```
A = {{a1}, {a2}, {a3}, {a4}};
B = {{b1}, {b2}, {b3}, {b4}};
Outer[Times, Transpose[A], B] // MatrixForm
Outer[Times, B, Transpose[A]] // MatrixForm
```

$$\begin{pmatrix} \begin{pmatrix} a1\,b1 \\ a1\,b2 \\ a1\,b3 \\ a1\,b4 \end{pmatrix} & \begin{pmatrix} a2\,b1 \\ a2\,b2 \\ a2\,b3 \\ a2\,b4 \end{pmatrix} & \begin{pmatrix} a3\,b1 \\ a3\,b2 \\ a3\,b3 \\ a3\,b4 \end{pmatrix} & \begin{pmatrix} a4\,b1 \\ a4\,b2 \\ a4\,b3 \\ a4\,b4 \end{pmatrix} \end{pmatrix}$$

$$\begin{pmatrix} \begin{pmatrix} a1\,b1 & a2\,b1 & a3\,b1 & a4\,b1 \end{pmatrix} \\ \begin{pmatrix} a1\,b2 & a2\,b2 & a3\,b2 & a4\,b2 \end{pmatrix} \\ \begin{pmatrix} a1\,b3 & a2\,b3 & a3\,b3 & a4\,b3 \end{pmatrix} \\ \begin{pmatrix} a1\,b4 & a2\,b4 & a3\,b4 & a4\,b4 \end{pmatrix} \end{pmatrix}$$

```
Transpose[A].B // MatrixForm
B.Transpose[A] // MatrixForm
```

$$\begin{pmatrix} a1\,b1 & a1\,b2 & a1\,b3 & a1\,b4 \\ a2\,b1 & a2\,b2 & a2\,b3 & a2\,b4 \\ a3\,b1 & a3\,b2 & a3\,b3 & a3\,b4 \\ a4\,b1 & a4\,b2 & a4\,b3 & a4\,b4 \end{pmatrix}$$

$$\begin{pmatrix} a1\,b1 + a2\,b2 + a3\,b3 + a4\,b4 \end{pmatrix}$$

Tensor product of some states

```
ψ = {{ⅈ}, {1}, {1}, {1}};
ϕ = {{ⅈ}, {1}, {1}, {1}};
(*ψ//MatrixForm
   ϕ//MatrixForm*)

ϕ.Transpose[ψ] // MatrixForm
ψ.Transpose[ϕ] // MatrixForm
```

$$\begin{pmatrix} -1 & i & i & i \\ i & 1 & 1 & 1 \\ i & 1 & 1 & 1 \\ i & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & i & i & i \\ i & 1 & 1 & 1 \\ i & 1 & 1 & 1 \\ i & 1 & 1 & 1 \end{pmatrix}$$

```
ϕ.Transpose[ψ] // MatrixForm
ψ.Transpose[ϕ] // MatrixForm

Transpose[ψ].ϕ // MatrixForm
Transpose[ϕ].ψ // MatrixForm
```

$$\begin{pmatrix} -1 & i & i & i \\ -i & -1 & -1 & -1 \\ -i & -1 & -1 & -1 \\ -i & -1 & -1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & -i & -i & -i \\ i & -1 & -1 & -1 \\ i & -1 & -1 & -1 \\ i & -1 & -1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} -4 \end{pmatrix}$$

$$\begin{pmatrix} -4 \end{pmatrix}$$

```
Transpose[ψ].ψ // MatrixForm
ψ.Transpose[ψ] // MatrixForm
```

$$\begin{pmatrix} -1 & i & i & i \\ i & 1 & 1 & 1 \\ i & 1 & 1 & 1 \\ i & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 \end{pmatrix}$$

```
Outer[Times, ψ, ψ] // MatrixForm
Outer[Times, ψ, Transpose[ψ]] // MatrixForm
```

$$\begin{pmatrix} \begin{pmatrix} -1 & i & i & i \end{pmatrix} & \begin{pmatrix} i & 1 & 1 & 1 \end{pmatrix} & \begin{pmatrix} i & 1 & 1 & 1 \end{pmatrix} & \begin{pmatrix} i & 1 & 1 & 1 \end{pmatrix} \end{pmatrix}$$

$$\begin{pmatrix} \begin{pmatrix} -1 \\ i \\ i \\ i \end{pmatrix} & \begin{pmatrix} i \\ 1 \\ 1 \\ 1 \end{pmatrix} & \begin{pmatrix} i \\ 1 \\ 1 \\ 1 \end{pmatrix} & \begin{pmatrix} i \\ 1 \\ 1 \\ 1 \end{pmatrix} \end{pmatrix}$$

differential component tensors

```
dψ = {{ⅈ dt1}, {dx1}, {dy1}, {dz1}};
dϕ = {{ⅈ dt2}, {dx2}, {dy2}, {dz2}};
(*ψ//MatrixForm
   ϕ//MatrixForm*)

dϕ.Transpose[dψ] // MatrixForm
dψ.Transpose[dϕ] // MatrixForm
```

$$\begin{pmatrix} -dt_1\, dt_2 & i\, dt_2\, dx_1 & i\, dt_2\, dy_1 & i\, dt_2\, dz_1 \\ i\, dt_1\, dx_2 & dx_1\, dx_2 & dx_2\, dy_1 & dx_2\, dz_1 \\ i\, dt_1\, dy_2 & dx_1\, dy_2 & dy_1\, dy_2 & dy_2\, dz_1 \\ i\, dt_1\, dz_2 & dx_1\, dz_2 & dy_1\, dz_2 & dz_1\, dz_2 \end{pmatrix}$$

$$\begin{pmatrix} -dt_1\, dt_2 & i\, dt_1\, dx_2 & i\, dt_1\, dy_2 & i\, dt_1\, dz_2 \\ i\, dt_2\, dx_1 & dx_1\, dx_2 & dx_1\, dy_2 & dx_1\, dz_2 \\ i\, dt_2\, dy_1 & dx_2\, dy_1 & dy_1\, dy_2 & dy_1\, dz_2 \\ i\, dt_2\, dz_1 & dx_2\, dz_1 & dy_2\, dz_1 & dz_1\, dz_2 \end{pmatrix}$$

```
"Sum and Difference:"
(dϕ.Transpose[dψ]) + (dψ.Transpose[dϕ]) // MatrixForm
(dϕ.Transpose[dψ]) - (dψ.Transpose[dϕ]) // MatrixForm
((dϕ.Transpose[dψ]) + (dψ.Transpose[dϕ])) -
  ((dϕ.Transpose[dψ]) - (dψ.Transpose[dϕ])) // MatrixForm

Transpose[(dϕ.Transpose[dψ])] + (dψ.Transpose[dϕ]) // MatrixForm
Transpose[(dϕ.Transpose[dψ])] - (dψ.Transpose[dϕ]) // MatrixForm
```

Sum and Difference:

$$\begin{pmatrix} -2\,dt_1\,dt_2 & i\,dt_2\,dx_1 + i\,dt_1\,dx_2 & i\,dt_2\,dy_1 + i\,dt_1\,dy_2 & i\,dt_2\,dz_1 + i\,dt_1\,dz_2 \\ i\,dt_2\,dx_1 + i\,dt_1\,dx_2 & 2\,dx_1\,dx_2 & dx_2\,dy_1 + dx_1\,dy_2 & dx_2\,dz_1 + dx_1\,dz_2 \\ i\,dt_2\,dy_1 + i\,dt_1\,dy_2 & dx_2\,dy_1 + dx_1\,dy_2 & 2\,dy_1\,dy_2 & dy_2\,dz_1 + dy_1\,dz_2 \\ i\,dt_2\,dz_1 + i\,dt_1\,dz_2 & dx_2\,dz_1 + dx_1\,dz_2 & dy_2\,dz_1 + dy_1\,dz_2 & 2\,dz_1\,dz_2 \end{pmatrix}$$

$$\begin{pmatrix} 0 & i\,dt_2\,dx_1 - i\,dt_1\,dx_2 & i\,dt_2\,dy_1 - i\,dt_1\,dy_2 & i\,dt_2\,dz_1 - i\,dt_1\,dz_2 \\ -i\,dt_2\,dx_1 + i\,dt_1\,dx_2 & 0 & dx_2\,dy_1 - dx_1\,dy_2 & dx_2\,dz_1 - dx_1\,dz_2 \\ -i\,dt_2\,dy_1 + i\,dt_1\,dy_2 & -dx_2\,dy_1 + dx_1\,dy_2 & 0 & dy_2\,dz_1 - dy_1\,dz_2 \\ -i\,dt_2\,dz_1 + i\,dt_1\,dz_2 & -dx_2\,dz_1 + dx_1\,dz_2 & -dy_2\,dz_1 + dy_1\,dz_2 & 0 \end{pmatrix}$$

$$\begin{pmatrix} -2\,dt_1\,dt_2 & 2\,i\,dt_1\,dx_2 & 2\,i\,dt_1\,dy_2 & 2\,i\,dt_1\,dz_2 \\ 2\,i\,dt_2\,dx_1 & 2\,dx_1\,dx_2 & 2\,dx_1\,dy_2 & 2\,dx_1\,dz_2 \\ 2\,i\,dt_2\,dy_1 & 2\,dx_2\,dy_1 & 2\,dy_1\,dy_2 & 2\,dy_1\,dz_2 \\ 2\,i\,dt_2\,dz_1 & 2\,dx_2\,dz_1 & 2\,dy_2\,dz_1 & 2\,dz_1\,dz_2 \end{pmatrix}$$

$$\begin{pmatrix} -2\,dt_1\,dt_2 & 2\,i\,dt_1\,dx_2 & 2\,i\,dt_1\,dy_2 & 2\,i\,dt_1\,dz_2 \\ 2\,i\,dt_2\,dx_1 & 2\,dx_1\,dx_2 & 2\,dx_1\,dy_2 & 2\,dx_1\,dz_2 \\ 2\,i\,dt_2\,dy_1 & 2\,dx_2\,dy_1 & 2\,dy_1\,dy_2 & 2\,dy_1\,dz_2 \\ 2\,i\,dt_2\,dz_1 & 2\,dx_2\,dz_1 & 2\,dy_2\,dz_1 & 2\,dz_1\,dz_2 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
$Assumptions = {dt₁ ∈ Reals, dt₂ ∈ Reals,}
```

$\{dt_1 \in \text{Reals}, dt_2 \in \text{Reals}\}$

```
ConjugateTranspose[(dϕ.Transpose[dψ])] + (dψ.Transpose[dϕ]) // MatrixForm
```

$$\begin{pmatrix} -\text{Conjugate}[dt_1\,dt_2] - dt_1\,dt_2 & -i\,\text{Conjugate}[dt_1\,dx_2] + i\,dt_1\,dx_2 & -i\,\text{Conjugate}[dt_1\,dx_2] + i\,dt_1\,d \\ -i\,\text{Conjugate}[dt_2\,dx_1] + i\,dt_2\,dx_1 & \text{Conjugate}[dx_1\,dx_2] + dx_1\,dx_2 & \text{Conjugate}[dx_1\,dx_2] + dx_1\,dx_2 \\ -i\,\text{Conjugate}[dt_2\,dx_1] + i\,dt_2\,dx_1 & \text{Conjugate}[dx_1\,dx_2] + dx_1\,dx_2 & \text{Conjugate}[dx_1\,dx_2] + dx_1\,dx_2 \\ -i\,\text{Conjugate}[dt_2\,dx_1] + i\,dt_2\,dx_1 & \text{Conjugate}[dx_1\,dx_2] + dx_1\,dx_2 & \text{Conjugate}[dx_1\,dx_2] + dx_1\,dx_2 \end{pmatrix}$$

```
(*"--------------Dot Products:--------------------"
        ( dϕ.Transpose[dψ] ).( dψ.Transpose[dϕ] )//MatrixForm
        Transpose[( dϕ.Transpose[dψ] )].( dψ.Transpose[dϕ] )//MatrixForm
        ( dϕ.Transpose[dψ] ).Transpose[( dψ.Transpose[dϕ] )]//MatrixForm
        Transpose[( dϕ.Transpose[dψ] )].Transpose[( dψ.Transpose[dϕ] )]//MatrixForm

        "--------------Switched order of above Dot Products:--------------------"
        ( dψ.Transpose[dϕ] ).( dϕ.Transpose[dψ] )//MatrixForm
        ( dψ.Transpose[dϕ] ).Transpose[( dϕ.Transpose[dψ] )]//MatrixForm
        Transpose[( dψ.Transpose[dϕ] )].( dϕ.Transpose[dψ] )//MatrixForm
        Transpose[( dψ.Transpose[dϕ] )].Transpose[( dϕ.Transpose[dψ] )]//MatrixForm*)
```

Forms inspired by Fubini-Study form

ψ,ϕ, terms re-defined from other parts, run definitional part below

```
ψ = {{dt1}, {dx1}, {dy1}, {dz1}};
ϕ = {{dt2}, {dx2}, {dy2}, {dz2}};

δψ = {{ⅈ dt1}, {dx1}, {dy1}, {dz1}};
δϕ = {{ⅈ dt2}, {dx2}, {dy2}, {dz2}};

ψ.Transpose[ϕ] // MatrixForm
δψ.Transpose[δϕ] // MatrixForm
```

$$\begin{pmatrix} dt_1\,dt_2 & dt_1\,dx_2 & dt_1\,dy_2 & dt_1\,dz_2 \\ dt_2\,dx_1 & dx_1\,dx_2 & dx_1\,dy_2 & dx_1\,dz_2 \\ dt_2\,dy_1 & dx_2\,dy_1 & dy_1\,dy_2 & dy_1\,dz_2 \\ dt_2\,dz_1 & dx_2\,dz_1 & dy_2\,dz_1 & dz_1\,dz_2 \end{pmatrix}$$

$$\begin{pmatrix} -dt_1\,dt_2 & i\,dt_1\,dx_2 & i\,dt_1\,dy_2 & i\,dt_1\,dz_2 \\ i\,dt_2\,dx_1 & dx_1\,dx_2 & dx_1\,dy_2 & dx_1\,dz_2 \\ i\,dt_2\,dy_1 & dx_2\,dy_1 & dy_1\,dy_2 & dy_1\,dz_2 \\ i\,dt_2\,dz_1 & dx_2\,dz_1 & dy_2\,dz_1 & dz_1\,dz_2 \end{pmatrix}$$

```
M1 = ψ.Transpose[ϕ] + δψ.Transpose[δϕ];
M2 = ψ.Transpose[ϕ] - δψ.Transpose[δϕ];

M1 // MatrixForm
M2 // MatrixForm

(*(M1)-Transpose[(M1)]//MatrixForm*)
(*M1+M2//MatrixForm
   M1-M2//MatrixForm
   (M1+M2)-Transpose[(M1+M2)]//MatrixForm*)
```

$$\begin{pmatrix} 0 & (1+i)\, dt_1\, dx_2 & (1+i)\, dt_1\, dy_2 & (1+i)\, dt_1\, dz_2 \\ (1+i)\, dt_2\, dx_1 & 2\, dx_1\, dx_2 & 2\, dx_1\, dy_2 & 2\, dx_1\, dz_2 \\ (1+i)\, dt_2\, dy_1 & 2\, dx_2\, dy_1 & 2\, dy_1\, dy_2 & 2\, dy_1\, dz_2 \\ (1+i)\, dt_2\, dz_1 & 2\, dx_2\, dz_1 & 2\, dy_2\, dz_1 & 2\, dz_1\, dz_2 \end{pmatrix}$$

$$\begin{pmatrix} 2\, dt_1\, dt_2 & (1-i)\, dt_1\, dx_2 & (1-i)\, dt_1\, dy_2 & (1-i)\, dt_1\, dz_2 \\ (1-i)\, dt_2\, dx_1 & 0 & 0 & 0 \\ (1-i)\, dt_2\, dy_1 & 0 & 0 & 0 \\ (1-i)\, dt_2\, dz_1 & 0 & 0 & 0 \end{pmatrix}$$

```
ComplexExpand[M1 - Conjugate[M2]] // MatrixForm
```

$$\begin{pmatrix} -2\, dt_1\, dt_2 & 0 & 0 & 0 \\ 0 & 2\, dx_1\, dx_2 & 2\, dx_1\, dy_2 & 2\, dx_1\, dz_2 \\ 0 & 2\, dx_2\, dy_1 & 2\, dy_1\, dy_2 & 2\, dy_1\, dz_2 \\ 0 & 2\, dx_2\, dz_1 & 2\, dy_2\, dz_1 & 2\, dz_1\, dz_2 \end{pmatrix}$$

i.e.
ψ.Transpose[ϕ] + δψ.Transpose[δϕ] -
Conjugate[ψ.Transpose[ϕ] - δψ.Transpose[δϕ]]

gives a matrix which has not space - time mixing terms,
only the diagonal and spatial - spatial mixing terms

Therefore this can be rotated onto a standard positive -
definite Riemannian geometry via multiplication by the Minkoswki matrix

More directly, for B = δψ.Transpose[δϕ];

(B + $\eta_{\mu\nu}$. B . $\eta_{\mu\nu}$) . $\eta_{\mu\nu}$ is a positive -
definite Riemannian metric without spatial - temporal mixing terms
so that we may effectively separate it into a temporal 1 -
geometry and a sptial 3 - geometry

$$\begin{pmatrix} -2\,dt_1\,dt_2 & 0 & 0 & 0 \\ 0 & 2\,dx_1\,dx_2 & 2\,dx_1\,dy_2 & 2\,dx_1\,dz_2 \\ 0 & 2\,dx_2\,dy_1 & 2\,dy_1\,dy_2 & 2\,dy_1\,dz_2 \\ 0 & 2\,dx_2\,dz_1 & 2\,dy_2\,dz_1 & 2\,dz_1\,dz_2 \end{pmatrix} . \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \text{ // MatrixForm}$$

$$\begin{pmatrix} 2\,dt_1\,dt_2 & 0 & 0 & 0 \\ 0 & 2\,dx_1\,dx_2 & 2\,dx_1\,dy_2 & 2\,dx_1\,dz_2 \\ 0 & 2\,dx_2\,dy_1 & 2\,dy_1\,dy_2 & 2\,dy_1\,dz_2 \\ 0 & 2\,dx_2\,dz_1 & 2\,dy_2\,dz_1 & 2\,dz_1\,dz_2 \end{pmatrix}$$

This is interesting but we still need to construct a workable metric tensor

construction of a viable metric tensor

```
ψ = {{dt₁}, {dx₁}, {dy₁}, {dz₁}};
ϕ = {{dt₂}, {dx₂}, {dy₂}, {dz₂}};

δψ = {{ⅈ dt₁}, {dx₁}, {dy₁}, {dz₁}};
δϕ = {{ⅈ dt₂}, {dx₂}, {dy₂}, {dz₂}};

A = ψ.Transpose[ϕ];
B = δψ.Transpose[δϕ];
η = {{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}};

ψ.Transpose[ϕ] // MatrixForm
δψ.Transpose[δϕ] // MatrixForm
```

$$\begin{pmatrix} dt_1\,dt_2 & dt_1\,dx_2 & dt_1\,dy_2 & dt_1\,dz_2 \\ dt_2\,dx_1 & dx_1\,dx_2 & dx_1\,dy_2 & dx_1\,dz_2 \\ dt_2\,dy_1 & dx_2\,dy_1 & dy_1\,dy_2 & dy_1\,dz_2 \\ dt_2\,dz_1 & dx_2\,dz_1 & dy_2\,dz_1 & dz_1\,dz_2 \end{pmatrix}$$

$$\begin{pmatrix} -dt_1\,dt_2 & i\,dt_1\,dx_2 & i\,dt_1\,dy_2 & i\,dt_1\,dz_2 \\ i\,dt_2\,dx_1 & dx_1\,dx_2 & dx_1\,dy_2 & dx_1\,dz_2 \\ i\,dt_2\,dy_1 & dx_2\,dy_1 & dy_1\,dy_2 & dy_1\,dz_2 \\ i\,dt_2\,dz_1 & dx_2\,dz_1 & dy_2\,dz_1 & dz_1\,dz_2 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} -dt_1\,dt_2 & i\,dt_1\,dx_2 & i\,dt_1\,dy_2 & i\,dt_1\,dz_2 \\ i\,dt_2\,dx_1 & dx_1\,dx_2 & dx_1\,dy_2 & dx_1\,dz_2 \\ i\,dt_2\,dy_1 & dx_2\,dy_1 & dy_1\,dy_2 & dy_1\,dz_2 \\ i\,dt_2\,dz_1 & dx_2\,dz_1 & dy_2\,dz_1 & dz_1\,dz_2 \end{pmatrix} . \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \text{ // MatrixForm}$$

$$\begin{pmatrix} -dt_1\,dt_2 & -i\,dt_1\,dx_2 & -i\,dt_1\,dy_2 & -i\,dt_1\,dz_2 \\ -i\,dt_2\,dx_1 & dx_1\,dx_2 & dx_1\,dy_2 & dx_1\,dz_2 \\ -i\,dt_2\,dy_1 & dx_2\,dy_1 & dy_1\,dy_2 & dy_1\,dz_2 \\ -i\,dt_2\,dz_1 & dx_2\,dz_1 & dy_2\,dz_1 & dz_1\,dz_2 \end{pmatrix}$$

$$\begin{pmatrix} -dt_1\,dt_2 & i\,dt_1\,dx_2 & i\,dt_1\,dy_2 & i\,dt_1\,dz_2 \\ i\,dt_2\,dx_1 & dx_1\,dx_2 & dx_1\,dy_2 & dx_1\,dz_2 \\ i\,dt_2\,dy_1 & dx_2\,dy_1 & dy_1\,dy_2 & dy_1\,dz_2 \\ i\,dt_2\,dz_1 & dx_2\,dz_1 & dy_2\,dz_1 & dz_1\,dz_2 \end{pmatrix} +$$

$$\begin{pmatrix} -dt_1\,dt_2 & -i\,dt_1\,dx_2 & -i\,dt_1\,dy_2 & -i\,dt_1\,dz_2 \\ -i\,dt_2\,dx_1 & dx_1\,dx_2 & dx_1\,dy_2 & dx_1\,dz_2 \\ -i\,dt_2\,dy_1 & dx_2\,dy_1 & dy_1\,dy_2 & dy_1\,dz_2 \\ -i\,dt_2\,dz_1 & dx_2\,dz_1 & dy_2\,dz_1 & dz_1\,dz_2 \end{pmatrix} \text{ // MatrixForm}$$

$$\begin{pmatrix} -2\,dt_1\,dt_2 & 0 & 0 & 0 \\ 0 & 2\,dx_1\,dx_2 & 2\,dx_1\,dy_2 & 2\,dx_1\,dz_2 \\ 0 & 2\,dx_2\,dy_1 & 2\,dy_1\,dy_2 & 2\,dy_1\,dz_2 \\ 0 & 2\,dx_2\,dz_1 & 2\,dy_2\,dz_1 & 2\,dz_1\,dz_2 \end{pmatrix}$$

```
(A + η.A.η )   // MatrixForm
(B + η.B.η ).η // MatrixForm
```

$$\begin{pmatrix} 2\,dt_1\,dt_2 & 0 & 0 & 0 \\ 0 & 2\,dx_1\,dx_2 & 2\,dx_1\,dy_2 & 2\,dx_1\,dz_2 \\ 0 & 2\,dx_2\,dy_1 & 2\,dy_1\,dy_2 & 2\,dy_1\,dz_2 \\ 0 & 2\,dx_2\,dz_1 & 2\,dy_2\,dz_1 & 2\,dz_1\,dz_2 \end{pmatrix}$$

$$\begin{pmatrix} 2\,dt_1\,dt_2 & 0 & 0 & 0 \\ 0 & 2\,dx_1\,dx_2 & 2\,dx_1\,dy_2 & 2\,dx_1\,dz_2 \\ 0 & 2\,dx_2\,dy_1 & 2\,dy_1\,dy_2 & 2\,dy_1\,dz_2 \\ 0 & 2\,dx_2\,dz_1 & 2\,dy_2\,dz_1 & 2\,dz_1\,dz_2 \end{pmatrix}$$

SVD

```
η = {{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}};
```

```
SingularValueDecomposition[η][[1]] // MatrixForm
SingularValueDecomposition[η][[2]] // MatrixForm
SingularValueDecomposition[η][[3]] // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

ADM metric tensor

$$ds^2 = -\left(\alpha^2 - \beta_i\,\beta^i\right) dt^2 + 2\,\beta_i\,dx^i\,dt + \gamma_{ij}\,dx^i\,dx^j$$

```
η = {{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}};
```

$$\begin{pmatrix} -\sqrt{\left(\alpha^2 - \left((\beta_1)^2 + (\beta_2)^2 + (\beta_3)^2\right)\right)} & \beta_1 & \beta_2 & \beta_3 \\ \beta_1 & \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \beta_2 & \gamma_{21} & \gamma_{22} & \gamma_{23} \\ \beta_3 & \gamma_{31} & \gamma_{32} & \gamma_{33} \end{pmatrix}$$

$$\left\{\left\{-\sqrt{\alpha^2 - \beta_1^2 - \beta_2^2 - \beta_3^2}, \beta_1, \beta_2, \beta_3\right\}, \{\beta_1, \gamma_{11}, \gamma_{12}, \gamma_{13}\}, \{\beta_2, \gamma_{21}, \gamma_{22}, \gamma_{23}\}, \{\beta_3, \gamma_{31}, \gamma_{32}, \gamma_{33}\}\right\}$$

ADMmetric =

$$\left\{\left\{-\sqrt{\alpha^2 - \beta_1^2 - \beta_2^2 - \beta_3^2}, \beta_1, \beta_2, \beta_3\right\}, \{\beta_1, \gamma_{11}, \gamma_{12}, \gamma_{13}\}, \{\beta_2, \gamma_{21}, \gamma_{22}, \gamma_{23}\}, \{\beta_3, \gamma_{31}, \gamma_{32}, \gamma_{33}\}\right\};$$

```
ADMsvd = SingularValueDecomposition[ADMmetric]
```

```
$Aborted
```

```
$Aborted
```

```
$Aborted
```