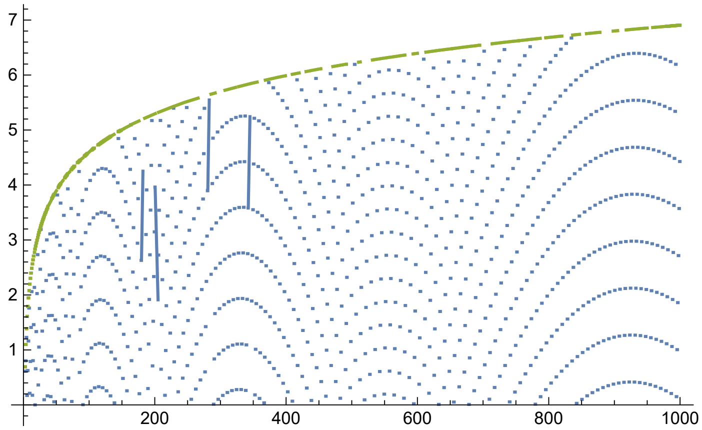
Spectral Pattern of the Log Modulo

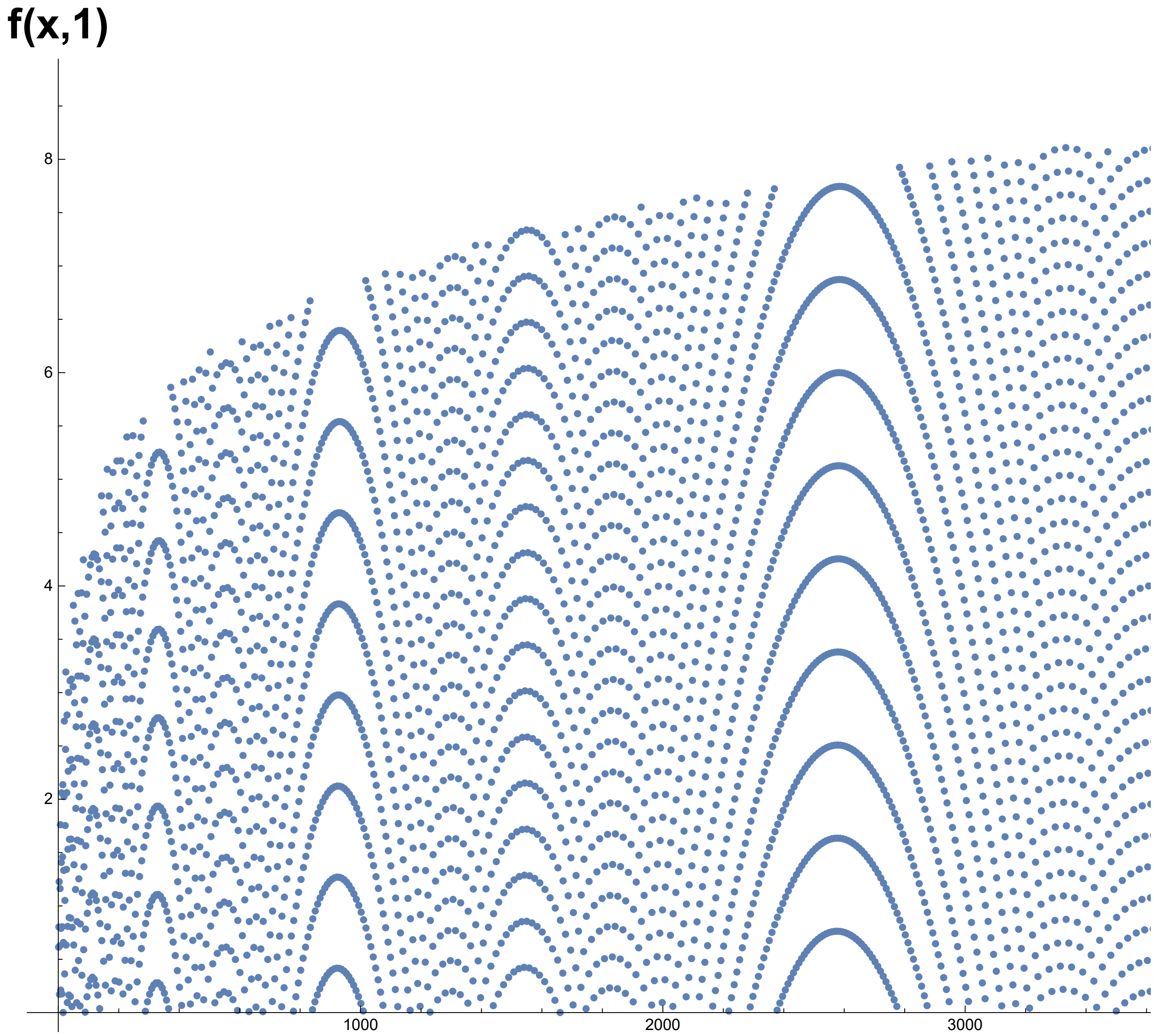
```
Cell[BoxData[

 RowBox[{"Plot", "[",

  RowBox[{
,


owBox[{"{",

    RowBox[{

     RowBox[{"Mod", "[",

      RowBox[{

       RowBox[{"Floor",

}, Open  ]],


Cell[CellGroupData[{


g", "[",

        RowBox[{"Floor", "[", "x", "]"}], "]"}]}],


}], ",",

     RowBox[{"Log", "[",

      RowBox[{"Floor", "[", "x", "

}, Open  ]],


Cell[CellGroupData[{


    RowBox[{"(",

       RowBox[{"SawtoothWave", "[",

        FractionBox[

         RowBox[{"Log", "[",

          RowBox[{"Floor", "[", "x", "]"}], "]"}], "x"], "]"}], ")"}]}]}]}],
```

```
   "}"}], ",",
  RowBox[{"{",
   RowBox[{"x", ",", "2", ",", "1000"}], "}"}]}], "]"}]], "Input",
CellChangeTimes->{{3.711753181792426*^9, 3.711753221376954*^9}}]
```

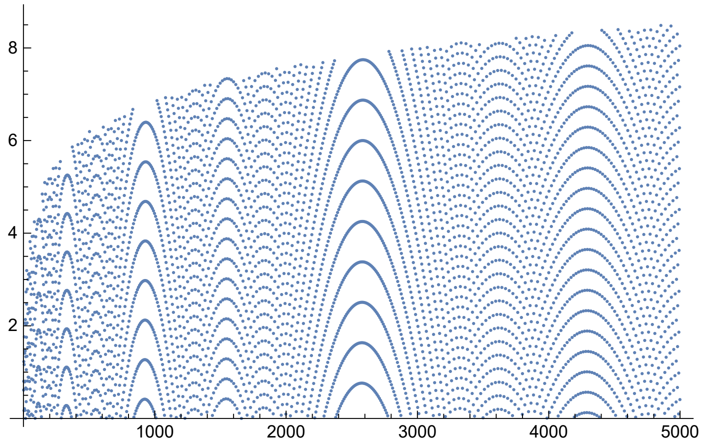


Plot[{Mod[Floor[x], Log[Floor[x]]], Log[Floor[x]]}, {x, 2, 500}]

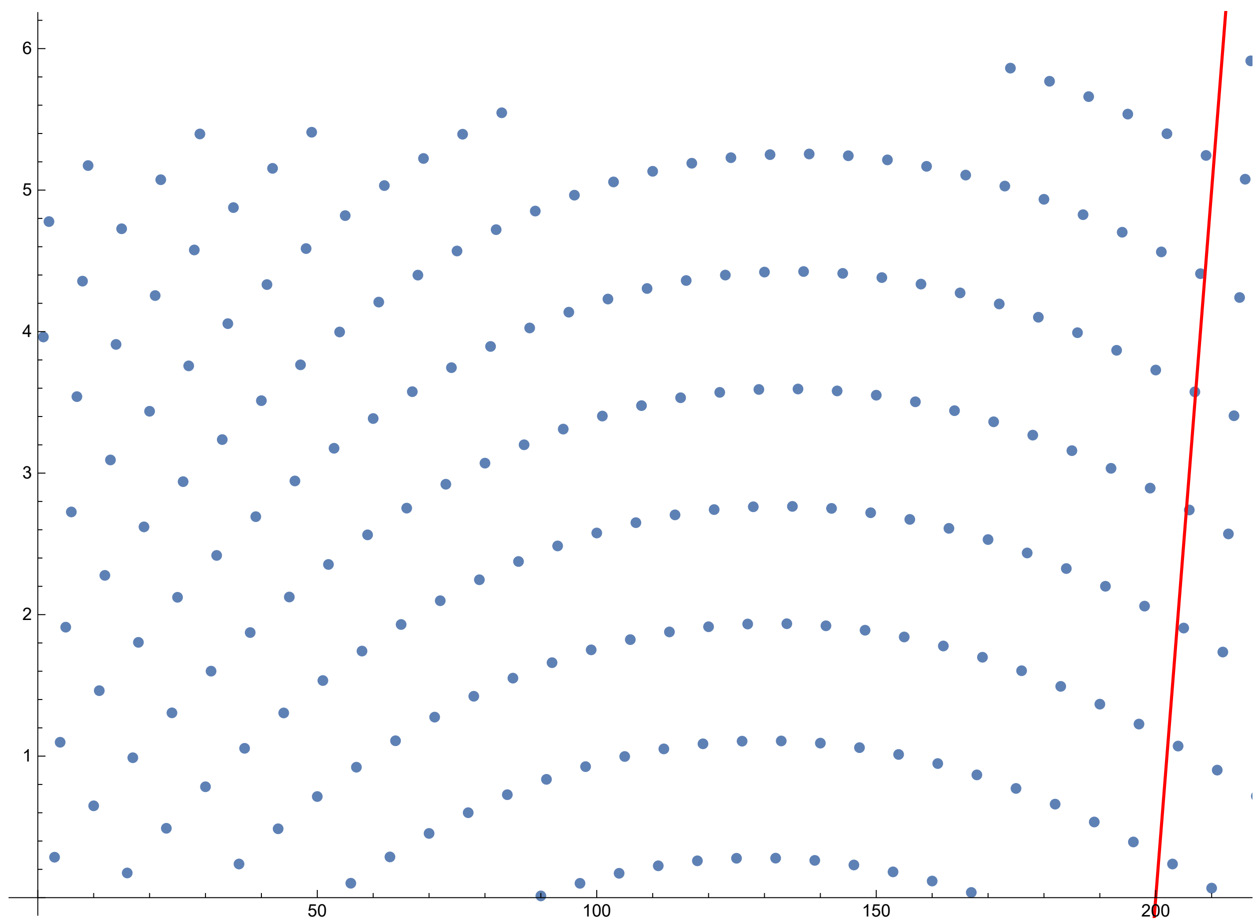
```
ListPlot[Table[Mod[Floor[x], Log[Floor[x]]], {x, 2, 5000}],
 AxesLabel → {Style[Null ], Style["f(x,1)", Large, Bold]}]
```



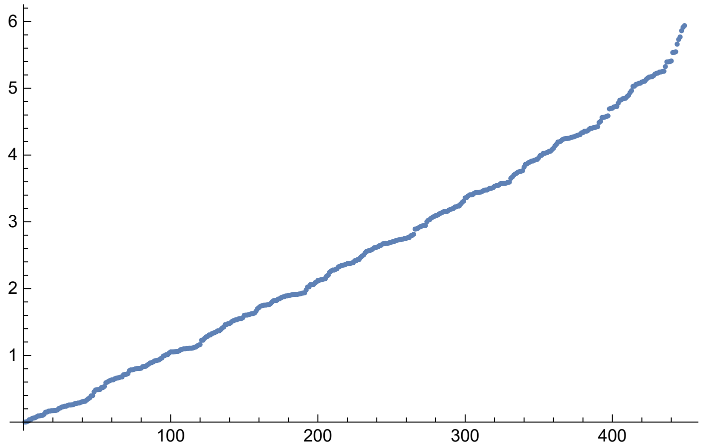
```
ListPlot[Table[Mod[x, Log[x]], {x, 2, 5000}]]
```



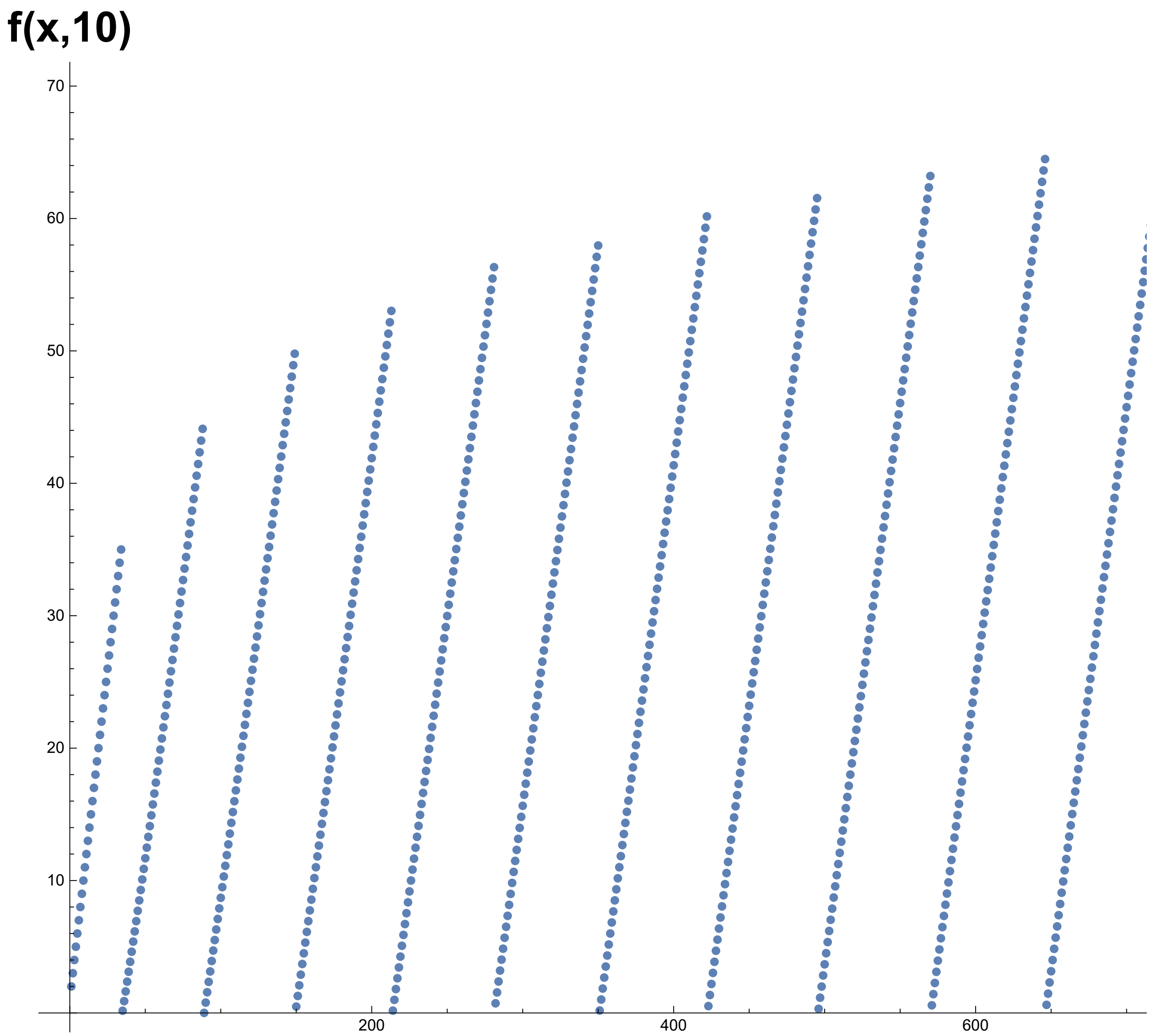

```
Show[
 ListPlot[Table[Mod[x, Log[x]], {x, 200, 450}]],
 Plot[{.5 (x) - 100}, {x, 0, 250}, PlotStyle → Red]
]
```



```
ListPlot[Union[Table[Mod[x, Log[x]], {x, 2, 450}] // N]]
```

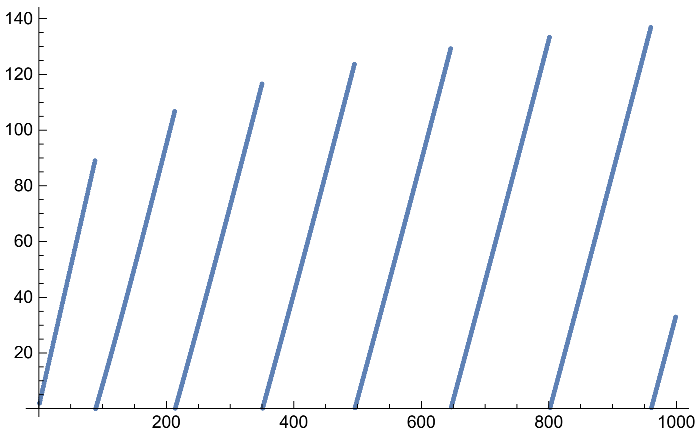


```
ListPlot[Table[Mod[x, Log[x^10]], {x, 2, 1000}],
 AxesLabel → {Style["x", Large, Bold], Style["f(x,10)", Large, Bold]}]
```

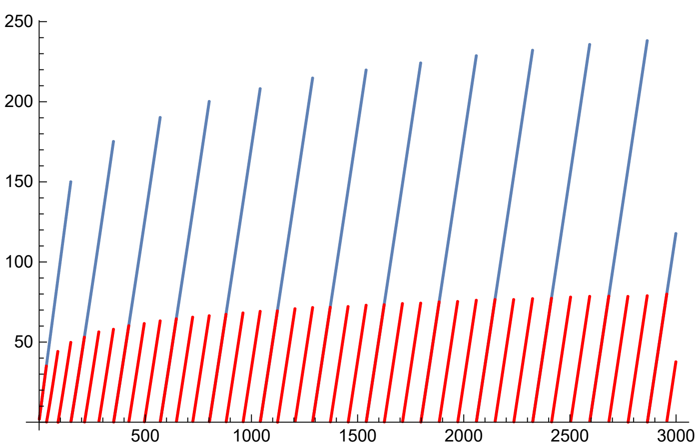


```
ListPlot[GradientFilter[ Table[Mod[x, Log[x^10]], {x, 2, 5000}] // N, 1]]
```

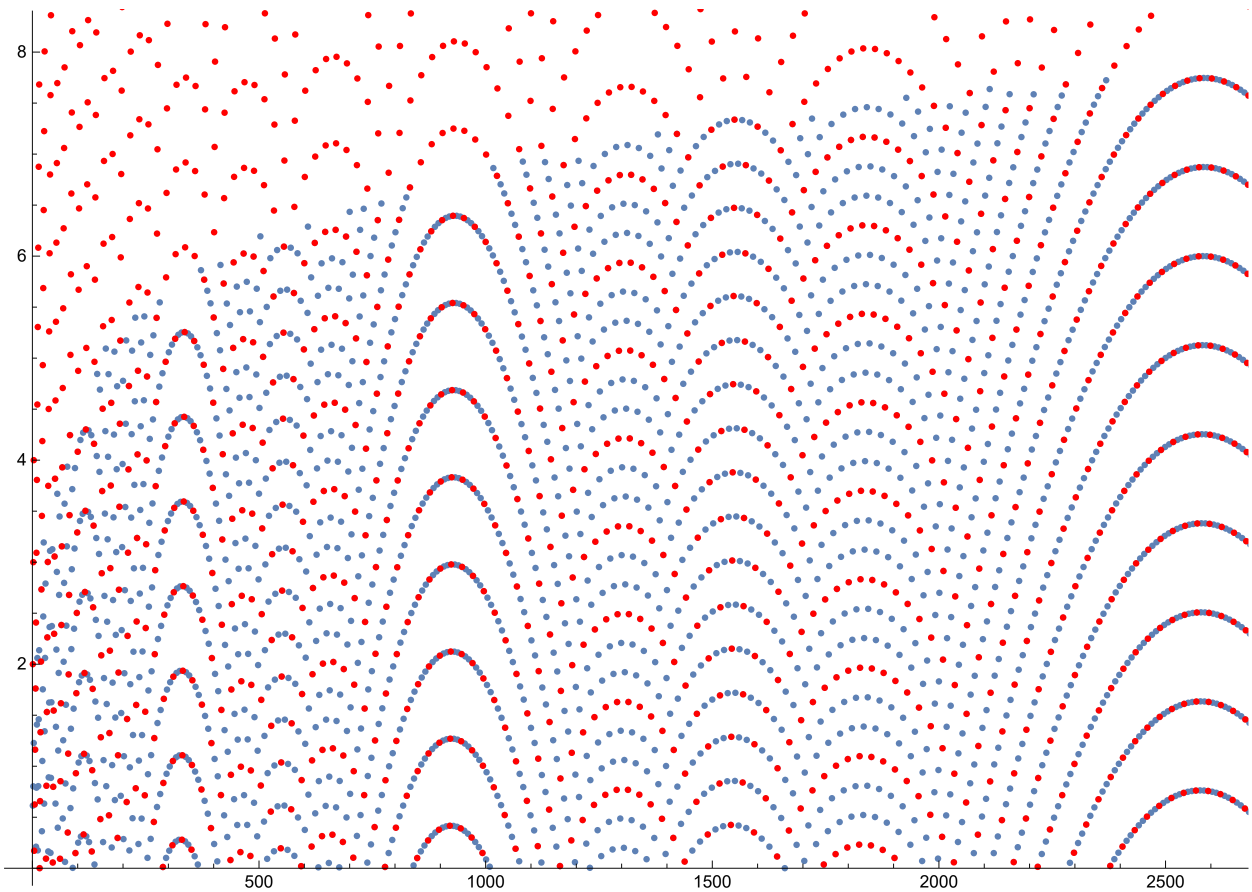

```
ListPlot[Table[Mod[x, Log[x^20]], {x, 2, 1000}]]
```




```
Show[
 ListPlot[Table[Mod[x, Log[x^30]], {x, 2, 3000}]],
 ListPlot[Table[Mod[x, Log[x^10]], {x, 2, 3000}], PlotStyle → Red]
]
```



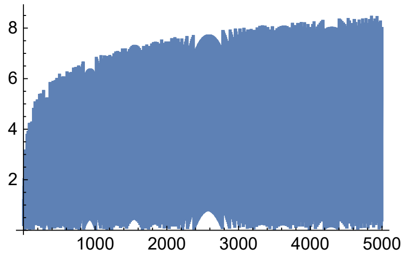
```
Show[
 ListPlot[Table[Mod[x, Log[x]], {x, 2, 3000}]],
 ListPlot[Table[Mod[x, Log[x^3]], {x, 2, 3000}], PlotStyle → Red]
]
```



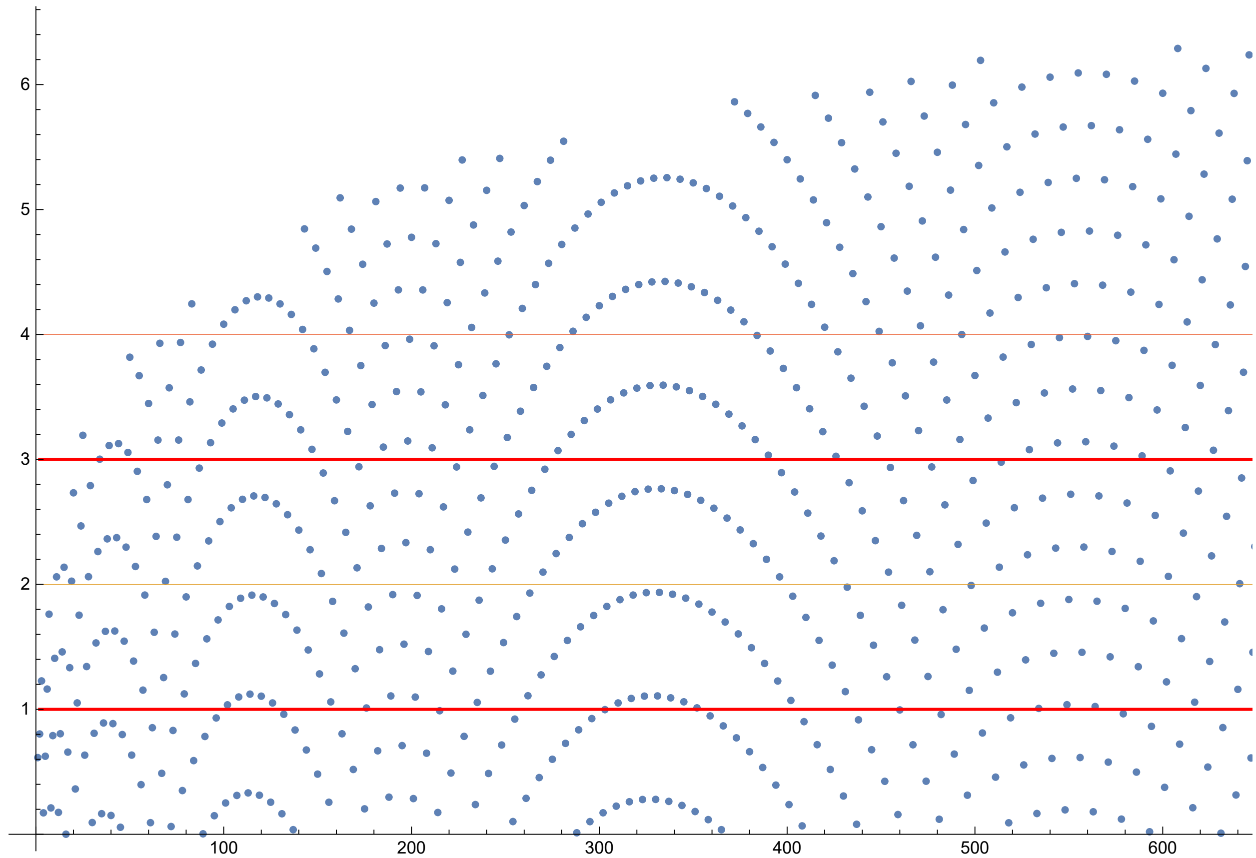
Note that although other patterns in number theory, many of them related to the logarithmic fn, appear to have many seemingly random properties, the log modulo here clearly displays a distinct and unique pattern which might go unnoticed in a line plot (as shown below)
Might this hint at greater structure? Namely Amongst the distribution of the primes and growth of prime counting fns, fns which are very closely related to the log?

At what x values do the poles (i.e. top of parabolas) occur?
And does this series of x values have any special property? Is it a known series? How is this series distributed?

```
ListLinePlot[Table[Mod[Floor[x], Log[Floor[x]]], {x, 2, 5000}]]
```




```
plotmax = 700;
Show[
 ListPlot[Table[Mod[Floor[x], Log[x]], {x, 2, plotmax}]],
 Plot[{1, 2, 3, 4}, {x, 2, plotmax}, PlotStyle → {Red, Thin}]
]
```




```
ListDensityPlot[Table[Mod[Floor[x], Log[x]], {x, 2, plotmax}]]
```

Can find the x values of these peaks by seing for what x values the elements of the set Table[Mod-[Floor[x],Log[x]],{x,2,plotmax}] start to decrease

```
jmax = 100;
A = Floor[Table[{j, Mod[Floor[j + 1], Log[j + 1]] - Mod[Floor[j], Log[j]]}, {j, 2, jmax}]];
PeakPts = Drop[Union[Table[If[A[[j, 2]] == 0, 0, j + 2], {j, 1, jmax - 1}]], 1]
 (*As we can see by comparison with A,
 this give the places at which (Mod[Floor[j],Log[j]] - Mod[Floor[j-1],Log[j-1]]) < 0,
 The drop function is used to remove 0 from this set  *)
```

```
{5, 9, 13, 17, 22, 27, 31, 36, 41, 46, 52, 57, 62, 68, 73, 79, 85, 90, 96}
```

```
jmax = 700;
A = Floor[Table[{j, Mod[Floor[j + 1], Log[j + 1]] - Mod[Floor[j], Log[j]]}, {j, 2, jmax}]];
PeakPts = Drop[Union[Table[If[A[[j, 2]] == 0, 0, j + 2], {j, 1, jmax - 1}]], 1]
```

```
{5, 9, 13, 17, 22, 27, 31, 36, 41, 46, 52, 57, 62, 68, 73, 79, 85, 90, 96, 102, 108,
 114, 120, 126, 132, 138, 145, 151, 157, 164, 170, 176, 183, 189, 196, 202, 209,
 215, 222, 229, 235, 242, 249, 255, 262, 269, 276, 283, 289, 296, 303, 310, 317, 324,
 331, 338, 345, 352, 359, 366, 374, 381, 388, 395, 402, 409, 417, 424, 431, 438, 446,
 453, 460, 468, 475, 482, 490, 497, 505, 512, 519, 527, 534, 542, 549, 557, 564, 572,
 579, 587, 594, 602, 610, 617, 625, 632, 640, 648, 655, 663, 671, 678, 686, 694}
```

This list gives vaues j for which
Mod[Floor[j], Log[j]] < Mod[Floor[j - 1], Log[j - 1]],
or (Mod[Floor[j], Log[j]] - Mod[Floor[j - 1], Log[j - 1]]) < 0
So it is between j and (j - 1) that we have the peaks
Confirm that j = 5 is one such number below

But actually need to narrow this list more

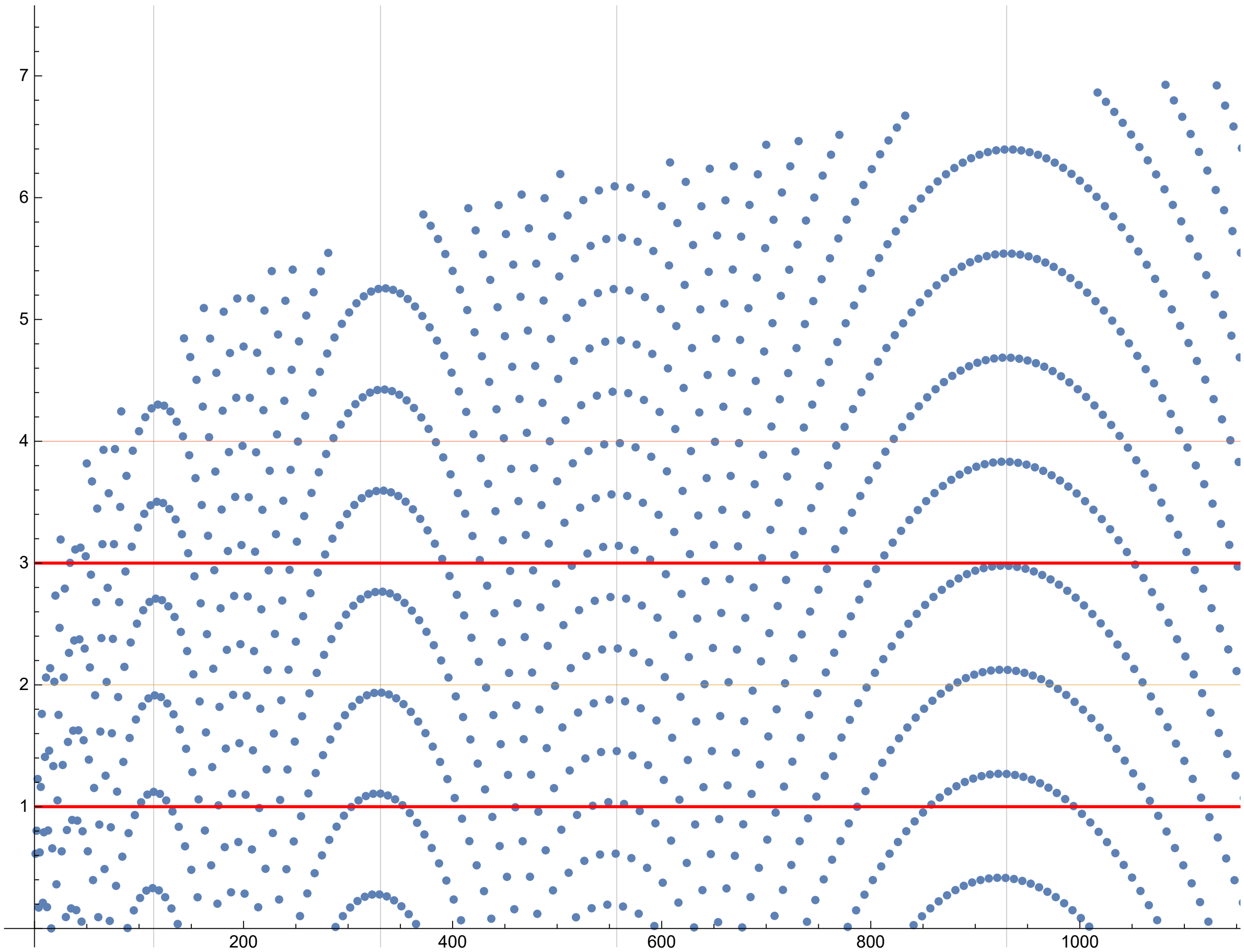
```
Mod[Floor[4], Log[4]] - Mod[Floor[3], Log[3]] // N
Mod[Floor[5], Log[5]] - Mod[Floor[4], Log[4]] // N
```

```
0.424636
```

```
-1.05573
```

```
plotmax = 1400;
A =
  Floor[Table[{j, Mod[Floor[j + 1], Log[j + 1]] - Mod[Floor[j], Log[j]]}, {j, 2, plotmax}]];
PeakPts = Drop[Union[Table[If[A[[j, 2]] == 0, 0, j + 2], {j, 1, plotmax - 1}]], 1]
 (*As we can see by comparison with A,
 this give the places at which (Mod[Floor[j],Log[j]] - Mod[Floor[j-1],Log[j-1]]) < 0,
 The drop function is used to remove 0 from this set  *)
{5, 9, 13, 17, 22, 27, 31, 36, 41, 46, 52, 57, 62, 68, 73, 79, 85, 90, 96, 102, 108, 114, 120, 126,
 132, 138, 145, 151, 157, 164, 170, 176, 183, 189, 196, 202, 209, 215, 222, 229, 235, 242,
 249, 255, 262, 269, 276, 283, 289, 296, 303, 310, 317, 324, 331, 338, 345, 352, 359, 366,
 374, 381, 388, 395, 402, 409, 417, 424, 431, 438, 446, 453, 460, 468, 475, 482, 490, 497,
 505, 512, 519, 527, 534, 542, 549, 557, 564, 572, 579, 587, 594, 602, 610, 617, 625, 632,
 640, 648, 655, 663, 671, 678, 686, 694, 702, 709, 717, 725, 733, 740, 748, 756, 764, 772,
 779, 787, 795, 803, 811, 819, 827, 835, 842, 850, 858, 866, 874, 882, 890, 898, 906, 914,
 922, 930, 938, 946, 954, 962, 970, 978, 986, 994, 1002, 1010, 1019, 1027, 1035, 1043,
 1051, 1059, 1067, 1075, 1084, 1092, 1100, 1108, 1116, 1124, 1133, 1141, 1149, 1157, 1165,
 1174, 1182, 1190, 1198, 1207, 1215, 1223, 1231, 1240, 1248, 1256, 1265, 1273, 1281,
 1290, 1298, 1306, 1315, 1323, 1331, 1340, 1348, 1356, 1365, 1373, 1382, 1390, 1398}
```

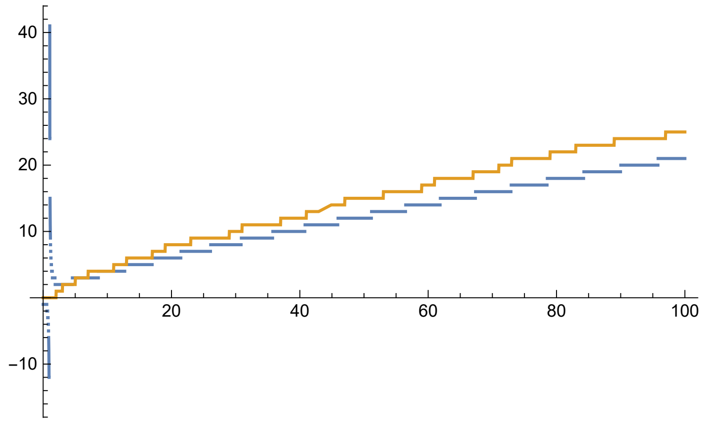
```
plotmax = 1400;
Show[
 ListPlot[Table[Mod[Floor[x], Log[x]], {x, 2, plotmax}],
  GridLines → {{114, 331, 557, 930}}],
 Plot[{1, 2, 3, 4}, {x, 2, plotmax}, PlotStyle → {Red, Thin}]
]
```



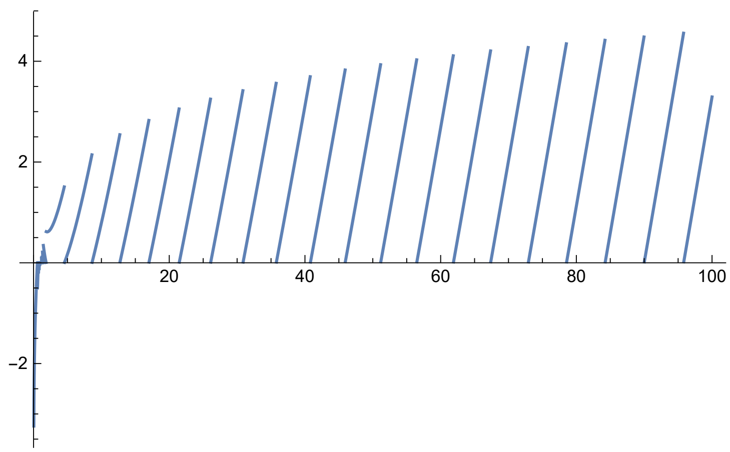
For a $\in$ Naturals,

$$\text{Mod}[a, \text{Log}[a]] = a - \left(\text{Floor}\left[\frac{a}{\text{Log}[a]}\right] \text{Log}[a]\right)$$

```
Plot[{(Floor[a/Log[a]]), PrimePi[a]}, {a, 0, 100}]
```



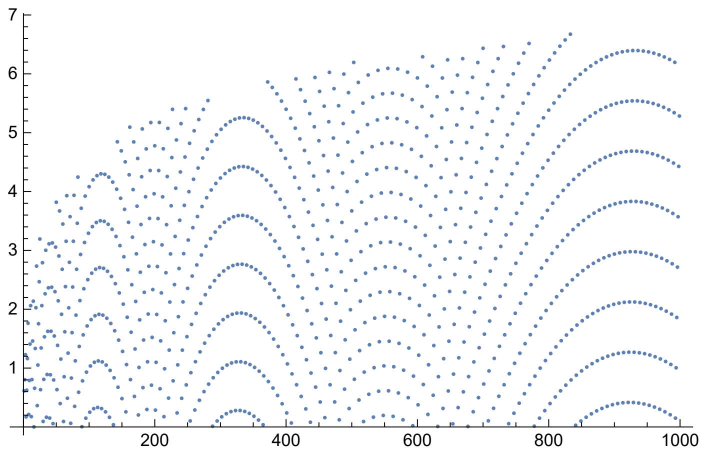

```
Plot[a - (Floor[a/Log[a]] Log[a]), {a, 0, 100}]
```



Means

Finding Peaks

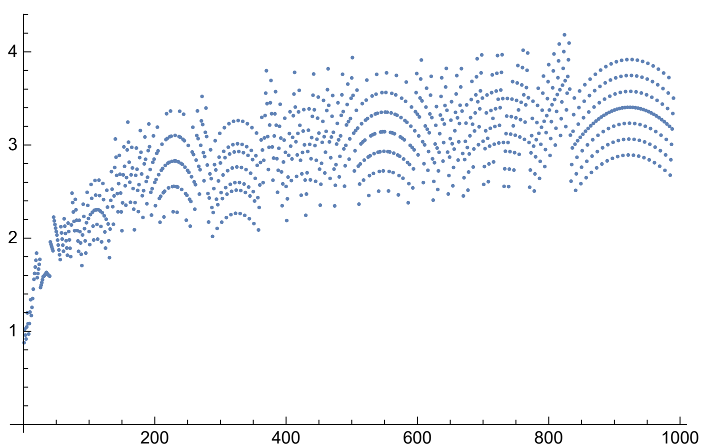
```
ListPlot[Table[Mod[Floor[x], Log[x]], {x, 2, 1000}]]
```




```
9 * Log[31] // N
```

30.9059

```
ListPlot[MovingAverage[Table[Mod[Floor[x], Log[Floor[x]]], {x, 2, 1000}], 10] // N]
```




```
Mean[Table[Mod[Floor[x], Log[Floor[x]]], {x, 200, 425}]] // N
```

2.81268

```
values = {2, 1, 3, 5, 6, 6, 4, 3, 2, 4, 7, 3, 2, 4, 2, 2, 1};
```

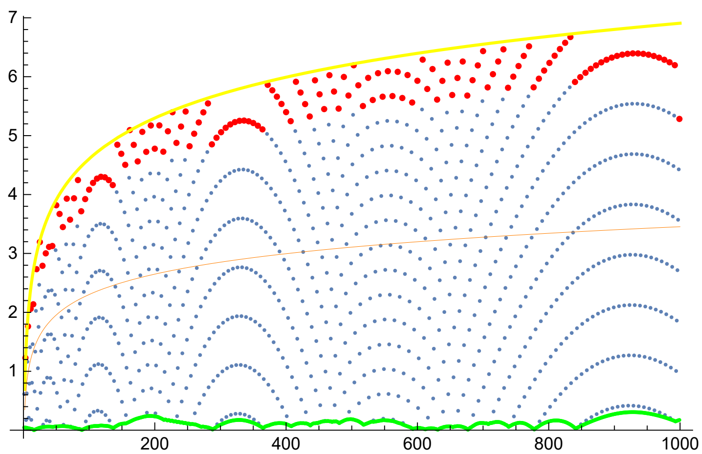
```
peaks = FindPeaks[values]
```

$\left\{\{1, 2\}, \left\{\frac{11}{2}, 6\right\}, \{11, 7\}, \{14, 4\}\right\}$

```
PEAKS = FindPeaks[Table[Mod[Floor[x], Log[x]], {x, 2, 1000}] // N]
```

```
{{3, 1.22741}, {7, 1.76168}, {11, 2.06037}, {15, 2.13706}, {20, 2.73287}, {25, 3.19332},
 {29, 2.79042}, {34, 3.00187}, {39, 3.11121}, {44, 3.12671}, {50, 3.81809}, {55, 3.67043},
 {60, 3.44777}, {66, 3.92961}, {71, 3.57334}, {77, 3.93595}, {83, 4.2453}, {88, 3.71591},
 {94, 3.92246}, {100, 4.08247}, {106, 4.19777}, {112, 4.27008}, {118, 4.30104},
 {124, 4.29216}, {130, 4.24487}, {136, 4.16052}, {143, 4.84523}, {149, 4.69158},
 {155, 4.50432}, {162, 5.09374}, {168, 4.84324}, {174, 4.56206}, {181, 5.06377},
 {187, 4.72453}, {194, 5.17202}, {200, 4.77772}, {207, 5.17355}, {213, 4.72694},
 {220, 5.07349}, {227, 5.39683}, {233, 4.87651}, {240, 5.15373}, {247, 5.40914},
 {253, 4.81996}, {260, 5.03206}, {267, 5.22361}, {274, 5.39499}, {281, 5.54655},
 {287, 4.85198}, {294, 4.96426}, {301, 5.0578}, {308, 5.13291}, {315, 5.18992},
 {322, 5.22912}, {329, 5.25081}, {336, 5.25527}, {343, 5.24278}, {350, 5.21361},
 {357, 5.16802}, {364, 5.10626}, {372, 5.86214}, {379, 5.76921}, {386, 5.66082},
 {393, 5.53719}, {400, 5.39855}, {407, 5.2451}, {415, 5.9134}, {422, 5.73132},
 {429, 5.53504}, {436, 5.32474}, {444, 5.93865}, {451, 5.7012}, {458, 5.45028},
 {466, 6.02531}, {473, 5.74824}, {480, 5.45822}, {488, 5.99573}, {495, 5.6805},
 {503, 6.1939}, {510, 5.85406}, {517, 5.50203}, {525, 5.98}, {532, 5.6042},
 {540, 6.05936}, {547, 5.66033}, {555, 6.09316}, {562, 5.67139}, {570, 6.08236},
 {577, 5.63835}, {585, 6.0279}, {592, 5.56212}, {600, 5.93067}, {608, 6.28908},
 {615, 5.79154}, {623, 6.12956}, {630, 5.61133}, {638, 5.92936}, {646, 6.23772},
 {653, 5.68926}, {661, 5.97818}, {669, 6.25767}, {676, 5.67986}, {684, 5.94044},
 {692, 6.19185}, {700, 6.43416}, {707, 5.81848}, {715, 6.04254}, {723, 6.25774},
 {731, 6.46414}, {738, 5.81193}, {746, 6.0007}, {754, 6.18089}, {762, 6.35258},
 {770, 6.51584}, {777, 5.81972}, {785, 5.96606}, {793, 6.10415}, {801, 6.23408},
 {809, 6.35589}, {817, 6.46966}, {825, 6.57544}, {833, 6.67329}, {840, 5.91063},
 {848, 5.9926}, {856, 6.06682}, {864, 6.13335}, {872, 6.19225}, {880, 6.24357},
 {888, 6.28736}, {896, 6.32368}, {904, 6.35259}, {912, 6.37413}, {920, 6.38836},
 {928, 6.39532}, {936, 6.39507}, {944, 6.38766}, {952, 6.37314}, {960, 6.35156},
 {968, 6.32295}, {976, 6.28738}, {984, 6.24489}, {992, 6.19552}, {999, 5.28324}}
```

```
xmax = 1000;
PEAKS = FindPeaks[Table[Mod[Floor[x], Log[x]], {x, 2, xmax}] // N];
Show[
 ListPlot[Table[Mod[Floor[x], Log[x]], {x, 2, xmax}]],
 ListPlot[PEAKS, PlotStyle → {Red}],
 ListPlot[EstimatedBackground[Table[Mod[Floor[x], Log[x]], {x, 2, xmax}] // N],
  PlotStyle → {Green}],
 Plot[{Log[x], 1/2 Log[x]}, {x, 2, xmax},
  PlotStyle → {{Yellow}, {Orange, Thin}, {Magenta, Thin}}]
]
```




```
EstimatedBackground[Table[Mod[Floor[x], Log[x]], {x, 2, 1000}] // N];
FindPeaks[EstimatedBackground[Table[Mod[Floor[x], Log[x]], {x, 2, 1000}] // N]];
```

```
Mod[5, 4]
```

1

$$\text{Floor}\left[\frac{7}{\text{Log}[7]}\right] \text{ // N}$$

$$\text{Floor}\left[\frac{8}{\text{Log}[8]}\right] \text{ // N}$$

$$\text{Floor}\left[\frac{9}{\text{Log}[9]}\right] \text{ // N}$$

3.

3.

4.

```
Table[Mod[Floor[x], Log[x]], {x, 2, 40}]
```

```
{2 - 2 Log[2], 3 - 2 Log[3], 4 - 2 Log[4], 5 - 3 Log[5], 6 - 3 Log[6], 7 - 3 Log[7],
 8 - 3 Log[8], 9 - 4 Log[9], 10 - 4 Log[10], 11 - 4 Log[11], 12 - 4 Log[12], 13 - 5 Log[13],
 14 - 5 Log[14], 15 - 5 Log[15], 16 - 5 Log[16], 17 - 6 Log[17], 18 - 6 Log[18], 19 - 6 Log[19],
 20 - 6 Log[20], 21 - 6 Log[21], 22 - 7 Log[22], 23 - 7 Log[23], 24 - 7 Log[24], 25 - 7 Log[25],
 26 - 7 Log[26], 27 - 8 Log[27], 28 - 8 Log[28], 29 - 8 Log[29], 30 - 8 Log[30],
 31 - 9 Log[31], 32 - 9 Log[32], 33 - 9 Log[33], 34 - 9 Log[34], 35 - 9 Log[35],
 36 - 10 Log[36], 37 - 10 Log[37], 38 - 10 Log[38], 39 - 10 Log[39], 40 - 10 Log[40]}
```

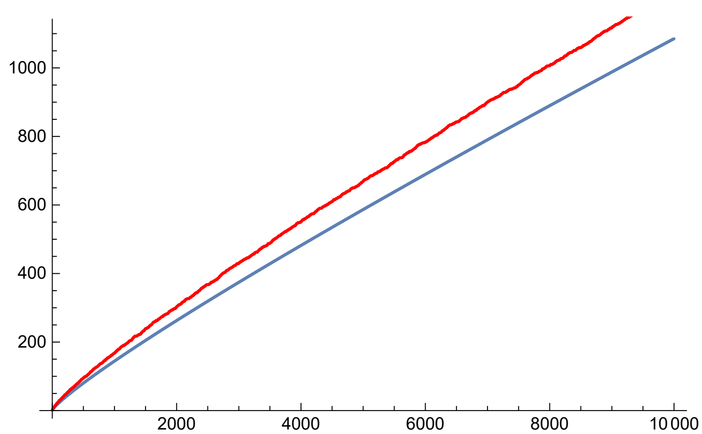
$$\text{Table}\left[\frac{-(\text{Mod}[\text{Floor}[x], \text{Log}[x]] - x)}{\text{Log}[x]}, \{x, 2, 100\}\right]$$

```
{2, 2, 2, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 6, 6, 6, 6, 6, 7, 7, 7, 7, 7, 8, 8, 8, 8, 9, 9, 9,
 9, 9, 10, 10, 10, 10, 10, 11, 11, 11, 11, 11, 12, 12, 12, 12, 12, 12, 13, 13, 13, 13, 13,
 14, 14, 14, 14, 14, 15, 15, 15, 15, 15, 15, 16, 16, 16, 16, 16, 17, 17, 17, 17, 17, 17,
 18, 18, 18, 18, 18, 18, 19, 19, 19, 19, 19, 20, 20, 20, 20, 20, 20, 21, 21, 21, 21, 21}
```

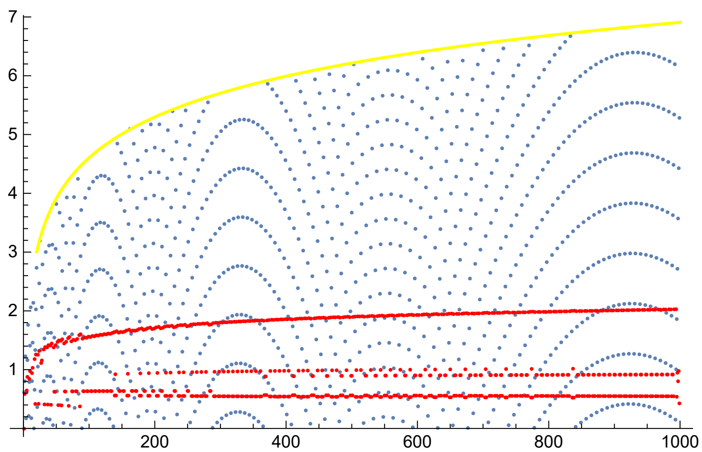
$$\text{Union}\left[\text{Table}\left[\frac{-(\text{Mod}[\text{Floor}[x], \text{Log}[x]] - x)}{\text{Log}[x]}, \{x, 2, 1000\}\right]\right]$$

```
{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25,
 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47,
 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69,
 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91,
 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110,
 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127,
 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144}
```

```
XMAX = 10 000;
Show[
 ListPlot[Table[-(Mod[Floor[x], Log[x]] - x)/Log[x], {x, 2, XMAX}]],
 ListPlot[Table[PrimePi[x], {x, 2, XMAX}], PlotStyle → {Red}]
]
```



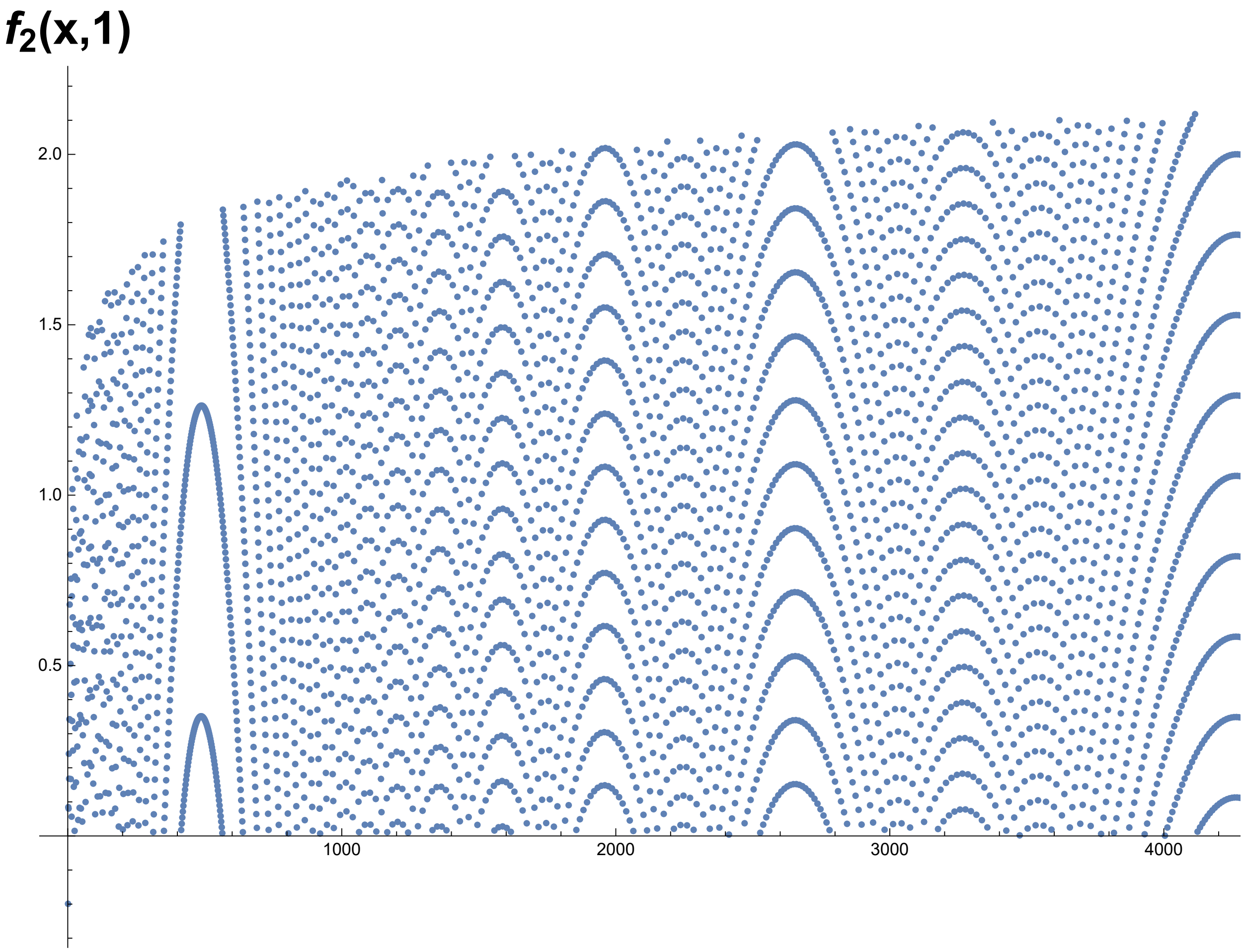
```
xmax = 1000;
PEAKS = FindPeaks[Table[Mod[Floor[x], Log[x]], {x, 2, xmax}] // N];
Show[
 ListPlot[Table[Mod[Floor[x], Log[x]], {x, 2, xmax}]],
 ListPlot[DerivativeFilter[Table[Mod[Floor[x], Log[x]], {x, 2, xmax}] // N, {1}],
  PlotStyle → {Red}],
 Plot[{Log[x]}, {x, 2, xmax}, PlotStyle → {{Yellow}}]
]
```



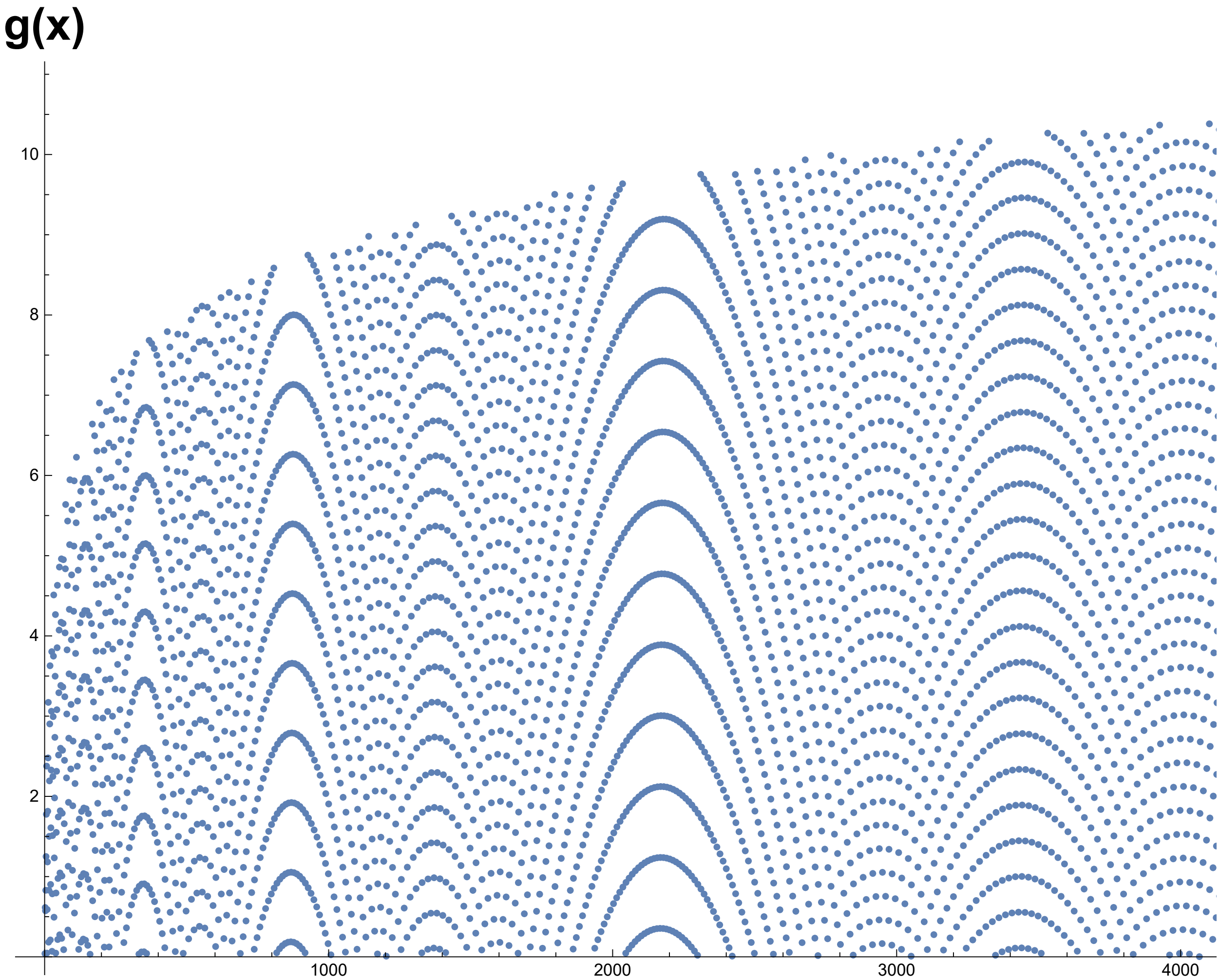
Misc 1

Logarithmic Variants and Dushert's ineq relation

```
ListPlot[Table[Mod[x, Log[Log[x]]], {x, 2, 5000}], AxesLabel →
  {Style["x", Large, Bold], Style["f₂(x,1)", Large, Bold]}] (*Log Log version *)
```



```
ListPlot[Table[Mod[Floor[x], (Log[x] + Log[Log[x]]) ], {x, 2, 5000}],
 AxesLabel → {Style["x", Large, Bold], Style["g(x)", Large, Bold]}]
(* (Log[x]+ Log[Log[x]]) *)
```



Note that in the above plot a spectral pattern using a (Log[x] + Log[Log[x]]) modulo
This function is the upper bound of Dushert ' s inequality
Prime[x] < x Log[x] + x Log[Log[x]], for x > 6, which gives us

$$\frac{\text{Prime}[x]}{x} < \text{Log}[x] + \text{Log}[\text{Log}[x]]$$

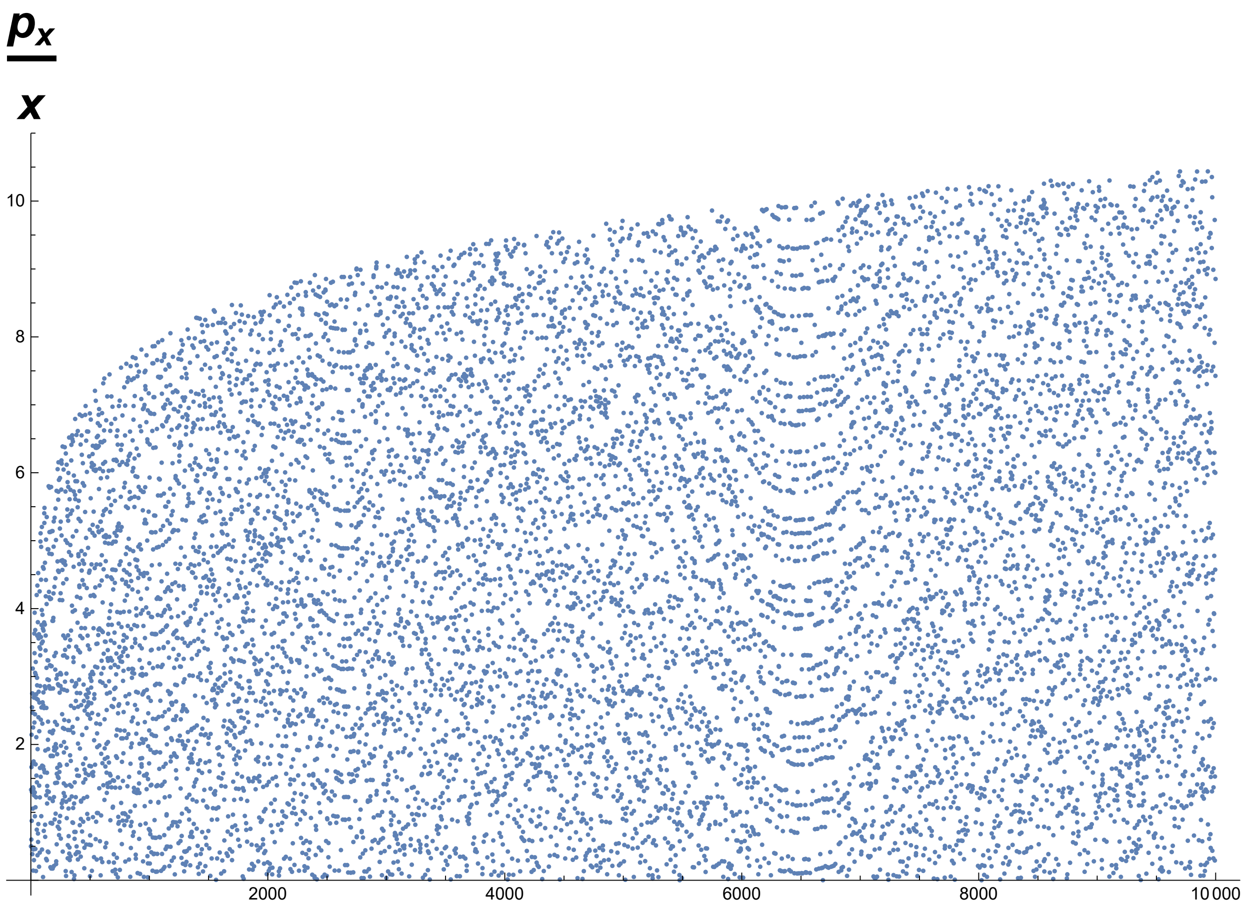
The fact that this bound has a spectral pattern may signal something abouth the primes

Indeed we can see some bands in the $\left(\frac{\text{Prime}[x]}{x}\right)$ modulo which might
we might not have seen if it were ' n ' t for the logarithmic banding

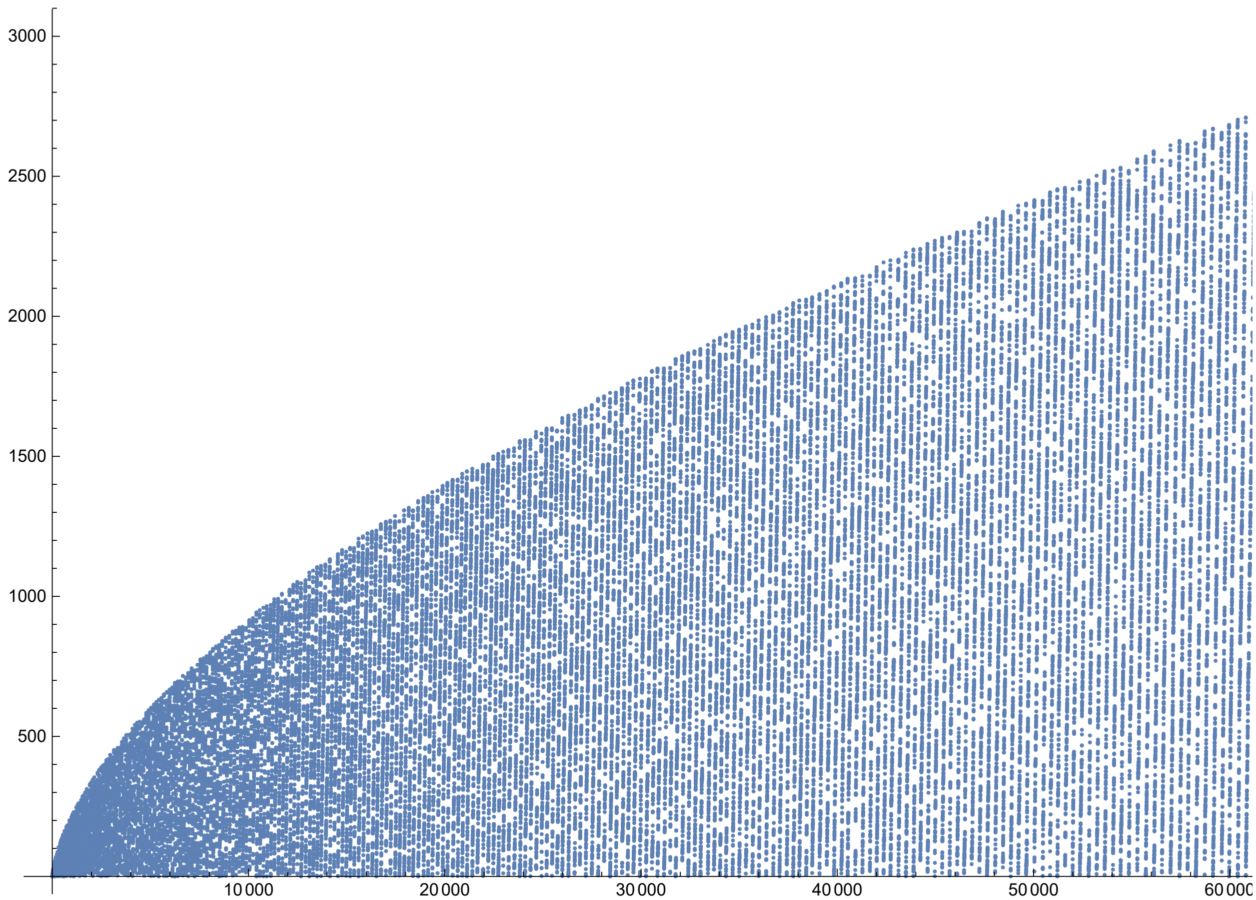
Banding in $\left(\frac{\text{Prime}[x]}{x}\right)$ modulo is far more apparent for large xmax

(~70 000) whereas it appears relatively random for low xmax (~1000)

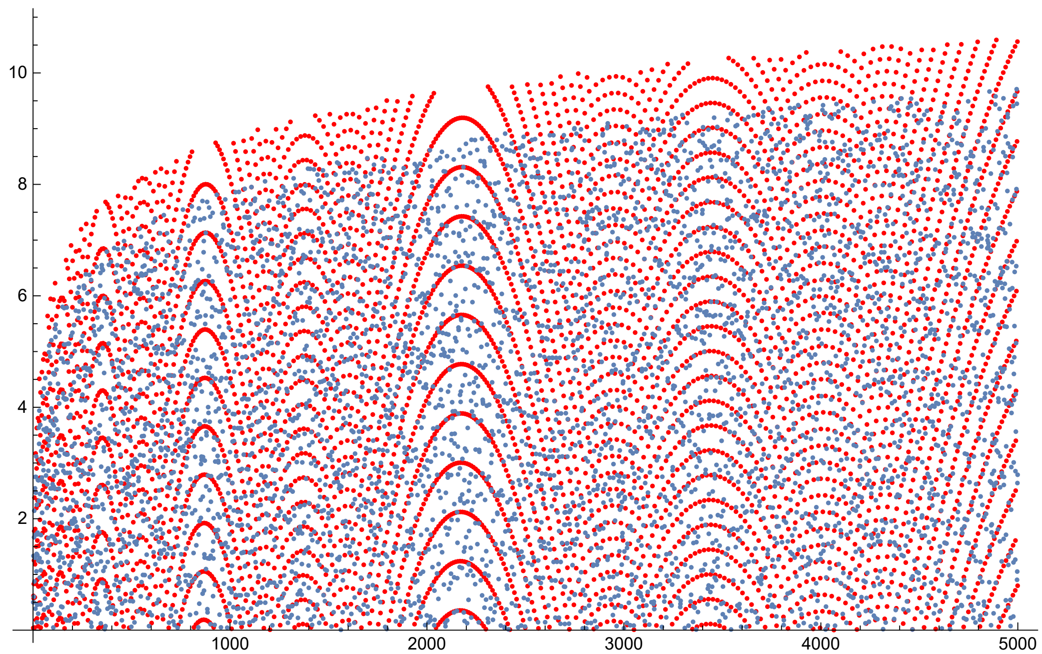
```
xmax = 10 000;
ListPlot[Table[Mod[Floor[x], (Prime[x]/x)], {x, 2, xmax}],
 AxesLabel → {Style["x", Large, Bold], Style["p_x/x", Large, Bold]}]
```



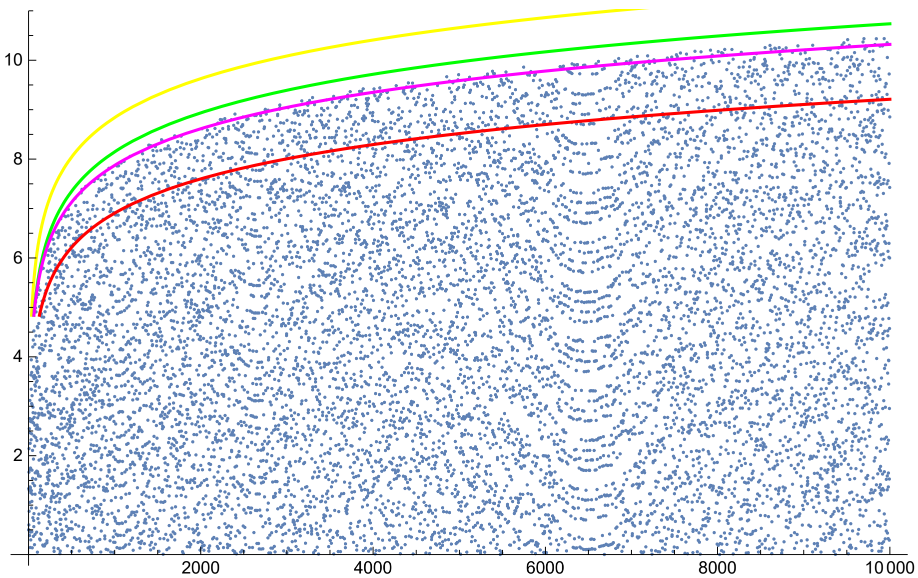
```
ListPlot[Table[Mod[Prime[x], √x Log[x]], {x, 2, 70000}]]
```



```
Show[
 ListPlot[Table[Mod[Floor[x], (Log[x] + Log[Log[x]]) ], {x, 2, 5000}], PlotStyle → Red] ,
 ListPlot[Table[Mod[Floor[x], (Prime[x]/x) ], {x, 2, 5000}]]
]
```



```
xmax = 10 000;
Show[
 ListPlot[Table[Mod[Floor[x], (Prime[x]/x)], {x, 2, xmax}]],
 Plot[
  {Log[x], (Log[x] + Log[Log[x]]), (Log[x] + Log[Log[√x]]), (Log[x] + 1/2 Log[Log[x]])},
  {x, 2, xmax}, PlotStyle → {{Red}, {Yellow}, {Green}, {Magenta}}]
]
```




```
Table[Mod[Floor[x], (Log[x] + Log[Log[x]])], {x, 2, 100}]
```

{2 - 6 (Log[2] + Log[Log[2]]), 3 - 2 (Log[3] + Log[Log[3]]), 4 - 2 (Log[4] + Log[Log[4]]),
 5 - 2 (Log[5] + Log[Log[5]]), 6 - 2 (Log[6] + Log[Log[6]]), 7 - 2 (Log[7] + Log[Log[7]]),
 8 - 2 (Log[8] + Log[Log[8]]), 9 - 3 (Log[9] + Log[Log[9]]), 10 - 3 (Log[10] + Log[Log[10]]),
 11 - 3 (Log[11] + Log[Log[11]]), 12 - 3 (Log[12] + Log[Log[12]]),
 13 - 3 (Log[13] + Log[Log[13]]), 14 - 3 (Log[14] + Log[Log[14]]),
 15 - 4 (Log[15] + Log[Log[15]]), 16 - 4 (Log[16] + Log[Log[16]]),
 17 - 4 (Log[17] + Log[Log[17]]), 18 - 4 (Log[18] + Log[Log[18]]),
 19 - 4 (Log[19] + Log[Log[19]]), 20 - 4 (Log[20] + Log[Log[20]]),
 21 - 5 (Log[21] + Log[Log[21]]), 22 - 5 (Log[22] + Log[Log[22]]),
 23 - 5 (Log[23] + Log[Log[23]]), 24 - 5 (Log[24] + Log[Log[24]]),
 25 - 5 (Log[25] + Log[Log[25]]), 26 - 5 (Log[26] + Log[Log[26]]),
 27 - 6 (Log[27] + Log[Log[27]]), 28 - 6 (Log[28] + Log[Log[28]]),
 29 - 6 (Log[29] + Log[Log[29]]), 30 - 6 (Log[30] + Log[Log[30]]),
 31 - 6 (Log[31] + Log[Log[31]]), 32 - 6 (Log[32] + Log[Log[32]]),
 33 - 6 (Log[33] + Log[Log[33]]), 34 - 7 (Log[34] + Log[Log[34]]),
 35 - 7 (Log[35] + Log[Log[35]]), 36 - 7 (Log[36] + Log[Log[36]]),
 37 - 7 (Log[37] + Log[Log[37]]), 38 - 7 (Log[38] + Log[Log[38]]),
 39 - 7 (Log[39] + Log[Log[39]]), 40 - 8 (Log[40] + Log[Log[40]]),
 41 - 8 (Log[41] + Log[Log[41]]), 42 - 8 (Log[42] + Log[Log[42]]),
 43 - 8 (Log[43] + Log[Log[43]]), 44 - 8 (Log[44] + Log[Log[44]]),

```
45 - 8 (Log[45] + Log[Log[45]]), 46 - 8 (Log[46] + Log[Log[46]]),
47 - 9 (Log[47] + Log[Log[47]]), 48 - 9 (Log[48] + Log[Log[48]]),
49 - 9 (Log[49] + Log[Log[49]]), 50 - 9 (Log[50] + Log[Log[50]]),
51 - 9 (Log[51] + Log[Log[51]]), 52 - 9 (Log[52] + Log[Log[52]]),
53 - 9 (Log[53] + Log[Log[53]]), 54 - 10 (Log[54] + Log[Log[54]]),
55 - 10 (Log[55] + Log[Log[55]]), 56 - 10 (Log[56] + Log[Log[56]]),
57 - 10 (Log[57] + Log[Log[57]]), 58 - 10 (Log[58] + Log[Log[58]]),
59 - 10 (Log[59] + Log[Log[59]]), 60 - 10 (Log[60] + Log[Log[60]]),
61 - 11 (Log[61] + Log[Log[61]]), 62 - 11 (Log[62] + Log[Log[62]]),
63 - 11 (Log[63] + Log[Log[63]]), 64 - 11 (Log[64] + Log[Log[64]]),
65 - 11 (Log[65] + Log[Log[65]]), 66 - 11 (Log[66] + Log[Log[66]]),
67 - 11 (Log[67] + Log[Log[67]]), 68 - 12 (Log[68] + Log[Log[68]]),
69 - 12 (Log[69] + Log[Log[69]]), 70 - 12 (Log[70] + Log[Log[70]]),
71 - 12 (Log[71] + Log[Log[71]]), 72 - 12 (Log[72] + Log[Log[72]]),
73 - 12 (Log[73] + Log[Log[73]]), 74 - 12 (Log[74] + Log[Log[74]]),
75 - 12 (Log[75] + Log[Log[75]]), 76 - 13 (Log[76] + Log[Log[76]]),
77 - 13 (Log[77] + Log[Log[77]]), 78 - 13 (Log[78] + Log[Log[78]]),
79 - 13 (Log[79] + Log[Log[79]]), 80 - 13 (Log[80] + Log[Log[80]]),
81 - 13 (Log[81] + Log[Log[81]]), 82 - 13 (Log[82] + Log[Log[82]]),
83 - 14 (Log[83] + Log[Log[83]]), 84 - 14 (Log[84] + Log[Log[84]]),
85 - 14 (Log[85] + Log[Log[85]]), 86 - 14 (Log[86] + Log[Log[86]]),
87 - 14 (Log[87] + Log[Log[87]]), 88 - 14 (Log[88] + Log[Log[88]]),
89 - 14 (Log[89] + Log[Log[89]]), 90 - 14 (Log[90] + Log[Log[90]]),
91 - 15 (Log[91] + Log[Log[91]]), 92 - 15 (Log[92] + Log[Log[92]]),
93 - 15 (Log[93] + Log[Log[93]]), 94 - 15 (Log[94] + Log[Log[94]]),
95 - 15 (Log[95] + Log[Log[95]]), 96 - 15 (Log[96] + Log[Log[96]]),
97 - 15 (Log[97] + Log[Log[97]]), 98 - 16 (Log[98] + Log[Log[98]]),
99 - 16 (Log[99] + Log[Log[99]]), 100 - 16 (Log[100] + Log[Log[100]])}
```

LogIntegral Variants

Misc. Variants

Comparison To Riemann Zeroes

Sequence of integers nearest to Riemann Zeta zeros is located at: http://oeis.org/A002410

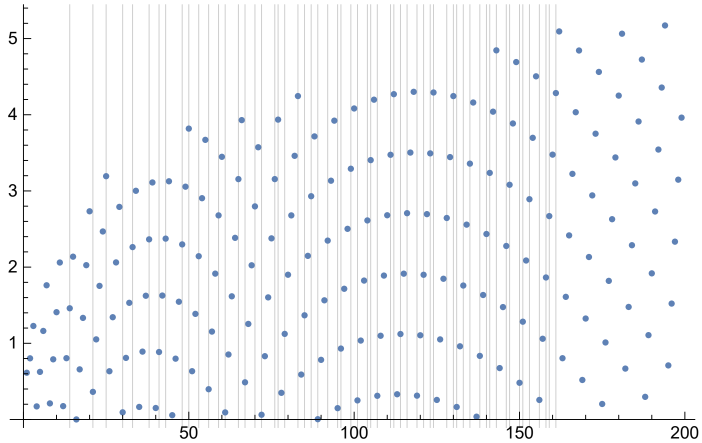
```
RieZetaZeros = Table[Round[Im[ZetaZero[n]]], {n, 59}]
```

```
{14, 21, 25, 30, 33, 38, 41, 43, 48, 50, 53, 56, 59, 61, 65, 67, 70, 72, 76, 77, 79, 83, 85,
 87, 89, 92, 95, 96, 99, 101, 104, 105, 107, 111, 112, 114, 116, 119, 121, 123, 124, 128,
 130, 131, 133, 135, 138, 140, 141, 143, 146, 147, 150, 151, 153, 156, 158, 159, 161}
```

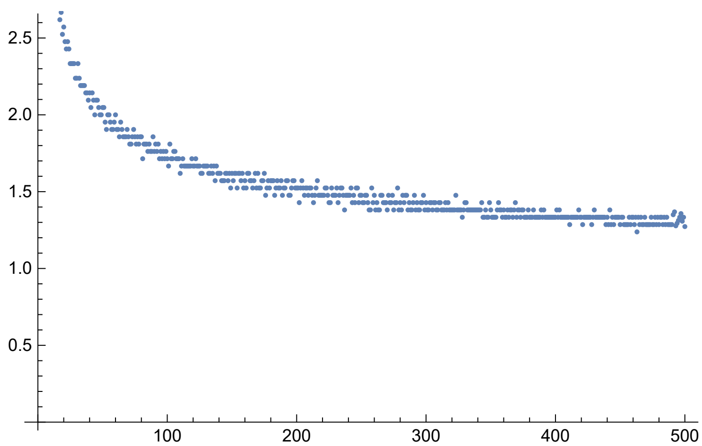
Also given exactly (but not nec. in real integer form) by the function ZetaZero

```
ZetaZero
```

```
ListPlot[Table[Mod[Floor[x], Log[x]], {x, 2, 200}], GridLines → {RieZetaZeros}]
```

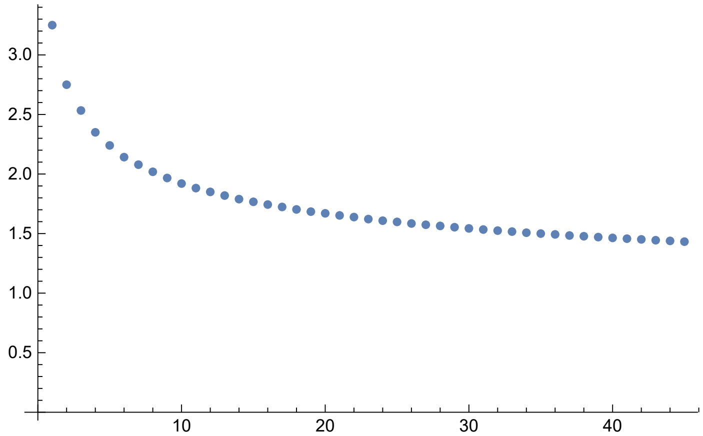



```
ListPlot[
 MeanFilter[Table[Round[Im[ZetaZero[n + 1]]] - Round[Im[ZetaZero[n]]], {n, 500}], 10]]
```



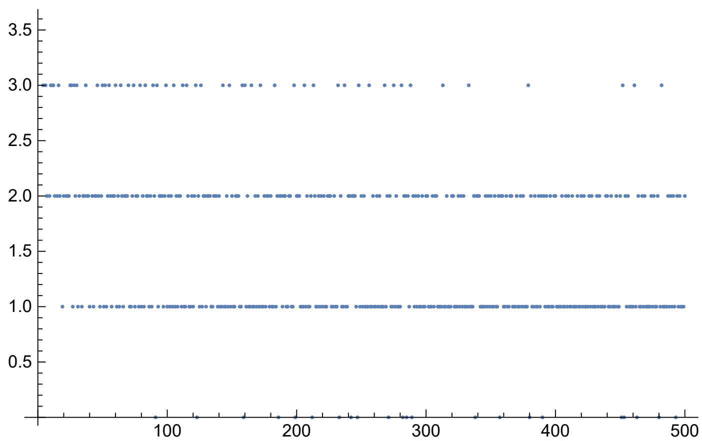
```
Table[Mean[Table[Round[Im[ZetaZero[n + 1]]] - Round[Im[ZetaZero[n]]], {n, 20 k}]],
  {k, 1, 45}] // N
ListPlot[Table[Mean[Table[Round[Im[ZetaZero[n + 1]]] - Round[Im[ZetaZero[n]]], {n, 20 k}]],
   {k, 1, 45}] // N]
```

```
{3.25, 2.75, 2.53333, 2.35, 2.24, 2.14167, 2.07857, 2.01875, 1.96667, 1.92,
 1.88182, 1.85, 1.81923, 1.78929, 1.76667, 1.74375, 1.72353, 1.70278, 1.68421,
 1.67, 1.65238, 1.63864, 1.62174, 1.60833, 1.598, 1.58462, 1.57407, 1.56429,
 1.55345, 1.54333, 1.53387, 1.525, 1.51667, 1.50735, 1.5, 1.49306, 1.48378,
 1.47763, 1.47051, 1.46375, 1.45732, 1.45119, 1.44419, 1.43864, 1.43222}
```

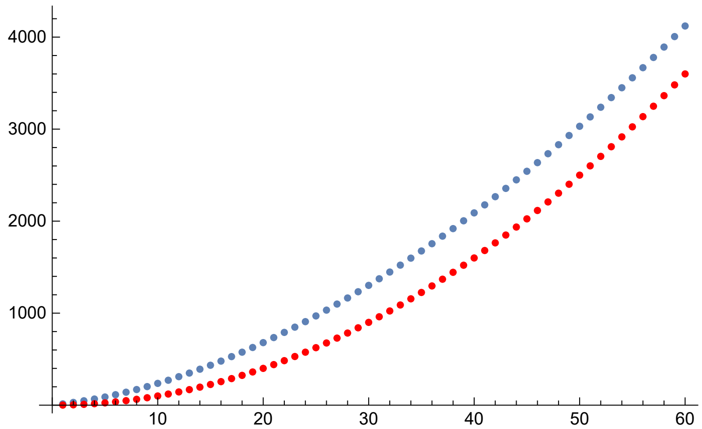



```
1.73245
```

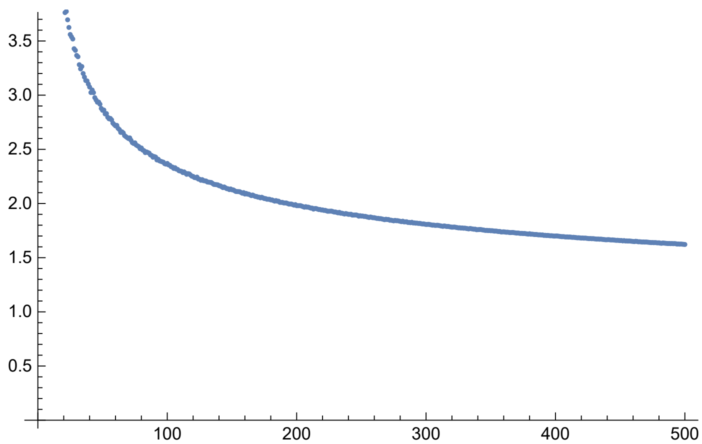
```
ListPlot[Table[Round[Im[ZetaZero[n + 1]]] - Round[Im[ZetaZero[n]]], {n, 500}]]
```



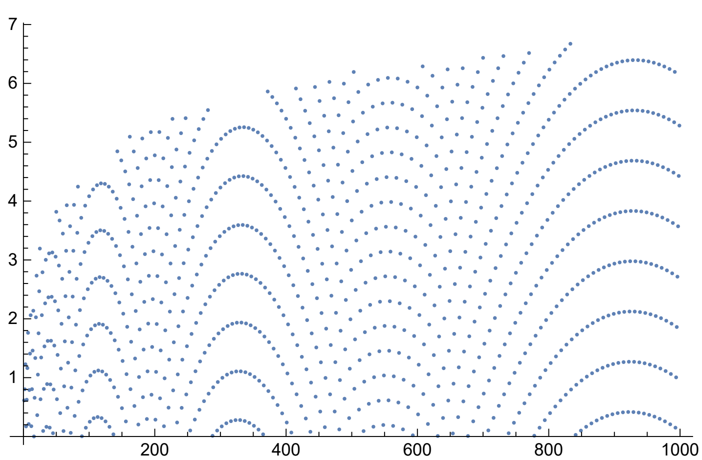
```
Show[
 ListPlot[Table[Round[Im[ZetaZero[n^2]]], {n, 100}]],
 ListPlot[Table[n^2, {n, 60}], PlotStyle → Red]
]
```



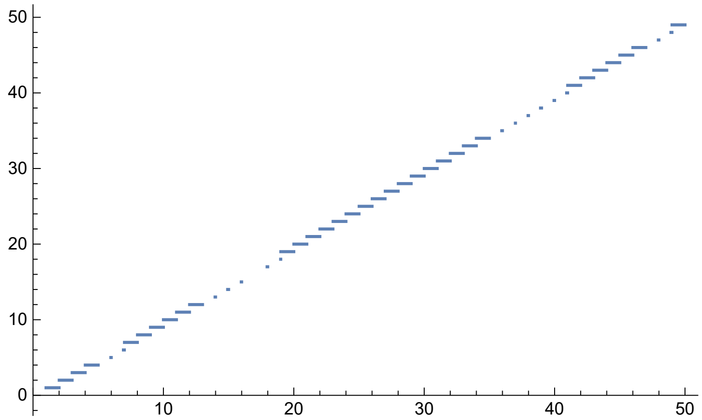
```
ListPlot[Table[(Round[Im[ZetaZero[n]]] / n), {n, 500}]]
```



```
ListPlot[Table[Mod[Floor[x], Log[x]], {x, 2, 1000}]]
```

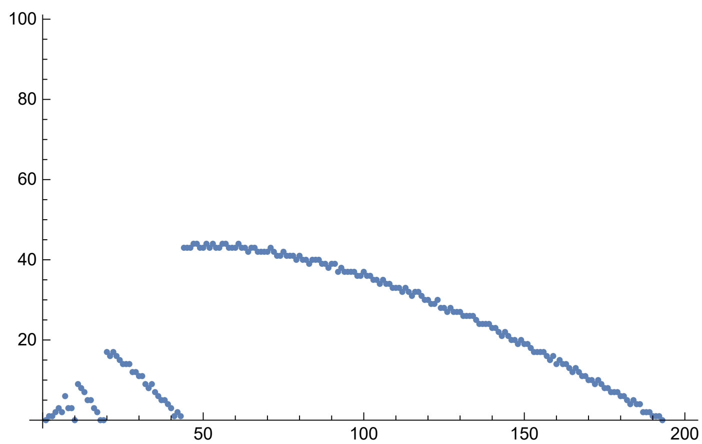


```
Plot[Mod[Floor[x], Round[Im[ZetaZero[Floor[x]]]]], {x, 1, 50}]
```

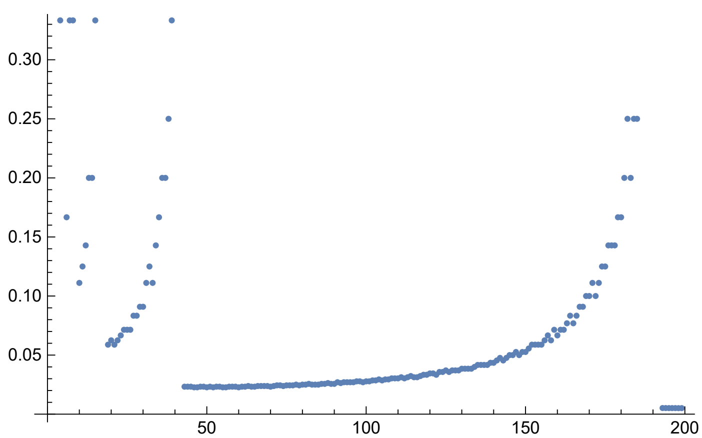


```
Mod[Floor[x], Round[Im[ZetaZero[Floor[x]]]]]
```

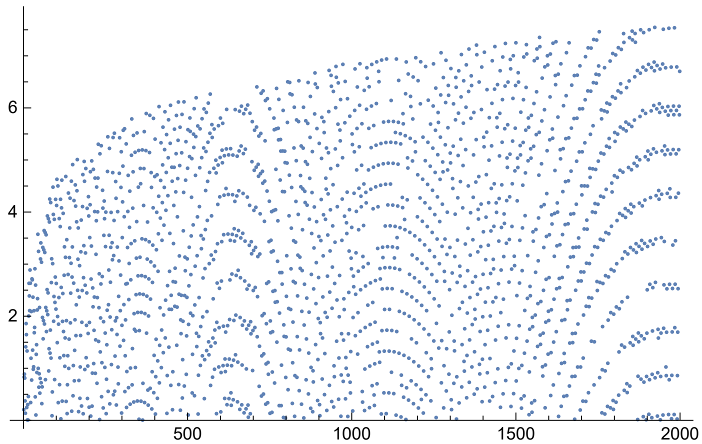
```
ListPlot[Table[Mod[Round[Im[ZetaZero[x]]], x], {x, 200}]]
```



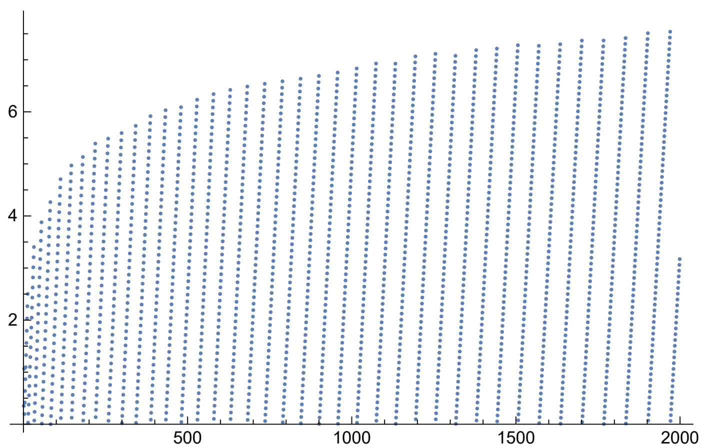
```
ListPlot[Table[1/Mod[Round[Im[ZetaZero[x]]], x], {x, 2, 200}]]
```



```
ListPlot[Table[Mod[Round[Im[ZetaZero[x]]], Log[x]], {x, 2, 2000}]]
```




```
ListPlot[Table[Mod[LogIntegral[x], Log[x]], {x, 2, 2000}]]
```

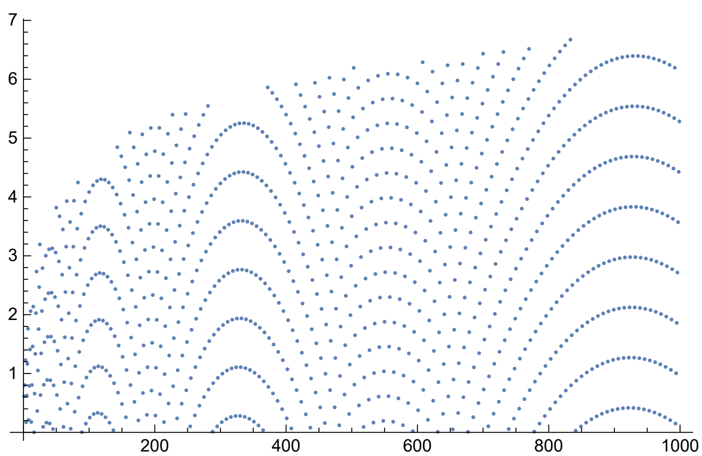


```
Table[Mod[Round[Im[ZetaZero[x]]], Log[x]], {x, 2, 200}] // N
```

```
{0.205585, 0.83053, 0.887818, 0.811242, 0.373051, 0.135887, 1.41117, 1.85828, 1.64571,
 0.246304, 1.33205, 0.00616478, 0.301681, 0.00679517, 0.457871, 2.00288, 2.63108,
 2.38903, 2.10669, 2.88694, 2.6329, 0.341656, 1.19255, 2.09035, 0.773297, 2.71657,
 2.69827, 1.34842, 2.36528, 0.980384, 1.02792, 2.10477, 1.68282, 1.78421, 2.91091,
 0.450627, 2.59724, 0.102466, 1.26698, 1.45212, 0.919233, 2.1192, 2.33755, 3.57348,
 0.997551, 3.24483, 0.636764, 0.894469, 2.16717, 0.522452, 0.803982, 3.0992, 3.40759,
 0.721339, 3.03664, 0.321001, 0.642723, 1.97604, 3.32056, 1.56505, 1.91462, 3.27461,
 3.64468, 1.85012, 3.22416, 3.6076, 0.780676, 2.16753, 3.5632, 1.70477, 2.10336, 2.51024,
 3.9252, 2.03052, 2.44773, 3.87256, 0.948103, 1.37485, 3.8088, 4.24979, 1.29091,
 1.73333, 4.18243, 1.19539, 2.64568, 3.10232, 0.0878329, 0.545454, 3.00914, 4.47874,
 3.95415, 1.90262, 2.37856, 3.86003, 0.78259, 2.26445, 2.75163, 4.24401, 2.13632,
 2.62885, 4.12639, 4.62882, 1.49167, 1.99406, 4.50117, 0.340072, 1.84704, 2.35856,
 3.87454, 0.685369, 1.20106, 3.72106, 4.24528, 0.0287329, 2.55254, 4.08043, 4.61235,
 0.369084, 1.90046, 2.43573, 3.97482, 1.70549, 1.24395, 2.78612, 3.33193, 1.03715,
 1.58224, 3.13088, 4.683, 0.363358, 1.91469, 3.4694, 0.129612, 0.683513, 1.24071,
 2.80114, 4.36478, 0.99709, 1.55981, 3.12565, 3.69455, 4.26648, 1.87158, 2.44251,
 3.01639, 4.59318, 0.175627, 2.75138, 3.32998, 4.91137, 0.471647, 1.05197, 2.63503,
 4.22079, 0.75936, 1.34402, 1.93132, 4.52123, 4.1137, 1.62731, 2.21864, 3.81249, 4.40882,
 5.0076, 2.49682, 3.09441, 3.69441, 0.166887, 0.765706, 2.36688, 2.97038, 0.42288,
 1.02518, 1.62977, 3.2366, 3.84567, 0.275151, 1.88299, 2.49302, 4.10521, 4.71953,
 0.126483, 2.73956, 3.35474, 4.97199, 4.59129, 0.976179, 2.59421, 3.21427, 4.83631,
 1.20284, 1.82362, 2.44635, 4.07103, 0.419515, 1.04292, 1.66824, 4.29544, 3.92451}
```

Entropy

```
ListPlot[Table[Mod[Floor[x], Log[x]], {x, 2, 1000}]]
```




```
Entropy[Table[Mod[Floor[x], Log[x]], {x, 2, 10000}]] // N
Entropy[Table[Round[Mod[Floor[x], Log[x]]], {x, 2, 10000}]] // N
```

```
9.21024
```

```
2.23358
```

The above shows that rounding the values greatly reduces the en

```
Entropy[Table[Prime[x], {x, 2, 10000}]]
```

```
Log[9999]
```

```
Entropy[RandomInteger[1, 10000]] // N
Log[2] // N
(* Tend to Log[2] since it's a random list of zeros and ones (2 digits ) *)
```

```
0.693147
```

```
0.693147
```

```
Entropy[{0, 1, 2, 3}] // N
Entropy[{0, 1, 2, 2}] // N
Entropy[{0, 1, 1, 1}] // N
Entropy[{1, 1, 1, 1}]
```

```
1.38629
```

```
1.03972
```

```
0.562335
```

```
0
```

```
xmax = 10000;
Entropy[Table[-(Mod[Floor[x], Log[x]] - x)/Log[x], {x, 2, xmax}]] // N
Entropy[Table[PrimePi[x], {x, 2, xmax}]] // N
```

```
6.97982
```

```
6.88431
```

^ These two have roughly the same entropies, which is interesting

```
Table[((Entropy[Table[-(Mod[Floor[x], Log[x]] - x)/Log[x], {x, 2, k}]]) -
   (Entropy[Table[PrimePi[x], {x, 2, k}]])), {k, 2, 10002, 1000}] // N
```

```
{0., -0.00632258, 0.0426423, 0.0522947, 0.0691163,
 0.0783841, 0.0903942, 0.0871925, 0.0957175, 0.0981665, 0.0957306}
```

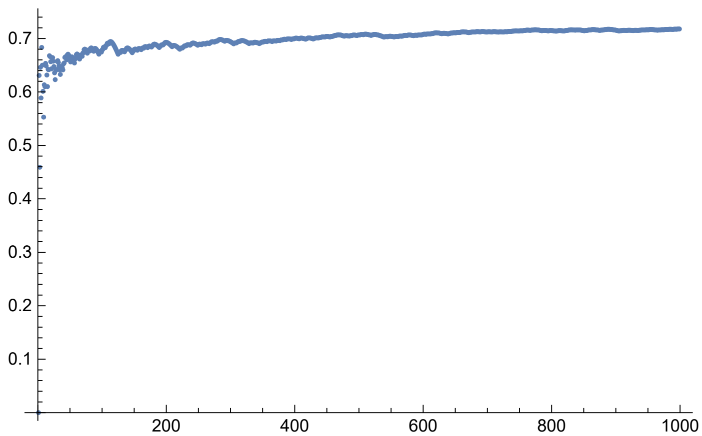
```
Log[2] // N
```

```
0.693147
```

```
Table[Entropy[k, Table[PrimePi[x], {x, 2, k}]] // N, {k, 2, 100}]
```

```
{0., 0.63093, 0.459148, 0.646015, 0.588762, 0.683311, 0.650071, 0.601162, 0.552869, 0.613375,
 0.609966, 0.653516, 0.648658, 0.631633, 0.610056, 0.642208, 0.641666, 0.667759,
 0.666255, 0.656643, 0.643245, 0.664274, 0.664193, 0.657537, 0.647568, 0.635798,
 0.623059, 0.640302, 0.642107, 0.657335, 0.658518, 0.65503, 0.648982, 0.641383, 0.63281,
 0.645716, 0.647515, 0.645506, 0.641329, 0.652619, 0.654228, 0.664571, 0.665841,
 0.663961, 0.660258, 0.669499, 0.670669, 0.669016, 0.665709, 0.661317, 0.656174,
 0.664403, 0.665719, 0.664587, 0.661999, 0.658436, 0.654185, 0.661572, 0.662934,
 0.669884, 0.671068, 0.670189, 0.668068, 0.665102, 0.661528, 0.667849, 0.669051,
 0.668409, 0.666648, 0.672504, 0.673606, 0.679174, 0.680157, 0.679483, 0.677798,
 0.675415, 0.672521, 0.677662, 0.678651, 0.678133, 0.67669, 0.681508, 0.682426,
 0.681926, 0.680552, 0.678569, 0.676134, 0.680621, 0.681532, 0.681138, 0.679937,
 0.678171, 0.675983, 0.673468, 0.670695, 0.674864, 0.675818, 0.675595, 0.674638}
```

```
ListPlot[Table[Entropy[k, Table[PrimePi[x], {x, 2, k}]] // N, {k, 2, 1000}]]
```




```
kk = 50001;
Entropy[kk, Table[PrimePi[x], {x, 2, kk}]] // N
```

```
0.766345
```

The above shows that Entropy[k, Table[PrimePi[x], {x, 2, k}]] ≈ Log[2] in this k range. And this function seems to grow very very slowly

```
Table[Entropy[k, Table[Floor[LogIntegral[x]], {x, 2, k}]] // N, {k, 2, 1000}]
```