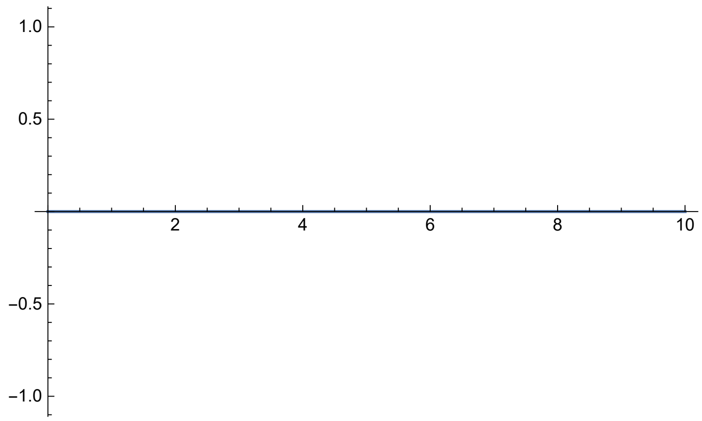
Definitional Function

```
a = 10000; k = 1000000000;
```

$$\text{Plot}\left[\text{Sum}\left[\frac{k}{\left(1+\text{Cosh}\left[2\,k\,\left(x-\text{Prime}[n]\right)\right]\right)},\ \{n, 1, a\}\right],\ \{x, 0, 10\}\right]$$



$$\text{Sum}\left[\frac{k}{\left(1+\text{Cosh}\left[2\,k\,\left(5.5-\text{Prime}[n]\right)\right]\right)},\ \{n, 1, a\}\right]\ //\ N$$

$2.499069\times 10^{-434\,294\,473}$

$$\text{Sum}\left[\frac{k}{\left(1+\text{Cosh}\left[2\,k\,\left(6-\text{Prime}[n]\right)\right]\right)},\ \{n, 1, a\}\right]\ //\ N$$

$6.245343586820707\times 10^{-868\,588\,955}$

$$\text{Sum}\left[\frac{k}{\left(1+\text{Cosh}\left[2\,k\,\left(5-\text{Prime}[n]\right)\right]\right)},\ \{n, 1, a\}\right]\ //\ N$$

$5.\times 10^{8}$

$$\text{Sum}\left[\frac{k}{\left(1+\text{Cosh}\left[2\,k\,\left(3-\text{Prime}[n]\right)\right]\right)},\ \{n, 1, a\}\right]\ //\ N$$

$5.\times 10^{8}$

$$\text{Sum}\left[\frac{k}{\left(1+\text{Cosh}\left[2\,k\,\left(25\,666-\text{Prime}[n]\right)\right]\right)},\ \{n, 1, a\}\right]\ //\ N$$

$3.122671793410354\times 10^{-868\,588\,955}$

$$\text{Sum}\left[\frac{k}{\left(1+\text{Cosh}\left[2\,k\,\left(25\,666.9-\text{Prime}[n]\right)\right]\right)},\ \{n, 1, a\}\right]\ //\ N$$

$8.349176\times 10^{-86\,858\,888}$

```
Sum[k / (1 + Cosh[2 k (25667 - Prime[n])]), {n, 1, a}] // N
```

$5. \times 10^{8}$

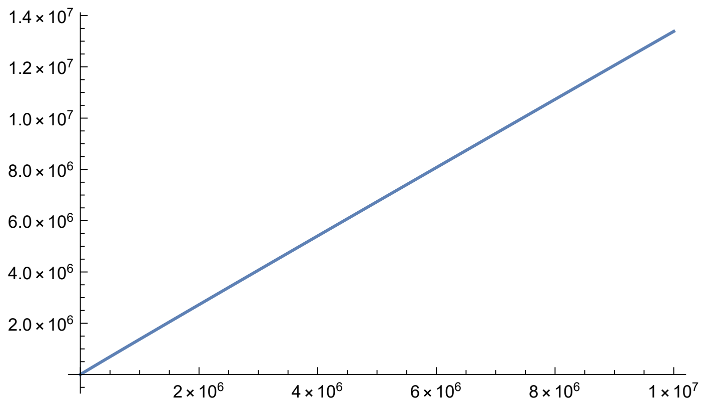
Spikes to infinity at primes, accurate way of predicting whether or not a number is prime

Mersenne Numbers

Functions

Key Relation

```
Plot[{Prime[Floor[x]] / Log[PrimePi[x]]}, {x, 1, 10000000}]
```



$$\frac{\text{Prime[Floor[x]]}}{\text{Log[PrimePi[x]]}} \approx ax \text{ !!!!!!!!}$$

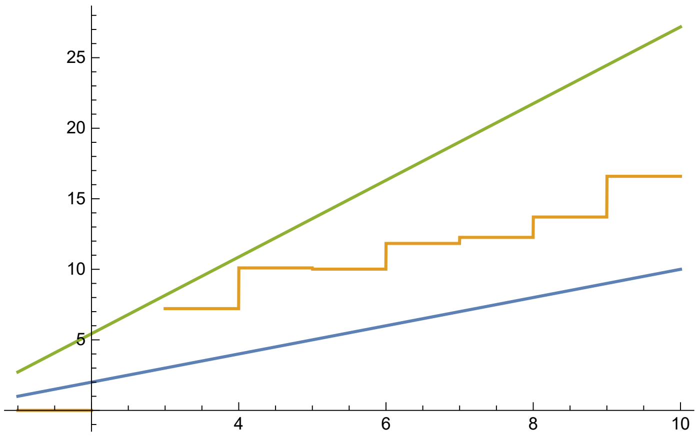
```
(Prime[Floor[4]] / Log[PrimePi[4]]) / 4
```

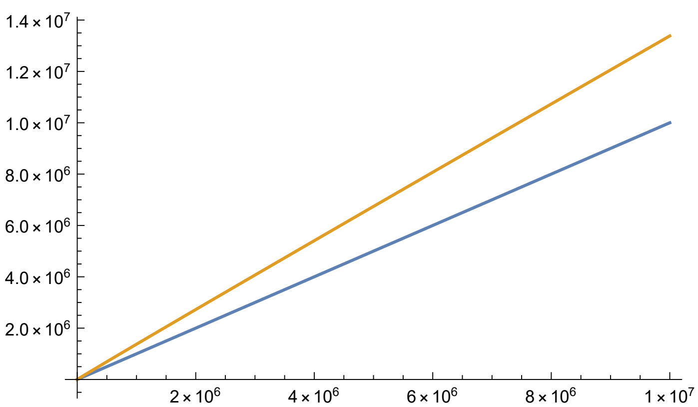
$\frac{7}{4 \text{ Log}[2]}$

```
7 / (4 Log[2]) // N
```

2.52472

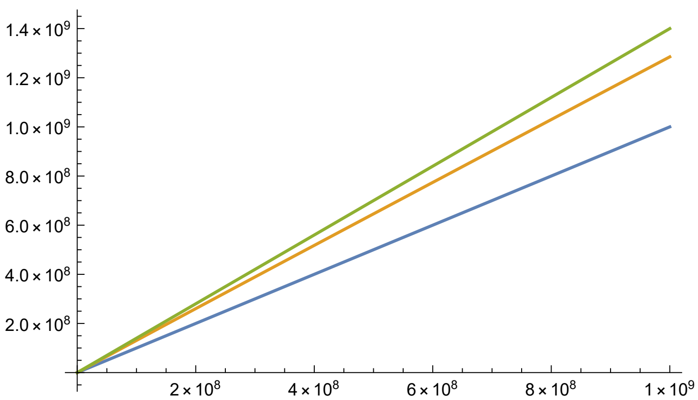
```
Plot[{x, Prime[Floor[x]]/Log[PrimePi[x]], E x}, {x, 1, 10}]
Plot[{x, Prime[Floor[x]]/Log[PrimePi[x]]}, {x, 1, 10 000 000}]
```



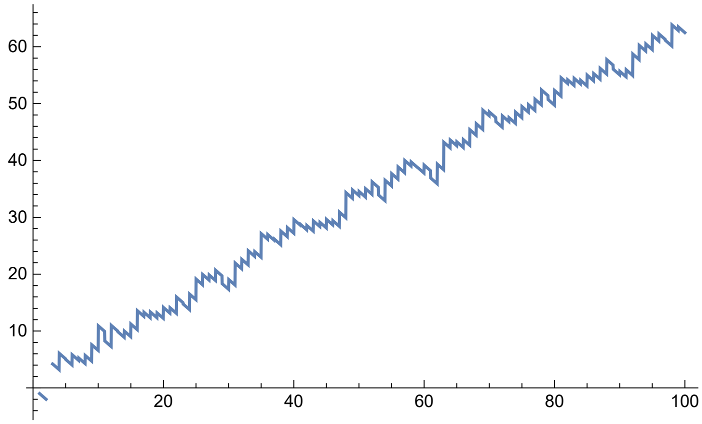


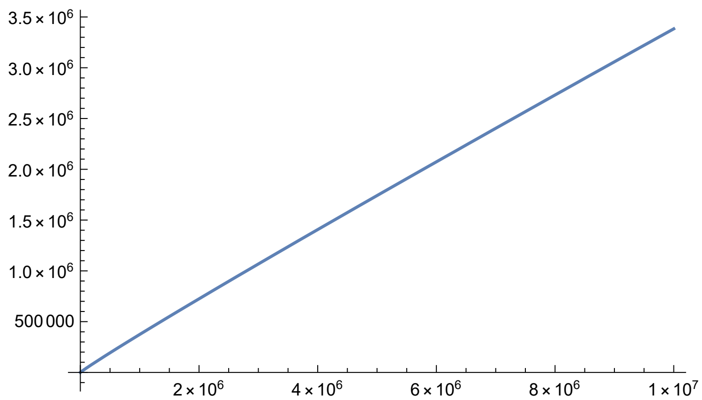
Goes closer to x as range for x increases

```
Plot[{x, Prime[Floor[x]]/Log[PrimePi[x]], 1.4 x}, {x, 1, 1 000 000 000}]
```

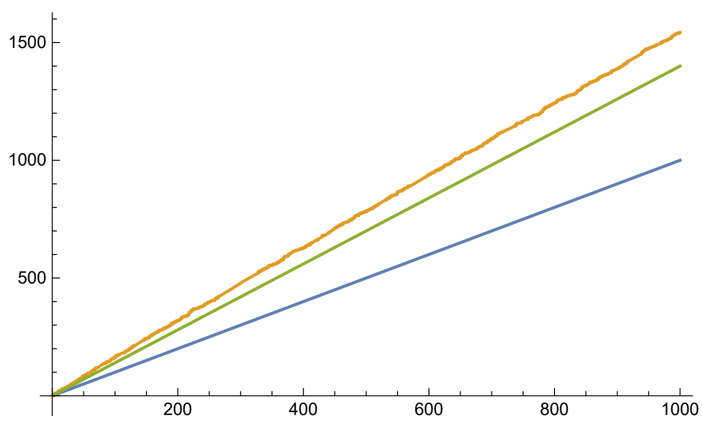


```
Plot[{Prime[Floor[x]]/Log[PrimePi[x]] - x}, {x, 1, 100}]
Plot[{Prime[Floor[x]]/Log[PrimePi[x]] - x}, {x, 1, 10000000}]
```

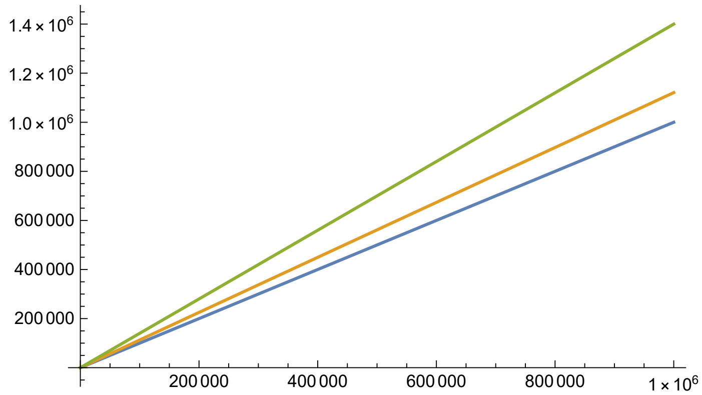





```
Plot[{x, Prime[Floor[x]]/Log[PrimePi[x]], 1.4 x}, {x, 1, 1000}]
```



$$\texttt{Plot}\left[\left\{x, \frac{\texttt{Prime[Floor[x]]}}{\texttt{Log[x]}}, 1.4\,x\right\}, \{x, 1, 1\,000\,000\}\right]$$



From Dusert ' s inequality we have that

$\text{Prime}[x] < x\,\text{Log}[x] + x\,\text{Log}[\text{Log}[x]]$, for $x > 6$, which gives us appx

$$\frac{\text{Prime}[x]}{\text{Log}\left[\frac{x}{\text{Log}[x]}\right]} < x + \left(2\,x\,\frac{\text{Log}[\text{Log}[x]]}{\text{Log}\left[\frac{x}{\text{Log}[x]}\right]}\right)$$

$$\frac{\text{Prime}[x]}{\text{Log}[\text{PrimePi}[x]]} < x + \left(2\,x\,\frac{\text{Log}[\text{Log}[x]]}{\text{Log}\left[\frac{x}{\text{Log}[x]}\right]}\right) \approx x + \left(2\,x\,\frac{\text{Log}[\text{Log}[x]]}{\text{Log}[\text{PrimePi}[x]]}\right)$$

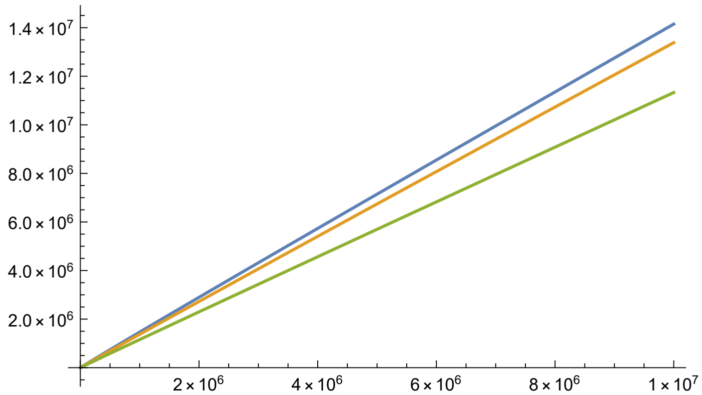
for large x

having used $\dfrac{x}{\text{Log}[x]} \approx \text{PrimePi}[x]$

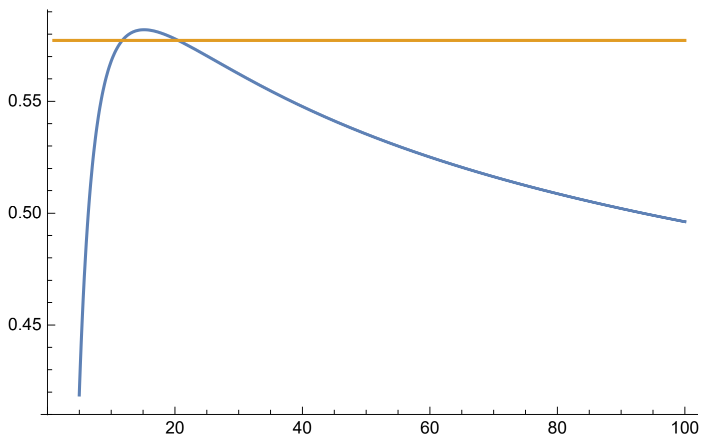
$$\frac{\text{Log}[\text{Log}[x]]}{\text{Log}\left[\frac{x}{\text{Log}[x]}\right]} = \frac{\text{Log}[\text{Log}[x]]}{\text{Log}[x] - \text{Log}[\text{Log}[x]]} = \frac{1}{\frac{\text{Log}[x]}{\text{Log}[\text{Log}[x]]} - 1}$$

And also for the earlier appx, $\dfrac{\text{Log}[\text{Log}[x]]}{\text{Log}[\text{PrimePi}[x]]} \to 0$ as $x \to \infty$, so

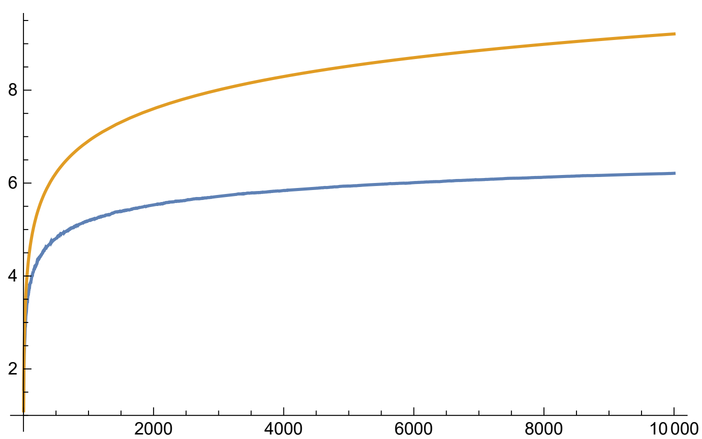
$$\text{Plot}\left[\left\{x + \left(2\,x\,\frac{\text{Log}[\text{Log}[x]]}{\text{Log}[\text{PrimePi}[x]]}\right),\ \frac{\text{Prime}[\text{Floor}[x]]}{\text{Log}[\text{PrimePi}[x]]},\ x + 2\,\text{PrimePi}[x]\right\},\ \{x, 1, 10\,000\,000\}\right]$$



$$\text{Plot}\left[\left\{1\Big/\left(\frac{\text{Log}[x]}{\text{Log}[\text{Log}[x]]} - 1\right),\ \text{EulerGamma}\right\},\ \{x, 1, 100\}\right]$$



$$\text{Plot}\left[\left\{\text{Log}\left[\frac{\text{Prime}[\text{Floor}[x]]}{\text{LogIntegral}[\text{PrimePi}[x]]}\right],\ \text{Log}[x]\right\},\ \{x, 3, 10\,000\}\right]$$



$$\texttt{FF1[x\_] := }\frac{\texttt{Prime[Floor[x]]}}{\texttt{Log[PrimePi[x]]}}$$

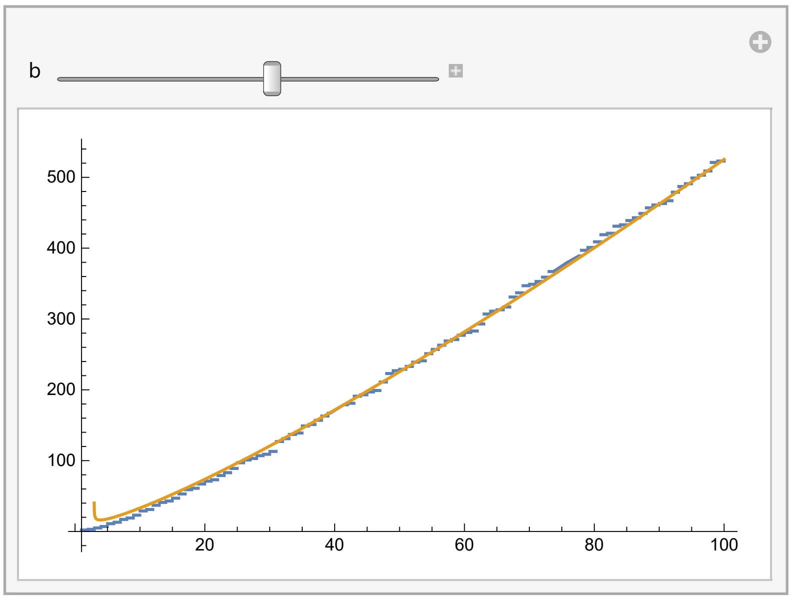
$$\texttt{FF2[x\_] := }\frac{\texttt{Prime[Floor[x]]}}{\texttt{LogIntegral[PrimePi[x]]}}$$

```
FF1[100 000 000 000] // N
```

1.24702×10^{11}

$$\texttt{(* b x}\left(\texttt{Log}\left[\frac{\texttt{x}}{\texttt{(Log[x]-1.084)}}\right]\right)\texttt{ = b x}\left(\texttt{Log[x]-Log[Log[x]-1.084]}\right)\texttt{*)}$$

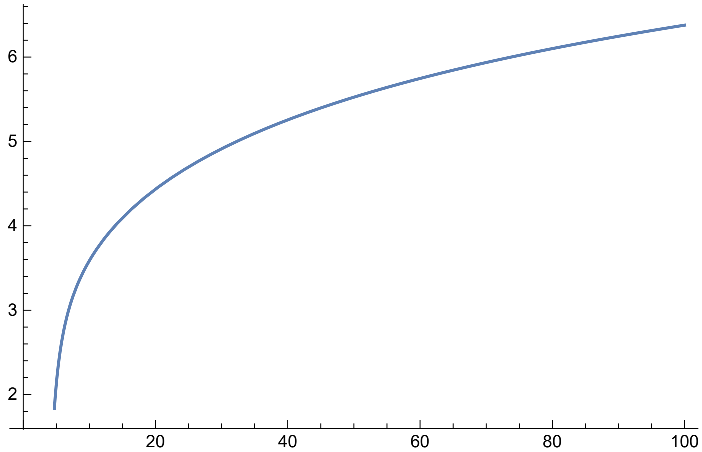
$$\texttt{Manipulate}\left[\texttt{Plot}\left[\left\{\texttt{Prime[Floor[x]], b x}\left(\texttt{Log}\left[\frac{\texttt{x}}{\left(\texttt{Log[x] - 1.084}\right)}\right]\right)\right\}\texttt{, \{x, 1, 100\}}\right]\texttt{, \{b, 1, 2\}}\right]$$



$$\texttt{D}\left[\texttt{b x}\left(\texttt{Log}\left[\frac{\texttt{x}}{\left(\texttt{Log[x] - 1.084}\right)}\right]\right)\texttt{, x}\right]\texttt{ // FullSimplify}$$

$$\texttt{b}\left(\frac{-2.084+\texttt{Log[x]}}{-1.084+\texttt{Log[x]}}+\texttt{Log}\left[\frac{\texttt{x}}{-1.084+\texttt{Log[x]}}\right]\right)$$

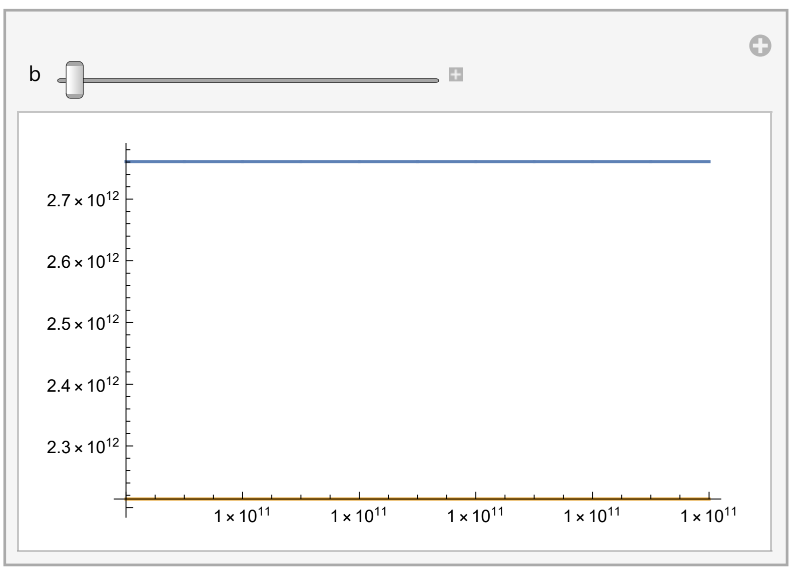
```
Plot[(1.57) ((-2.084` + Log[x])/(-1.084` + Log[x]) + Log[x/(-1.084` + Log[x])]), {x, 0, 100}]
(* since b ~ 1.57 for this scale *)
```




```
10^20
```

100 000 000 000 000 000 000

```
Manipulate[Plot[{Prime[Floor[x]], b x (Log[x/(Log[x] - 1.084)])},
  {x, 100 000 000 000, 100 000 000 010}], {b, 1, 2}]
```



b ~ 1.374 for {x,1000000,1000100}

b ~ 1.3092 for {x,100000000,100000100}

b ~ 1.248 for {x,100000000000,100000000100}

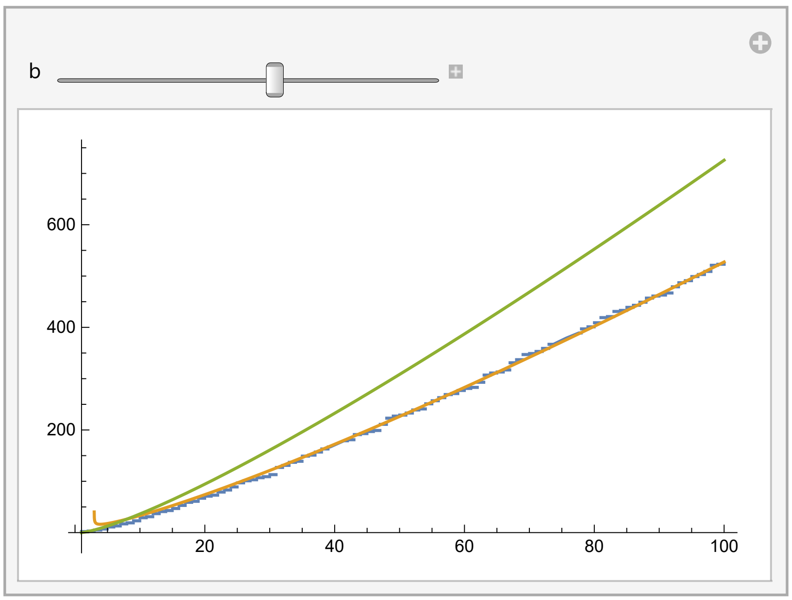
```
1/Log[2] + 1.08 // N
7 / 4 // N
1/Log[2] // N
```

2.5227

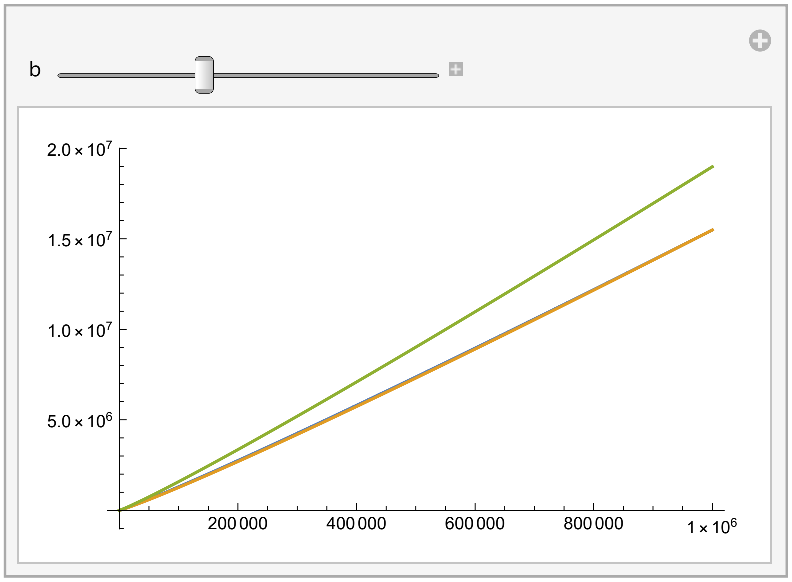
1.75

1.4427

```
Manipulate[
 Plot[{Prime[Floor[x]], b x (Log[x / (Log[x] - 1.084)]), b x Log[x]}, {x, 1, 100}], {b, 1, 2}]
```

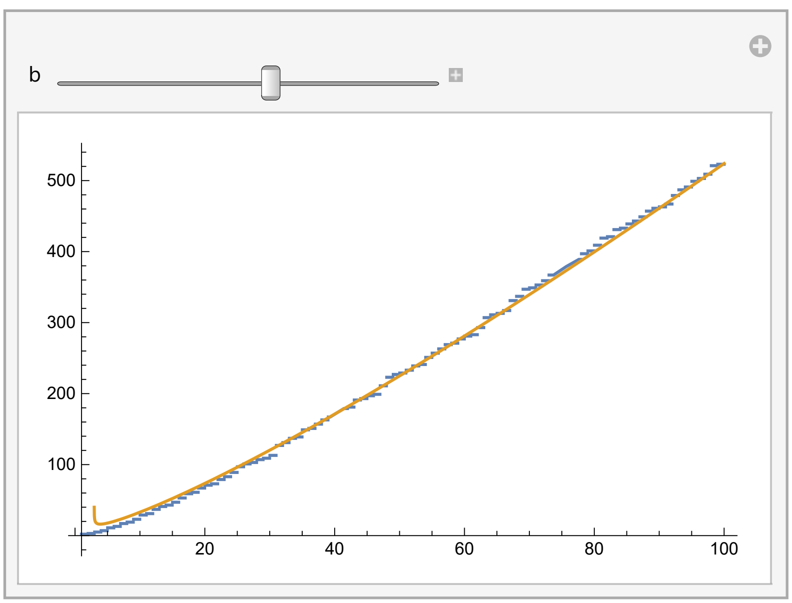


```
Manipulate[Plot[
  {Prime[Floor[x]], b x (Log[x / (Log[x] - 1.084)]), b x Log[x]}, {x, 1, 1000000}], {b, 1, 2}]
```



b

```
Manipulate[Plot[{Prime[Floor[x]], b x (Log[x / (Log[x] - 1.084)])}, {x, 1, 100}], {b, 1, 2}]
```




```
f[x_, b_] := b x (Log[x / (Log[x] - 1.084)]);
```

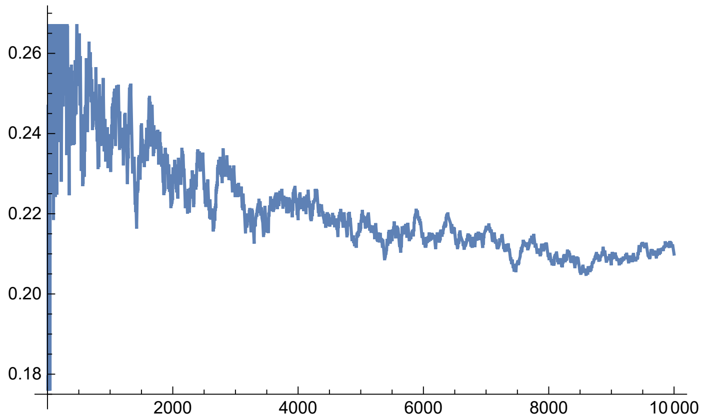
```
Prime[Floor[50]]
```

229

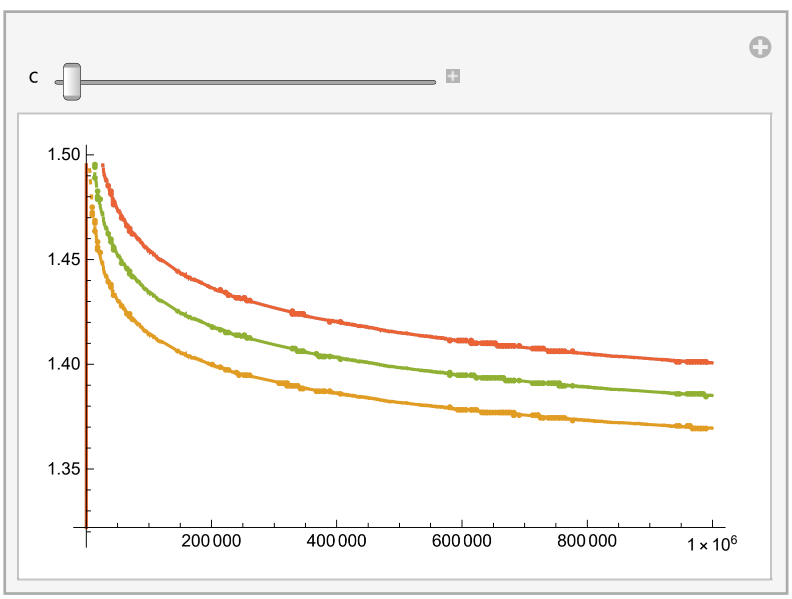
```
f[10000, 1.566]
```

111424.

$$\text{Plot}\left[\left\{\frac{\text{PrimePi}[x]}{\left(\frac{x}{\text{Log}[x]}\right)} - 0.922\right\}, \{x, 1, 10000\}\right]$$



$$\text{Manipulate}\left[\text{Plot}\left[\left\{\frac{\text{Prime}[\text{Floor}[x]]}{x}\left(\left(\text{Log}\left[\left(\frac{7}{8}\right)\left(\frac{x}{\text{Log}[x]}\right)\right]\right)^{\wedge}(-1)\right), \frac{\text{Prime}[\text{Floor}[x]]}{x}\left(\left(\text{Log}\left[\left(\frac{9}{8}\right)\left(\frac{x}{\text{Log}[x]}\right)\right]\right)^{\wedge}(-1)\right), \left(\left(\frac{\text{Prime}[\text{Floor}[x]]}{x}\left(\left(\text{Log}\left[\left(\frac{7}{8}\right)\left(\frac{x}{\text{Log}[x]}\right)\right]\right)^{\wedge}(-1)\right)\right) + \left(\frac{\text{Prime}[\text{Floor}[x]]}{x}\left(\left(\text{Log}\left[\left(\frac{9}{8}\right)\left(\frac{x}{\text{Log}[x]}\right)\right]\right)^{\wedge}(-1)\right)\right)\right)\Big/ 2, \frac{\text{Prime}[\text{Floor}[x]]}{x}\left(\left(\text{Log}\left[(c)\left(\frac{x}{\text{Log}[x]}\right)\right]\right)^{\wedge}(-1)\right)\right\}, \{x, 1, 1000000\}\right], \left\{c, \left(\frac{7}{8}\right), \left(\frac{9}{8}\right)\right\}\right]$$



$$\left(\left(\frac{\text{Prime}[\text{Floor}[x]]}{x}\left(\left(\text{Log}\left[\left(\frac{7}{8}\right)\left(\frac{x}{\text{Log}[x]}\right)\right]\right)\text{^}(-1)\right)\right) + \left(\frac{\text{Prime}[\text{Floor}[x]]}{x}\left(\left(\text{Log}\left[\left(\frac{9}{8}\right)\left(\frac{x}{\text{Log}[x]}\right)\right]\right)\text{^}(-1)\right)\right)\right)\Big/ 2\ \text{// FullSimplify}$$

$$\frac{\left(\frac{1}{\text{Log}\left[\frac{7\,x}{8\,\text{Log}[x]}\right]}+\frac{1}{\text{Log}\left[\frac{9\,x}{8\,\text{Log}[x]}\right]}\right)\text{Prime}[\text{Floor}[x]]}{2\,x}$$

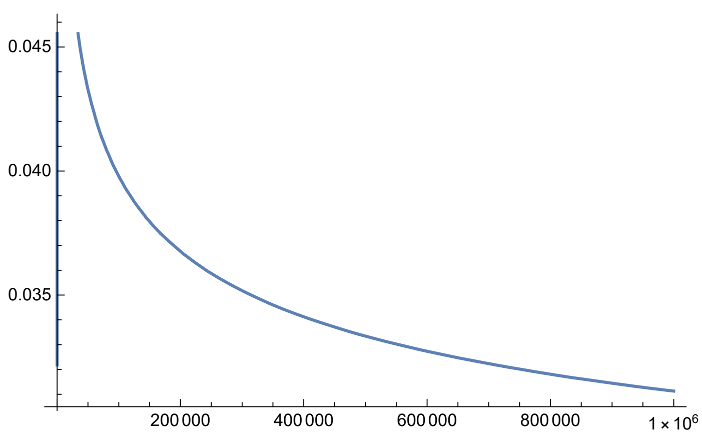
$$\left(\frac{\text{Prime}[\text{Floor}[x]]}{x}\left(\left(\text{Log}\left[\left(\frac{7}{8}\right)\left(\frac{x}{\text{Log}[x]}\right)\right]\right)\text{^}(-1)\right)\right) - \left(\frac{\text{Prime}[\text{Floor}[x]]}{x}\left(\left(\text{Log}\left[\left(\frac{9}{8}\right)\left(\frac{x}{\text{Log}[x]}\right)\right]\right)\text{^}(-1)\right)\right)\ \text{// FullSimplify}$$

$$\frac{\left(\frac{1}{\text{Log}\left[\frac{7\,x}{8\,\text{Log}[x]}\right]}-\frac{1}{\text{Log}\left[\frac{9\,x}{8\,\text{Log}[x]}\right]}\right)\text{Prime}[\text{Floor}[x]]}{x}$$

$$\text{Limit}\left[\left(\frac{1}{\text{Log}\left[\frac{7\,x}{8\,\text{Log}[x]}\right]}-\frac{1}{\text{Log}\left[\frac{9\,x}{8\,\text{Log}[x]}\right]}\right),\ x\to\text{Infinity}\right]$$

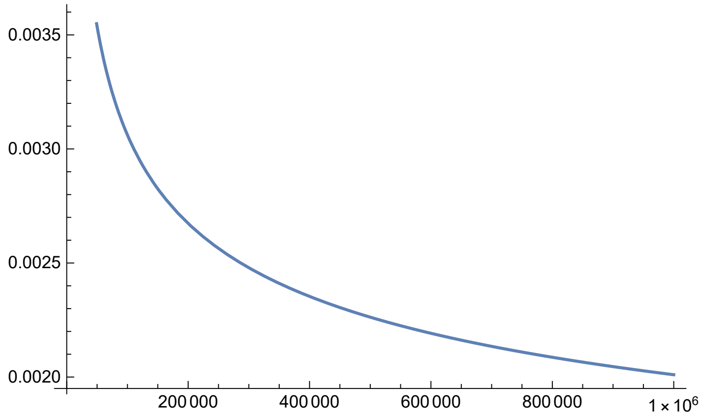
0

$$\text{Plot}\left[\left\{\frac{\text{Prime}[\text{Floor}[x]]}{x}\left(\left(\text{Log}\left[\left(\frac{7}{8}\right)\left(\frac{x}{\text{Log}[x]}\right)\right]\right)\text{^}(-1)\right) - \frac{\text{Prime}[\text{Floor}[x]]}{x}\left(\left(\text{Log}\left[\left(\frac{9}{8}\right)\left(\frac{x}{\text{Log}[x]}\right)\right]\right)\text{^}(-1)\right)\right\},\ \{x, 1, 1\,000\,000\}\right]$$



```
Plot[{1/Log[7 x/(8 Log[x])] - 1/Log[9 x/(8 Log[x])]}, {x, 1, 1000000}]
```

$$\text{Plot}\left[\left\{\frac{1}{\text{Log}\left[\frac{7x}{8\,\text{Log}[x]}\right]}-\frac{1}{\text{Log}\left[\frac{9x}{8\,\text{Log}[x]}\right]}\right\},\{x,1,1000000\}\right]$$



Legr Conj

Log Integral

Appxs

Test to find large primes ???