Basic Construction

```
V = {{v₁[x₁, x₂, x₃, t], v₂[x₁, x₂, x₃, t], v₃[x₁, x₂, x₃, t]}};

v₁ = v1[x₁, x₂, x₃, t];
v₂ = v2[x₁, x₂, x₃, t];
v₃ = v3[x₁, x₂, x₃, t];

COOD = {x₁, x₂, x₃};

D[Sum[vᵢ, {i, 1, 3}], t]
```

$$v1^{(0,0,0,1)}[x_1, x_2, x_3, t] + v2^{(0,0,0,1)}[x_1, x_2, x_3, t] + v3^{(0,0,0,1)}[x_1, x_2, x_3, t]$$

```
Sum[vⱼ (D[Sum[vᵢ, {i, 1, 3}], COOD[[j]]]), {j, 1, 3}];

Sum[D[Sum[vᵢ, {i, 1, 3}], {COOD[[j]], 2}], {j, 1, 3}]
```

$$v1^{(0,0,2,0)}[x_1, x_2, x_3, t] + v2^{(0,0,2,0)}[x_1, x_2, x_3, t] + v3^{(0,0,2,0)}[x_1, x_2, x_3, t] + v1^{(0,2,0,0)}[x_1, x_2, x_3, t] + v2^{(0,2,0,0)}[x_1, x_2, x_3, t] + v3^{(0,2,0,0)}[x_1, x_2, x_3, t] + v1^{(2,0,0,0)}[x_1, x_2, x_3, t] + v2^{(2,0,0,0)}[x_1, x_2, x_3, t] + v3^{(2,0,0,0)}[x_1, x_2, x_3, t]$$

All the Velocity-dependent terms of NS eqns are then:

```
D[Sum[vᵢ, {i, 1, 3}], t] + Sum[vⱼ (D[Sum[vᵢ, {i, 1, 3}], COOD[[j]]]), {j, 1, 3}] -
  (b * (Sum[D[Sum[vᵢ, {i, 1, 3}], {COOD[[j]], 2}], {j, 1, 3}]))
```

$$\begin{aligned}
&v1^{(0,0,0,1)}[x_1, x_2, x_3, t] + v2^{(0,0,0,1)}[x_1, x_2, x_3, t] + v3^{(0,0,0,1)}[x_1, x_2, x_3, t] + \\
&v3[x_1, x_2, x_3, t]\left(v1^{(0,0,1,0)}[x_1, x_2, x_3, t] + v2^{(0,0,1,0)}[x_1, x_2, x_3, t] + v3^{(0,0,1,0)}[x_1, x_2, x_3, t]\right) + \\
&v2[x_1, x_2, x_3, t]\left(v1^{(0,1,0,0)}[x_1, x_2, x_3, t] + v2^{(0,1,0,0)}[x_1, x_2, x_3, t] + v3^{(0,1,0,0)}[x_1, x_2, x_3, t]\right) + \\
&v1[x_1, x_2, x_3, t]\left(v1^{(1,0,0,0)}[x_1, x_2, x_3, t] + v2^{(1,0,0,0)}[x_1, x_2, x_3, t] + v3^{(1,0,0,0)}[x_1, x_2, x_3, t]\right) - \\
&b\left(v1^{(0,0,2,0)}[x_1, x_2, x_3, t] + v2^{(0,0,2,0)}[x_1, x_2, x_3, t] + v3^{(0,0,2,0)}[x_1, x_2, x_3, t] + \right. \\
&\quad v1^{(0,2,0,0)}[x_1, x_2, x_3, t] + v2^{(0,2,0,0)}[x_1, x_2, x_3, t] + v3^{(0,2,0,0)}[x_1, x_2, x_3, t] + \\
&\quad \left. v1^{(2,0,0,0)}[x_1, x_2, x_3, t] + v2^{(2,0,0,0)}[x_1, x_2, x_3, t] + v3^{(2,0,0,0)}[x_1, x_2, x_3, t]\right)
\end{aligned}$$

Where b is the viscosity, assumed to be a constant such that b>0 in general NS formulation

Test with defined v-fields

For now define the v_i in terms of single variable x_i associated with that v_i
(Really they're multivariate but can restrict it to single variable)

```
(*Do[vₖ=Exp[xₖ],{k,1,3}]*)
(*Do[vₖ=LogIntegral[xₖ],{k,1,3}]*)
Do[vₖ = xₖ^2, {k, 1, 3}]

v₁
```

x_1^2

Then the v-field terms become:

```
D[Sum[vi, {i, 1, 3}], t] + Sum[vj (D[Sum[vi, {i, 1, 3}], COOD[[j]]]), {j, 1, 3}] -
  (b * (Sum[D[Sum[vi, {i, 1, 3}], {COOD[[j]], 2}], {j, 1, 3}]))
```

$x_1 + x_2 + x_3$

Series

```
Do[vk = (xk^-4) + (xk^-3) + (xk^-2) + (xk^-1), {k, 1, 3}]
D[Sum[vi, {i, 1, 3}], t] + Sum[vj (D[Sum[vi, {i, 1, 3}], COOD[[j]]]), {j, 1, 3}] -
  (b * (Sum[D[Sum[vi, {i, 1, 3}], {COOD[[j]], 2}], {j, 1, 3}]))
```

$$\left(-\frac{4}{x_1^5}-\frac{3}{x_1^4}-\frac{2}{x_1^3}-\frac{1}{x_1^2}\right)\left(\frac{1}{x_1^4}+\frac{1}{x_1^3}+\frac{1}{x_1^2}+\frac{1}{x_1}\right)+\left(-\frac{4}{x_2^5}-\frac{3}{x_2^4}-\frac{2}{x_2^3}-\frac{1}{x_2^2}\right)\left(\frac{1}{x_2^4}+\frac{1}{x_2^3}+\frac{1}{x_2^2}+\frac{1}{x_2}\right)-$$
$$b\left(\frac{20}{x_1^6}+\frac{12}{x_1^5}+\frac{6}{x_1^4}+\frac{2}{x_1^3}+\frac{20}{x_2^6}+\frac{12}{x_2^5}+\frac{6}{x_2^4}+\frac{2}{x_2^3}+\frac{20}{x_3^6}+\frac{12}{x_3^5}+\frac{6}{x_3^4}+\frac{2}{x_3^3}\right)+$$
$$\left(-\frac{4}{x_3^5}-\frac{3}{x_3^4}-\frac{2}{x_3^3}-\frac{1}{x_3^2}\right)\left(\frac{1}{x_3^4}+\frac{1}{x_3^3}+\frac{1}{x_3^2}+\frac{1}{x_3}\right)$$

```
Table[2 (x^2) - 4, {x, 0, 15}]
```

{-4, -2, 4, 14, 28, 46, 68, 94, 124, 158, 196, 238, 284, 334, 388, 446}

```
Table[(x^2), {x, 0, 15}]
```

{0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225}

```
Intersection[{0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225},
  {-3, -1, 5, 15, 29, 47, 69, 95, 125, 159, 197, 239, 285, 335, 389, 447}]
```

{}