} \left(\pi^2 - 6 \text{Zeta}[3]\right)$$

$$-\frac{\pi^4}{15} + 6 \text{Zeta}[3]$$

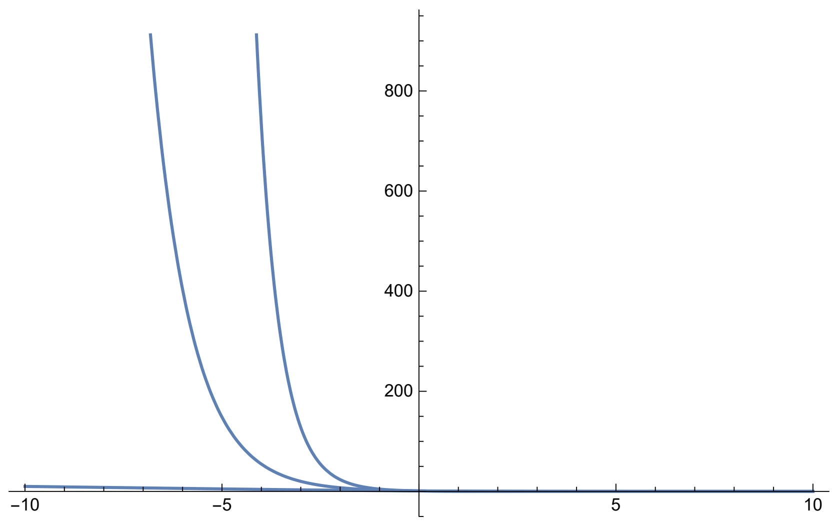
Compare to putting E^(-x) in bottom

```
b = 7;
Int1 = Integrate[((x^b) (E^(-x)))/(1 - E^(-x))^b, {x, 0, Infinity}] // FullSimplify;
Int2 = Integrate[(x^b)/(E^(x) - 1)^(b), {x, 0, Infinity}] // FullSimplify;
Int2 - Int1 // FullSimplify
N[Int2 - Int1]
```

$7\pi^4 + 10\pi^6 + \frac{8\pi^8}{15} - 84\,(3\,\mathrm{Zeta}[3] + 80\,\mathrm{Zeta}[5] + 157\,\mathrm{Zeta}[7])$

-5212.88

```
Plot[Table[((x^k) (E^(-x)))/(1 - E^(-x))^k, {k, -1, 1}], {x, -10, 10}]
```




```
Integrate[((x^n) (E^(-x)))/(1 - E^(-x))^n, {x, 0, Infinity}] // FullSimplify
```

Gen Hurwitz Zeta

```
n1 = -1;
Integrate[((x^n1) (E^(-q x)))/((1 - E^(-x))^(n1)), x] // FullSimplify
Integrate[((x^n1 - 1) (E^(-q x)))/((1 - E^(-x))^(n1)), x] // FullSimplify
Integrate[((x^n1) (E^(-q x)))/((1 - E^(-x))^(n1 + 1)), x] // FullSimplify
```

ExpIntegralEi[-q x] - ExpIntegralEi[-(1 + q) x]

$$\frac{1}{q\,(1+q)}\,e^{-(1+2q)x}\left(-e^{q x}\,q + e^{(1+q)x}\,(1+q)\,\left(1 + e^{q x}\,q\,(\text{ExpIntegralEi}[-q\,x] - \text{ExpIntegralEi}[-(1+q)\,x])\right)\right)$$

ExpIntegralEi[-q x]

```
n2 = -1;
Integrate[((x^n2) (E^(-q x)))/((1 - E^(-x))^(n2)), {x, 0, Infinity}] // FullSimplify
Integrate[((x^n2 - 1) (E^(-q x)))/((1 - E^(-x))^(n2)), {x, 0, Infinity}] // FullSimplify
Integrate[((x^n2) (E^(-q x)))/((1 - E^(-x))^(n2 + 1)), {x, 0, Infinity}] // FullSimplify
```

ConditionalExpression[-Log[q] + Log[1 + q], Re[q] > 0]

$$\text{ConditionalExpression}\left[-\frac{1}{q+q^2} - \text{Log}[q] + \text{Log}[1+q],\ \text{Re}[q] > 0\right]$$

Integrate::idiv : Integral of $\frac{e^{-q x}}{x}$ does not converge on $\{0, \infty\}$. ≫

$$\int_0^\infty \frac{e^{-q\,x}}{x}\,dx$$