Explore some functions Relevant to Quantum Statistics, their Generalizations , and Mathematical functions with similar properties

Part 1

$$\sum_{N=0}^{r} ((E^{\wedge}(-N\,b)))$$

$$\frac{e^{-b\,r}\left(-1+e^{b+b\,r}\right)}{-1+e^{b}}$$

$$\text{Assuming}\left[b>0,\ \text{Limit}\left[\frac{1}{-1+e^{b}}-\frac{1+r}{-1+e^{b\,(1+r)}},\ r\to\text{Infinity}\right]\right]$$

$$\text{Assuming}\left[b<0,\ \text{Limit}\left[\frac{1}{-1+e^{b}}-\frac{1+r}{-1+e^{b\,(1+r)}},\ r\to\text{Infinity}\right]\right]$$

$$\frac{1}{-1+e^{b}}$$

$$\infty$$

$$\left(\frac{1}{\left(\beta\,\frac{-1+e^{-(1+r)\,\beta\,(e0-\mu)}}{-1+e^{-\beta\,(e0-\mu)}}\right)}\right)\left(\partial_{\mu}\left(\frac{-1+e^{-(1+r)\,\beta\,(e0-\mu)}}{-1+e^{-\beta\,(e0-\mu)}}\right)\right)\ //\ \text{FullSimplify}$$

$$\frac{1}{-1+e^{\beta\,(e0-\mu)}}-\frac{1+r}{-1+e^{(1+r)\,\beta\,(e0-\mu)}}$$

$$\frac{1}{-1+e^{\beta\,(e0-\mu)}}-\frac{1+r}{-1+e^{(1+r)\,\beta\,(e0-\mu)}}\ /.\ (\beta\,(e0-\mu))\to b$$

$$\frac{1}{-1+e^{b}}-\frac{1+r}{-1+e^{b\,(1+r)}}$$

$$\text{Assuming}\left[b>0,\ \text{Limit}\left[\frac{-1+e^{-(1+r)\,b}}{-1+e^{-b}},\ r\to\text{Infinity}\right]\right]$$

$$\text{Assuming}\left[b<0,\ \text{Limit}\left[\frac{-1+e^{-(1+r)\,b}}{-1+e^{-b}},\ r\to\text{Infinity}\right]\right]$$

$$\frac{e^{b}}{-1+e^{b}}$$

$$\frac{\infty}{-1+e^{-b}}$$

$$\sum_{N=1}^{r} (N\wedge k) \text{ // FullSimplify}$$

```
HarmonicNumber[r, -k]
```

```
FunctionExpand[HarmonicNumber[r, -k]] // FullSimplify
```

```
-HurwitzZeta[-k, 1 + r] + Zeta[-k]
```

HarmonicNumber[r,-k] = Zeta[-k] - Zeta[-k,1+r]

```
HarmonicNumber[r, -k]; (* is equivalent to: *)
Zeta[-k] - Zeta[-k, 1 + r];
```

```
HarmonicNumber[r, 1]
Zeta[1] - Zeta[1, 1 + r]
```

```
HarmonicNumber[r]
```

Infinity::indet : Indeterminate expression ComplexInfinity + ComplexInfinity encountered. ≫

```
Indeterminate
```

```
Zeta[-k] - Zeta[-k, 1 + r] // FullSimplify
```

```
Zeta[-k] - Zeta[-k, 1 + r]
```

```
Zeta[-k] - Zeta[-k, 2]
```

```
1
```

```
Solve[Zeta[-k] - Zeta[-k, q] == Zeta[k], q]
```

Solve::nsmet : This system cannot be solved with the methods available to Solve. ≫

```
Solve[Zeta[-k] - Zeta[-k, q] == Zeta[k], q]
```

```
HarmonicNumber[1, -k]
```

```
1
```

```
TrigToExp[Sinh[b N]]
```

$$-\frac{1}{2} e^{-b N} + \frac{e^{b N}}{2}$$

$$\sum_{N=1}^{\infty} (\text{Exp[bN] - Exp[-bN]}) \text{ // FullSimplify}$$

Sum::div : Sum does not converge. ≫

$$\sum_{N=1}^{\infty} \left(-e^{-bN} + e^{bN}\right)$$

```
Assuming[b > 0, Sum[(Exp[b N] - Exp[-b N]), {N, 0, ∞}] // FullSimplify]
Assuming[b < 0, Sum[(Exp[b N] - Exp[-b N]), {N, 0, ∞}] // FullSimplify]
```

$$\text{Assuming}\left[b>0,\ \sum_{N=0}^{\infty}\left(\text{Exp}[b\,N]-\text{Exp}[-b\,N]\right)\ //\ \text{FullSimplify}\right]$$

$$\text{Assuming}\left[b<0,\ \sum_{N=0}^{\infty}\left(\text{Exp}[b\,N]-\text{Exp}[-b\,N]\right)\ //\ \text{FullSimplify}\right]$$

Simplify::timc : Number of seconds 300 is not a positive machine–sized number or Infinity. >>

Simplify::timc : Number of seconds 300 is not a positive machine–sized number or Infinity. >>

$$-\text{Coth}\left[\frac{b}{2}\right]$$

Simplify::timc : Number of seconds 300 is not a positive machine–sized number or Infinity. >>

Simplify::timc : Number of seconds 300 is not a positive machine–sized number or Infinity. >>

$$-\text{Coth}\left[\frac{b}{2}\right]$$

$$\text{TrigToExp}\left[-\text{Coth}\left[\frac{b}{2}\right]\right]$$

$$-\frac{e^{-b/2}+e^{b/2}}{-e^{-b/2}+e^{b/2}}$$

$$\text{Simplify}\left[-\frac{e^{-b/2}+e^{b/2}}{-e^{-b/2}+e^{b/2}}\right]$$

$$\frac{1+e^{b}}{1-e^{b}}$$

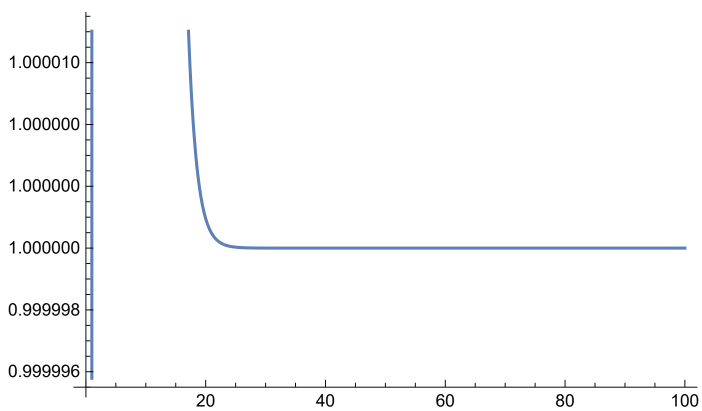
$$\sum_{N=0}^{\infty}\left(\text{Exp}[b\,N]\right)\ //\ \text{FullSimplify}$$

$$\sum_{N=0}^{\infty}\left(\text{Exp}[-b\,N]\right)\ //\ \text{FullSimplify}$$

$$\frac{1}{1-e^{b}}$$

$$1+\frac{1}{-1+e^{b}}$$

```
Plot[Zeta[z], {z, 0, 100}]
```

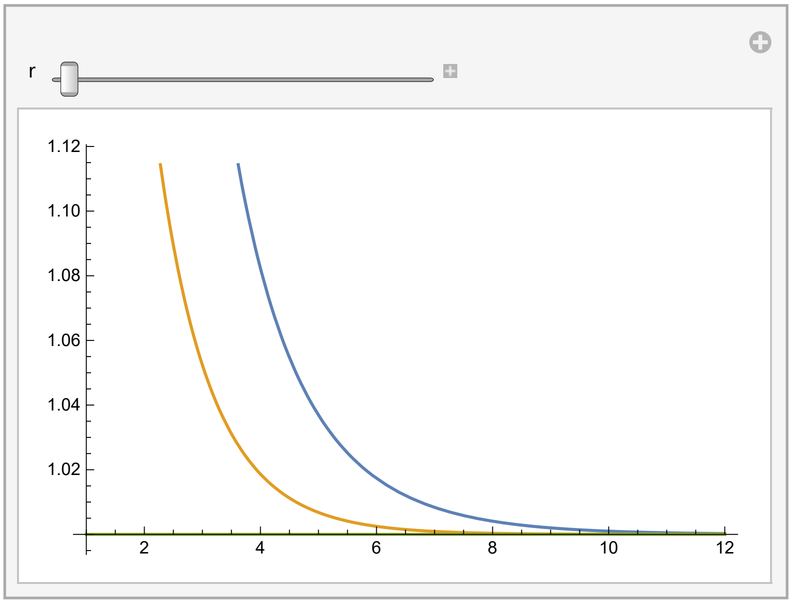


```
Solve[1/(1 - e^-b) == Zeta[b], b]
```

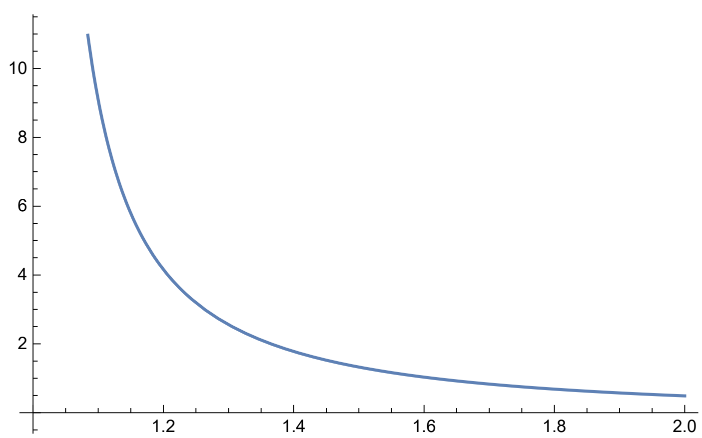
Solve::nsmet : This system cannot be solved with the methods available to Solve. >>

```
Solve[1/(1 - e^-b) == Zeta[b], b]
```

```
Manipulate[Plot[{Zeta[b], 1/(1 - e^-b), (-1 + e^-(1+r) b)/(-1 + e^-b)}, {b, 1, 12}], {r, 0, 100}]
```




```
Plot[Zeta[b] - 1/(1 - e^-b), {b, 1, 2}]
```




```
Zeta[b] - (-1 + e^-(1+r) b)/(-1 + e^-b)
```

$$-\frac{-1+e^{b\,(-1-r)}}{-1+e^{-b}}+\text{Zeta}[b]$$

(*$\partial_b\left(-\frac{-1+e^{b\ (-1-r)}}{-1+e^{-b}}+\text{Zeta}[b]\right)$ = $\sum_{k=1}^{\infty}\left(\left(\frac{(((-b)\text{^}k)\ (\text{HarmonicNumber}[r,-k]))}{((k-1)\ !)}\right)-\left(\frac{(\text{Log}[k])}{(k\text{^}b)}\right)\right)$*)
(*VALID ONLY FOR b>1*)

$$\sum_{k=1}^{\infty}\left(\left(\frac{\left(\left(\left(-b\right)\text{^}k\right)\ \left(\text{HarmonicNumber}[r,-k]\right)\right)}{\left(\left(k-1\right)\ !\right)}\right)-\left(\frac{\left(\text{Log}[k]\right)}{\left(k\text{^}b\right)}\right)\right)$$

$Aborted

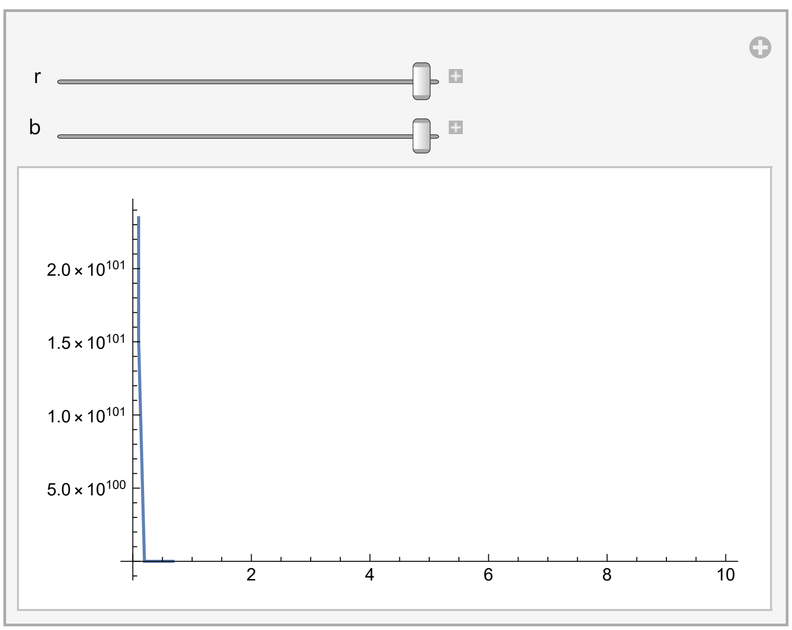
$$\left(\left(\frac{\left(\left(\left(-b\right)\text{^}k\right)\ \left(\text{HarmonicNumber}[r,-k]\right)\right)}{\left(\left(k-1\right)\ !\right)}\right)-\left(\frac{\left(\text{Log}[k]\right)}{\left(k\text{^}b\right)}\right)\right)\ \text{// FullSimplify}$$

$$\frac{(-b)^k\ \text{HarmonicNumber}[r,-k]}{\text{Gamma}[k]}-k^{-b}\ \text{Log}[k]$$

$$\int\left(\frac{(-b)^k\ \text{HarmonicNumber}[r,-k]}{\text{Gamma}[k]}-k^{-b}\ \text{Log}[k]\right)dr\ \text{// FullSimplify}$$

$$-k^{-b}\ r\ \text{Log}[k]+\frac{(-b)^k\ \left(-\text{HurwitzZeta}[-1-k,1+r]+(1+k)\ r\ \text{Zeta}[-k]\right)}{(1+k)\ \text{Gamma}[k]}$$

$$\text{Manipulate}\left[\text{Plot}\left[\frac{(-b)^k\ \text{HarmonicNumber}[r,-k]}{\text{Gamma}[k]}-k^{-b}\ \text{Log}[k],\ \{k,0,10\}\right],\ \{r,0,10,1\},\ \{b,1,100\}\right]$$



```
fddelb1 = Table[((-b)^k HarmonicNumber[1, -k])/Gamma[k] - k^-b Log[k], {k, 1, 20}] // MatrixForm;
fddelb2 = Table[N[((-b)^k HarmonicNumber[1, -k])/Gamma[k] - k^-b Log[k]], {k, 1, 20}] // MatrixForm;
bedelb1 = Table[((-b)^k Zeta[-k])/Gamma[k] - k^-b Log[k], {k, 1, 20}] // MatrixForm;
bedelb2 = Table[N[((-b)^k Zeta[-k])/Gamma[k] - k^-b Log[k]], {k, 1, 20}] // MatrixForm;
```

$$\text{fddelb1 = Table}\left[\frac{(-b)^k\ \text{HarmonicNumber}[1, -k]}{\text{Gamma}[k]} - k^{-b}\ \text{Log}[k],\ \{k, 1, 20\}\right] \text{ // MatrixForm;}$$

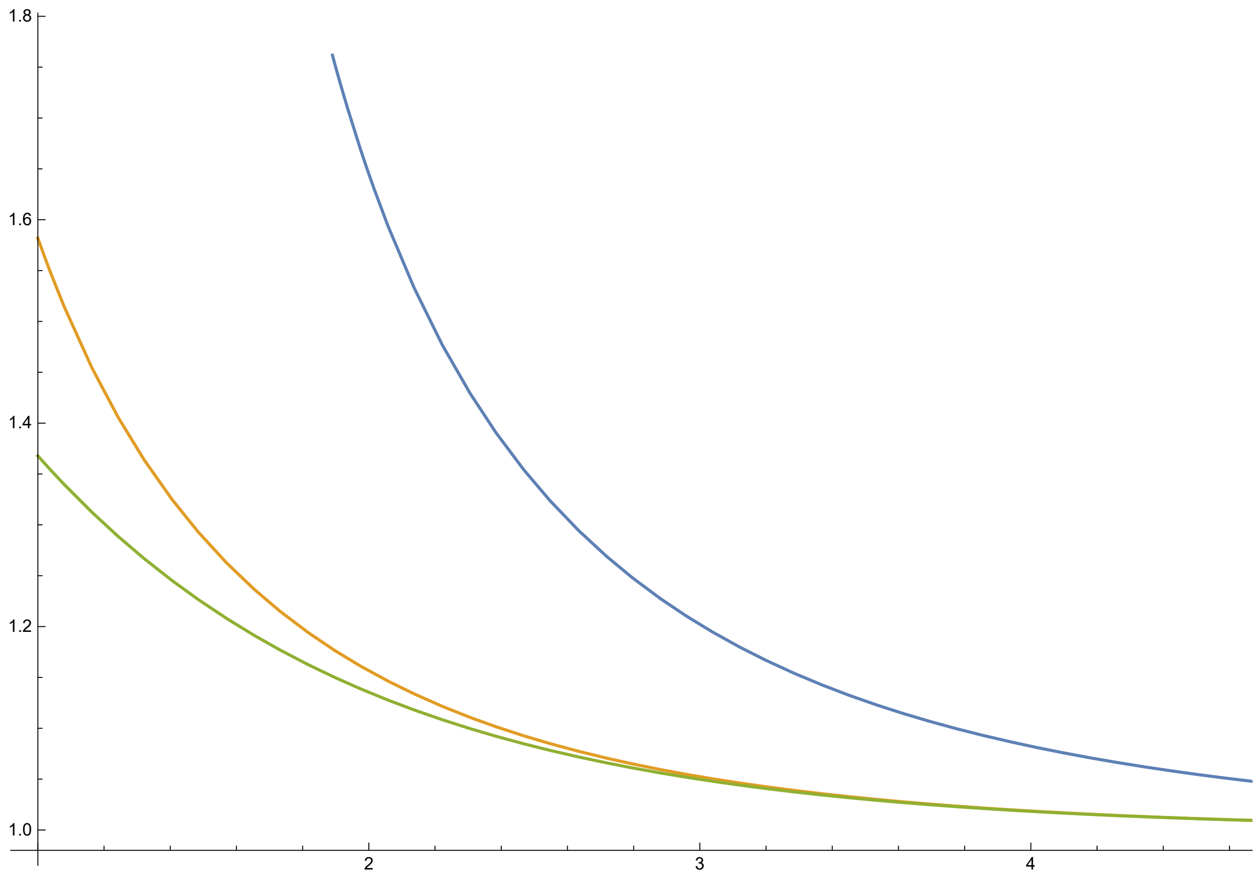
$$\text{fddelb2 = Table}\left[\text{N}\left[\frac{(-b)^k\ \text{HarmonicNumber}[1, -k]}{\text{Gamma}[k]} - k^{-b}\ \text{Log}[k]\right],\ \{k, 1, 20\}\right] \text{ // MatrixForm;}$$

$$\text{bedelb1 = Table}\left[\frac{(-b)^k\ \text{Zeta}[-k]}{\text{Gamma}[k]} - k^{-b}\ \text{Log}[k],\ \{k, 1, 20\}\right] \text{ // MatrixForm;}$$

$$\text{bedelb2 = Table}\left[\text{N}\left[\frac{(-b)^k\ \text{Zeta}[-k]}{\text{Gamma}[k]} - k^{-b}\ \text{Log}[k]\right],\ \{k, 1, 20\}\right] \text{ // MatrixForm;}$$

```
Total[fddelb1] // FullSimplify;
Total[fddelb2] // FullSimplify;
Total[bedelb1] // FullSimplify;
Total[bedelb2] // FullSimplify;
```

$$\text{Plot}\left[\left\{\text{Zeta}[b],\ \frac{1}{1 - e^{-b}},\ 1 + e^{-b}\right\},\ \{b, 1, 5\}\right] \text{ (* } b = \epsilon - \mu \text{ here *)}$$



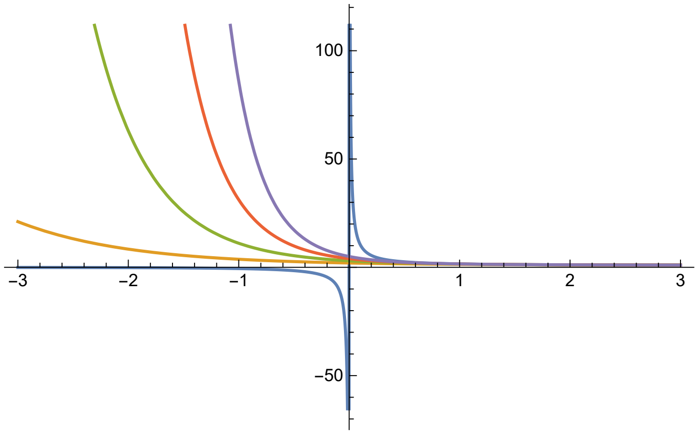
$$\text{(*plot1 = Animate}\left[\text{Plot}\left[\left\{\frac{-1}{-1+e^{-b}},\frac{-1+e^{-(1+r)\ b}}{-1+e^{-b}}\right\},\{b,-12,12\}\right],\{r,0,2\}\right]\text{*)}$$

$$\text{(*Animate}\left[\text{Plot}\left[\left\{\text{Zeta}[b],\frac{-1+e^{-(1+r)\ b}}{-1+e^{-b}}\right\},\{b,1,12\}\right],\{r,0,2\}\right]\text{*)}$$

$$\text{(*plot3=Animate}\left[\text{Plot}\left[\left\{\text{Zeta}[-b]-\text{Zeta}[-b,1+r],\frac{-1+e^{-(1+r)\ b}}{-1+e^{-b}}\right\},\{b,1,12\}\right],\{r,0,3\}\right]\text{*)}$$

```
plotimg1 =
```

$$\text{Plot}\left[\left\{\frac{-1}{-1+e^{-b}}, \frac{-1+e^{-(1+1)\,b}}{-1+e^{-b}}, \frac{-1+e^{-(1+2)\,b}}{-1+e^{-b}}, \frac{-1+e^{-(1+3)\,b}}{-1+e^{-b}}, \frac{-1+e^{-(1+4)\,b}}{-1+e^{-b}}\right\}, \{b, -3, 3\}\right]$$

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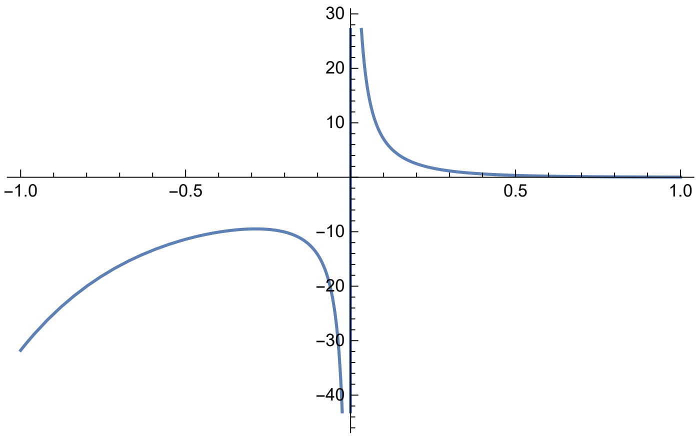
```

$$\text{Plot}\left[\left\{\frac{-1}{-1+e^{-b}}-\frac{-1+e^{-(1+3)\,b}}{-1+e^{-b}}\right\},\ \{b,-1,1\}\right]$$

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```
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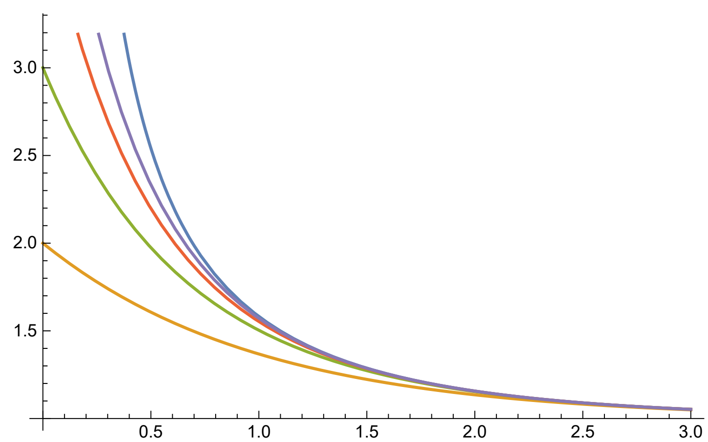

$$\sum_{N=0}^{\infty}\left(\left(E^{\wedge}\left(-N\, b\right)\right)\right)$$

$$\frac{e^{b}}{-1+e^{b}}$$

$$\sum_{N=0}^{r-1} \left(\left(E \wedge (-N\, b)\right)\right) \text{ // FullSimplify}$$

$$\frac{e^{b} - e^{b-b\, r}}{-1 + e^{b}}$$

$$b - b\, r \;=\; b\, (1 - r) \;=\; -b\, (r - 1)$$

Part 2

$$\sum_{i=0}^{r-1} \left(E \wedge (-b\, i)\right)$$

$$\frac{e^{b-b\, r}\left(-1 + e^{b\, r}\right)}{-1 + e^{b}}$$

$$\text{Simplify}\left[\frac{e^{b-b\, r}\left(-1 + e^{b\, r}\right)}{-1 + e^{b}}\right]$$

$$\frac{e^{b} - e^{b-b\, r}}{-1 + e^{b}}$$

$$\frac{1 - e^{-b\, r}}{1 - e^{-b}}$$

$$\text{Plot}\left[\left\{\frac{1 - e^{-b\, 2}}{1 - e^{-b}},\ \frac{1 - e^{-b\, 3}}{1 - e^{-b}},\ \frac{1 - e^{-b\, 4}}{1 - e^{-b}},\ \frac{1}{1 - e^{-b}}\right\},\ \{b,\ -1,\ 1\}\right]$$