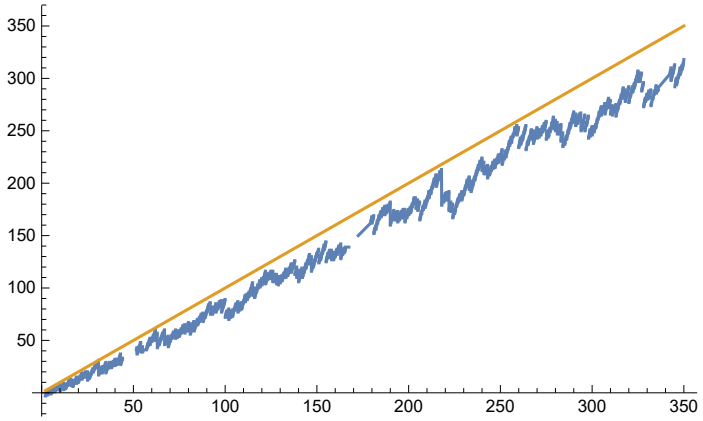
From Dusert's inequality we have that
Prime[x] < x Log[x] + x Log[Log[x]], for x > 6

```
Plot[{(Floor[x Log[x] + x Log[Log[x]]] - Prime[Floor[x]]), x}, {x, 2, 350}]
```



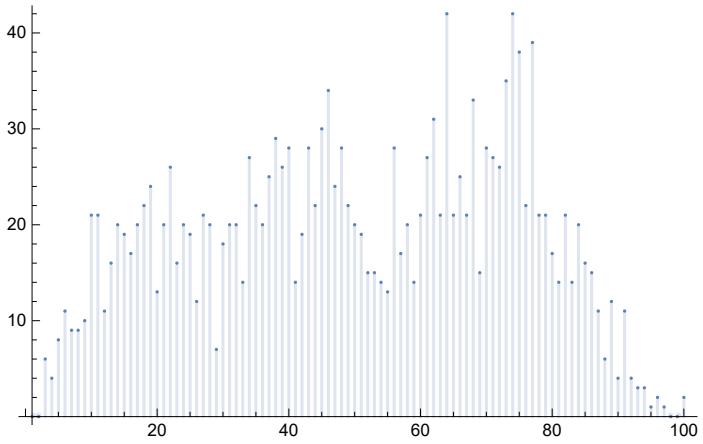

```
F[x_] := x - (Floor[x Log[x] + x Log[Log[x]]] - Prime[Floor[x]])

DusList = Table[F[i], {i, 2, 4000}];

Table[Count[DusList, Prime[i]], {i, 1, 50}]
```

{0, 0, 6, 4, 8, 11, 9, 9, 10, 21, 21, 11, 16, 20, 19, 17, 20, 22, 24, 13, 20, 26, 16, 20, 19, 12, 21, 20, 7, 18, 14, 17, 13, 25, 21, 20, 24, 25, 21, 23, 11, 14, 14, 14, 13, 16, 12, 11, 6, 5}

```
xMAX = 10 000;
DusList = Table[F[i], {i, 2, xMAX}];
DiscretePlot[Count[DusList, Prime[i]], {i, 1, 100}]
```



```
iMAX = 4000;
DusList = Table[F[i], {i, 2, iMAX}];
Table[{i, Count[DusList, i]}, {i, 1, 50}]
(* Gives a list of {i, n} which means that integer i appears n times in DusList *)
```

```
{{1, 0}, {2, 0}, {3, 0}, {4, 0}, {5, 6}, {6, 4}, {7, 4}, {8, 6}, {9, 6}, {10, 2},
 {11, 8}, {12, 6}, {13, 11}, {14, 9}, {15, 7}, {16, 12}, {17, 9}, {18, 12},
 {19, 9}, {20, 9}, {21, 19}, {22, 5}, {23, 10}, {24, 11}, {25, 15}, {26, 12},
 {27, 13}, {28, 16}, {29, 21}, {30, 16}, {31, 21}, {32, 17}, {33, 17}, {34, 4},
 {35, 14}, {36, 9}, {37, 11}, {38, 7}, {39, 12}, {40, 7}, {41, 16}, {42, 9},
 {43, 20}, {44, 18}, {45, 22}, {46, 12}, {47, 19}, {48, 20}, {49, 17}, {50, 18}}
```

```
{5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15} - {5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}
{5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15} [[-1]] (*Gives last element of the list *)
(*Just to show what these operations do on lists *)
```

```
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

```
15
```

```
Union[Table[F[i], {i, 2, 400}]] (*List of the integers created by this function *)
```

```
{5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25,
 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45,
 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 66, 69, 72}
```

```
Range[5, 200];
```

```
iMAX = 4000;
maxint = Union[Table[F[i], {i, 2, iMAX}]][[-1]];
(*Gives last element of this list *)
Length[Union[Table[F[i], {i, 2, iMAX}]]] + 4
Union[Table[F[i], {i, 2, iMAX}]]
```

```
281
```

```
{5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27,
 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49,
 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71,
 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93,
 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111,
 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128,
 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145,
 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162,
 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179,
 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196,
 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213,
 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230,
 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 244, 245, 246, 247, 248,
 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 263, 264, 265, 268,
 269, 270, 272, 274, 276, 282, 283, 284, 285, 287, 288, 289, 290, 291, 292, 295, 296}
```

Most Integers >5 seem to occur in this list, not clear if all of them do, but we can see that raising the imax often fills iqn some missing integers

i.e. 47 doesn't appear in Union[Table[F[i],{i,2,400}]] but does appear once we go to 4000

Union[Table[F[i],{i,2,4000}]]

```
Union[Table[F[i], {i, 2, 5000}]] - Range[5, 340]
Union[Union[Table[F[i], {i, 2, 5000}]] - Range[5, 340]]
(*So Union[Table[F[i],{i,2,5000}]] contains all integers from 5 to 340
 iff this last list only consists of a single zero, which it does so check*)
```

```
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

```
{0}
```

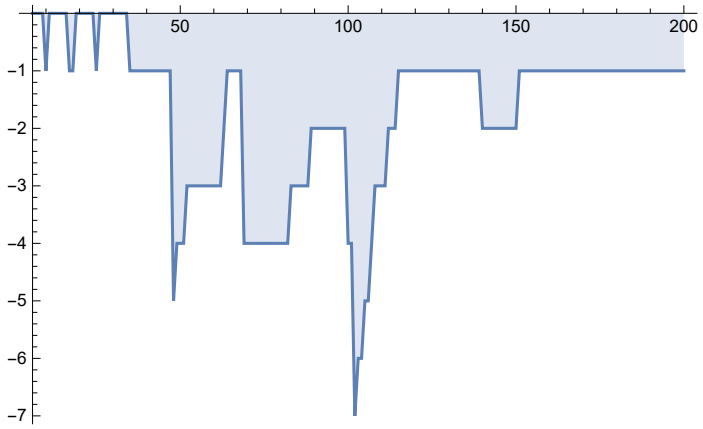
```
iMAX = 5000;
(Length[Union[Table[F[i], {i, 2, iMAX}]]] + 4) - (Union[Table[F[i], {i, 2, iMAX}]][[-1]])
 (*All integers above 5 included in this list only if and only if this =0*)
```

```
0
```

Note that the +4 is included since Union[Table[F[i],{i,2,iMAX}]] starts at 5

From the way
(Length[Union[Table[F[i], {i, 2, iMAX}]]] + 4) -
 (Union[Table[F[i], {i, 2, iMAX}]][[-1]])
is structured the function Union[Table[F[i], {i, 2, iMAX}]]
 contains all integers up to its last element if and only if
(Length[Union[Table[F[i], {i, 2, iMAX}]]] + 4) -
 (Union[Table[F[i], {i, 2, iMAX}]][[-1]]) = 0
Which we can see is true for instances such as iMAX = 5000

```
DiscretePlot[(Length[Union[Table[F[i], {i, 2, k}]]] + 4) -
  (Union[Table[F[i], {i, 2, k}]][[-1]]), {k, 6, 200}]
```




```
iMAX = 150 000;
(Length[Union[Table[F[i], {i, 2, iMAX}]]] + 4) - (Union[Table[F[i], {i, 2, iMAX}]][[-1]])
 (*All integers above 5 included in this list only if and only if this =0*)
```

-41

```
kmax = 700;
AAA1 = Table[Union[Table[F[i], {i, 2, k}]][[-1]], {k, 2, kmax}]
(*Gives a list of the highest integer in the list Table[F[i],{i,2,k}] for a given k *)
AAA2 = Union[Table[Union[Table[F[i], {i, 2, k}]][[-1]], {k, 2, kmax}]]
 (*Compactifies the above list *)
```

```
{5, 5, 5, 6, 6, 6, 6, 6, 8, 8, 9, 9, 9, 9, 9, 11, 11, 11, 11, 11, 11, 11, 11, 13, 13, 13, 13, 13,
 13, 14, 14, 14, 14, 16, 16, 16, 16, 16, 16, 16, 16, 16, 16, 16, 16, 16, 21, 21, 21, 21, 21,
 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 25, 25, 25, 25, 25, 25, 25,
 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25,
 25, 28, 28, 32, 32, 32, 32, 32, 32, 32, 32, 32, 32, 32, 32, 32, 32, 32, 32, 32, 32, 32, 32,
 32, 32, 32, 32, 32, 32, 32, 32, 32, 32, 32, 32, 32, 32, 32, 32, 32, 32, 34, 34, 34, 34, 34,
 34, 34, 34, 34, 34, 34, 34, 34, 34, 34, 34, 34, 34, 34, 34, 34, 34, 34, 35, 35, 35, 35, 35,
 35, 35, 35, 35, 35, 35, 35, 35, 35, 35, 35, 35, 35, 35, 35, 35, 35, 35, 35, 35, 35, 35, 35,
 35, 35, 35, 35, 35, 35, 35, 35, 35, 35, 35, 35, 35, 35, 35, 41, 41, 41, 41, 41, 41, 41, 41,
 41, 41, 41, 41, 41, 41, 41, 41, 48, 50, 57, 57, 57, 57, 57, 57, 57, 57, 57, 57, 57, 57, 57,
 57, 57, 57, 57, 57, 57, 57, 57, 57, 57, 57, 57, 57, 57, 57, 57, 57, 57, 57, 57, 57, 57, 57,
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 58, 58, 58, 58, 58, 58, 61, 61, 63, 63, 63, 63, 69, 72, 72, 72, 72, 72, 72, 72, 72, 72,
 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72,
 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72, 72,
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 74, 74, 74, 74, 81, 81, 81, 81, 81, 81, 81, 81, 81, 81, 81, 81, 81, 81, 81, 81, 81, 81,
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 87, 89, 89, 89, 89, 89, 89, 89, 89, 89, 89, 89, 89, 89, 89, 89, 89, 89, 89, 89, 89, 89,
 89, 89, 89, 89, 89, 89, 89, 89, 89, 89, 89, 89, 89, 89, 89, 89, 89, 89, 89, 89, 89, 89}
```

```
{5, 6, 8, 9, 11, 13, 14, 16, 21, 25, 28, 32,
 34, 35, 41, 48, 50, 57, 58, 61, 63, 69, 72, 74, 81, 87, 89}
```

```
ListPlot[AAA1]
ListPlot[AAA2]
```