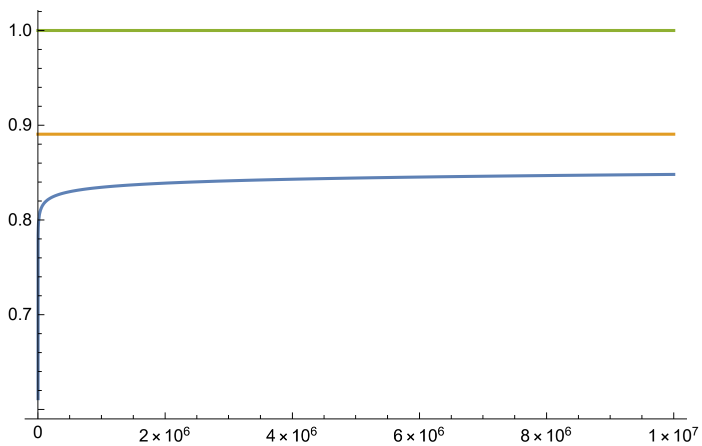
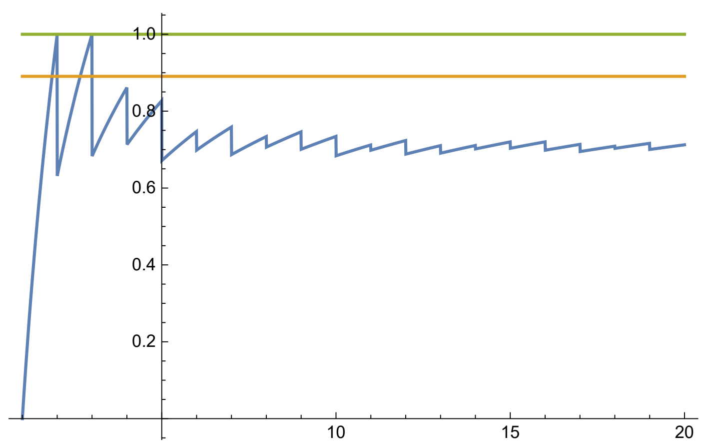
```
N[E^EulerGamma / 2]
Table[{n, Prime[n], N[(Log[n] / (Log[Prime[n]]))], N[(Log[n] / (Log[Prime[Floor[n]]]))]}, {n, 1, 10}] //
 MatrixForm
Plot[{(Log[n] / (Log[Prime[Floor[n]]])), E^EulerGamma / 2, 1}, {n, 1, 10000000}]
Plot[{(Log[n] / (Log[Prime[Floor[n]]])), E^EulerGamma / 2, 1}, {n, 1, 20}]
```

0.890536

1	2	0.	0.
2	3	0.63093	0.63093
3	5	0.682606	0.682606
4	7	0.712414	0.712414
5	11	0.671188	0.671188
6	13	0.698555	0.698555
7	17	0.686821	0.686821
8	19	0.706227	0.706227
9	23	0.700759	0.700759
10	29	0.683808	0.683808





Function Definitions

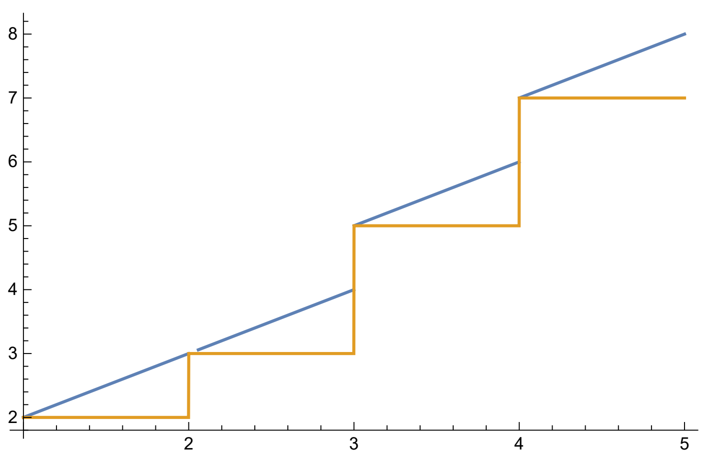
```
Li[x_] := LogIntegral[x] - LogIntegral[2]

♉[x_]  := Abs[-1 + Abs[Sign[x]]] + Floor[x] - Sign[-SawtoothWave[x]]/2 - (1/2)

♉1[x_]  := Abs[-1 + Abs[Sign[x]]] + Floor[x] - Sign[-SawtoothWave[x]]/2

Li[x_] := LogIntegral[x] - LogIntegral[2]

RHPi[x_] := Abs[Li[x] - PrimePi[x]]/(Log[x] Sqrt[x])
```

p_x

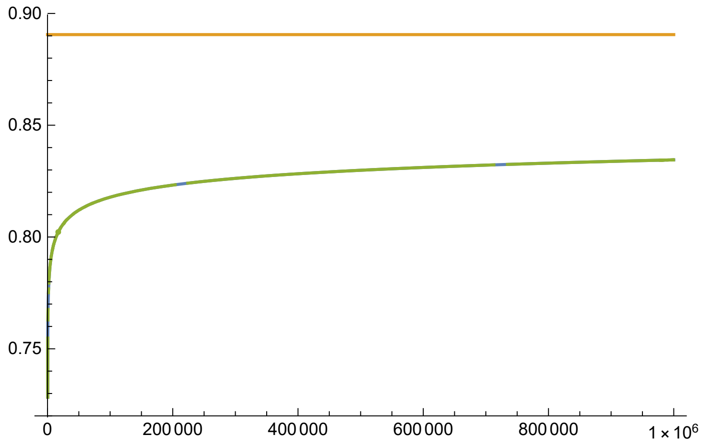
Floor used, but can also do denominator Log[Prime[Floor[n]] + (n-Floor[n])] to get the decimal in the log, thus basically generalize p_n, n Natural, to p_x, x real and x>0

```
px = Prime[Floor[x]] + (x - Floor[x])
x - Floor[x] + Prime[Floor[x]]
```

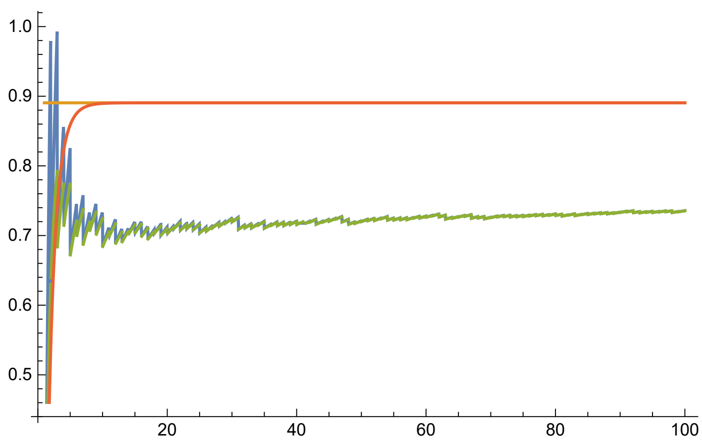
```
Plot[{Prime[Floor[x]] + (x - Floor[x]), Prime[Floor[x]]}, {x, 1, 5}]
```



```
Plot[{(Log[x]/(Log[Prime[Floor[x]]])), E^EulerGamma/2,
  Log[x]/(Log[Prime[Floor[x]] + (x - Floor[x])])}, {x, 1, 1000000}]
```

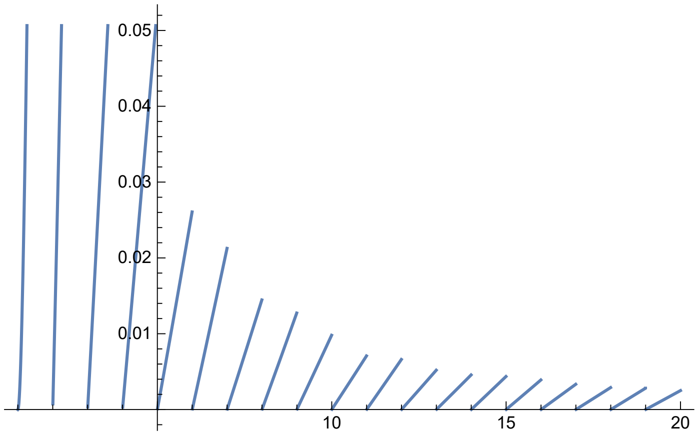



```
Plot[{(Log[x]/(Log[Prime[Floor[x]]])), E^EulerGamma/2,
  Log[x]/(Log[Prime[Floor[x]] + (x - Floor[x])]), (E^EulerGamma/2)/Zeta[x]}, {x, 1, 100}]
```



Maybe (E^EulerGamma /2)-(E^EulerGamma /2)/Zeta[x] is the difference b/w this estimate and an important fn

```
Plot[{(Log[x] / (Log[Prime[Floor[x]]])) - (Log[x] / (Log[Prime[Floor[x]] + (x - Floor[x])]))}, {x, 1, 20}]
```

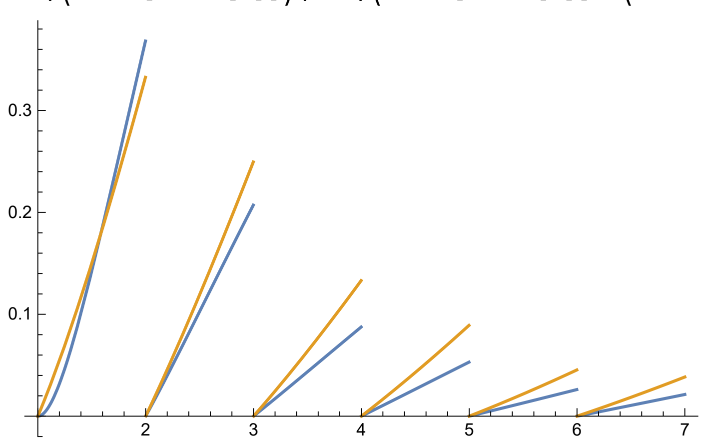


To get the approximate values of those peaks:

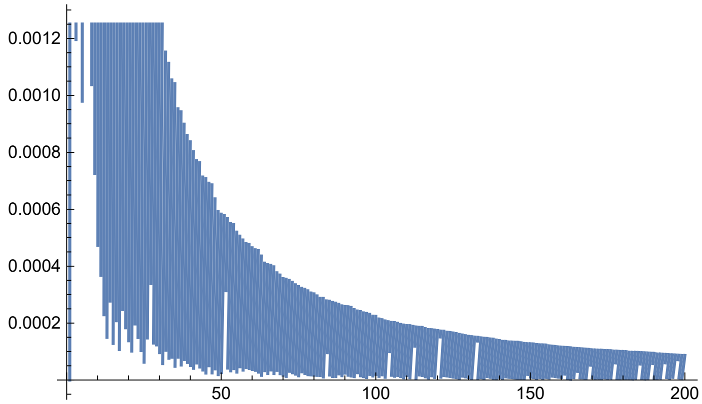
```
a = 0.99;
Table[{x, N[(Log[x + a] / (Log[Prime[Floor[x]]])) -
    (Log[x + a] / (Log[Prime[Floor[x]] + (x + a - Floor[x])]))]}, {x, 1, 10}] // MatrixForm
```

1	0.364492
2	0.205459
3	0.0867696
4	0.0525801
5	0.0258986
6	0.0210887
7	0.0143672
8	0.0126481
9	0.00973511
10	0.00702623

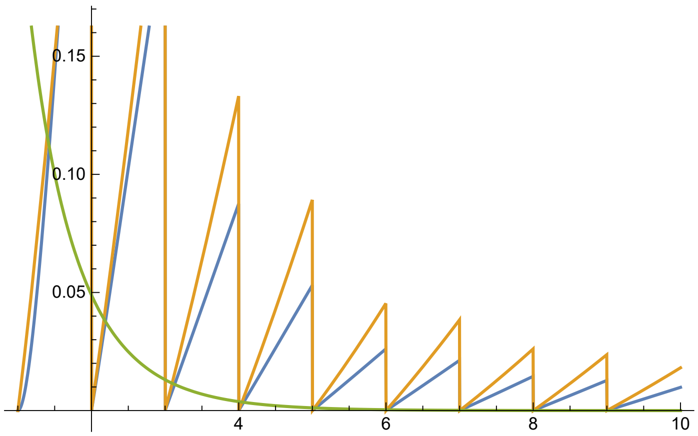
```
Plot[{(Log[x] / (Log[Prime[Floor[x]]])) - (Log[x] / (Log[Prime[Floor[x]] + (x - Floor[x])])),
  (x / (Prime[Floor[x]])) - (x / (Prime[Floor[x]] + (x - Floor[x])))}, {x, 1, 7}]
```



```
Plot[{(Log[x] / (Log[Prime[Floor[x]]])) - (Log[x] / (Log[Prime[Floor[x]] + (x - Floor[x])]))}, {x, 1, 200}]
```




```
Plot[{(Log[x] / (Log[Prime[Floor[x]]])) - (Log[x] / (Log[Prime[Floor[x]] + (x - Floor[x])])),
  (x / (Prime[Floor[x]])) - (x / (Prime[Floor[x]] + (x - Floor[x]))),
  ExpIntegralE[1, x]}, {x, 1, 10}, Exclusions → None]
```



x -> Floor[x] in numerators

Search for Similar Continuous Fn.

Exp Integral fns & others

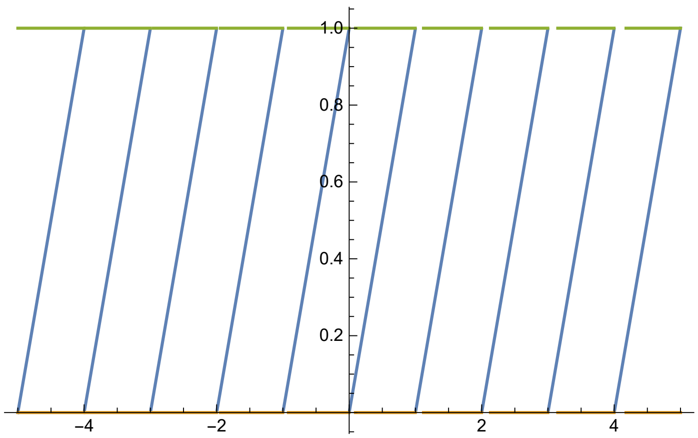
Bounds and Low n Estimates of π(n)

Comparisons to known bounds

Estimation of Prime Counting Fn for low and “mid” range n

Heaviside Fn, Dirac Delta

```
Plot[{x - Floor[x], DiracDelta[x - Floor[x]], HeavisideTheta[x - Floor[x]]}, {x, -5, 5}]
```



$$\sum_{n=0}^{\infty} \text{HeavisideTheta}[x - n]$$

$$\sum_{n=0}^{\infty} \text{HeavisideTheta}[x + n]$$

$$\begin{cases} 1 + \text{Floor}[x] & x \geq 0 \\ 0 & \text{True} \end{cases}$$

Sum::div : Sum does not converge. ≫

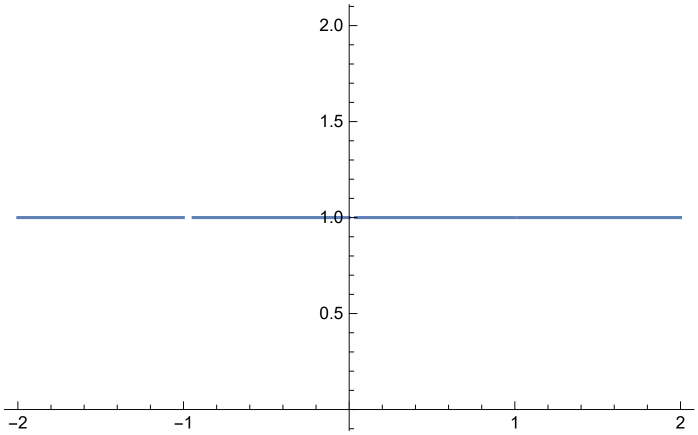
$$\sum_{n=0}^{\infty} \text{HeavisideTheta}[n + x]$$

$$\sum_{n=-\infty}^{\infty} \text{HeavisideTheta}[x - n]$$

Sum::div : Sum does not converge. ≫

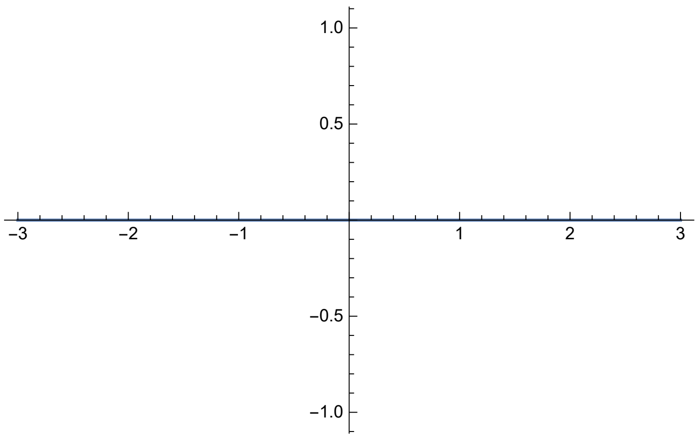
$$\sum_{n=-\infty}^{\infty} \text{HeavisideTheta}[-n + x]$$

```
Plot[HeavisideTheta[SawtoothWave[x]], {x, -2, 2}]
```



$-\text{HeavisideTheta}\left[-\frac{1}{2}+x\right]+\text{HeavisideTheta}\left[\frac{1}{2}+x\right]$ = HeavisidePi[x]

```
Plot[{Abs[Abs[Sign[x]] - 1]}, {x, -3, 3}, Exclusions → None]
```




```
Abs[Abs[Sign[0]] - 1]
```

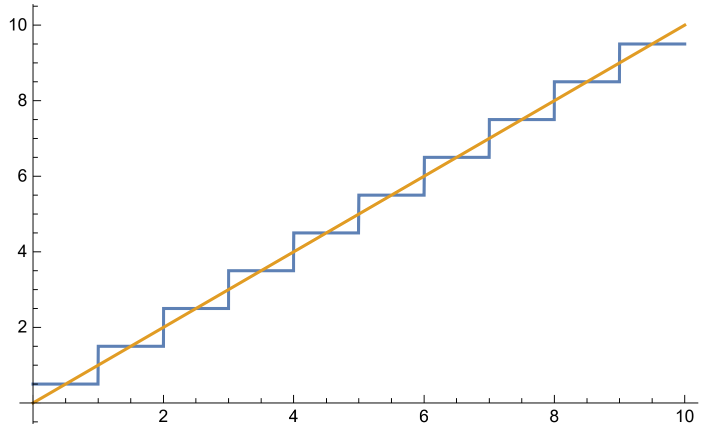
1

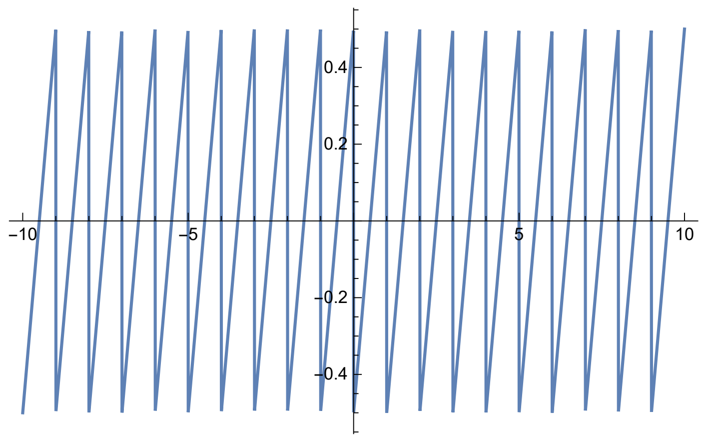
$$\text{ɤ1}[x\_] := \text{Abs}[-1+\text{Abs}[\text{Sign}[x]]]+\text{Floor}[x]-\frac{\text{Sign}[-\text{SawtoothWave}[x]]}{2}$$

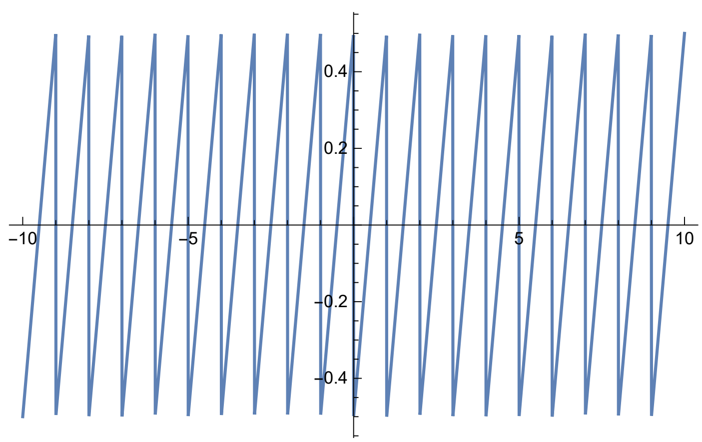
Note that $\left(\frac{\text{Sign}[x]+1}{2}\right)$ = HeavisideTheta[x], but this way of writing makes it 1/2 at x=0 in mathematica

$\left(\frac{\text{Sign}[-\text{SawtoothWave}[x]]+1}{2}\right)$ = HeavisideTheta[-SawtoothWave[x]]

```
Plot[{♉1[x], x}, {x, 0, 10}, Exclusions → None]
Plot[{(x - ♉1[x])}, {x, -10, 10}, Exclusions → None]
Plot[{SawtoothWave[x] - (1/2)}, {x, -10, 10}, Exclusions → None]
```







So (x-♉1[x]) =SawtoothWave[x]-(1/2)

```
DiracDelta[-(0 - Floor[0])]
DiracComb[0]
DiracDelta[-(.54 - Floor[.54])]
DiracComb[.54]
```

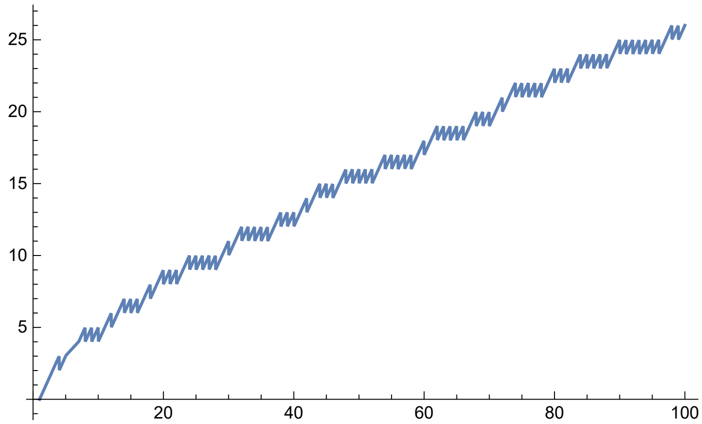
```
DiracDelta[0]
```

```
DiracComb[0]
```

```
0
```

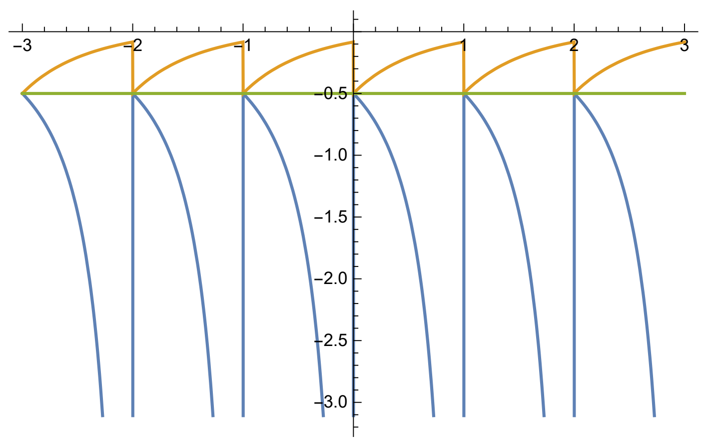
```
0
```

```
Plot[{PrimePi[Floor[x]] + (x - Floor[x])}, {x, 1, 100}, Exclusions → None]
```



Zeta Fn & Sawtooth & Heaviside Fns

```
Plot[{Zeta[SawtoothWave[x]], Zeta[-SawtoothWave[x]],
  Zeta[-2 HeavisideTheta[-SawtoothWave[x]]]}, {x, -3, 3}, Exclusions → None]
```



```
NumberForm[Zeta[-SawtoothWave[.99]], 50]
NumberForm[Zeta[-SawtoothWave[.999]], 50]
NumberForm[Zeta[-SawtoothWave[.9999]], 50]
NumberForm[Zeta[-SawtoothWave[.99999999]], 50]
N[Zeta[-1]]
```

-0.0850001190598196

-0.0834988796430323

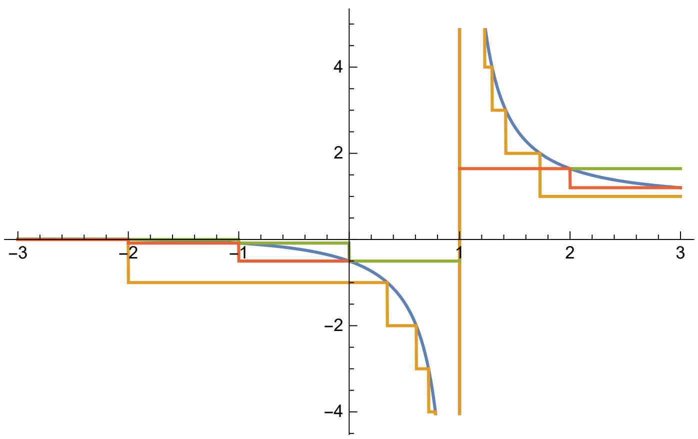
-0.0833498766987893

-0.0833333349875448

-0.0833333

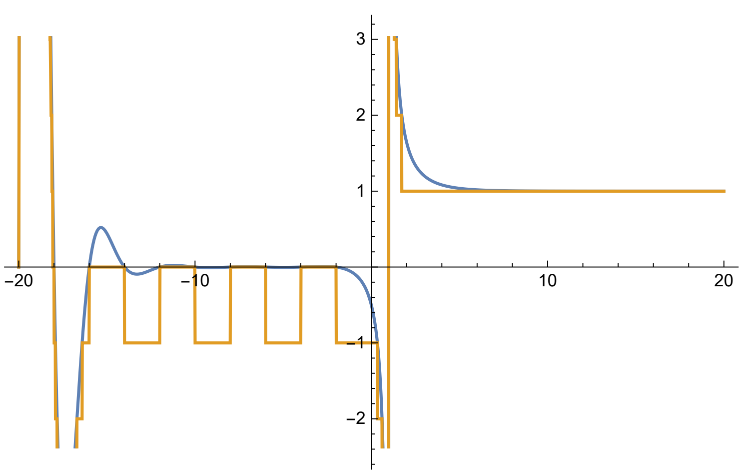
So the Limit of Zeta[SawtoothWave[x]] as x approaches the integers from the left is Zeta[-1]

```
b = 3;
Plot[{Zeta[x], Floor[Zeta[x]], Zeta[Floor[x]], Zeta[Ceiling[x]]},
 {x, -b, b}, Exclusions → None]
```

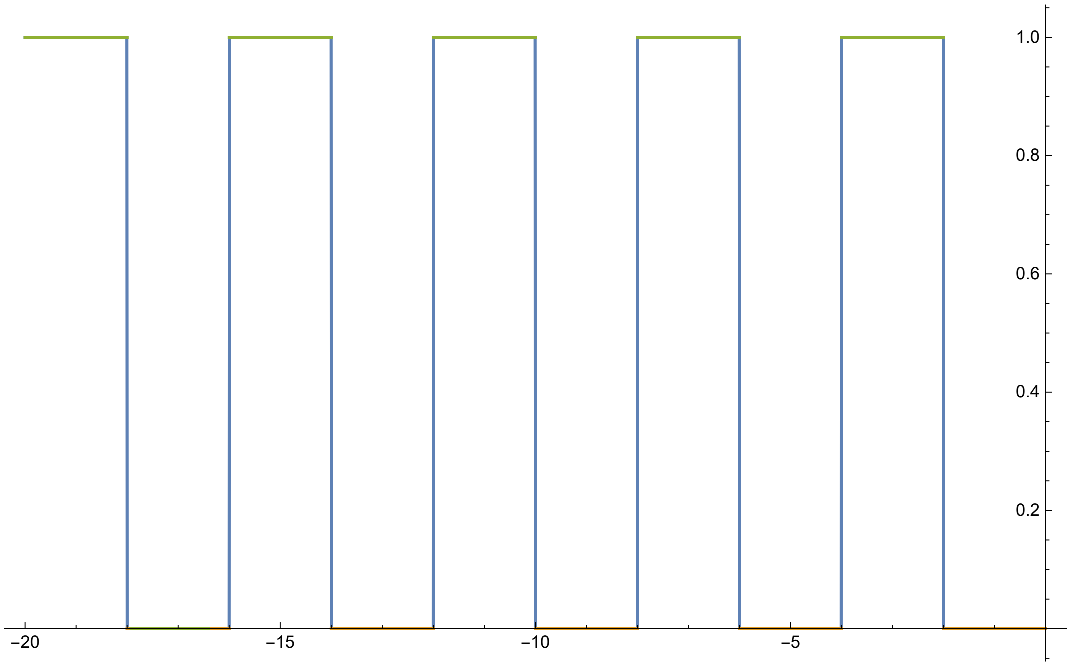


Zeta[Floor[x]], Zeta[Ceiling[x]] bound Zeta[x], by defn, let them be variable heaviside fns

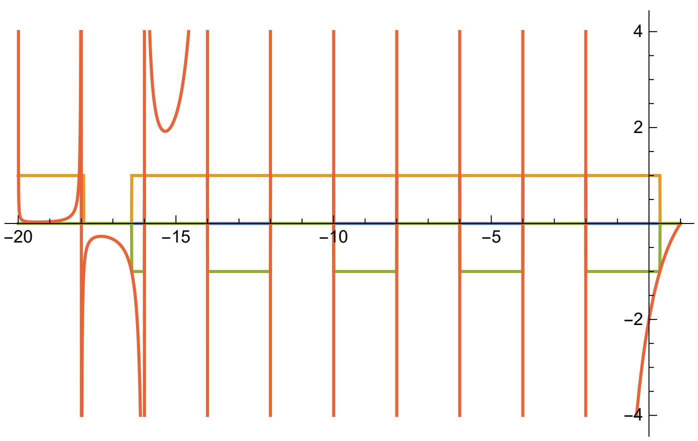
```
b1 = 20;
Plot[{Zeta[x], Floor[Zeta[x]]}, {x, -b1, b1}, Exclusions → None]
```



```
b2 = 20;
Plot[{HeavisideTheta[Zeta[x]], HeavisideTheta[Floor[Zeta[x]]],
  HeavisideTheta[Floor[1 + Zeta[x]]]}, {x, -b2, 0}, Exclusions → None]
```




```
b3 = 20;
Plot[{HeavisideTheta[Zeta[x]], HeavisideTheta[1 + Zeta[x]],
  (HeavisideTheta[Zeta[x]] - HeavisideTheta[1 + Zeta[x]]),
  1 / Zeta[x]}, {x, -b3, 1}, Exclusions → None]
```



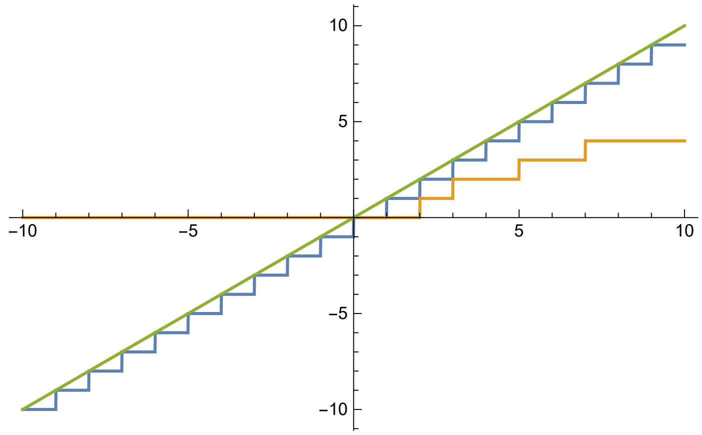
Red asymptotes are the zeroes of the zeta fn. Notice that alot of stuff goes on around where the gap between fn 1 & fn 2 occurs, the asymptote near that happens to be after the first non-trivial zero, and unique stuff seems to occur about other non-trivial zeroes.

Note that HeavisideTheta[x] not defined in Mathematica

Prime Counting Function

$$♉[x\_] := \text{Abs}[-1 + \text{Abs}[\text{Sign}[x]]] + \text{Floor}[x] - \frac{\text{Sign}[-\text{SawtoothWave}[x]]}{2} - (1/2)$$

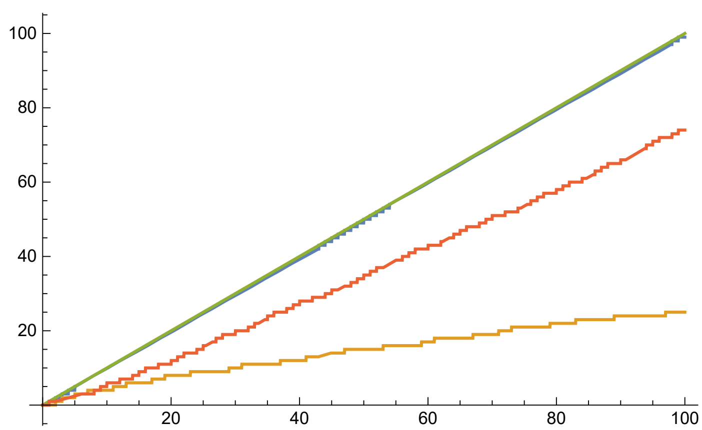
```
Plot[{♉[x], PrimePi[x], x}, {x, -10, 10}, Exclusions → None]
Table[♉[i], {i, 0, 20}]
```



$$\left\{\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}, \frac{11}{2}, \frac{13}{2}, \frac{15}{2}, \frac{17}{2}, \frac{19}{2}, \frac{21}{2}, \frac{23}{2}, \frac{25}{2}, \frac{27}{2}, \frac{29}{2}, \frac{31}{2}, \frac{33}{2}, \frac{35}{2}, \frac{37}{2}, \frac{39}{2}\right\}$$

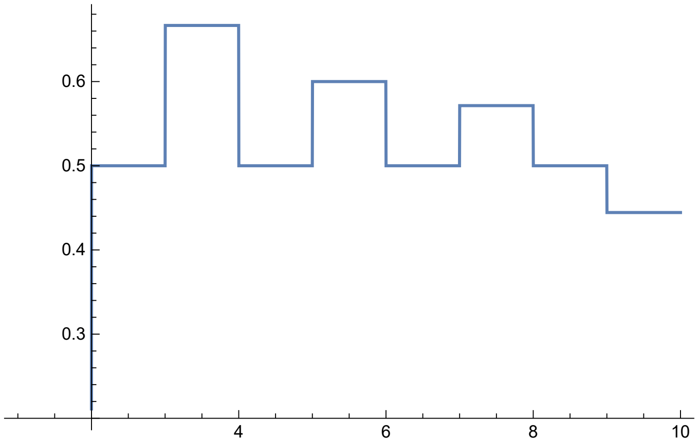
So ♉[x] counts the number of integers less than or equal to x for x>0, not sgnificant in itself, as this is obvious, but significant in that it gives an analytic function for this. Expect π[x] = PrimePi[x] to take on a similar analytic form, but with jumps at the primes.

```
Plot[{♉[x], PrimePi[x], x, ♉[x] - PrimePi[x]}, {x, 0, 100}, Exclusions → None]
```



For ♉[x]-PrimePi[x], there’s no spikes when x is prime, ♉[x]-PrimePi[x] is bounded above by ♉[x] and below by PrimePi[x]

```
Plot[{PrimePi[x]/♉[x]}, {x, 1, 10}, Exclusions → None]
Table[{i, PrimePi[i]/♉[i]}, {i, 0, 20}]
```



$\{\{0, 0\}, \{1, 0\}, \{2, \frac{2}{3}\}, \{3, \frac{4}{5}\}, \{4, \frac{4}{7}\}, \{5, \frac{2}{3}\}, \{6, \frac{6}{11}\},$
$\{7, \frac{8}{13}\}, \{8, \frac{8}{15}\}, \{9, \frac{8}{17}\}, \{10, \frac{8}{19}\}, \{11, \frac{10}{21}\}, \{12, \frac{10}{23}\}, \{13, \frac{12}{25}\},$
$\{14, \frac{4}{9}\}, \{15, \frac{12}{29}\}, \{16, \frac{12}{31}\}, \{17, \frac{14}{33}\}, \{18, \frac{2}{5}\}, \{19, \frac{16}{37}\}, \{20, \frac{16}{39}\}\}$

$\frac{\text{PrimePi}[x]}{♉[x]}$ is the number of primes at or below x, divided by the number of integers at or below x

```
Table[N[Sum[PrimePi[i]/♉[i], {i, 1, imax}]], {imax, 1, 100}]
Table[N[Sum[♉[i]/PrimePi[i], {i, 2, imax}]], {imax, 1, 100}]
```

```
{0., 0.666667, 1.46667, 2.0381, 2.70476, 3.25022, 3.8656, 4.39893, 4.86952, 5.29058,
 5.76677, 6.20155, 6.68155, 7.12599, 7.53979, 7.92688, 8.35113, 8.75113, 9.18356,
 9.59381, 9.98406, 10.3562, 10.7562, 11.1391, 11.5065, 11.8594, 12.199, 12.5263,
 12.8772, 13.2162, 13.5768, 13.926, 14.2645, 14.5929, 14.9117, 15.2216, 15.5503,
 15.8703, 16.182, 16.4858, 16.8068, 17.12, 17.4495, 17.7713, 18.0859, 18.3936,
 18.7162, 19.032, 19.3412, 19.6443, 19.9413, 20.2326, 20.5373, 20.8364, 21.13,
 21.4183, 21.7014, 21.9797, 22.2703, 22.556, 22.8535, 23.1462, 23.4342, 23.7177,
 23.9968, 24.2716, 24.5573, 24.8388, 25.1161, 25.3895, 25.6732, 25.9529, 26.2426,
 26.5283, 26.8102, 27.0883, 27.3628, 27.6338, 27.9141, 28.1908, 28.4641, 28.734,
 29.0128, 29.2883, 29.5604, 29.8294, 30.0953, 30.3582, 30.6294, 30.8975, 31.1627,
 31.425, 31.6845, 31.9412, 32.1951, 32.4465, 32.7055, 32.9619, 33.2157, 33.467}
```

```
{0., 1.5, 2.75, 4.5, 6., 7.83333, 9.45833, 11.3333, 13.4583, 15.8333, 17.9333,
 20.2333, 22.3167, 24.5667, 26.9833, 29.5667, 31.9238, 34.4238, 36.7363,
 39.1738, 41.7363, 44.4238, 46.9238, 49.5349, 52.2571, 55.0905, 58.0349, 61.0905,
 63.9405, 66.8905, 69.6632, 72.5268, 75.4814, 78.5268, 81.6632, 84.8905, 87.9321,
 91.0571, 94.2655, 97.5571, 100.673, 103.865, 106.901, 110.008, 113.186, 116.436,
 119.536, 122.703, 125.936, 129.236, 132.603, 136.036, 139.318, 142.661, 146.068,
 149.536, 153.068, 156.661, 160.102, 163.602, 166.964, 170.38, 173.852, 177.38,
 180.964, 184.602, 188.102, 191.655, 195.26, 198.918, 202.443, 206.018, 209.471,
 212.971, 216.518, 220.113, 223.756, 227.447, 231.015, 234.629, 238.288, 241.992,
 245.579, 249.21, 252.884, 256.601, 260.362, 264.166, 267.854, 271.583, 275.354,
 279.166, 283.02, 286.916, 290.854, 294.833, 298.693, 302.593, 306.533, 310.513}
```

$$\sum_{i=1}^{\infty}\left(\frac{1}{2} + \mathrm{Abs}[-1 + \mathrm{Abs}[\mathrm{Sign}[i]]] - \frac{\mathrm{Sign}[i]}{2}\right)$$

$$\sum_{i=1}^{\infty}\left(\mathrm{Abs}[-1 + \mathrm{Abs}[\mathrm{Sign}[i]]]\right)$$

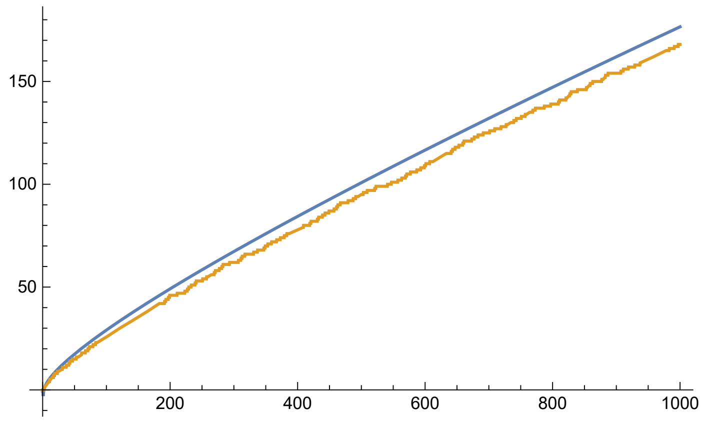
```
0
```

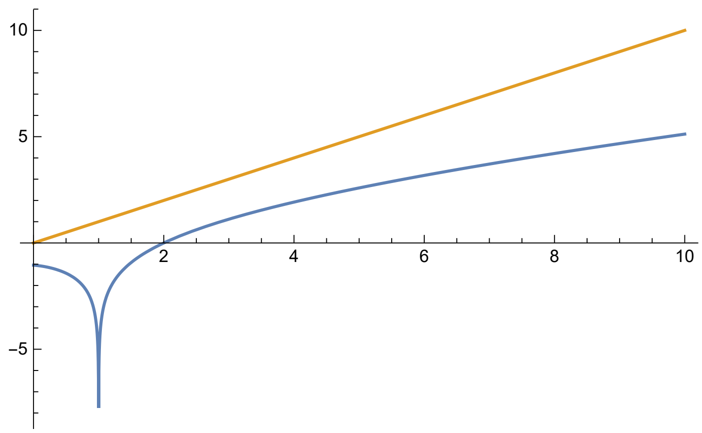
```
0
```

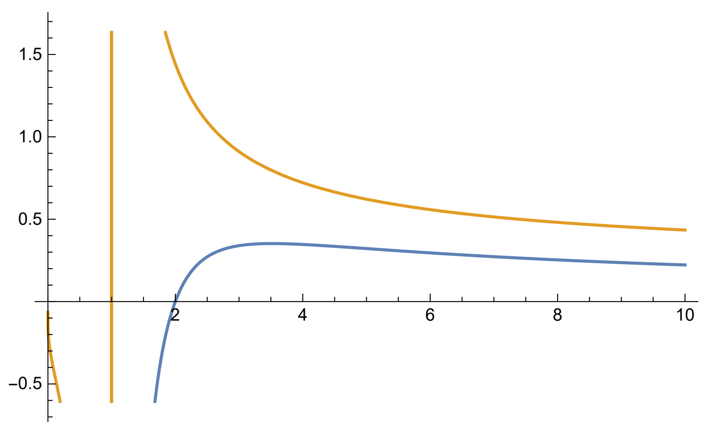
```
Plot[{Log[♉[x]], Log[PrimePi[x]], Log[x], Log[♉[x] - PrimePi[x]]},
 {x, 0, 100}, Exclusions → None]
```

```
Li[x_] := LogIntegral[x] - LogIntegral[2]
```

```
Plot[{Li[x], PrimePi[x]}, {x, 0, 1000}, Exclusions → None]
Plot[{Li[x], x}, {x, 0, 10}, Exclusions → None]
Plot[{Li[x]/(x Log[x]), x/(x Log[x])}, {x, 0, 10}, Exclusions → None]
```







Chebyshev Fns

Prime Counting RH equivalency

```
RHPi[x_] := Abs[Li[x] - PrimePi[x]] / (Log[x] Sqrt[x])
```

```
RHPi[0.1]
RHPi[.5]
N[RHPi[π]]
N[RHPi[E]]
N[RHPi[E^(EulerGamma / 2)]]
Table[N[RHPi[i]], {i, 2, 10}]
```

-1.47987

-2.90502

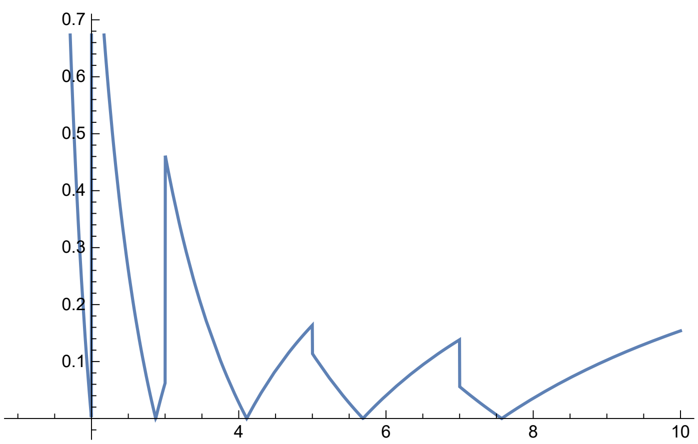
0.372277

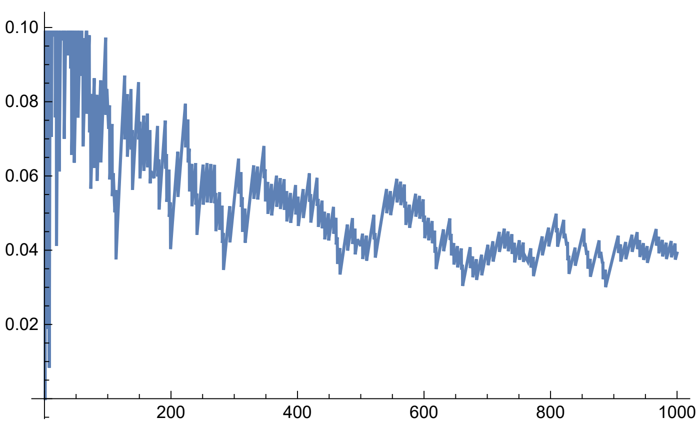
0.0910075

4.19841

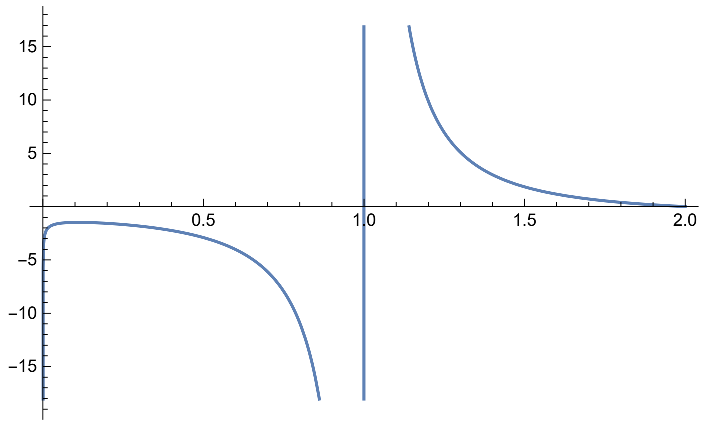
{1.02014, 0.463291, 0.0279806, 0.114086, 0.0403424, 0.0559615, 0.0354591, 0.102565, 0.153876}

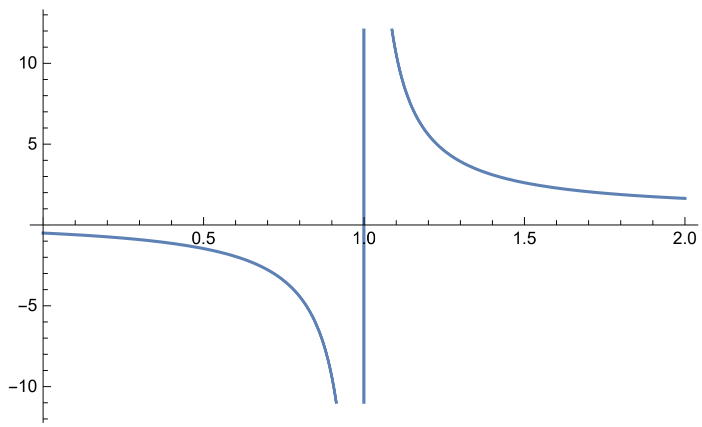
```
Plot[{RHPi[x]}, {x, 1, 10}, Exclusions → None]
Plot[{RHPi[x]}, {x, 0, 1000}, Exclusions → None]
```





```
Plot[{RHPi[x]}, {x, 0, 2}, Exclusions → None]
Plot[{Zeta[x]}, {x, 0, 2}, Exclusions → None]
```

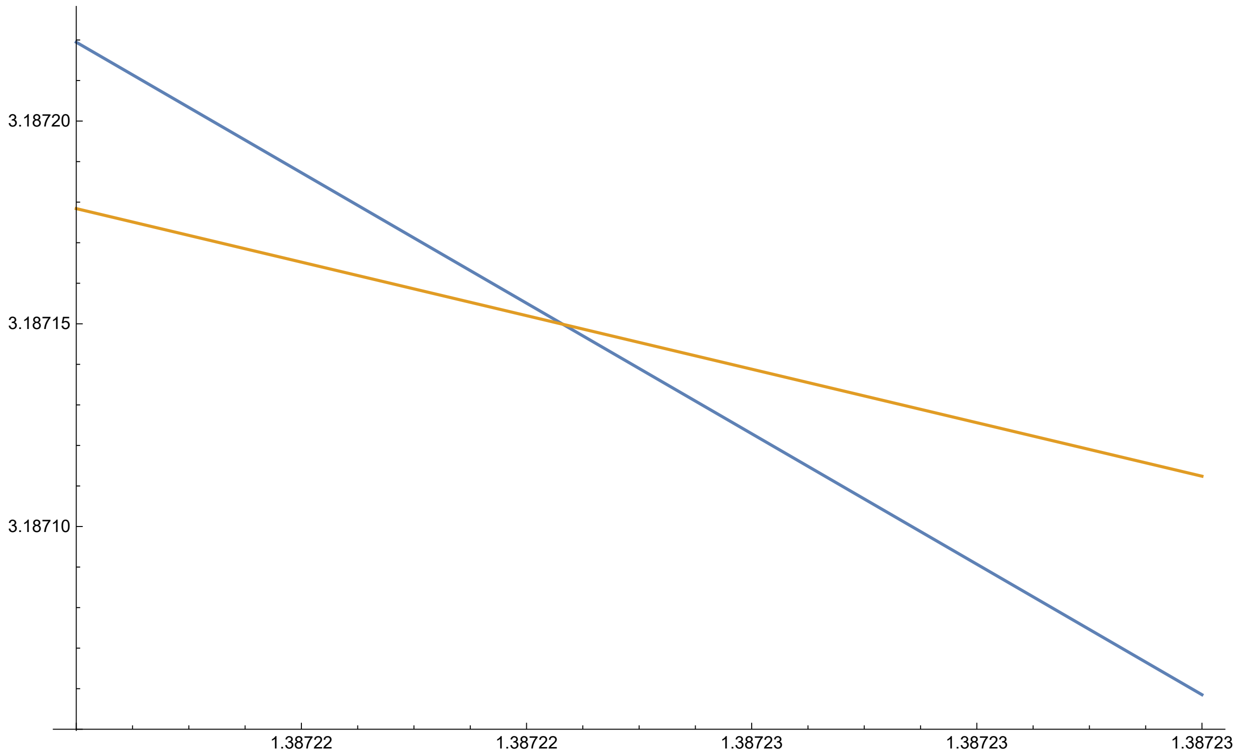





```
N[2 Log[2]]
```

1.38629

```
Plot[{RHPi[x], Zeta[x]}, {x, 1.38722, 1.38723},
 GridLines → {{2 Log[2]}, {}}, Exclusions → None]
```




```
RHPi[x] - Zeta[x]
```

$$\frac{1}{\sqrt{x}\ \mathrm{Log}[x]}\mathrm{Abs}[-\mathrm{LogIntegral}[2]+\mathrm{LogIntegral}[x]-\mathrm{PrimePi}[x]]-\mathrm{Zeta}[x]$$

```
Li[1.38]
```

```
-1.2509
```

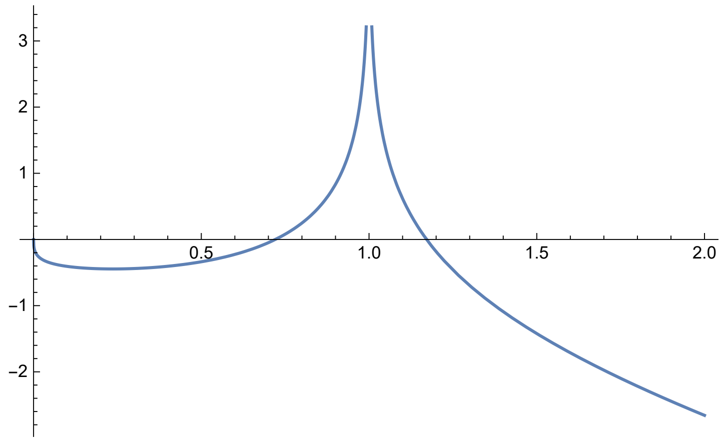
RHPi[x]-Zeta[x] = 0

-Li[x] - PrimePi[x] = $\sqrt{x}$ Log[x] Zeta[x]

Know for the above graph PrimePi[~1.38] = 0, giving

-Li[x] = $\sqrt{x}$ Log[x] Zeta[x]

```
Plot[-LogIntegral[x] - Sqrt[x] Log[x] Zeta[x] , {x, 0, 2}]
```



RHPi[x] has spikes at the primes, as can be told from its derivative

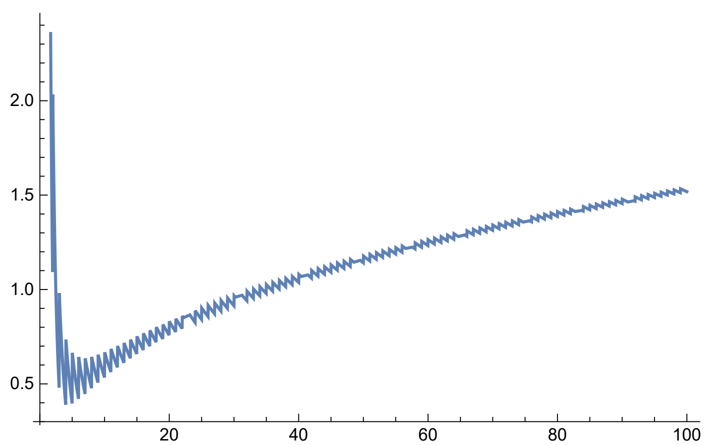
```
D[Abs[Li[x] - PrimePi[x]] / (Log[x] Sqrt[x]), x] // FullSimplify
```

$$\left(-\text{Abs}[\text{LogIntegral}[2]-\text{LogIntegral}[x]+\text{PrimePi}[x]]\ (2+\text{Log}[x])-2\,x\,\text{Abs}'[-\text{LogIntegral}[2]+\text{LogIntegral}[x]-\text{PrimePi}[x]]\ (-1+\text{Log}[x]\ \text{PrimePi}'[x])\right)\Big/\left(2\,x^{3/2}\,\text{Log}[x]^2\right)$$

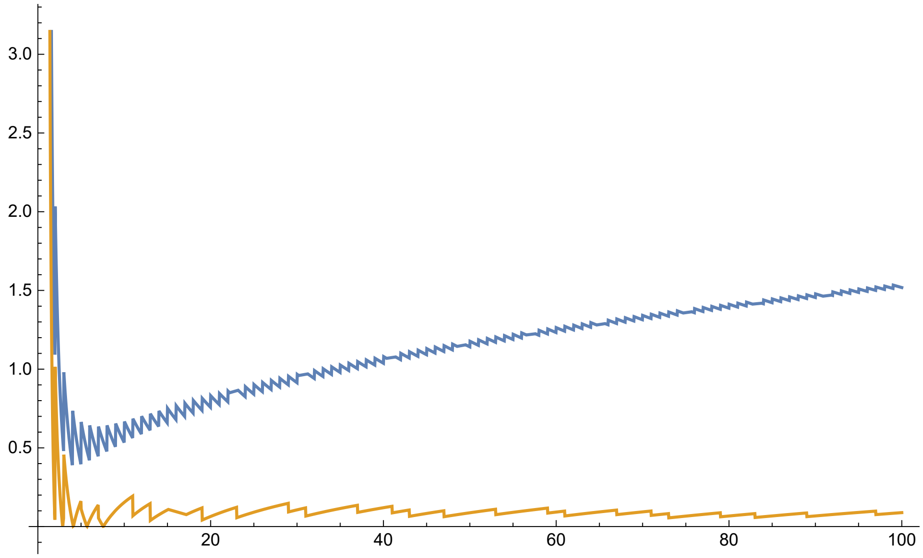
```
D[Li[x], x] // FullSimplify
```

$$\frac{1}{\text{Log}[x]}$$

```
Plot[{Abs[Li[x] - ♉[x]] / (Log[x] Sqrt[x])}, {x, 1, 100}, Exclusions → None]
```

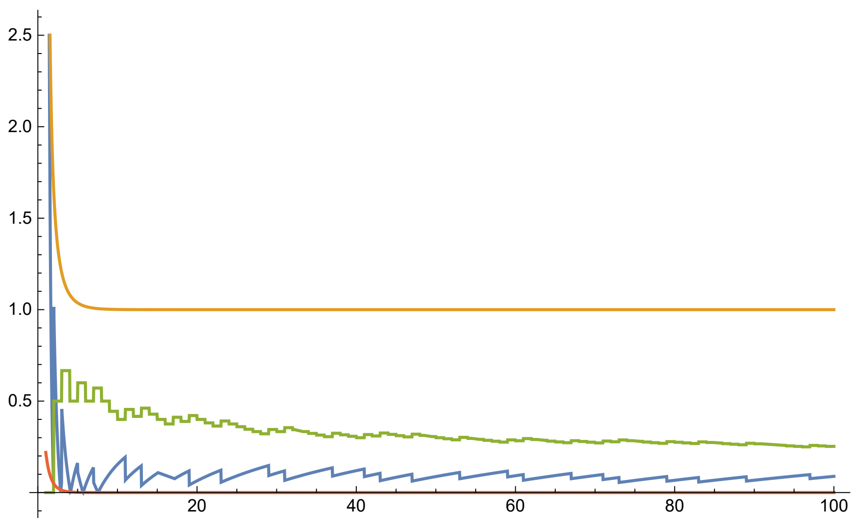


$$\texttt{Plot}\left[\left\{\frac{\texttt{Abs[Li[x] - ♉[x]]}}{\left(\texttt{Log[x]}\ \sqrt{\texttt{x}}\right)},\ \texttt{RHPi[x]}\right\},\ \texttt{\{x, 1, 100\}, Exclusions} \to \texttt{None}\right]$$



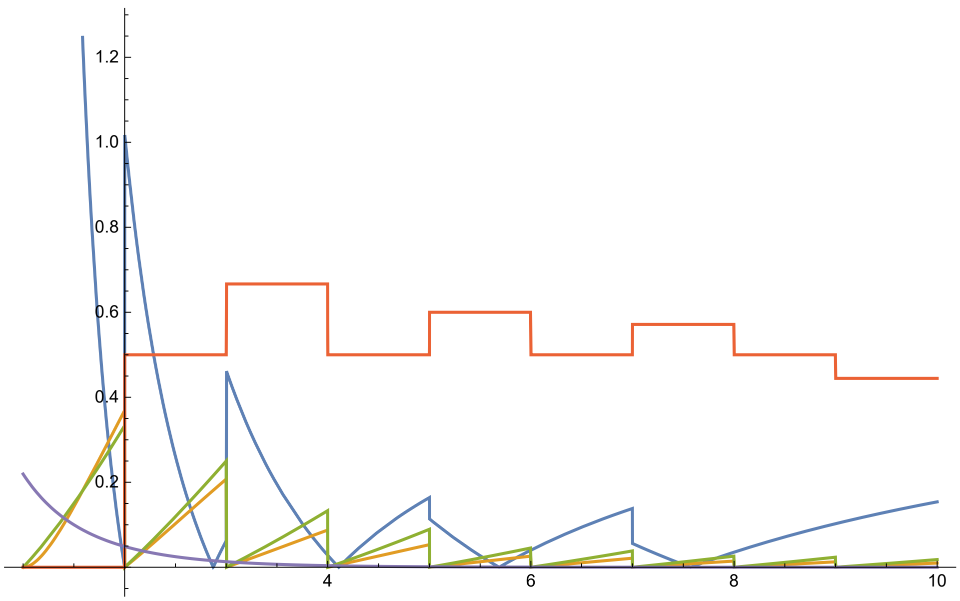
RHPi[x] has jumps at the primes, as expected, but also has changes in directions for other values (why???)

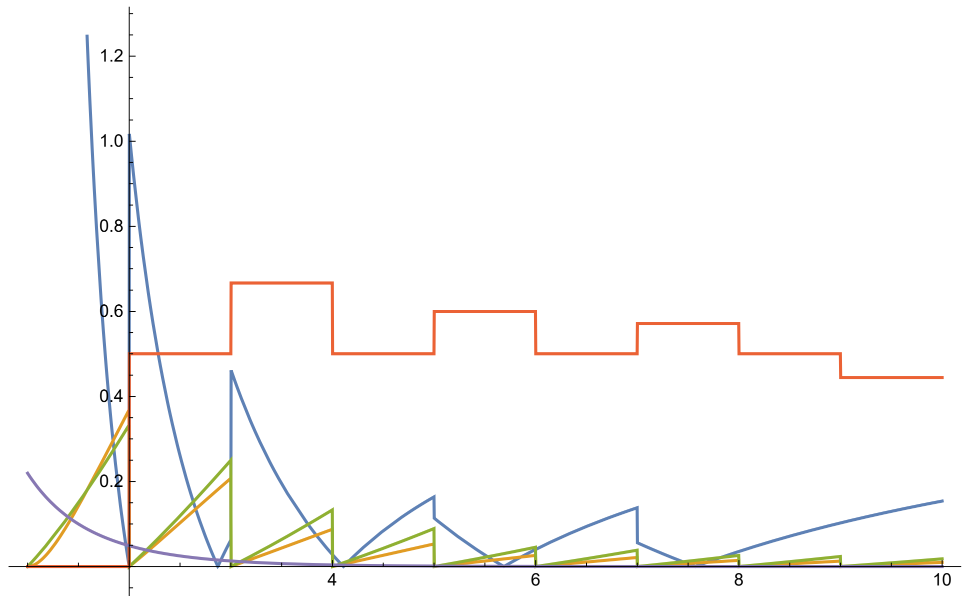
$$\texttt{Plot}\left[\left\{\texttt{RHPi[x], Zeta[x]},\ \frac{\texttt{PrimePi[x]}}{\texttt{♉[x]}},\ \texttt{ExpIntegralE[1, x]}\right\},\ \texttt{\{x, 1, 100\}, Exclusions} \to \texttt{None}\right]$$



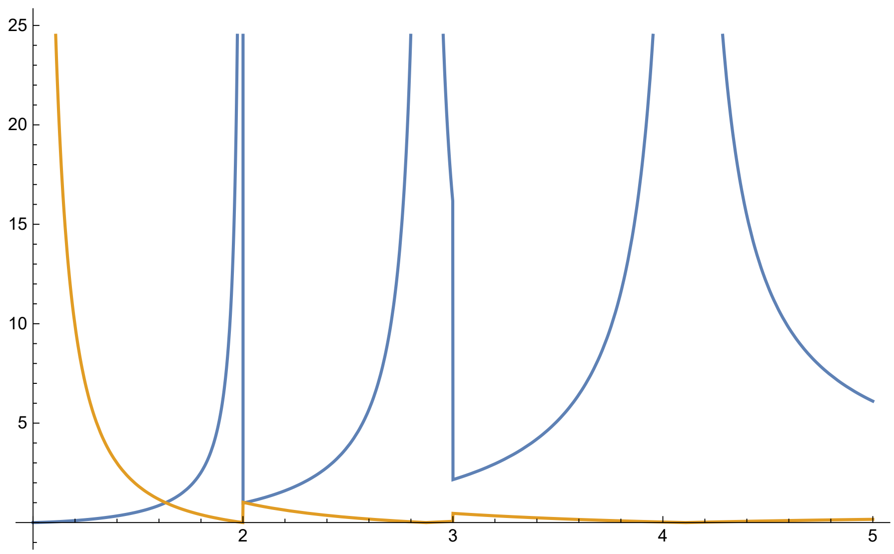
```
Plot[{RHPi[x], (Log[x] / (Log[Prime[Floor[x]]])) - (Log[x] / (Log[Prime[Floor[x]] + (x - Floor[x])])),
  (x / (Prime[Floor[x]])) - (x / (Prime[Floor[x]] + (x - Floor[x]))),
  PrimePi[x] / ɤ[x], ExpIntegralE[1, x]}, {x, 1, 10}, Exclusions → None]
```


```
Plot[{RHPi[x], (Log[x] / (Log[Prime[Floor[x]]])) - (Log[x] / (Log[Prime[Floor[x]] + (x - Floor[x])])),
  (x / (Prime[Floor[x]])) - (x / (Prime[Floor[x]] + (x - Floor[x]))),
  PrimePi[x] / ɤ[x], ExpIntegralE[1, x]}, {x, 1, 10}, Exclusions → None]
```

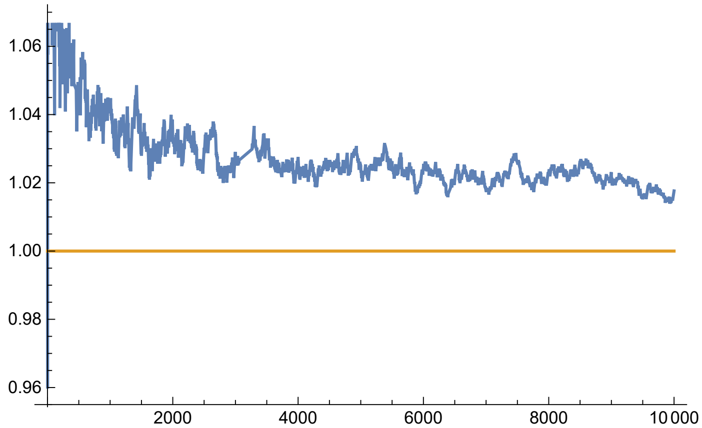



```
Plot[{1/(RHPi[x]), RHPi[x]}, {x, 1, 5}, Exclusions → None]
(*Shows where RHPi[x]-ExpIntegralE[1,x] =0*)
```

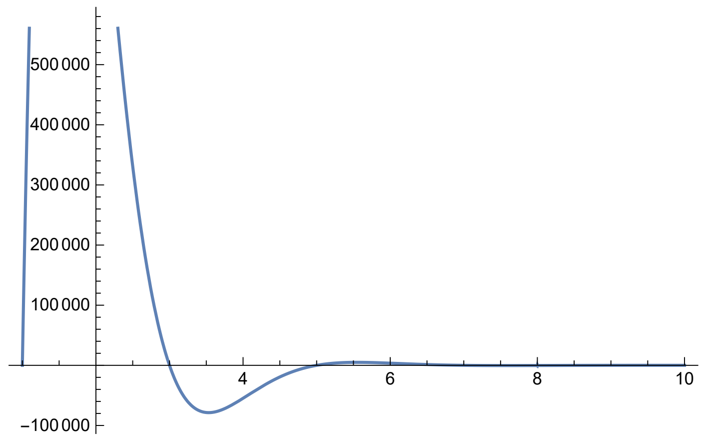


Notice the fn RHPi[x] has dips when

```
Plot[{E^RHPi[x], 1}, {x, 0, 10000}]
```




```
Plot[Sum[Zeta[x - i], {i, Table[Prime[n], {n, 1, 10}]}], {x, 1, 10}]
```



```
Sum[Zeta[k - i], {i, Table[Prime[n], {n, 1, 10}]}]
Table[{k, Sum[Zeta[k - i], {i, Table[Prime[n], {n, 1, 10}]}]}, {k, 1, 20}] // MatrixForm
```

Zeta[−29 + k] + Zeta[−23 + k] + Zeta[−19 + k] + Zeta[−17 + k] + Zeta[−13 + k] + Zeta[−11 + k] + Zeta[−7 + k] + Zeta[−5 + k] + Zeta[−3 + k] + Zeta[−2 + k]

$$\begin{pmatrix}
1 & -\frac{1}{12} \\
2 & \frac{181604178992477089}{186327165024} \\
3 & \text{ComplexInfinity} \\
4 & \text{ComplexInfinity} \\
5 & -\frac{1}{2}+\frac{\pi^2}{6}+\text{Zeta}[3] \\
6 & \text{ComplexInfinity} \\
7 & -\frac{1}{2}+\frac{\pi^2}{6}+\frac{\pi^4}{90}+\text{Zeta}[5] \\
8 & \text{ComplexInfinity} \\
9 & \frac{\pi^2}{6}+\frac{\pi^4}{90}+\frac{\pi^6}{945}+\text{Zeta}[7] \\
10 & \frac{347087}{13200}+\frac{\pi^8}{9450}+\text{Zeta}[3]+\text{Zeta}[5]+\text{Zeta}[7] \\
11 & -\frac{1}{2}+\frac{\pi^4}{90}+\frac{\pi^6}{945}+\frac{\pi^8}{9450}+\text{Zeta}[9] \\
12 & \text{ComplexInfinity} \\
13 & -\frac{1}{2}+\frac{\pi^2}{6}+\frac{\pi^6}{945}+\frac{\pi^8}{9450}+\frac{\pi^{10}}{93555}+\text{Zeta}[11] \\
14 & \text{ComplexInfinity} \\
15 & \frac{\pi^2}{6}+\frac{\pi^4}{90}+\frac{\pi^8}{9450}+\frac{\pi^{10}}{93555}+\frac{691\,\pi^{12}}{638512875}+\text{Zeta}[13] \\
16 & -\frac{37}{240}+\frac{2\,\pi^{14}}{18243225}+\text{Zeta}[3]+\text{Zeta}[5]+\text{Zeta}[9]+\text{Zeta}[11]+\text{Zeta}[13] \\
17 & -\frac{1}{2}+\frac{\pi^4}{90}+\frac{\pi^6}{945}+\frac{\pi^{10}}{93555}+\frac{691\,\pi^{12}}{638512875}+\frac{2\,\pi^{14}}{18243225}+\text{Zeta}[15] \\
18 & \text{ComplexInfinity} \\
19 & -\frac{1}{2}+\frac{\pi^2}{6}+\frac{\pi^6}{945}+\frac{\pi^8}{9450}+\frac{691\,\pi^{12}}{638512875}+\frac{2\,\pi^{14}}{18243225}+\frac{3617\,\pi^{16}}{325641566250}+\text{Zeta}[17] \\
20 & \text{ComplexInfinity}
\end{pmatrix}$$

```
Sum[Zeta[Prime[i]], {i, 1, 20}]
```

$$\frac{\pi^2}{6}+\text{Zeta}[3]+\text{Zeta}[5]+\text{Zeta}[7]+\text{Zeta}[11]+\text{Zeta}[13]+\text{Zeta}[17]+\text{Zeta}[19]+\text{Zeta}[23]+\text{Zeta}[29]+\text{Zeta}[31]+\text{Zeta}[37]+\text{Zeta}[41]+\text{Zeta}[43]+\text{Zeta}[47]+\text{Zeta}[53]+\text{Zeta}[59]+\text{Zeta}[61]+\text{Zeta}[67]+\text{Zeta}[71]$$

```
FourierTransform[HeavisideTheta[x - p], x, w]
FourierSequenceTransform[HeavisideTheta[x - p], x, w]
```

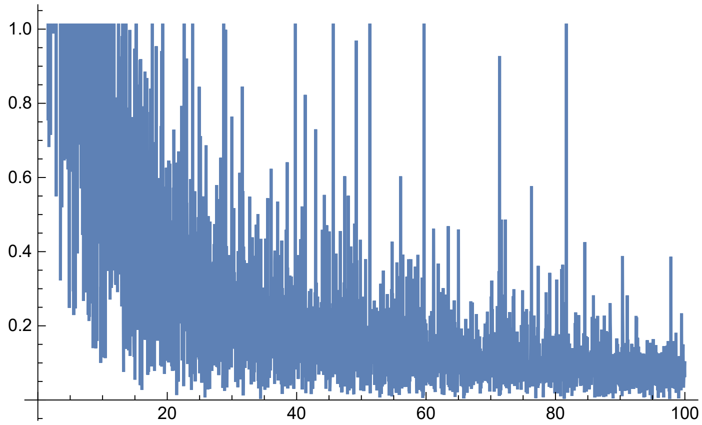
$$\frac{i\, e^{i p w}}{\sqrt{2\pi}\; w}+\sqrt{\frac{\pi}{2}}\ \text{DiracDelta}[w]$$

$$\begin{cases} \frac{e^{i w}}{-1+e^{i w}} & -1 < p \le 0 \\ \frac{e^{-i w\,(-1+\text{Ceiling}[p])}}{-1+e^{i w}} & \text{True} \end{cases}$$

```
Pi1 = FourierTransform[Sum[HeavisideTheta[x - p], {p, Table[Prime[n], {n, 1, 1000}]}], x, w];
```

```
Pi2 = FourierSequenceTransform[
   Sum[HeavisideTheta[x - p], {p, Table[Prime[n], {n, 1, 100}]}], x, w];
```

```
Plot[{Abs[Pi1]}, {w, 0, 100}]
```




```
FourierSequenceTransform[HeavisideTheta[x], x, w] // FullSimplify
```

$$1+\frac{1}{-1+e^{i w}}$$

$$\int\left(1+\frac{1}{-1+e^{i w}}\right)dw$$

$$-i \operatorname{Log}\left[1-e^{i w}\right]$$

```
FourierTransform[HeavisideTheta[x - n], x, w]
```

$$\frac{i e^{i n w}}{\sqrt{2\pi}\, w}+\sqrt{\frac{\pi}{2}}\ \text{DiracDelta}[w]$$

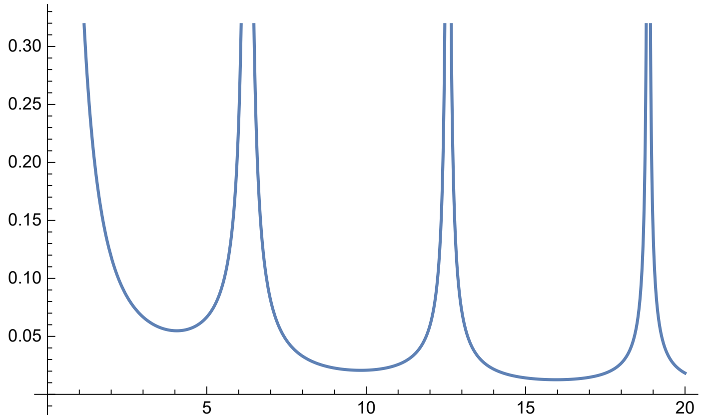
$$\sum_{n=0}^{\infty}\left(\frac{i e^{i n w}}{\sqrt{2\pi}\, w}\right)$$

$$\sum_{n=1}^{\infty}\left(\frac{i e^{i n w}}{\sqrt{2\pi}\, w}\right)$$

$$-\frac{i}{\left(-1+e^{i w}\right)\sqrt{2\pi}\, w}$$

$$-\frac{i e^{i w}}{\left(-1+e^{i w}\right)\sqrt{2\pi}\, w}$$

$$\text{Plot}\left[\left\{\text{Abs}\left[-\frac{i}{\left(-1+e^{i x}\right)\sqrt{2\pi}\ x}\right]\right\}, \{x, 0, 20\}\right]$$



$$\sum_{n=1}^{\infty}\text{HeavisideTheta}[x-n]$$

$$\begin{cases} \text{Floor}[x] & x \geq 1 \\ 0 & \text{True} \end{cases}$$

Note that this would really be
diff using the half maximum conv.

```
FourierTransform[DiracDelta[x - n], x, w] // FullSimplify
```

$$\frac{e^{i n w}}{\sqrt{2\pi}}$$

$$\text{FourierTransform}\left[\sum_{n=0}^{N}\text{DiracDelta}[x-n], x, w\right] \text{ // FullSimplify}$$

$$\frac{-1+e^{i(1+N)w}}{\left(-1+e^{i w}\right)\sqrt{2\pi}}$$

$$\text{FourierTransform}\left[\sum_{n=0}^{\infty}\text{DiracDelta}[x-n], x, w\right] \text{ // FullSimplify}$$

$$-\frac{1}{\left(-1+e^{i w}\right)\sqrt{2\pi}}$$

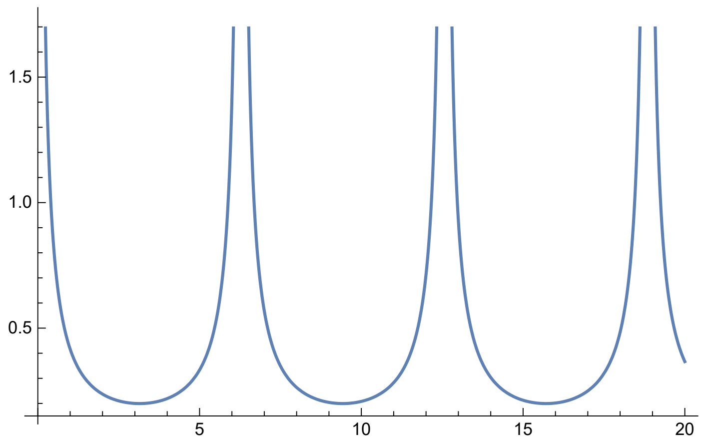
```
FourierTransform[DiracComb[x - n], x, w] // FullSimplify
```

$$\frac{e^{i n w}\,\text{DiracComb}\left[\frac{w}{2\pi}\right]}{\sqrt{2\pi}}$$

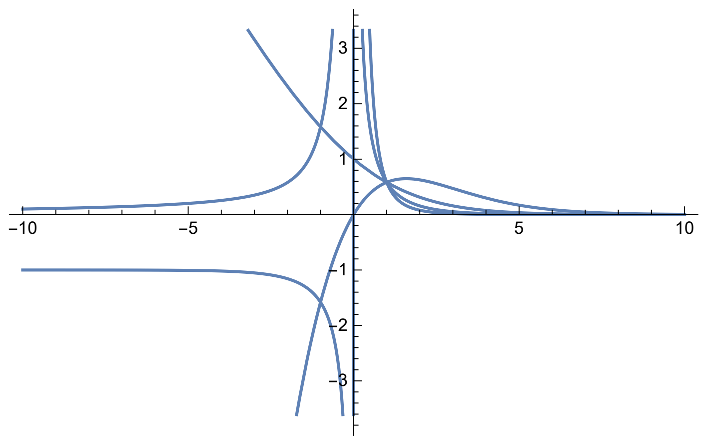
$$\text{Table}\left[\left\{\text{Abs}\left[\text{Exp}\left[i\left(\sum_{n=1}^{i}\text{Prime}[n]\right)\right]\right]\right\}, \{i, 1, 10\}\right]$$

```
{{1}, {1}, {1}, {1}, {1}, {1}, {1}, {1}, {1}, {1}}
```

$$\texttt{Plot}\left[\left\{\texttt{Abs}\left[-\frac{1}{\left(-1+e^{i x}\right)\sqrt{2\pi}}\right]\right\}, \{x, 0, 20\}\right]$$



$$\texttt{Plot}\left[\left\{\texttt{Table}\left[\frac{(x^{\wedge}(s-1))}{(E^{\wedge}x)-1}, \{s, 0, 3\}\right]\right\}, \{x, -10, 10\}\right]$$




```
FourierTransform[HeavisideTheta[x - n], x, w]
FourierTransform[DiracDelta[x - n], x, w] // FullSimplify
```

$$\frac{i\, e^{i n w}}{\sqrt{2\pi}\, w} + \sqrt{\frac{\pi}{2}}\ \texttt{DiracDelta}[w]$$

$$\frac{e^{i n w}}{\sqrt{2\pi}}$$

$$\sum_{n=1}^{100}\left(\frac{i\, e^{i\, \texttt{Prime}[n]\, x}}{\sqrt{2\pi}\, x} + \sqrt{\frac{\pi}{2}}\ \texttt{DiracDelta}[x]\right);$$

$$\sum_{n=1}^{100}\left(\frac{e^{i\, \texttt{Prime}[n]\, x}}{\sqrt{2\pi}}\right);$$

$$\frac{i\, e^{2 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{3 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{5 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{7 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{11 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{13 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{17 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{19 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{23 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{29 i x}}{\sqrt{2\pi}\, x}+$$

$$\frac{i\, e^{31 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{37 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{41 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{43 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{47 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{53 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{59 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{61 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{67 i x}}{\sqrt{2\pi}\, x}+$$

$$\frac{i\, e^{71 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{73 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{79 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{83 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{89 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{97 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{101 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{103 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{107 i x}}{\sqrt{2\pi}\, x}+$$

$$\frac{i\, e^{109 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{113 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{127 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{131 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{137 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{139 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{149 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{151 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{157 i x}}{\sqrt{2\pi}\, x}+$$

$$\frac{i\, e^{163 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{167 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{173 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{179 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{181 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{191 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{193 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{197 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{199 i x}}{\sqrt{2\pi}\, x}+$$

$$\frac{i\, e^{211 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{223 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{227 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{229 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{233 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{239 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{241 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{251 i x}}{\sqrt{2\pi}\, x}+$$

$$\frac{i\, e^{257 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{263 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{269 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{271 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{277 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{281 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{283 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{293 i x}}{\sqrt{2\pi}\, x}+$$

$$\frac{i\, e^{307 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{311 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{313 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{317 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{331 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{337 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{347 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{349 i x}}{\sqrt{2\pi}\, x}+$$

$$\frac{i\, e^{353 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{359 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{367 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{373 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{379 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{383 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{389 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{397 i x}}{\sqrt{2\pi}\, x}+$$

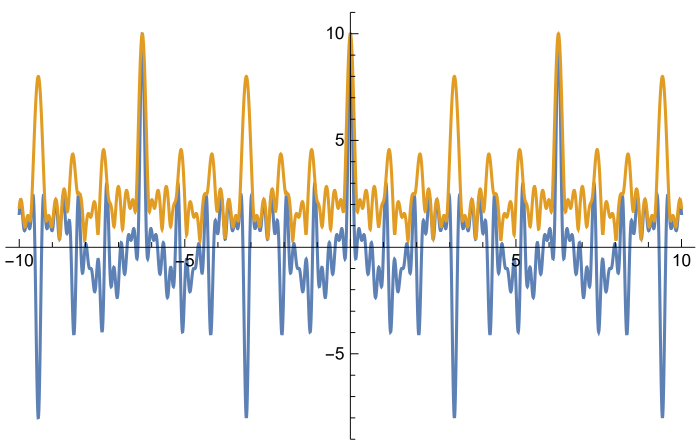
$$\frac{i\, e^{401 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{409 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{419 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{421 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{431 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{433 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{439 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{443 i x}}{\sqrt{2\pi}\, x}+$$

$$\frac{i\, e^{449 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{457 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{461 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{463 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{467 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{479 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{487 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{491 i x}}{\sqrt{2\pi}\, x}+$$

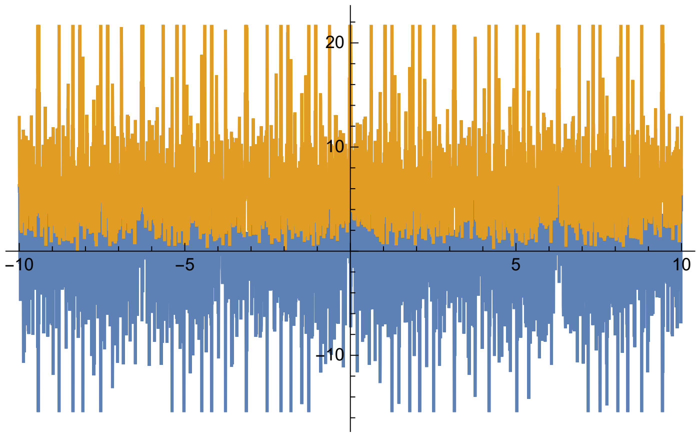
$$\frac{i\, e^{499 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{503 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{509 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{521 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{523 i x}}{\sqrt{2\pi}\, x}+\frac{i\, e^{541 i x}}{\sqrt{2\pi}\, x}+50\sqrt{2\pi}\,\mathrm{DiracDelta}[x]$$

$$\frac{e^{2 i x}}{\sqrt{2\pi}}+\frac{e^{3 i x}}{\sqrt{2\pi}}+\frac{e^{5 i x}}{\sqrt{2\pi}}+\frac{e^{7 i x}}{\sqrt{2\pi}}+\frac{e^{11 i x}}{\sqrt{2\pi}}+\frac{e^{13 i x}}{\sqrt{2\pi}}+\frac{e^{17 i x}}{\sqrt{2\pi}}+\frac{e^{19 i x}}{\sqrt{2\pi}}+\frac{e^{23 i x}}{\sqrt{2\pi}}+\frac{e^{29 i x}}{\sqrt{2\pi}}+$$
$$\frac{e^{31 i x}}{\sqrt{2\pi}}+\frac{e^{37 i x}}{\sqrt{2\pi}}+\frac{e^{41 i x}}{\sqrt{2\pi}}+\frac{e^{43 i x}}{\sqrt{2\pi}}+\frac{e^{47 i x}}{\sqrt{2\pi}}+\frac{e^{53 i x}}{\sqrt{2\pi}}+\frac{e^{59 i x}}{\sqrt{2\pi}}+\frac{e^{61 i x}}{\sqrt{2\pi}}+\frac{e^{67 i x}}{\sqrt{2\pi}}+\frac{e^{71 i x}}{\sqrt{2\pi}}+$$
$$\frac{e^{73 i x}}{\sqrt{2\pi}}+\frac{e^{79 i x}}{\sqrt{2\pi}}+\frac{e^{83 i x}}{\sqrt{2\pi}}+\frac{e^{89 i x}}{\sqrt{2\pi}}+\frac{e^{97 i x}}{\sqrt{2\pi}}+\frac{e^{101 i x}}{\sqrt{2\pi}}+\frac{e^{103 i x}}{\sqrt{2\pi}}+\frac{e^{107 i x}}{\sqrt{2\pi}}+\frac{e^{109 i x}}{\sqrt{2\pi}}+\frac{e^{113 i x}}{\sqrt{2\pi}}+$$
$$\frac{e^{127 i x}}{\sqrt{2\pi}}+\frac{e^{131 i x}}{\sqrt{2\pi}}+\frac{e^{137 i x}}{\sqrt{2\pi}}+\frac{e^{139 i x}}{\sqrt{2\pi}}+\frac{e^{149 i x}}{\sqrt{2\pi}}+\frac{e^{151 i x}}{\sqrt{2\pi}}+\frac{e^{157 i x}}{\sqrt{2\pi}}+\frac{e^{163 i x}}{\sqrt{2\pi}}+\frac{e^{167 i x}}{\sqrt{2\pi}}+\frac{e^{173 i x}}{\sqrt{2\pi}}+$$
$$\frac{e^{179 i x}}{\sqrt{2\pi}}+\frac{e^{181 i x}}{\sqrt{2\pi}}+\frac{e^{191 i x}}{\sqrt{2\pi}}+\frac{e^{193 i x}}{\sqrt{2\pi}}+\frac{e^{197 i x}}{\sqrt{2\pi}}+\frac{e^{199 i x}}{\sqrt{2\pi}}+\frac{e^{211 i x}}{\sqrt{2\pi}}+\frac{e^{223 i x}}{\sqrt{2\pi}}+\frac{e^{227 i x}}{\sqrt{2\pi}}+\frac{e^{229 i x}}{\sqrt{2\pi}}+$$
$$\frac{e^{233 i x}}{\sqrt{2\pi}}+\frac{e^{239 i x}}{\sqrt{2\pi}}+\frac{e^{241 i x}}{\sqrt{2\pi}}+\frac{e^{251 i x}}{\sqrt{2\pi}}+\frac{e^{257 i x}}{\sqrt{2\pi}}+\frac{e^{263 i x}}{\sqrt{2\pi}}+\frac{e^{269 i x}}{\sqrt{2\pi}}+\frac{e^{271 i x}}{\sqrt{2\pi}}+\frac{e^{277 i x}}{\sqrt{2\pi}}+\frac{e^{281 i x}}{\sqrt{2\pi}}+$$
$$\frac{e^{283 i x}}{\sqrt{2\pi}}+\frac{e^{293 i x}}{\sqrt{2\pi}}+\frac{e^{307 i x}}{\sqrt{2\pi}}+\frac{e^{311 i x}}{\sqrt{2\pi}}+\frac{e^{313 i x}}{\sqrt{2\pi}}+\frac{e^{317 i x}}{\sqrt{2\pi}}+\frac{e^{331 i x}}{\sqrt{2\pi}}+\frac{e^{337 i x}}{\sqrt{2\pi}}+\frac{e^{347 i x}}{\sqrt{2\pi}}+\frac{e^{349 i x}}{\sqrt{2\pi}}+$$
$$\frac{e^{353 i x}}{\sqrt{2\pi}}+\frac{e^{359 i x}}{\sqrt{2\pi}}+\frac{e^{367 i x}}{\sqrt{2\pi}}+\frac{e^{373 i x}}{\sqrt{2\pi}}+\frac{e^{379 i x}}{\sqrt{2\pi}}+\frac{e^{383 i x}}{\sqrt{2\pi}}+\frac{e^{389 i x}}{\sqrt{2\pi}}+\frac{e^{397 i x}}{\sqrt{2\pi}}+\frac{e^{401 i x}}{\sqrt{2\pi}}+\frac{e^{409 i x}}{\sqrt{2\pi}}+$$
$$\frac{e^{419 i x}}{\sqrt{2\pi}}+\frac{e^{421 i x}}{\sqrt{2\pi}}+\frac{e^{431 i x}}{\sqrt{2\pi}}+\frac{e^{433 i x}}{\sqrt{2\pi}}+\frac{e^{439 i x}}{\sqrt{2\pi}}+\frac{e^{443 i x}}{\sqrt{2\pi}}+\frac{e^{449 i x}}{\sqrt{2\pi}}+\frac{e^{457 i x}}{\sqrt{2\pi}}+\frac{e^{461 i x}}{\sqrt{2\pi}}+\frac{e^{463 i x}}{\sqrt{2\pi}}+$$
$$\frac{e^{467 i x}}{\sqrt{2\pi}}+\frac{e^{479 i x}}{\sqrt{2\pi}}+\frac{e^{487 i x}}{\sqrt{2\pi}}+\frac{e^{491 i x}}{\sqrt{2\pi}}+\frac{e^{499 i x}}{\sqrt{2\pi}}+\frac{e^{503 i x}}{\sqrt{2\pi}}+\frac{e^{509 i x}}{\sqrt{2\pi}}+\frac{e^{521 i x}}{\sqrt{2\pi}}+\frac{e^{523 i x}}{\sqrt{2\pi}}+\frac{e^{541 i x}}{\sqrt{2\pi}}$$

Plot$\left[\left\{\mathbf{Re}\left[\sum_{n=1}^{10}\left(e^{i\,\mathrm{Prime}[n]\,x}\right)\right], \mathbf{Abs}\left[\sum_{n=1}^{10}\left(e^{i\,\mathrm{Prime}[n]\,x}\right)\right]\right\}, \{x, -10, 10\}\right]$



```
Plot[{Re[∑_{n=1}^{100} (ⅇ^(ⅈ Prime[n] x))], Abs[∑_{n=1}^{100} (ⅇ^(ⅈ Prime[n] x))]}, {x, -10, 10}]
```



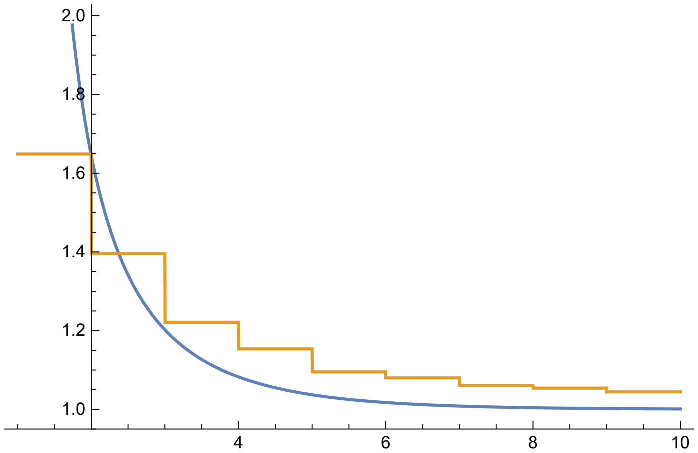

```
xx = 89
∑_{n=1}^{100} (ⅇ^(ⅈ Prime[n] xx)) // N
Abs[∑_{n=1}^{100} (ⅇ^(ⅈ Prime[n] xx)) // N]
```

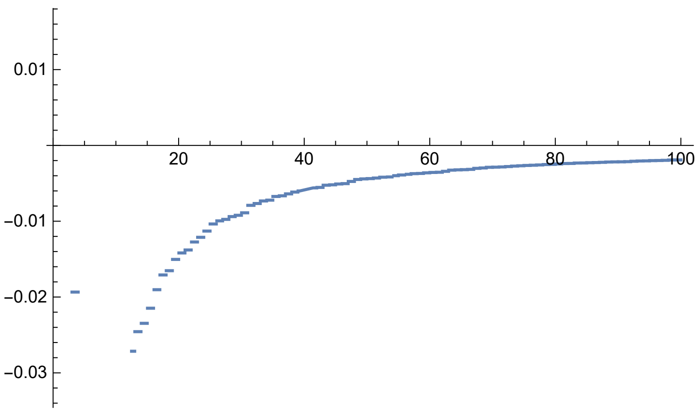
89

0.77673 − 3.33179 ⅈ

3.42113

```
Plot[{Zeta[x], (E)^(1/(Prime[Floor[x]]))}, {x, 1, 10}]
Plot[{Zeta[Floor[x]] - (E)^(1/(Prime[Floor[x]]))}, {x, 1, 100}]
```






```
Zeta[2] // N
E^(1/2) // N
```

1.64493

1.64872

```
Solve[Zeta[x] - (E)^(1/3) == 0, x]
```

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. ≫

$\{\{x \to \text{Root}[\{-e^{1/3} + \text{Zeta}[\#1]\ \&,\ 2.37342902687398411976097535667\}]\}\}$

$$\sum_{n=1}^{10} (\text{Zeta}[\text{Prime}[n]])$$

$$\frac{\pi^2}{6} + \text{Zeta}[3] + \text{Zeta}[5] + \text{Zeta}[7] + \text{Zeta}[11] + \text{Zeta}[13] + \text{Zeta}[17] + \text{Zeta}[19] + \text{Zeta}[23] + \text{Zeta}[29]$$

```
Zeta[9]
```

Zeta[9]

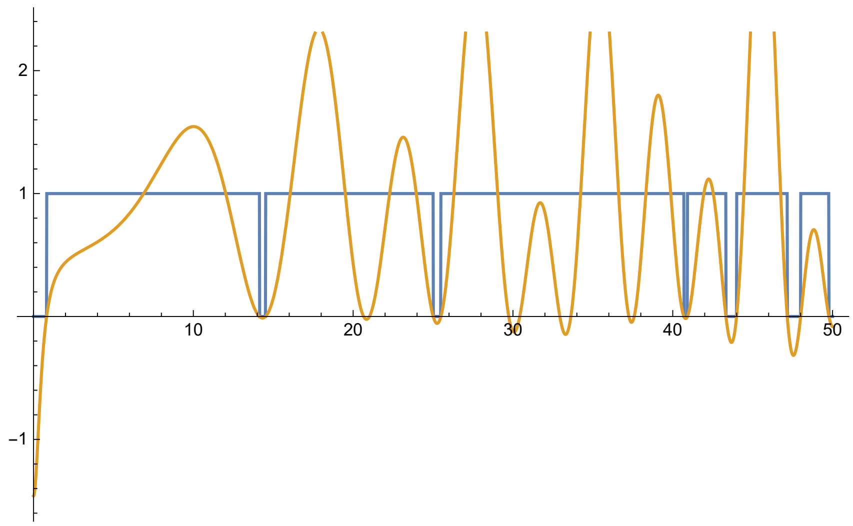
```
N[Zeta[9]]
```

```
1.00201
```

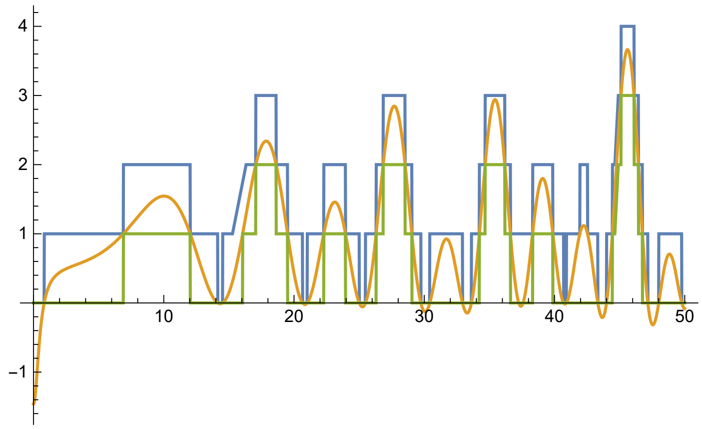
Zeta Prime Counting

Heaviside Zeta

```
Plot[{HeavisideTheta[Re[Zeta[(1/2) + ⅈ x]]], Re[Zeta[(1/2) + ⅈ x]]},
 {x, 0, 50}, Exclusions → None]
```




```
Plot[{Sum[HeavisideTheta[Re[Zeta[(1/2) + ⅈ x]] - n], {n, 0, 10}], Re[Zeta[(1/2) + ⅈ x]],
  Sum[HeavisideTheta[Re[Zeta[(1/2) + ⅈ x]] - n], {n, 1, 10}]}, {x, 0, 50}, Exclusions → None]
```



```
Plot[{HeavisideTheta[Re[Zeta[(-2) + ⅈ x]]], HeavisideTheta[Re[Zeta[(-4) + ⅈ x]]],
  HeavisideTheta[Re[Zeta[(-6) + ⅈ x]]]}, {x, 0, 50}, Exclusions → None]
```