Want to examine $\sum_{a=a0}^{af} \sum_{b=b0}^{bf}$ `((((q + a + (b i))) ^ (-s)) * (MatrixPower[X, f (a, bi)]))`

```
X = {{x11, x12, x13, x14}, {x21, x22, x23, x24}, {x31, x32, x33, x34}, {x41, x42, x43, x44}};
(* 4x4 tensor b/c in physics deal with 4D space-time *)
```

Want to generalize zeta fn to a polynomial of tensors, where the 1x1 tensor case would be a polynomial of numbers.

Number Case

X is 1x1 so a number, x

`(*` $\sum_{a=-\infty}^{\infty}$ `((((q+a))^(-s)))` = $(-1)^{-s}$ `HurwitzZeta[s,1-q]+HurwitzZeta[s,q] *)`

$\sum_{a=a0}^{af}$ `((((q + a)) ^ (-s)) * (x^a))`

x^{a0} `LerchPhi[x, s, a0 + q]` $- x^{1+af}$ `LerchPhi[x, s, 1 + af + q]`

∂_x $\left(x^{a0}\right.$ `LerchPhi[x, s, a0 + q]` $- x^{1+af}$ `LerchPhi[x, s, 1 + af + q]`$\left.\right)$ `// FullSimplify`

x^{-1+a0} `(LerchPhi[x, -1 + s, a0 + q] - q LerchPhi[x, s, a0 + q])` +
x^{af} `(-LerchPhi[x, -1 + s, 1 + af + q] + q LerchPhi[x, s, 1 + af + q])`

`N[`2^{-10} `LerchPhi[2, 3, -10 + 1]` $- 2^{1+10}$ `LerchPhi[2, 3, 1 + 10 + 1]]`

`4.137 + 0. i`

Tensor Case

Simplest Case

f(a,b) = a,

$\sum_{a=-3}^{3}$ `((((q + a)) ^ (-s)) * (MatrixPower[A, a]))` -
$\sum_{a=-2}^{2}$ `((((q + a)) ^ (-s)) * (MatrixPower[A, a]))` `// FullSimplify // MatrixForm`

$$\begin{pmatrix} -\frac{\left(d^3+b\,c\,(-3+2\,d)\right)\,(-3+q)^{-s}}{(b\,c+3\,d)^3} + \left(27+b\,c\,(6+d)\right)\,(3+q)^{-s} & \frac{b\,(9+b\,c+(-3+d)\,d)\,(-3+q)^{-s}}{(b\,c+3\,d)^3} + b\,\left(9+b\,c+d\,(3+d)\right)\,(3 \\ \frac{c\,(9+b\,c+(-3+d)\,d)\,(-3+q)^{-s}}{(b\,c+3\,d)^3} + c\,\left(9+b\,c+d\,(3+d)\right)\,(3+q)^{-s} & \frac{(27-b\,c\,(-6+d))\,(-3+q)^{-s}}{(b\,c+3\,d)^3} + \left(d^3+b\,c\,(3+2\,d)\right)\,(3+ \end{pmatrix}$$

`((((q + 3)) ^ (-s)) * (MatrixPower[A, 3]))` `// FullSimplify // MatrixForm`

$$\begin{pmatrix} \left(a^3+2\,a\,b\,c+b\,c\,d\right)\,(3+q)^{-s} & b\,\left(a^2+b\,c+a\,d+d^2\right)\,(3+q)^{-s} \\ c\,\left(a^2+b\,c+a\,d+d^2\right)\,(3+q)^{-s} & \left(a\,b\,c+2\,b\,c\,d+d^3\right)\,(3+q)^{-s} \end{pmatrix}$$

```
PolyLog[s, -1]
```

$-\left(1-2^{1-s}\right) \mathrm{Zeta}[s]$