Prime Construction

Imagine an infinite set of natural numbers $\mathcal{B}$ that satisfies the following condition:

Upon factoring the components of this set, every prime number appears as a factor at least once
(Want a set $\mathcal{B}$ other than the set of primes, which would satisfy this trivially)

i.e. for any $b \in \mathcal{B}$ such that it has prime decomposition of form b =
(Prime[1]^a_1) (Prime[2]^a_2) .... (Prime[k]^a_k) ....

Then want $\mathcal{B}$ such that every a_k is non-zero for at least one b

Is the set of integers the minimal set that satisfies this condition?? If so it would imply that primes are baically necessary to construct the set of integers.

If not, what is the minimal such set? Is it special in any other way

Zeta Factorial Sum

Noting that the total number of totient numbers (see wiki) up to a given limit x is

$$\text{Abs}[\{n \text{ such that } \text{EulerPhi}[n] \le x\}] = \left(\frac{\text{Zeta}[2]\,\text{Zeta}[3]}{\text{Zeta}[6]}\right) x + R[x]$$

Where R the error term is at most $\frac{x}{(\text{Log}[x])\text{^}k}$ for any positive k

Side Note [
(which looks like alot of terms related to prime counting fn) ×
and in fact the asymptotic expansion
of LogIntegral[x] consists of terms of form $\frac{x}{(\text{Log}[x])\text{^}k}$,
so we might say that $\text{LogIntegral}[x] \sim \frac{x}{\text{Log}[x]} \sum_{k=0}^{\infty} \frac{k!}{(\text{Log}[x])\text{^}k} =$

$$R[x, 1] \sum_{k=1}^{\infty} (k!)\, R[x, k]$$ ×

R[x, k] being the maximal error R[x] at x,

such that $R[x, k] \ge \text{Abs}[\{n \text{ such that } \text{EulerPhi}[n] \le x\}] - \left(\frac{\text{Zeta}[2]\,\text{Zeta}[3]}{\text{Zeta}[6]}\right) x$
]

So we want to construct a function and / or series of the form ,
for any a (especially h = 0 case),

$$\left(\frac{\text{Zeta}[2]\ \text{Zeta}[3]}{\text{Zeta}[2*3]}\right)(x\verb|^|h)$$

$$\to (x\verb|^|h) \sum_{n=2}^{N} \frac{\prod_{k=2}^{n} \text{Zeta}[k]}{\text{Zeta}[n!]} = (x\verb|^|h) \sum_{n=2}^{N} \frac{\text{Zeta}[2]\ \text{Zeta}[3]\ \ldots\ \text{Zeta}[n]}{\text{Zeta}[n!]}$$

$$\prod_{k=2}^{n} \text{Zeta}[k]$$

And a function of the form
for b = $(\text{Prime}[1]\verb|^|a_1)\ (\text{Prime}[2]\verb|^|a_2) \ldots.(\text{Prime}[k]\verb|^|a_k)$
(x^h)

$$\sum_{k=2}^{n} \frac{1}{\text{Zeta}[k]} (\text{Zeta}[\text{Prime}[1]] * a_1)\ (\text{Zeta}[\text{Prime}[2]] * a_2) \ldots (\text{Zeta}[\text{Prime}[i]] * a_i)$$

where $a_k \geq 1$ is the number of times the factor Prime[i] goes into k

Product[Zeta[k], {k, 2, 9}]
Product[Zeta[k], {k, 2, 19}] // N

$$\frac{\pi^{20}\ \text{Zeta}[3]\ \text{Zeta}[5]\ \text{Zeta}[7]\ \text{Zeta}[9]}{4\,822\,335\,000}$$

2.29485

$$\frac{\pi^{20}}{4\,822\,335\,000} \ // \ N$$
Zeta[3] Zeta[5] Zeta[7] Zeta[9] // N

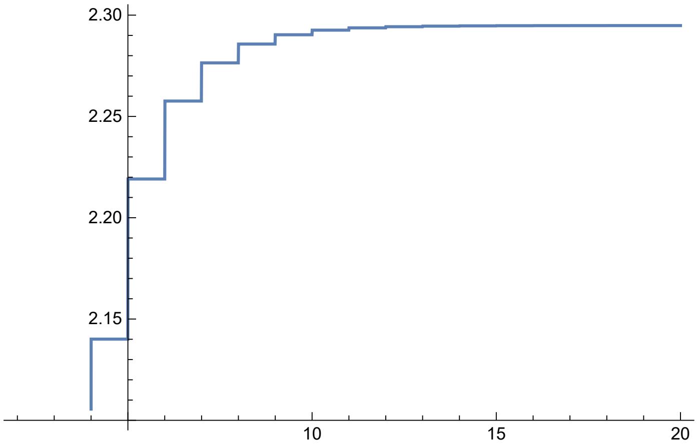
1.81861

1.25938

Table[Zeta[2 k], {k, 1, 20}]

$$\left\{\frac{\pi^2}{6}, \frac{\pi^4}{90}, \frac{\pi^6}{945}, \frac{\pi^8}{9450}, \frac{\pi^{10}}{93\,555}, \frac{691\,\pi^{12}}{638\,512\,875}, \frac{2\,\pi^{14}}{18\,243\,225}, \frac{3617\,\pi^{16}}{325\,641\,566\,250}, \frac{43\,867\,\pi^{18}}{38\,979\,295\,480\,125}, \frac{174\,611\,\pi^{20}}{1\,531\,329\,465\,290\,625}, \frac{155\,366\,\pi^{22}}{13\,447\,856\,940\,643\,125}, \frac{236\,364\,091\,\pi^{24}}{201\,919\,571\,963\,756\,521\,875}, \frac{1\,315\,862\,\pi^{26}}{11\,094\,481\,976\,030\,578\,125}, \frac{6\,785\,560\,294\,\pi^{28}}{564\,653\,660\,170\,076\,273\,671\,875}, \frac{6\,892\,673\,020\,804\,\pi^{30}}{5\,660\,878\,804\,669\,082\,674\,070\,015\,625}, \frac{7\,709\,321\,041\,217\,\pi^{32}}{62\,490\,220\,571\,022\,341\,207\,266\,406\,250}, \frac{151\,628\,697\,551\,\pi^{34}}{12\,130\,454\,581\,433\,748\,587\,292\,890\,625}, \frac{26\,315\,271\,553\,053\,477\,373\,\pi^{36}}{20\,777\,977\,561\,866\,588\,586\,487\,628\,662\,044\,921\,875}, \frac{308\,420\,411\,983\,322\,\pi^{38}}{2\,403\,467\,618\,492\,375\,776\,343\,276\,883\,984\,375}, \frac{261\,082\,718\,496\,449\,122\,051\,\pi^{40}}{20\,080\,431\,172\,289\,638\,826\,798\,401\,128\,390\,556\,640\,625}\right\}$$

```
Plot[Product[ Zeta[k], {k, 2, x}], {x, 2, 20}]
```




```
ConZetaProd = Product[ Zeta[k], {k, 2, 1000}] // N
(*Seems to converge to ~ 2.29486...,
which is near the universal parabolic contant Log[1+√2]+√2 = 2.295587... *)
```

2.29486

So $\prod_{k=2}^{n}$ Zeta[k] converges to ≈ 2.29486, is there anything special about this number?

```
ConZetaProd
1/ConZetaProd
Exp[ConZetaProd]
Exp[1/ConZetaProd]
```

2.29486

0.435757

9.92301

1.54613

```
(Product[ Zeta[2 k], {k, 1, nmax}] + Product[ Zeta[(2 k) + 1], {k, 1, nmax}]) / 2 // N
```

1.54061

```
nmax = 100;
Product[ Zeta[2 k], {k, 1, nmax}] // N
Product[ Zeta[(2 k) + 1], {k, 1, nmax}] // N
(Product[ Zeta[2 k], {k, 1, nmax}] + Product[ Zeta[(2 k) + 1], {k, 1, nmax}]) // N
(Product[ Zeta[2 k], {k, 1, nmax}] - Product[ Zeta[(2 k) + 1], {k, 1, nmax}]) // N
```

1.82101745149876

1.26021

3.08122

0.560812

$$\sum_{k=2}^{4} \frac{\texttt{Zeta[2] Zeta[3] ... Zeta[k]}}{\texttt{Zeta[k!]}}$$

$$\left(\frac{1}{6}\pi^2\,\text{Zeta}[3]\,\ldots\right) + \frac{4\,487\,101\,599\,194\,589\,375\left(\frac{1}{6}\pi^2\,\text{Zeta}[3]\,\ldots\right)}{472\,728\,182\,\pi^{20}} + \frac{945\left(\frac{1}{6}\pi^2\,\text{Zeta}[3]\,\ldots\right)\text{Zeta}[3]}{\pi^6}$$

```
ZPconst1 = (Product[Zeta[2 k], {k, 1, 100}] - Product[Zeta[(2 k) + 1], {k, 1, 100}]) // N
```

0.560812

```
nmax = 10;
Product[Zeta[k], {k, 2, nmax}] // N
Product[Zeta[(2 k) + 1], {k, 1, nmax}] // N
```

2.2926

1.26021

Numerical similarities of ZPconst1 to EulerGamma

```
1 / 0.560811740793514`
```

1.78313

```
ZPconst1 = (Product[Zeta[2 k], {k, 1, 100}] - Product[Zeta[(2 k) + 1], {k, 1, 100}]) // N
```

0.560812

```
EulerGamma // N
Exp[EulerGamma] // N
Exp[EulerGamma] ^ (-1) // N
Exp[0.560811740793514`] ^ (-1)
```

0.577216

1.78107

0.561459

0.570746

```
Exp[EulerGamma] ^ (-1) + Exp[ZPconst1] ^ (-1)
((EulerGamma) + (1 / Exp[EulerGamma])) // N
((ZPconst1) + (1 / Exp[ZPconst1])) // N
ZPconst1 + EulerGamma
```

1.13221

1.13868

1.13156

1.13803

```
(ZPconst1 + EulerGamma)/2
```

0.569014

```
zp1 = Nest[1/Exp[#] &, 0.560811740793514`, 200] (*latter term above
  Converges to this value which happens to be very close to EulerGamma *)
```

0.567143

Althogh we can replace 0.560811740793514` with other numbers and it'll converge to the same thing, so

```
zp1
EulerGamma // N
Exp[zp1] // N
Exp[EulerGamma] // N
zp1 - EulerGamma // N
Exp[zp1] - Exp[EulerGamma] // N
Exp[zp1 - EulerGamma]
1 - Exp[zp1 - EulerGamma]
```

0.567143

0.577216

1.76322

1.78107

-0.0100724

-0.0178496

0.989978

0.0100218

```
zp1 + EulerGamma
(Exp[zp1] + Exp[EulerGamma]) // N
(-(zp1 + EulerGamma) + (Exp[zp1] + Exp[EulerGamma])) // N
((zp1 + EulerGamma) + (Exp[zp1] + Exp[EulerGamma])) / 2 // N
```

1.14436

3.5443

2.39994

2.34433

Note that ((Exp[zp1] + Exp[EulerGamma]) - (zp1 + EulerGamma))
  is nearly the golden angle (in radians) :

```
π (3 - √5) // N
((Exp[zp1] + Exp[EulerGamma]) - (zp1 + EulerGamma))
π (3 - √5) - ((Exp[zp1] + Exp[EulerGamma]) - (zp1 + EulerGamma))
π (3 - √5) - ((Exp[ZPconst1] + Exp[EulerGamma]) - (ZPconst1 + EulerGamma))
((ZPconst1) + (Exp[ZPconst1])) // N
((zp1) + (Exp[zp1])) // N
((EulerGamma) + (1 / Exp[EulerGamma])) // N
```

2.39996

2.39994

0.0000269327

0.00482405

2.31291

2.33037

1.13868

```
π (3 - √5) // N
```

Sum of Prime Factors

```
Table[{i, FactorInteger[i]}, {i, 1, 25}] // MatrixForm
```

$$\begin{pmatrix} 1 & \{\{1, 1\}\} \\ 2 & \{\{2, 1\}\} \\ 3 & \{\{3, 1\}\} \\ 4 & \{\{2, 2\}\} \\ 5 & \{\{5, 1\}\} \\ 6 & \{\{2, 1\}, \{3, 1\}\} \\ 7 & \{\{7, 1\}\} \\ 8 & \{\{2, 3\}\} \\ 9 & \{\{3, 2\}\} \\ 10 & \{\{2, 1\}, \{5, 1\}\} \\ 11 & \{\{11, 1\}\} \\ 12 & \{\{2, 2\}, \{3, 1\}\} \\ 13 & \{\{13, 1\}\} \\ 14 & \{\{2, 1\}, \{7, 1\}\} \\ 15 & \{\{3, 1\}, \{5, 1\}\} \\ 16 & \{\{2, 4\}\} \\ 17 & \{\{17, 1\}\} \\ 18 & \{\{2, 1\}, \{3, 2\}\} \\ 19 & \{\{19, 1\}\} \\ 20 & \{\{2, 2\}, \{5, 1\}\} \\ 21 & \{\{3, 1\}, \{7, 1\}\} \\ 22 & \{\{2, 1\}, \{11, 1\}\} \\ 23 & \{\{23, 1\}\} \\ 24 & \{\{2, 3\}, \{3, 1\}\} \\ 25 & \{\{5, 2\}\} \end{pmatrix}$$

```
(* FactorInteger[20!];
FactorInteger[100!];
FactorInteger[1270!]; *)
```

```
FactorInteger[360]
FactorInteger[360][[2, 2]]
```

```
{{2, 3}, {3, 2}, {5, 1}}
```

```
2
```

```
(2 + 2 + 2) + (3 + 3) + 5
```

```
17
```

```
(FactorInteger[360][[1, 1]]) (FactorInteger[360][[1, 2]])
```

```
6
```

```
PrimeNu[360] (*Number of distinct primes*)
Sum[(FactorInteger[360][[i, 1]]) (FactorInteger[360][[i, 2]]), {i, 1, PrimeNu[360]}]
```

```
3
```

```
17
```

```
FactorInteger[100]
```

```
{{2, 2}, {5, 2}}
```

```
SumOfEachPrimeFactorOnce[Integer_] :=
 Sum[(FactorInteger[Integer][[i, 1]]), {i, 1, PrimeNu[Integer]}]
```

```
SumOfEachPrimeFactorOnce[10]
SumOfEachPrimeFactorOnce[100]
SumOfEachPrimeFactorOnce[360]
```

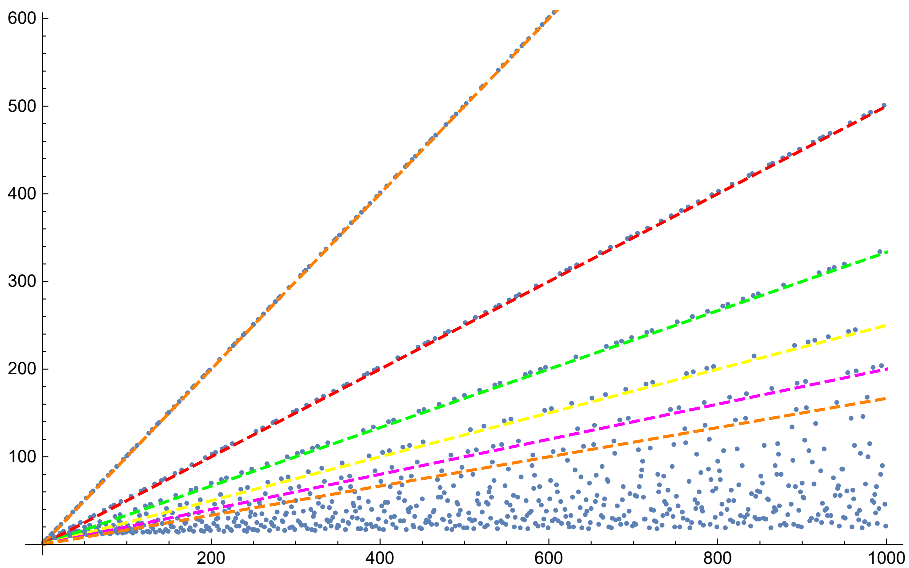
```
7
```

```
7
```

```
10
```

```
PrimeFactorSum[Integer_] :=
 Sum[(FactorInteger[Integer][[i, 1]]) (FactorInteger[Integer][[i, 2]]),
  {i, 1, PrimeNu[Integer]}]
```

```
Show[
 ListPlot[Table[PrimeFactorSum[k], {k, 2, 1000}]] ,
 Plot[{k, 1/2 k, 1/3 k, 1/4 k, 1/5 k, 1/6 k}, {k, 2, 1000}, PlotStyle →
   {{Orange, Dashed}, {Red, Dashed}, {Green, Dashed}, {Yellow, Dashed}, {Magenta, Dashed}}]
]
```



Mean and Appx of Log fns to mean

```
FactorInteger[1]
PrimeFactorSum[1](* 0 since 1 has no prime factors *)
```

{{1, 1}}

0

Note that under definition of PrimeFactorSum[k]

```
Mean[Table[PrimeFactorSum[k], {k, 1, 10000}]]
Mean[Table[PrimeFactorSum[k], {k, 1, 10000}]] // N
```

$$\frac{5121523}{5000}$$

1024.3

```
Mean[Table[PrimeFactorSum[k]/k, {k, 1, 10000}]] // N
Mean[Table[PrimeFactorSum[k]/k, {k, 1, 100000}]] // N
```

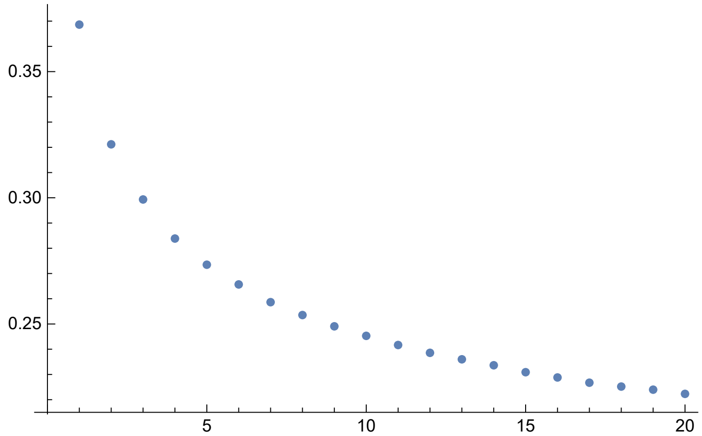
0.222307

0.168969

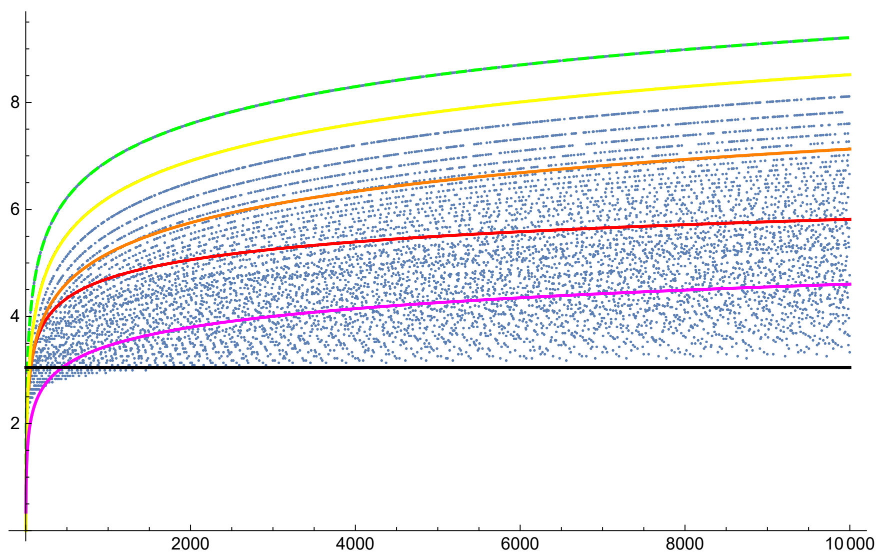
```
Table[j, {j, 500, 10 000, 500}]
```

```
{500, 1000, 1500, 2000, 2500, 3000, 3500, 4000, 4500,
 5000, 5500, 6000, 6500, 7000, 7500, 8000, 8500, 9000, 9500, 10 000}
```

```
ListPlot[Table[Mean[Table[PrimeFactorSum[k]/k, {k, 1, j}]] // N, {j, 500, 10 000, 500}]]
```




```
Show[
 ListPlot[Table[Log[PrimeFactorSum[k]], {k, 2, 10 000}]] ,
 Plot[{LogIntegral[Log[x]], Log[LogIntegral[x]]},
  {x, 2, 10 000}, PlotStyle → {{Red}, {Orange}}],
 Plot[{Log[x], Log[1/2 x], Log[√x], Log[21]}, {x, 2, 10 000},
  PlotStyle → {{Green, Dashed}, {Yellow}, {Magenta}, {Black}}]
]
```




```
Min[Table[Log[PrimeFactorSum[k]], {k, 2000, 10 000}]]
```

```
Log[21]
```

Although $\text{Log}\left[\frac{1}{n} x\right]$ approximates the stratificcations
well for low n (particularly n = 1, 2, 3, 4, 5),
it seems to break down as a good appx of the lowest layers
It seems need a fn that increases much faster from low k to the Log of the minimum
In the example above should go to Log[21] much faster than $\text{Log}\left[\frac{1}{21} x\right]$ does,
but also plateau at around Log[21], which $\text{Log}\left[\frac{1}{21} x\right]$ does not do

```
LogIntegral[Log[10000]] // N
Log[LogIntegral[10000]] // N
Mean[Table[Log[PrimeFactorSum[k]], {k, 2, 10000}]] // N
Mean[Table[Log[PrimeFactorSum[k]], {k, 9000, 10000}]] // N
Mean[Table[Log[PrimeFactorSum[k]], {k, 9900, 10000}]] // N
Mean[Table[Log[PrimeFactorSum[k]], {k, 9990, 10000}]] // N
```

5.81647

7.1278

5.602

6.19038

6.24162

5.88064

LogIntegral[Log[kMAX]] is a fair approximation of
the mean of the set Table[Log[PrimeFactorSum[k]], {k, 2, kMAX}]]

```
KMAX = 80000;
LogIntegral[Log[KMAX]] // N
Mean[Table[Log[PrimeFactorSum[k]], {k, 2, KMAX}]] // N
```

6.7112

6.88184

```
Exp[6.711204309228512`]
Exp[6.8818409266037355`]
```

821.559

974.419

```
KMAX = 80000;
Exp[LogIntegral[Log[KMAX]]] // N
Exp[Mean[Table[Log[PrimeFactorSum[k]], {k, 2, KMAX}]]] // N
```

821.559

974.419

```
KMAX = 70 000;
LogIntegral[KMAX] // N
LogIntegral[KMAX] - LogIntegral[LogIntegral[KMAX]] // N
LogIntegral[KMAX] - Sum[Nest[LogIntegral, KMAX, i], {i, 2, 10}] // N
Sum[(((-1)^(i - 1)) Nest[LogIntegral, KMAX, i]), {i, 1, 10}] // N
Mean[Table[PrimeFactorSum[k], {k, 2, KMAX}]] // N
```

```
6985.29

6072.62

5823.99

6204.49

5756.75
```

So Although some of these appx the non -
logarithm mean of the set to a fiar degree none works perfectly well at all scales

But `LogIntegral[KMAX] - Sum[Nest[LogIntegral, KMAX, i], {i, 2, 10}] //`
`N` seems to be the best of them, and even when it becomes complex for imax >
10 adding the real and imaginary parts yields a good appx

```
Min[Table[Log[PrimeFactorSum[k]], {k, 5000, 10 000}]]
Max[Table[Log[PrimeFactorSum[k]], {k, 5000, 10 000}]]
```

```
Log[24]

Log[9973]
```

```
Log[24] + Log[9973] // N
```

$\frac{\text{Log[9973]}}{\text{Log[24]}}$ // N

```
12.3857
```

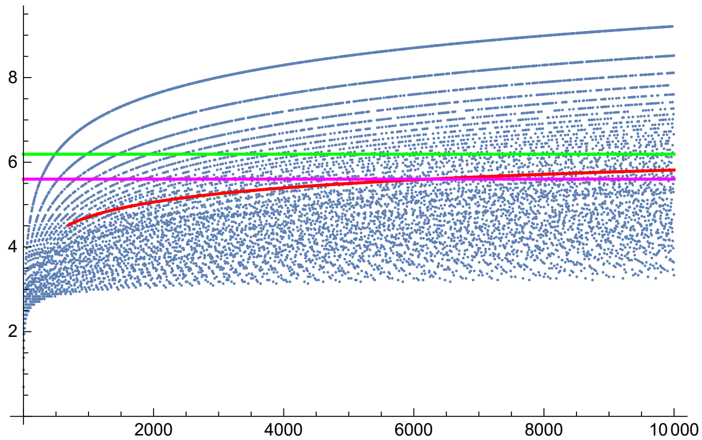
```
(* Mean[Table[Log[PrimeFactorSum[k]],{k,2,10000}]]*)
(* Can be used to Tell how many of each PrimeFactorSum[k] there are,
i.e. appearance of 2 Log[6] says PrimeFactorSum[k] = 6 twice *)
Mean[Table[Log[PrimeFactorSum[k]], {k, 2, 10 000}]] // N
Exp[%]
```

```
5.602

270.967
```

```
Mean[Table[Log[PrimeFactorSum[k]], {k, 9000, 10 000}]] // N
Exp[%]
```

```
6.19038

488.031
```

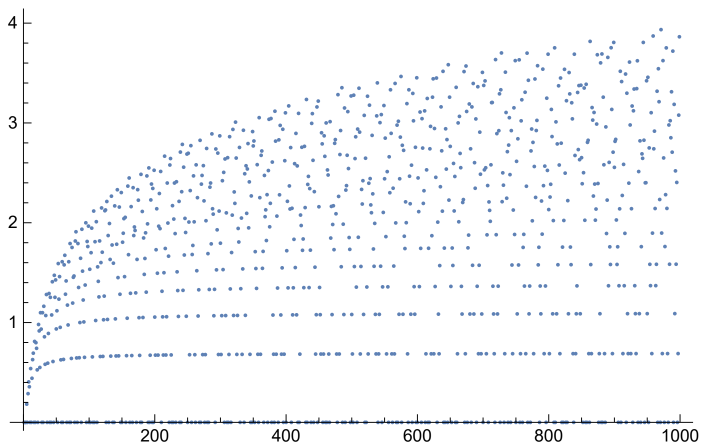
```
Show[
 ListPlot[Table[Log[PrimeFactorSum[k]], {k, 2, 10000}]] ,
 Plot[{LogIntegral[Log[x]], 5.601995661967257`, 6.190377911195523`},
  {x, 2, 10000}, PlotStyle → {{Red}, {Magenta}, {Green}}]
]
```



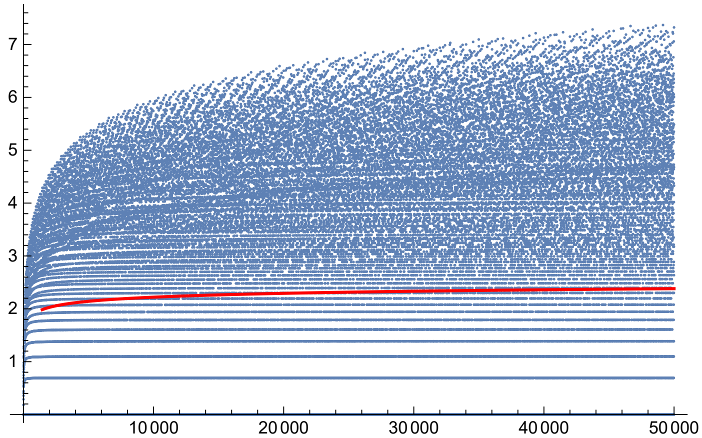

```
Log[1000] // N
```

6.90776

```
ListPlot[Table[Log[k / PrimeFactorSum[k]], {k, 2, 1000}]]
```



```
Show[
 ListPlot[Table[Log[k / PrimeFactorSum[k]], {k, 2, 50000}]],
 Plot[Log[Log[x]], {x, 2, 50000}, PlotStyle → Red]]
```




```
Log[Log[30000]] // N
LogIntegral[Log[30000]] // N
Mean[Table[Log[k / PrimeFactorSum[k]], {k, 2, 30000}]] // N
```

2.33301

6.29889

3.03324

Other Properties of PrimeFactorSum[k] fn

```
Length[Table[PrimeFactorSum[k], {k, 2, 10000}]]
Length[Union[Table[PrimeFactorSum[k], {k, 2, 10000}]]]
(* <9999 b/c union deletes duplicates *)
```

9999

3000

```
Union[Table[PrimeFactorSum[k], {k, 2, 10000}]]
 (*Just orders the set and deletes duplicates *);
```

Seems that (Length[Union[Table[PrimeFactorSum[k], {k, 2, Kmax}]]] / Length[Table[PrimeFactorSum[k], {k, 2, Kmax}]]) → 0 as Kmax → ∞

i.e. the number of unique valus of the set of PrimeFactorSum[k] as a proportion of the total continually decreases

```
PrimePi[1000]
```

168

```
(* Show[
 ListPlot[Sort[Table[PrimeFactorSum[k],{k,2,20000}]]],
 Plot[{k},{k,2,20000},PlotStyle→ {{Orange,Thin}}]
] *)
```

The function PrimeFactorSum[Integer_] gives the total sum of the prime factors such that for 100 = (2^2)(5^2),
PrimeFactorSum[100] = 2 + 2 + 5 +5 = 14,
i.e. even if there’s more than one of a given prime all of them are included in the sum

SumOfEachPrimeFactorOnce[Integer_] does the sum which includes each prime only once, so it has the property that very different numbers can give the same result after applying it,
e.g. SumOfEachPrimeFactorOnce[10] = SumOfEachPrimeFactorOnce[100] = SumOfEachPrimeFactorOnce[1000] = SumOfEachPrimeFactorOnce[10^n] = 7 for any n ∈ Naturals

```
PrimeFactorSum[100]
PrimeFactorSum[360]
```

```
14
```

```
17
```

```
Nest[PrimeFactorSum, k, 3] = PrimeFactorSum[PrimeFactorSum[PrimeFactorSum[k]]]
```

```
Nest[PrimeFactorSum, k, 0] (* "Does" the fn 0 times, so returns k *)
```

```
k
```

```
(* Table[Nest[PrimeFactorSum,k,i],{i,0,4}] Used below *)
```

```
Table[Table[Nest[PrimeFactorSum, k, i], {i, 0, 5}], {k, 2, 20}] // MatrixForm
```

$$\begin{pmatrix} 2 & 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 & 4 & 4 \\ 5 & 5 & 5 & 5 & 5 & 5 \\ 6 & 5 & 5 & 5 & 5 & 5 \\ 7 & 7 & 7 & 7 & 7 & 7 \\ 8 & 6 & 5 & 5 & 5 & 5 \\ 9 & 6 & 5 & 5 & 5 & 5 \\ 10 & 7 & 7 & 7 & 7 & 7 \\ 11 & 11 & 11 & 11 & 11 & 11 \\ 12 & 7 & 7 & 7 & 7 & 7 \\ 13 & 13 & 13 & 13 & 13 & 13 \\ 14 & 9 & 6 & 5 & 5 & 5 \\ 15 & 8 & 6 & 5 & 5 & 5 \\ 16 & 8 & 6 & 5 & 5 & 5 \\ 17 & 17 & 17 & 17 & 17 & 17 \\ 18 & 8 & 6 & 5 & 5 & 5 \\ 19 & 19 & 19 & 19 & 19 & 19 \\ 20 & 9 & 6 & 5 & 5 & 5 \end{pmatrix}$$

```
Table[Table[Nest[PrimeFactorSum, k, i], {i, 0, 5}], {k, 2, 100}][[99, -4]]
(*Corresponds to 4th from last element of row for k =
 100 since we start at k = 2 in the list above *)
```

9

```
PrimeQ[Table[Table[Nest[PrimeFactorSum, k, i], {i, 0, 5}], {k, 2, 100}]] // MatrixForm;
```

```
PrimeQ[Table[Table[Nest[PrimeFactorSum, k, i], {i, 0, 8}], {k, 2, 100}]] /. True → 0 //
  MatrixForm;(*Just turns the trues to 0's in order
 to make it a bit easier to look at and see what's what *)
```

```
AAAA =
  PrimeQ[Table[Table[Nest[PrimeFactorSum, k, i], {i, 0, 12}], {k, 2, 10000}]][[All, -1]] /.
   True → 0;
```

```
Count[AAAA, False]
Position[AAAA, False]
Position[AAAA, False] + 1
(* +1 included since Position[AAAA,False][[j,1]]] is just
   the position in the AAAA list, whereas Position[AAAA,False][[j,1]]]+
 1 is the number having the property described below,
this is because AAAA starts at position k = 2 so it's shifted,
need to adjust that *)
```

1

{{3}}

{{4}}

Note that for 4

PrimeFactorSum[4] = 4, so it remains 4 no matter how many times fn is applied

$4 = 2\char`^2 = (2 * 2)$

Want to see if any other numbers have this property :

For integer n, integer a_k (can have $a_k = 0$ and most often do),

$n = (\text{Prime}[1]\char`^a_1)(\text{Prime}[2]\char`^a_2)\ldots.(\text{Prime}[k]\char`^a_k)$ by definition

Has the property iff

$(\text{Prime}[1] * a_1) + (\text{Prime}[2] * a_2) + \ldots + (\text{Prime}[k] * a_k) = n$

$\Rightarrow (\text{Prime}[1] * a_1) + (\text{Prime}[2] * a_2) + \ldots + (\text{Prime}[k] * a_k) =$
$(\text{Prime}[1]\char`^a_1)(\text{Prime}[2]\char`^a_2)\ldots.(\text{Prime}[k]\char`^a_k) = n$

$\Rightarrow ((\text{Prime}[1] * a_1) + (\text{Prime}[2] * a_2) + \ldots + (\text{Prime}[k] * a_k)) /$
$(\text{Prime}[1]\char`^a_1)(\text{Prime}[2]\char`^a_2)\ldots.(\text{Prime}[k]\char`^a_k) = 1$

$$\Rightarrow \frac{\sum_{i=1}^{k}(\text{Prime}[i] * a_i)}{\prod_{i=1}^{k}(\text{Prime}[i]\char`^a_i)} = 1$$

Note that numerator is *times* a_i whereas denominator is *to the power* of a_i, which are different, e.g. for n = 8 :

$$\frac{(\text{Prime}[1]*3)}{(\text{Prime}[1]\text{^}3)} = \frac{(2*3)}{(2\text{^}3)} = \frac{6}{8} = 0.75 \neq 1$$

Can observe from searching for Falses in

```
(* PrimeQ[Table[Table[Nest[PrimeFactorSum,k,i],{i,0,12}],{k,2,10000}]][[
   All,-1]]/.True→ 0//MatrixForm *)
```

that this occurs only once, at n = 4, for n up to 10 000
(If we only go up to i = 8 appear to have 6 of these numbers but going to i = 12 removes all but n = 4)

From Dusert ' s inequality we have that
Prime[x] < x Log[x] + x Log[Log[x]], for x > 6, which gives us

$$\frac{\text{Prime}[x]}{(\text{Log}[x] + \text{Log}[\text{Log}[x]])} < x$$

$$\frac{(\text{Log}[x] + \text{Log}[\text{Log}[x]])}{\text{Prime}[x]} > \frac{1}{x}$$

$$\frac{(\text{Log}[x] + \text{Log}[\text{Log}[x]])}{\text{Prime}[x]} \sum_{i=1}^{k} (\text{Prime}[i]*a_i) > \frac{\sum_{i=1}^{k} (\text{Prime}[i]*a_i)}{x} = \frac{\sum_{i=1}^{k} (\text{Prime}[i]*a_i)}{\prod_{i=1}^{k} (\text{Prime}[i]\text{^}a_i)}$$

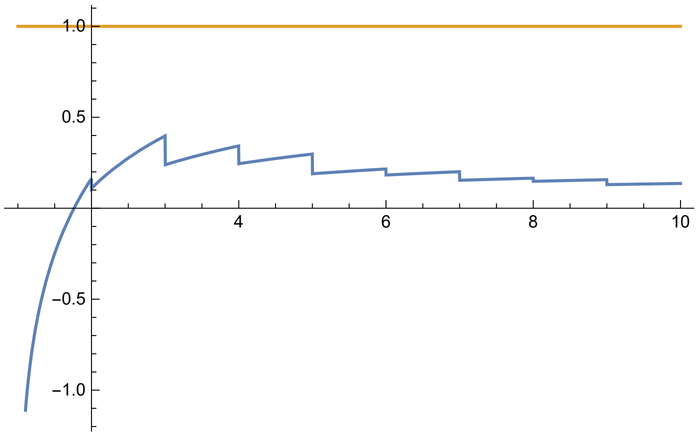
$$\frac{(\text{Log}[x] + \text{Log}[\text{Log}[x]])}{\text{Prime}[x]} > \frac{1}{x} = \frac{1}{\prod_{i=1}^{k} (\text{Prime}[i]\text{^}a_i)}$$

Then $\frac{\sum_{i=1}^{k} (\text{Prime}[i]*a_i)}{\prod_{i=1}^{k} (\text{Prime}[i]\text{^}a_i)} = 1 \Rightarrow \frac{1}{\prod_{i=1}^{k} (\text{Prime}[i]\text{^}a_i)} = \frac{1}{\sum_{i=1}^{k} (\text{Prime}[i]*a_i)}$

$\Rightarrow \frac{(\text{Log}[x] + \text{Log}[\text{Log}[x]])}{\text{Prime}[x]} > \frac{1}{\prod_{i=1}^{k} (\text{Prime}[i]\text{^}a_i)} = \frac{1}{\sum_{i=1}^{k} (\text{Prime}[i]*a_i)}$ for $x > 6$,

But note that since $\frac{(\text{Log}[x] + \text{Log}[\text{Log}[x]])}{\text{Prime}[x]} < 1$ for all x also, then

```
Plot[{(Prime[Floor[x]] / (Log[x] + Log[Log[x]]))^(-1), 1}, {x, 1, 10}]
```




```
(* Plot[{Prime[Floor[x]],x(Log[x] + Log[Log[x]]-Nest[Log,x,3]+Nest[Log,x,4]),
  LogIntegral[x]},{x,1,40}] (*Similar shape just not size as logintegral*) *)
```

```
Table[PrimeFactorSum[k] / k, {k, 2, 100}]
```

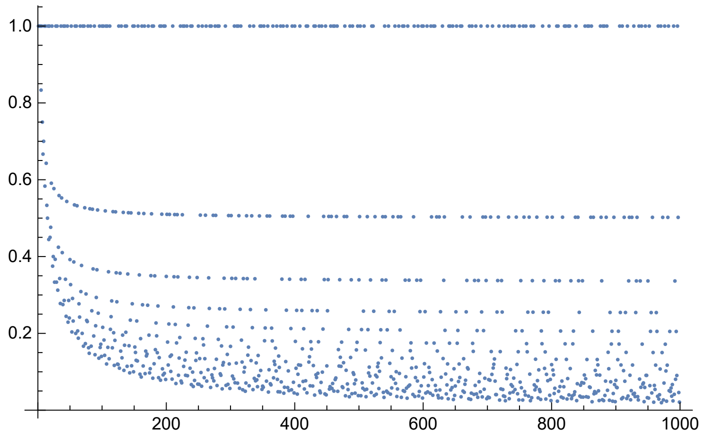
$$\Big\{1, 1, 1, 1, \tfrac{5}{6}, 1, \tfrac{3}{4}, \tfrac{2}{3}, \tfrac{7}{10}, 1, \tfrac{7}{12}, 1, \tfrac{9}{14}, \tfrac{8}{15}, \tfrac{1}{2}, 1, \tfrac{4}{9}, 1, \tfrac{9}{20}, \tfrac{10}{21}, \tfrac{13}{22},$$
$$1, \tfrac{3}{8}, \tfrac{2}{5}, \tfrac{15}{26}, \tfrac{1}{3}, \tfrac{11}{28}, 1, \tfrac{1}{3}, 1, \tfrac{5}{16}, \tfrac{14}{33}, \tfrac{19}{34}, \tfrac{12}{35}, \tfrac{5}{18}, 1, \tfrac{21}{38}, \tfrac{16}{39}, \tfrac{11}{40}, 1, \tfrac{2}{7}, 1,$$
$$\tfrac{15}{44}, \tfrac{11}{45}, \tfrac{25}{46}, 1, \tfrac{11}{48}, \tfrac{2}{7}, \tfrac{6}{25}, \tfrac{20}{51}, \tfrac{17}{52}, 1, \tfrac{11}{54}, \tfrac{16}{55}, \tfrac{13}{56}, \tfrac{22}{57}, \tfrac{31}{58}, 1, \tfrac{1}{5}, 1, \tfrac{33}{62},$$
$$\tfrac{13}{63}, \tfrac{3}{16}, \tfrac{18}{65}, \tfrac{8}{33}, 1, \tfrac{21}{68}, \tfrac{26}{69}, \tfrac{1}{5}, 1, \tfrac{1}{6}, 1, \tfrac{39}{74}, \tfrac{13}{75}, \tfrac{23}{76}, \tfrac{18}{77}, \tfrac{3}{13}, 1, \tfrac{13}{80}, \tfrac{4}{27},$$
$$\tfrac{43}{82}, 1, \tfrac{1}{6}, \tfrac{22}{85}, \tfrac{45}{86}, \tfrac{32}{87}, \tfrac{17}{88}, 1, \tfrac{13}{90}, \tfrac{20}{91}, \tfrac{27}{92}, \tfrac{34}{93}, \tfrac{49}{94}, \tfrac{24}{95}, \tfrac{13}{96}, 1, \tfrac{8}{49}, \tfrac{17}{99}, \tfrac{7}{50}\Big\}$$

```
Table[Floor[PrimeFactorSum[k] / k], {k, 2, 100}]
(*Leaves only primes and numbers which have the same property as 4 described above *)
```

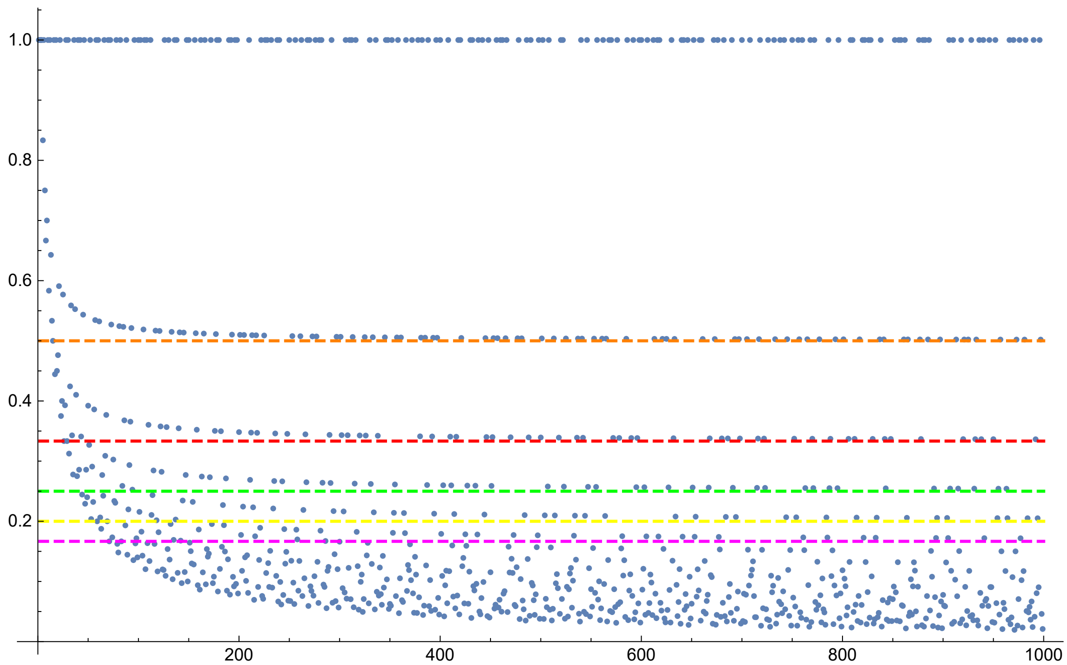
```
{1, 1, 1, 1, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0,
 0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0,
 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}
```

Stratification

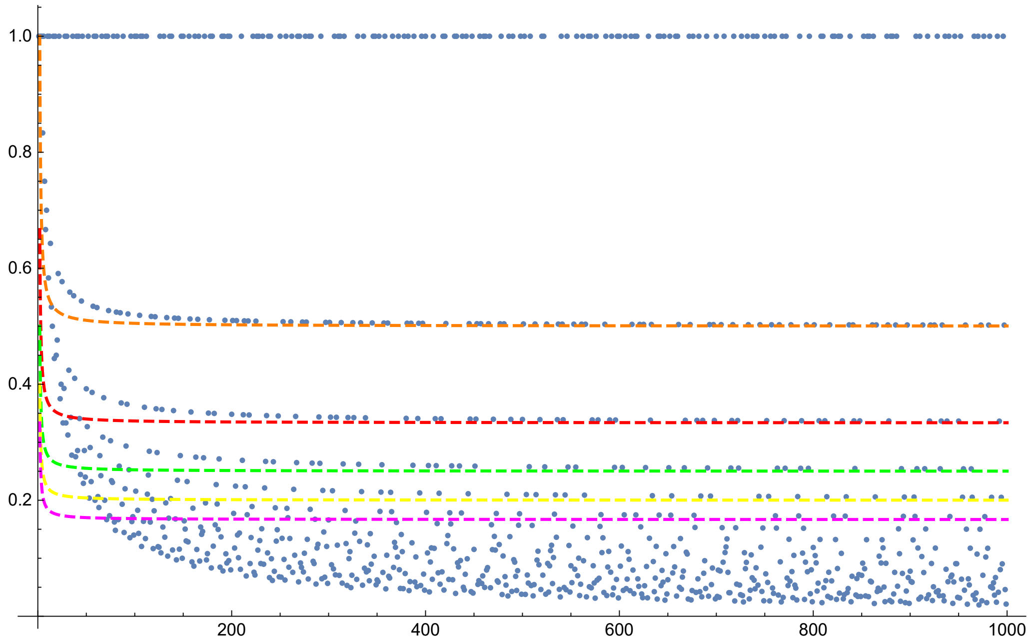
$$\text{ListPlot}\left[\text{Table}\left[\frac{\text{PrimeFactorSum}[k]}{k}, \{k, 2, 1000\}\right]\right]$$



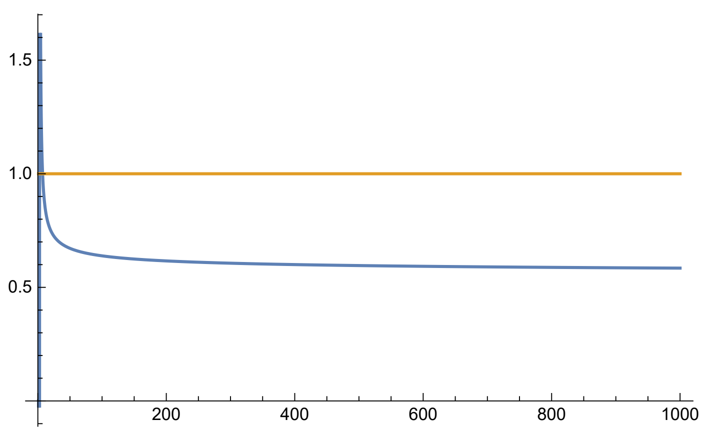
$$\text{Show}\left[\text{ListPlot}\left[\text{Table}\left[\frac{\text{PrimeFactorSum}[k]}{k}, \{k, 2, 1000\}\right]\right],\ \text{Plot}\left[\left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}\right\}, \{k, 2, 1000\}, \text{PlotStyle} \to \{\{\text{Orange}, \text{Dashed}\}, \{\text{Red}, \text{Dashed}\}, \{\text{Green}, \text{Dashed}\}, \{\text{Yellow}, \text{Dashed}\}, \{\text{Magenta}, \text{Dashed}\}\}\right]\right]$$



```
Show[ListPlot[Table[PrimeFactorSum[k]/k, {k, 2, 1000}]],
 Plot[{(1/2)/(1 - 1/k), (1/3)/(1 - 1/k), (1/4)/(1 - 1/k), (1/5)/(1 - 1/k), (1/6)/(1 - 1/k)},
  {k, 2, 1000}, PlotStyle → {{Orange, Dashed}, {Red, Dashed},
    {Green, Dashed}, {Yellow, Dashed}, {Magenta, Dashed}}]]
(*Tiny bit better of a fit of the fns the pts actually tend towards,
but still not perfect *)
```

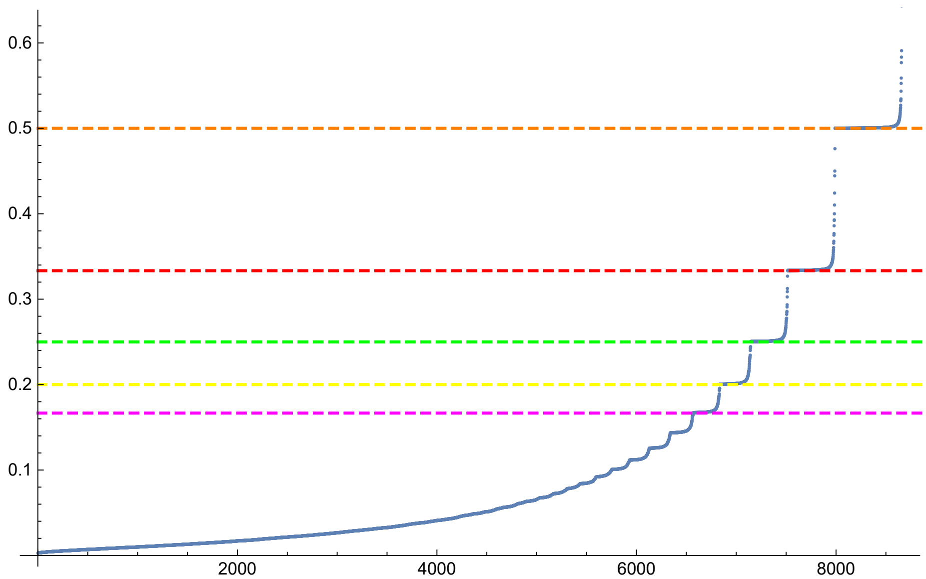



```
Plot[{(1/2)/(1 - 1/Log[x]), 1}, {x, 1, 1000}]
```

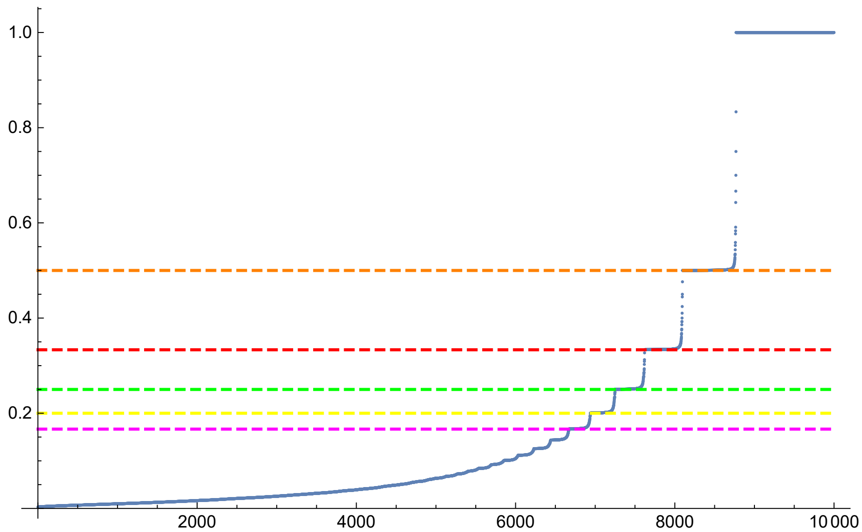


Note that in the above some values do go above the previous line
This most likely means that instead of tending towards $\frac{1}{n}$exactly the points actually tend towards functions which tend towards $\frac{1}{n}$but happen to be over

```
(* PrimeFactorSum[Integer_]:=
 Sum[(FactorInteger[Integer][[i,1]])(FactorInteger[Integer][[i,2]]),
  {i,1,PrimeNu[Integer]}] *)

Show[ListPlot[Union[Table[PrimeFactorSum[k]/k, {k, 2, 10000}]]],
 Plot[{1/2, 1/3, 1/4, 1/5, 1/6}, {k, 2, 10000}, PlotStyle → {{Orange, Dashed}, {Red, Dashed},
   {Green, Dashed}, {Yellow, Dashed}, {Magenta, Dashed}}]](* Orders the list *)
```



```
Show[ListPlot[Sort[Table[PrimeFactorSum[k]/k, {k, 2, 10000}]]],
 Plot[{1/2, 1/3, 1/4, 1/5, 1/6}, {k, 2, 10000}, PlotStyle → {{Orange, Dashed},
    {Red, Dashed}, {Green, Dashed}, {Yellow, Dashed}, {Magenta, Dashed}}]]
(* This version keeps duplicated values, maily keeps the 1's  *)
```




```
PrimeFactorSum[Integer_] :=
 Sum[(FactorInteger[Integer][[i, 1]]) (FactorInteger[Integer][[i, 2]]),
  {i, 1, PrimeNu[Integer]}]
SumOfEachPrimeFactorOnce[Integer_] :=
 Sum[(FactorInteger[Integer][[i, 1]]), {i, 1, PrimeNu[Integer]}]
```

```
PFSnTerms[n_, kmax_] :=
 Select[Table[{k, Floor[((n - 1) n (PrimeFactorSum[k]/k) + (2 - n))]}, {k, 2, kmax}],
   #[[2]] == 1 &][[All, 1]] (*Gives the k values for which
 the function PrimeFactorSum[k]/k lies in [1/n, 1/(n-1)] and/or tends to 1/n,
although some values given by this might actually be part of the set
 of points tending towards 1/(n + 1) as elaborated upon in the orange
 box and subsection "Finding Fns pts actually tend towards" *)
PFSnTermsList[n_, kmax_] :=
 Table[PrimeFactorSum[PFSnTerms[n, kmax][[j]]]/PFSnTerms[n, kmax][[j]], {j, 1, Length[PFSnTerms[n, kmax]]}]
 (*Gives the actual values of the function  PrimeFactorSum[k]/k  for
 the k produced in the previous fn PFSnTerms[n_,kmax_] *)
PFSnPlotList[n_, kmax_] := Table[{PFSnTerms[n, kmax][[j]], PFSnTermsList[n, kmax][[j]]},
  {j, 1, Length[PFSnTerms[n, kmax]]}]
(*Puts the previous two fns in a format easier to get some plots in *)
```

```
(* Do[LenPFSn=Length[PFSnTerms[n,kmax]], *)
(* LenPFSn(kmax)=Length[PFSnTerms[n,kmax]]*)
```

```
Table[Length[PFSnTerms[i, 1000]], {i, 2, 15}]
```

```
{99, 73, 57, 54, 45, 40, 37, 42, 33, 35, 23, 25, 22, 23}
```

```
(* ListPlot[PFSnTerms[15,2000]] (*Sort of has steps *) *)
```

```
PFSnTerms[2, 1000]
PFSnTerms[3, 1000]
PFSnTerms[4, 1000]
PFSnTerms[5, 1000]
PFSnTerms[6, 1000]
```

```
{6, 8, 9, 10, 12, 14, 15, 16, 22, 26, 34, 38, 46, 58, 62, 74, 82, 86, 94, 106, 118, 122, 134, 142,
 146, 158, 166, 178, 194, 202, 206, 214, 218, 226, 254, 262, 274, 278, 298, 302, 314, 326, 334,
 346, 358, 362, 382, 386, 394, 398, 422, 446, 454, 458, 466, 478, 482, 502, 514, 526, 538, 542,
 554, 562, 566, 586, 614, 622, 626, 634, 662, 674, 694, 698, 706, 718, 734, 746, 758, 766, 778,
 794, 802, 818, 838, 842, 862, 866, 878, 886, 898, 914, 922, 926, 934, 958, 974, 982, 998}
```

```
{18, 20, 21, 24, 25, 27, 28, 30, 33, 35, 39, 44, 51, 57, 69, 87, 93, 111, 123, 129, 141, 159,
 177, 183, 201, 213, 219, 237, 249, 267, 291, 303, 309, 321, 327, 339, 381, 393, 411,
 417, 447, 453, 471, 489, 501, 519, 537, 543, 573, 579, 591, 597, 633, 669, 681, 687,
 699, 717, 723, 753, 771, 789, 807, 813, 831, 843, 849, 879, 921, 933, 939, 951, 993}
```

```
{32, 36, 40, 42, 49, 52, 55, 65, 68, 76, 85, 92, 95, 116, 124, 148, 164, 172, 188, 212, 236,
 244, 268, 284, 292, 316, 332, 356, 388, 404, 412, 428, 436, 452, 508, 524, 548, 556, 596,
 604, 628, 652, 668, 692, 716, 724, 764, 772, 788, 796, 844, 892, 908, 916, 932, 956, 964}
```

```
{45, 48, 50, 54, 56, 60, 63, 66, 70, 77, 78, 91, 102, 114, 115, 119, 138, 145, 155, 185,
 205, 215, 235, 265, 295, 305, 335, 355, 365, 395, 415, 445, 485, 505, 515, 535, 545,
 565, 635, 655, 685, 695, 745, 755, 785, 815, 835, 865, 895, 905, 955, 965, 985, 995}
```

```
{64, 72, 75, 84, 88, 99, 104, 121, 133, 136, 143, 161, 174, 186, 203,
 217, 222, 246, 258, 259, 282, 287, 318, 354, 366, 402, 426, 438, 474, 498,
 534, 582, 606, 618, 642, 654, 678, 762, 786, 822, 834, 894, 906, 942, 978}
```

```
PFSnPlotList[n_, kmax_] := Table[{PFSnTerms[n, kmax][[j]], PFSnTermsList[n, kmax][[j]]},
  {j, 1, Length[PFSnTerms[n, kmax]]}]
```

```
PFSnTerms[4, 1000]
PFSnTermsList[4, 1000]
PFSnPlotList[4, 1000]
```

{32, 36, 40, 42, 49, 52, 55, 65, 68, 76, 85, 92, 95, 116, 124, 148, 164, 172, 188, 212, 236, 244, 268, 284, 292, 316, 332, 356, 388, 404, 412, 428, 436, 452, 508, 524, 548, 556, 596, 604, 628, 652, 668, 692, 716, 724, 764, 772, 788, 796, 844, 892, 908, 916, 932, 956, 964}

$$\left\{\frac{5}{16}, \frac{5}{18}, \frac{11}{40}, \frac{2}{7}, \frac{2}{7}, \frac{17}{52}, \frac{16}{55}, \frac{18}{65}, \frac{21}{68}, \frac{23}{76}, \frac{22}{85}, \frac{27}{92}, \frac{24}{95}, \frac{33}{116}, \frac{35}{124}, \frac{41}{148}, \frac{45}{164}, \frac{47}{172}, \frac{51}{188}, \frac{57}{212}, \frac{63}{236}, \frac{65}{244}, \frac{71}{268}, \frac{75}{284}, \frac{77}{292}, \frac{83}{316}, \frac{87}{332}, \frac{93}{356}, \frac{101}{388}, \frac{105}{404}, \frac{107}{412}, \frac{111}{428}, \frac{113}{436}, \frac{117}{452}, \frac{131}{508}, \frac{135}{524}, \frac{141}{548}, \frac{143}{556}, \frac{153}{596}, \frac{155}{604}, \frac{161}{628}, \frac{167}{652}, \frac{171}{668}, \frac{177}{692}, \frac{183}{716}, \frac{185}{724}, \frac{195}{764}, \frac{197}{772}, \frac{201}{788}, \frac{203}{796}, \frac{215}{844}, \frac{227}{892}, \frac{231}{908}, \frac{233}{916}, \frac{237}{932}, \frac{243}{956}, \frac{245}{964}\right\}$$

$Aborted

```
Table[Length[PFSnTerms[i, 1000]], {i, 2, 15}]
LenPFS2(1000) = Length[PFSnTerms[2, 1000]]
```

{99, 73, 57, 54, 45, 40, 37, 42, 33, 35, 23, 25, 22, 23}

99

```
Table[Length[PFSnTerms[i, 1000]], {i, 2, 15}][[3 - 1]]
```

73

```
Do[LengthPFSfnk = Table[Length[PFSnTerms[i, 1000]], {i, 2, 15}][[k - 1]], {k, 2, 15}]
```

```
LengthPFSfn15
```

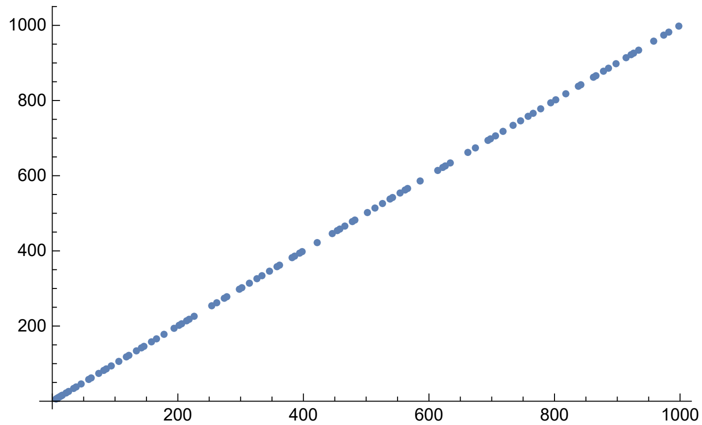
23

```
AA2 = Table[{PFSnTerms[2, 1000][[j]], PFSnTerms[2, 1000][[j]]}, {j, 1, 99}]
```

{{6, 6}, {8, 8}, {9, 9}, {10, 10}, {12, 12}, {14, 14}, {15, 15}, {16, 16}, {22, 22}, {26, 26}, {34, 34}, {38, 38}, {46, 46}, {58, 58}, {62, 62}, {74, 74}, {82, 82}, {86, 86}, {94, 94}, {106, 106}, {118, 118}, {122, 122}, {134, 134}, {142, 142}, {146, 146}, {158, 158}, {166, 166}, {178, 178}, {194, 194}, {202, 202}, {206, 206}, {214, 214}, {218, 218}, {226, 226}, {254, 254}, {262, 262}, {274, 274}, {278, 278}, {298, 298}, {302, 302}, {314, 314}, {326, 326}, {334, 334}, {346, 346}, {358, 358}, {362, 362}, {382, 382}, {386, 386}, {394, 394}, {398, 398}, {422, 422}, {446, 446}, {454, 454}, {458, 458}, {466, 466}, {478, 478}, {482, 482}, {502, 502}, {514, 514}, {526, 526}, {538, 538}, {542, 542}, {554, 554}, {562, 562}, {566, 566}, {586, 586}, {614, 614}, {622, 622}, {626, 626}, {634, 634}, {662, 662}, {674, 674}, {694, 694}, {698, 698}, {706, 706}, {718, 718}, {734, 734}, {746, 746}, {758, 758}, {766, 766}, {778, 778}, {794, 794}, {802, 802}, {818, 818}, {838, 838}, {842, 842}, {862, 862}, {866, 866}, {878, 878}, {886, 886}, {898, 898}, {914, 914}, {922, 922}, {926, 926}, {934, 934}, {958, 958}, {974, 974}, {982, 982}, {998, 998}}

```
Do[AAk = Table[{PFSnTerms[k, 1000][[j]], PFSnTerms[k, 1000][[j]]}, {j, 1, LengthPFSfnk}],
 {k, 3, 15}]
```

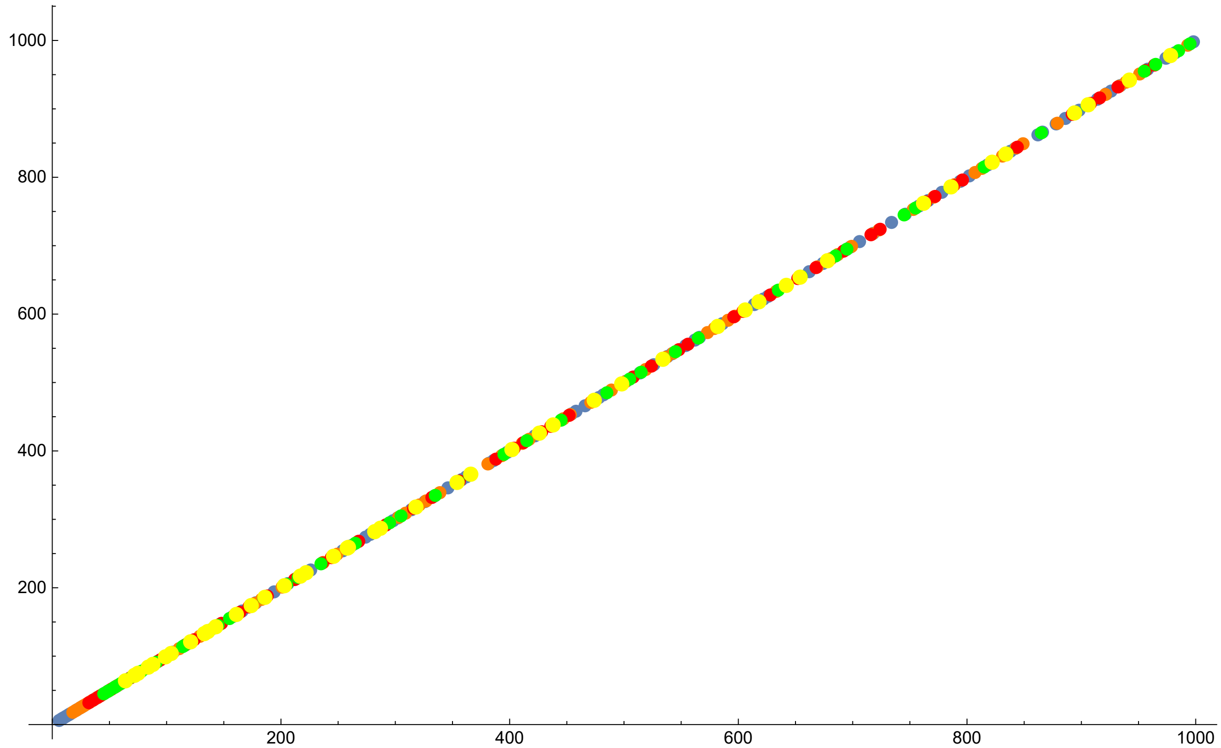
```
ListPlot[AA2]
```




```
AA6
```

```
{{64, 64}, {72, 72}, {75, 75}, {84, 84}, {88, 88}, {99, 99}, {104, 104},
 {121, 121}, {133, 133}, {136, 136}, {143, 143}, {161, 161}, {174, 174},
 {186, 186}, {203, 203}, {217, 217}, {222, 222}, {246, 246}, {258, 258},
 {259, 259}, {282, 282}, {287, 287}, {318, 318}, {354, 354}, {366, 366},
 {402, 402}, {426, 426}, {438, 438}, {474, 474}, {498, 498}, {534, 534}, {582, 582},
 {606, 606}, {618, 618}, {642, 642}, {654, 654}, {678, 678}, {762, 762}, {786, 786},
 {822, 822}, {834, 834}, {894, 894}, {906, 906}, {942, 942}, {978, 978}}
```

```
Show[
 ListPlot[AA₂],
 ListPlot[AA₃, PlotStyle → Orange],
 ListPlot[AA₄, PlotStyle → Red],
 ListPlot[AA₅, PlotStyle → Green],
 ListPlot[AA₆, PlotStyle → Yellow]
]
```



AA_n gives the set of points which tend to $\frac{1}{n}$ under the stratified graphs above.

e.g. a yellow dot at 642 means that given at k =
642 we have that $\frac{\text{PrimeFactorSum}[k]}{k}$ lies in $\left[\frac{1}{6}, \frac{1}{6-1}\right]$

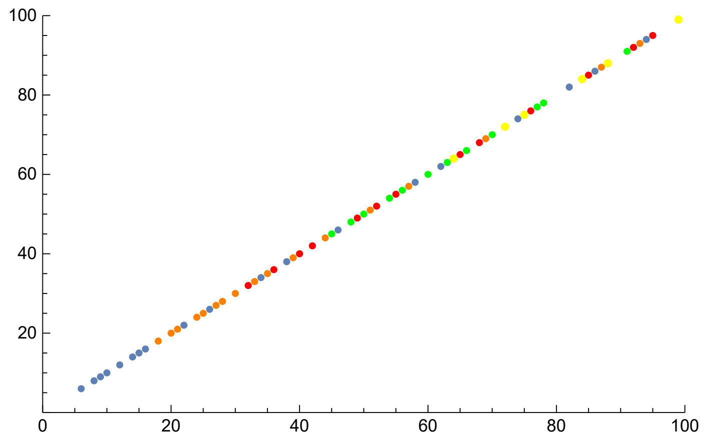
This plot is made to get a sense of how theyre distributed.

As we can see the dstribution is relatively random

Hypothesis: each class of these is randomly distributed similar to how the primes are randomly distributed

Zooming in:

```
Show[
 ListPlot[AA₂, PlotRange → {{0, 100}, {0, 100}}],
 ListPlot[AA₃, PlotStyle → Orange],
 ListPlot[AA₄, PlotStyle → Red],
 ListPlot[AA₅, PlotStyle → Green],
 ListPlot[AA₆, PlotStyle → Yellow]
]
```



Compare to the number of primes up to the same kmax:

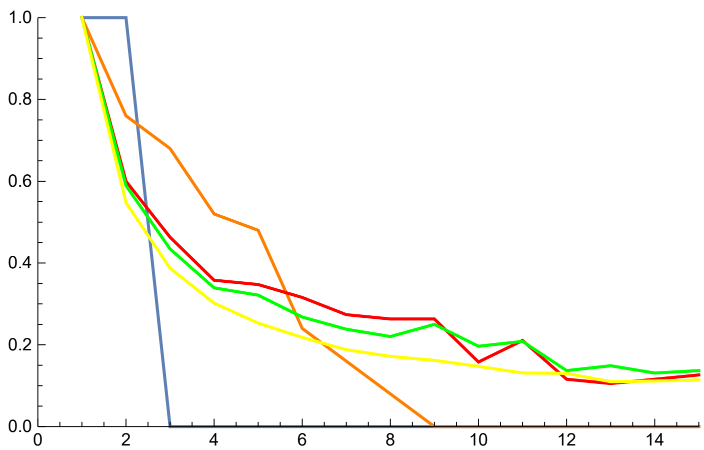
```
kmax = 1000;
Join[{PrimePi[kmax]}, Table[Length[PFSnTerms[i, kmax]], {i, 2, 15}]]
```

```
{168, 99, 73, 57, 54, 45, 40, 37, 42, 33, 35, 23, 25, 22, 23}
```

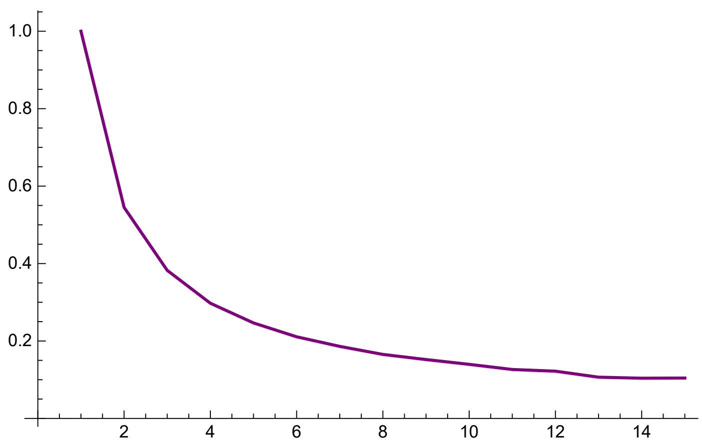
```
NUMeachClass[kmax_] :=
 Join[{PrimePi[kmax]}, Table[Length[PFSnTerms[i, kmax]], {i, 2, 15}]]
AdjustedNUMeachClass[kmax_] :=
 (Join[{PrimePi[kmax]}, Table[Length[PFSnTerms[i, kmax]], {i, 2, 15}]]) / PrimePi[kmax]
```

```
(* ListPlot[NUMeachClass[1000]]
 ListPlot[NUMeachClass[10000]] *)
```

```
Show[
 ListLinePlot[AdjustedNUMeachClass[10], PlotRange → {{0, 15}, {0, 1}}],
 ListLinePlot[AdjustedNUMeachClass[100], PlotStyle → Orange],
 ListLinePlot[AdjustedNUMeachClass[500], PlotStyle → Red],
 ListLinePlot[AdjustedNUMeachClass[1000], PlotStyle → Green],
 ListLinePlot[AdjustedNUMeachClass[10 000], PlotStyle → Yellow]
]
```



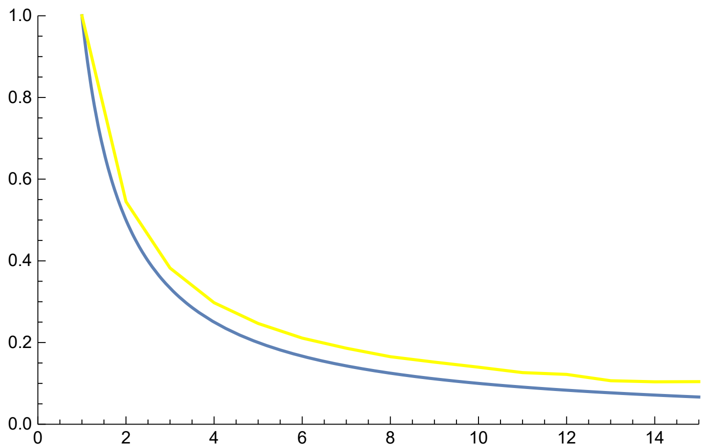

```
ListLinePlot[AdjustedNUMeachClass[20 000], PlotStyle → Purple]
```



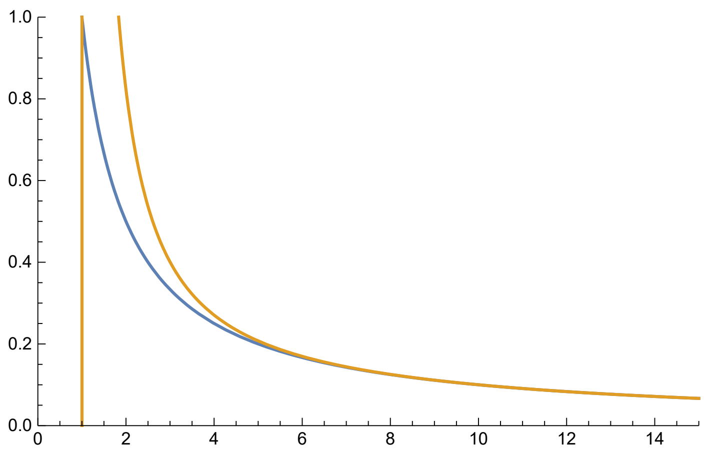
All (for which it doesn't yet drop to 0) seem to tend towards similar value, i.e. for kmax ≥ 500 in the plots aboves they all ≈ 0.2 at x=15

So as kmax → ∞, AdjustedNUMeachClass[kmax] probably tends towards some smooth function

```
Show[
 Plot[1/x, {x, 0, 15}, PlotRange → {{0, 15}, {0, 1}}],
 ListLinePlot[AdjustedNUMeachClass[20 000], PlotStyle → Yellow]
]
```



```
Plot[{1/x, Zeta[x]/x}, {x, 0, 15}, PlotRange → {{0, 15}, {0, 1}}] (*Should begin at 1 also*)
```



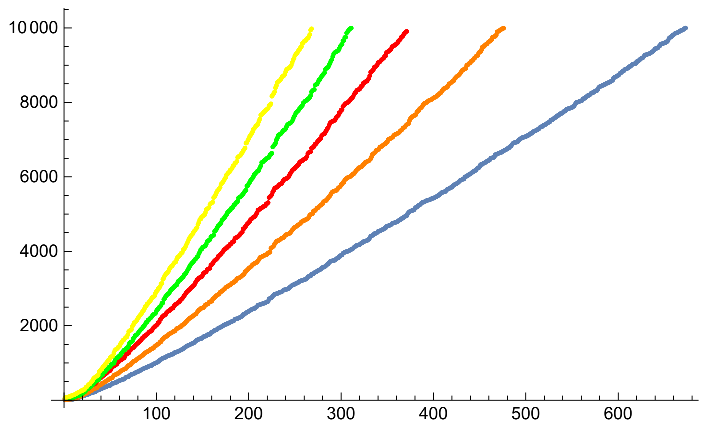
$$\frac{\text{NUMeachClass[100]}}{\text{PrimePi[100]}}$$

$$\frac{\text{NUMeachClass[1000]}}{\text{PrimePi[1000]}}$$

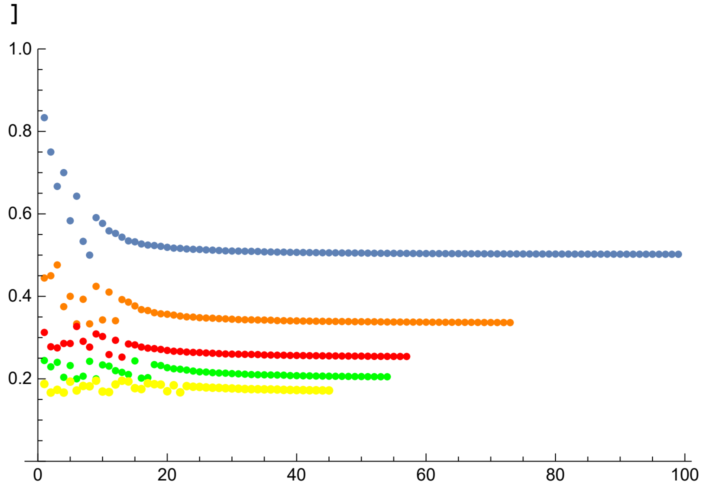
$$\left\{1, \frac{19}{25}, \frac{17}{25}, \frac{13}{25}, \frac{12}{25}, \frac{6}{25}, \frac{4}{25}, \frac{2}{25}, 0, 0, 0, 0, 0, 0, 0\right\}$$

$$\left\{1, \frac{33}{56}, \frac{73}{168}, \frac{19}{56}, \frac{9}{28}, \frac{15}{56}, \frac{5}{21}, \frac{37}{168}, \frac{1}{4}, \frac{11}{56}, \frac{5}{24}, \frac{23}{168}, \frac{25}{168}, \frac{11}{84}, \frac{23}{168}\right\}$$

```
kmax = 10000;
Show[
 ListPlot[PFSnTerms[2, kmax]],
 ListPlot[PFSnTerms[3, kmax], PlotStyle → Orange],
 ListPlot[PFSnTerms[4, kmax], PlotStyle → Red],
 ListPlot[PFSnTerms[5, kmax], PlotStyle → Green],
 ListPlot[PFSnTerms[6, kmax], PlotStyle → Yellow]
]
(* In these plots the y-
 axis is PFSnTerms[n,kmax] and the x-axis is PFSnTerms[n,kmax][[x]],
so basically plots how fast each of the fns increases *)
(* LABEL: PrimeFactorSum n vs. fn graph *)
```

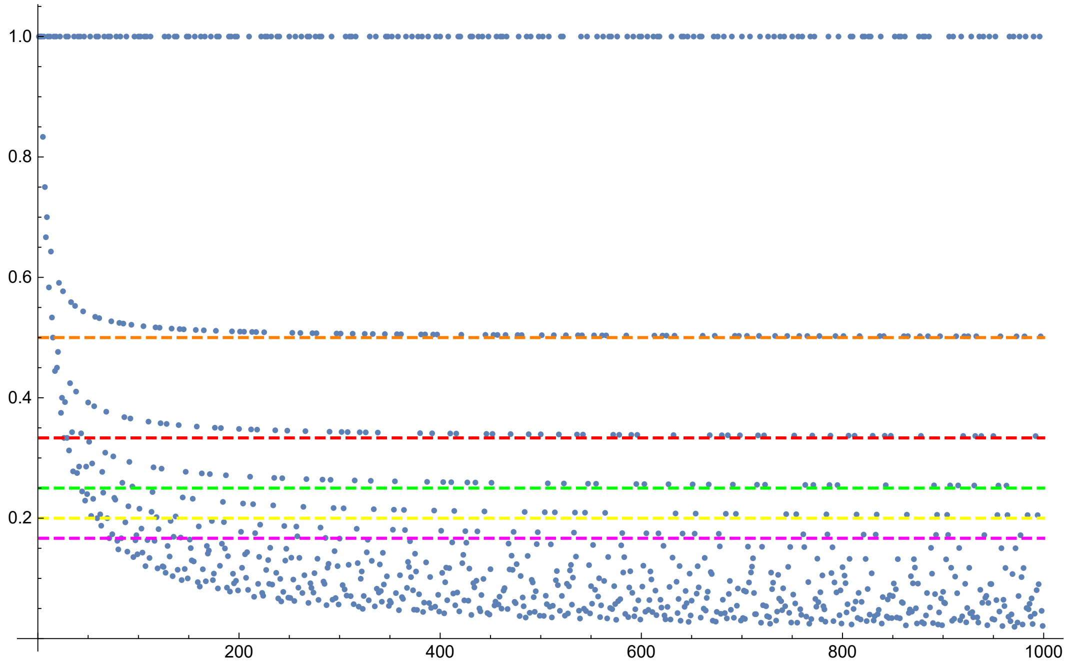



```
kmax = 1000;
Show[
 ListPlot[PFSnTermsList[2, kmax], PlotRange → {Automatic, {0, 1}}],
 ListPlot[PFSnTermsList[3, kmax], PlotStyle → Orange],
 ListPlot[PFSnTermsList[4, kmax], PlotStyle → Red],
 ListPlot[PFSnTermsList[5, kmax], PlotStyle → Green],
 ListPlot[PFSnTermsList[6, kmax], PlotStyle → Yellow]
]
```



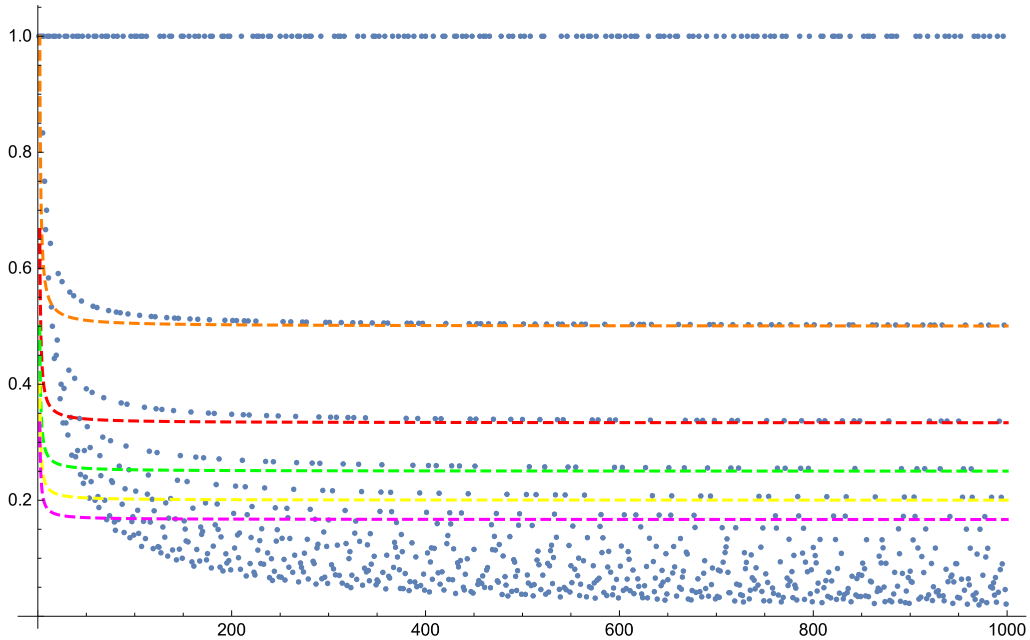
Finding Fns pts actually tend towards

```
Show[ListPlot[Table[PrimeFactorSum[k]/k, {k, 2, 1000}]],
 Plot[{1/2, 1/3, 1/4, 1/5, 1/6}, {k, 2, 1000}, PlotStyle →
   {{Orange, Dashed}, {Red, Dashed}, {Green, Dashed}, {Yellow, Dashed}, {Magenta, Dashed}}]]
```



Note that in the above some values do go above the previous line
This most likely means that instead of tending towards $\frac{1}{n}$exactly the points actually tend towards functions which tend towards $\frac{1}{n}$but happen to be over

```
Show[ListPlot[Table[PrimeFactorSum[k]/k, {k, 2, 1000}]],
 Plot[{(1/2)/(1 - 1/k), (1/3)/(1 - 1/k), (1/4)/(1 - 1/k), (1/5)/(1 - 1/k), (1/6)/(1 - 1/k)},
  {k, 2, 1000}, PlotStyle → {{Orange, Dashed}, {Red, Dashed},
    {Green, Dashed}, {Yellow, Dashed}, {Magenta, Dashed}}]]
(*Tiny bit better of a fit of the fns the pts actually tend towards,
but still not perfect *)
```




```
Plot[{(1/2)/(1 - 1/k), 1/2 + Log[k]/(2 k), 1/2 + 1/k, (2 Log[E - 1/k])^(-1), 1/2}, {k, 1, 1000},
 PlotStyle → {Blue, Yellow, Green, Red, Black}, PlotRange → {Automatic, {0.49, 0.52}}]
```