

```

For[npow = 1, npow < 1000, npow++, If[Length[M] < (2^(npow)), Break[]];
  FilledSize = 2^(npow + 1)];
Filler[vecvar1_] := Table[4, {i, 1, FilledSize - lengthofgeneitself}]
FilledVec[vecvar2_] := Join[Flatten[vecvar2], Filler[vecvar2]]

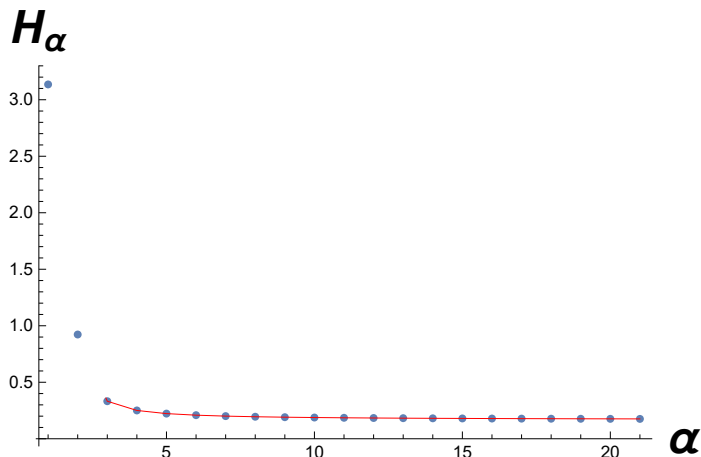
Filler[vecvar4_] := Table[4, {i, 1, FilledSize - lengthofgeneitself}]
FilledVec[vecvar5_] := Join[Flatten[vecvar5], Filler[vecvar5]]
For[npow = 1, npow < 1000, npow++, If[lengthofgeneitself ≤ (2^npow), Break[]]];
(* gives npow such that 2^npow > lengthofgeneitself > 2^(npow - 1) *)
FilledSize = 2^npow;
FilledM = FilledVec[M];
numrowsW = Sqrt[Length[FilledM]];

W = Table[Table[FilledM[[i]],
  {i, ((j - 1) * (numrowsW) + 1), (j * (numrowsW))}], {j, 1, numrowsW}];

ρ = (W.Transpose[W]); (* ρ as inner product *)
rhoEigens = Sort[DeleteCases[Eigenvalues[ρ] // N, 0.], Greater];
(*DeleteCases Removes 0's from the set of Eigenvalues,
Sort puts the list in order of greatest to least *)
set =  $\frac{\text{rhoEigens}}{\text{Total[rhoEigens]}}$ ;
(* This is the set of nonzero normalized eigenvalues in order of greatest to least *)
n = Length[set];
H[α_] :=  $\frac{1}{1 - \alpha} \text{Log}[2, \text{Sum}[(\text{set}[[i]])^\alpha, \{i, 1, n\}]] // N$ 
H0 = Log[n] // N; (* H0 = Hartley Entropy*)
H1 = -Sum[(set[[i]]) (Log[2, set[[i]])], {i, 1, n}] // N; (* H1 = Shannon Entropy*)
H2onward = Table[H[a], {a, 2, 20}] // N; (* H2 onward *)
RenyiEntropyofEigenvalues = Join[{H0}, {H1}, H2onward];

Print[Graphics[Show[
  ListPlot[RenyiEntropyofEigenvalues, PlotRange → All,
    AxesLabel → {Style["α", Large, Bold], Style["Hα", Large, Bold]}],
  ListLinePlot[RenyiEntropyofEigenvalues, PlotStyle → {Red, Thin}]
]]]

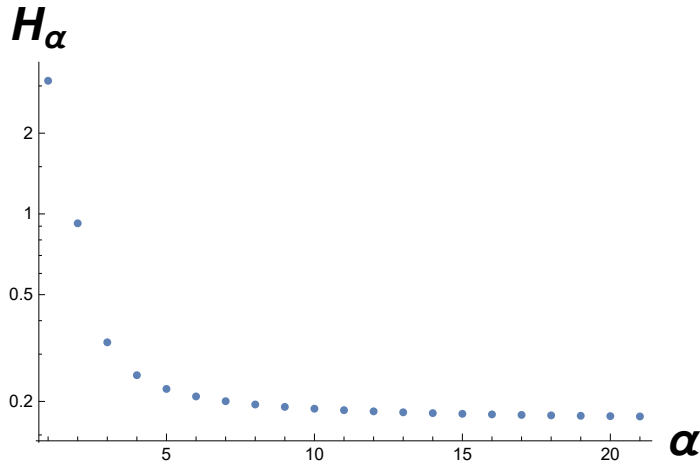
```



```
Print[Graphics[ListLogPlot[Table[{i, rhoEigens[[i]]}, {i, 2, Length[rhoEigens]}],
  AxesLabel → {Style["i", Medium, Bold], Style["Log[ $\lambda_i$ ]", Medium, Bold]},
  PlotRange → {{10, 2 * rhoEigens[[2]]}, PlotStyle → Red,
  PlotLabel → Style["Logplot of Eigenvalues, excluding  $\lambda_1$ ", Red, Bold, 16]] ]]
```

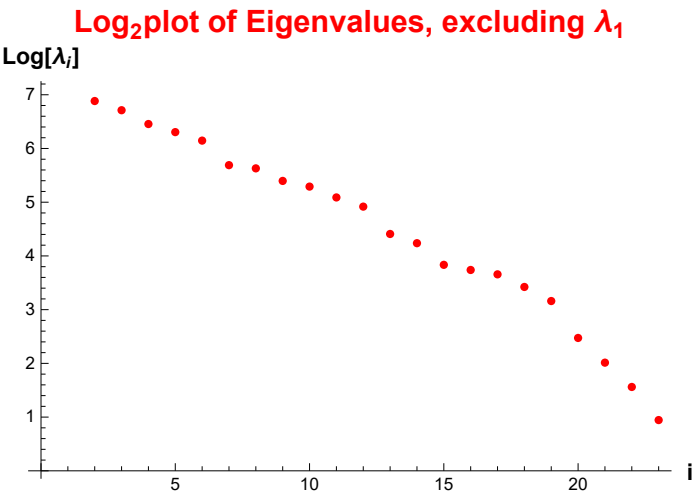
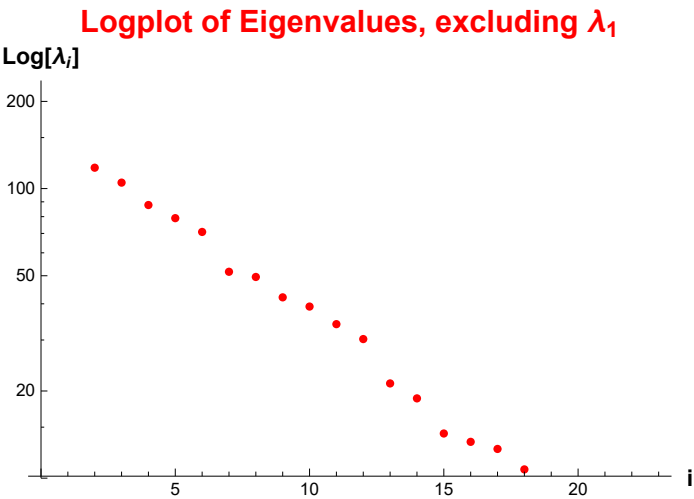
```
Print[Graphics[Show[
  ListPlot[RenyiEntropyofEigenvalues, PlotRange → All,
    AxesLabel → {Style[" $\alpha$ ", Large, Bold], Style[" $H_\alpha$ ", Large, Bold]}],
  ListLinePlot[RenyiEntropyofEigenvalues, PlotStyle → {Red, Thin}]
]]]
```

```
ListLogPlot[RenyiEntropyofEigenvalues, PlotRange → All,
  AxesLabel → {Style[" $\alpha$ ", Large, Bold], Style[" $H_\alpha$ ", Large, Bold]}]
```



```
Print[Graphics[ListLogPlot[Table[{i, rhoEigens[[i]]}, {i, 2, Length[rhoEigens]}],
  AxesLabel → {Style["i", Medium, Bold], Style["Log[ $\lambda_i$ ]", Medium, Bold]},
  PlotRange → {{10, 2 * rhoEigens[[2]]}, PlotStyle → Red,
  PlotLabel → Style["Logplot of Eigenvalues, excluding  $\lambda_1$ ", Red, Bold, 16]] ]]
```

```
Print[Graphics[ListPlot[Table[{i, Log[2, rhoEigens[[i]]}], {i, 2, Length[rhoEigens]}],
  AxesLabel → {Style["i", Medium, Bold], Style["Log[ $\lambda_i$ ]", Medium, Bold]}, PlotStyle → Red,
  PlotLabel → Style["Log2plot of Eigenvalues, excluding  $\lambda_1$ ", Red, Bold, 16]] ]]
```



Log[2, 1024]

10

```

For[npow = 1, npow < 1000, npow++, If[Length[M] < (2^(npow)), Break[]];
  FilledSize = 2^(npow + 1)];
Filler[vecvar1_] := Table[4, {i, 1, FilledSize - lengthofgeneitself}]
FilledVec[vecvar2_] := Join[Flatten[vecvar2], Filler[vecvar2]]

Filler[vecvar4_] := Table[4, {i, 1, FilledSize - lengthofgeneitself}]
FilledVec[vecvar5_] := Join[Flatten[vecvar5], Filler[vecvar5]]
For[npow = 1, npow < 1000, npow++, If[lengthofgeneitself ≤ (2^npow), Break[]]];
(* gives npow such that 2^npow > lengthofgeneitself > 2^(npow - 1) *)
FilledSize = 2^npow;
FilledM = FilledVec[M];
numrowsW = Sqrt[Length[FilledM]];

W = Table[Table[FilledM[[i]],
  {i, ((j - 1) * (numrowsW) + 1), (j * (numrowsW))}], {j, 1, numrowsW}];

ρ = (W.Transpose[W]); (* ρ as inner product *)
rhoEigens = Sort[DeleteCases[Eigenvalues[ρ] // N, 0.], Greater];
(*DeleteCases Removes 0's from the set of Eigenvalues,
Sort puts the list in order of greatest to least *)
set =  $\frac{\text{rhoEigens}}{\text{Total[rhoEigens]}}$ ;
(* This is the set of nonzero normalized eigenvalues in order of greatest to least *)
n = Length[set];
H[α_] :=  $\frac{1}{1 - \alpha} \text{Log}[2, \text{Sum}[(\text{set}[[i]])^\alpha, \{i, 1, n\}]] // N$ 
H0 = Log[n] // N; (* H0 = Hartley Entropy*)
H1 = -Sum[(set[[i]]) (Log[2, set[[i]]]), {i, 1, n}] // N; (* H1 = Shannon Entropy*)
H2onward = Table[H[a], {a, 2, 20}] // N; (* H2 onward *)
RenyiEntropyofEigenvalues = Join[{H0}, {H1}, H2onward];

RenyiEntropyofEigenvalues
{4.81218, 3.02888, 1.19163, 0.898593, 0.798784, 0.74886, 0.718906,
0.698936, 0.684672, 0.673974, 0.665653, 0.658997, 0.653551, 0.649012,
0.645172, 0.64188, 0.639027, 0.636531, 0.634329, 0.632371, 0.630619}

```

## Data

```

PCV1KP3373491RenyiEntropyofEigenvalues =
{3.1354942159291497`, 0.9224876815068266`, 0.3321644424748865`, 0.2504622262433977`,
0.2226432523739165`, 0.2087281509895171`, 0.20037902613420414`, 0.19481294208998814`,
0.1908371677618248`, 0.18785533701554916`, 0.18553613532399918`, 0.18368077397075921`,
0.18216275104538102`, 0.1808977319408992`, 0.17982733116018382`,
0.17890984477671348`, 0.17811468991103924`, 0.17741892940357426`,
0.17680502307345808`, 0.17625932855779922`, 0.1757710755701045`};
Print["For ", "PCV1 Genbank:KP337349.1"]

Table[{i, PCV1KP3373491RenyiEntropyofEigenvalues[[i]]},
  {i, 2, Length[PCV1KP3373491RenyiEntropyofEigenvalues]};

```

```

Table[{i, Log[PCV1KP3373491RenyiEntropyofEigenvalues[[i]]]},
  {i, 2, Length[PCV1KP3373491RenyiEntropyofEigenvalues]}};
Table[{Log[i], Log[PCV1KP3373491RenyiEntropyofEigenvalues[[i]]]},
  {i, 2, Length[PCV1KP3373491RenyiEntropyofEigenvalues]}};

Print["Values of  $H_\alpha$  ", PCV1KP3373491RenyiEntropyofEigenvalues]
ListPlot[Table[{i, PCV1KP3373491RenyiEntropyofEigenvalues[[i]]},
  {i, 1, Length[PCV1KP3373491RenyiEntropyofEigenvalues]}],
  AxesLabel → {Style[" $\alpha$ ", Large, Bold], Style[" $H_\alpha$ ", Large, Bold]},
  PlotLabel → Style["Plot of  $H_\alpha$ ", Black, Bold, 12]]
ListPlot[Table[{i, PCV1KP3373491RenyiEntropyofEigenvalues[[i]]},
  {i, 2, Length[PCV1KP3373491RenyiEntropyofEigenvalues]}],
  AxesLabel → {Style[" $\alpha$ ", Large, Bold], Style[" $H_\alpha$ ", Large, Bold]},
  PlotLabel → Style["Plot of  $H_\alpha$ , excluding  $H_1$ ", Black, Bold, 12]]

Print["Values of  $\text{Log}_2[H_\alpha]$  ", Log[PCV1KP3373491RenyiEntropyofEigenvalues]]
ListPlot[Table[{i, Log[2, PCV1KP3373491RenyiEntropyofEigenvalues[[i]]]},
  {i, 1, Length[PCV1KP3373491RenyiEntropyofEigenvalues]}],
  AxesLabel → {Style[" $\alpha$ ", Large, Bold], Style[" $\text{Log}_2[H_\alpha]$ ", Large, Bold]},
  PlotLabel → Style["Plot of  $\text{Log}_2[H_\alpha]$ ", Black, Bold, 12]]
ListPlot[Table[{i, Log[2, PCV1KP3373491RenyiEntropyofEigenvalues[[i]]]},
  {i, 1, Length[PCV1KP3373491RenyiEntropyofEigenvalues]}],
  AxesLabel → {Style[" $\alpha$ ", Large, Bold], Style[" $\text{Log}_2[H_\alpha]$ ", Large, Bold]},
  PlotLabel → Style["Plot of  $\text{Log}_2[H_\alpha]$ ", Black, Bold, 12]]

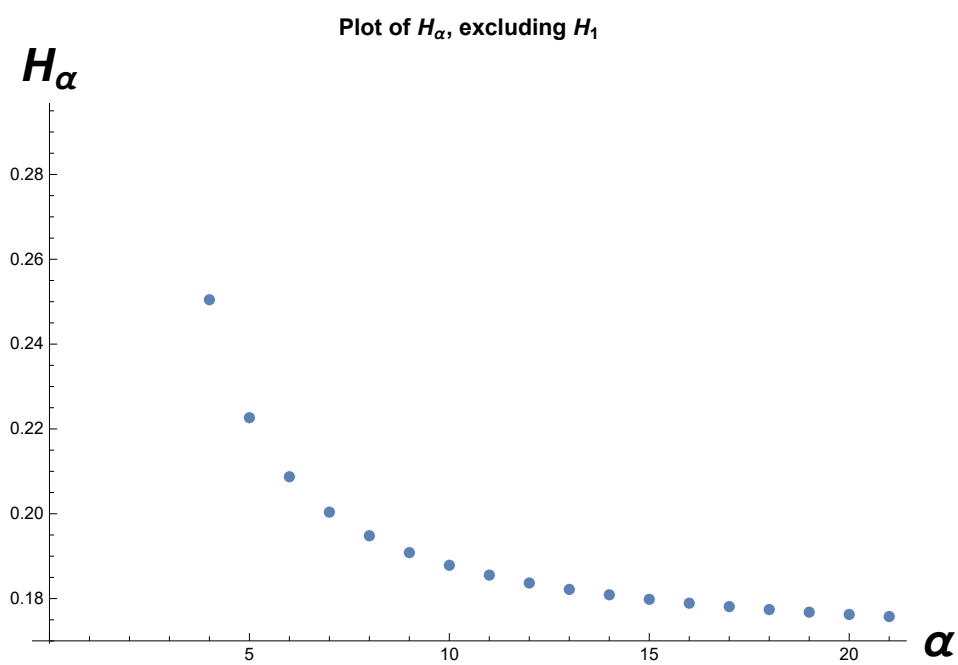
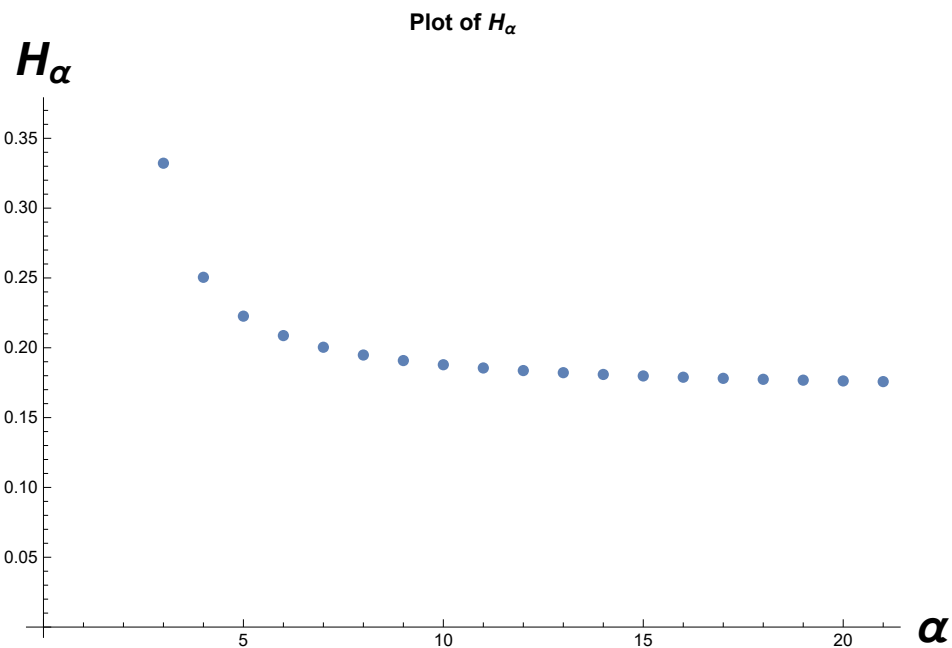
ListPlot[Table[{i, Log[2, PCV1KP3373491RenyiEntropyofEigenvalues[[i]]]},
  {i, 2, Length[PCV1KP3373491RenyiEntropyofEigenvalues]}],
  AxesLabel → {Style[" $\alpha$ ", Large, Bold], Style[" $\text{Log}_2[H_\alpha]$ ", Large, Bold]},
  PlotLabel → Style["Logplot of  $H_\alpha$ , excluding  $H_1$ ", Black, Bold, 12]]
ListPlot[Table[{i, Log[2, PCV1KP3373491RenyiEntropyofEigenvalues[[i]]]},
  {i, 3, Length[PCV1KP3373491RenyiEntropyofEigenvalues]}],
  AxesLabel → {Style[" $\alpha$ ", Large, Bold], Style[" $\text{Log}_2[H_\alpha]$ ", Large, Bold]},
  PlotLabel → Style["Logplot of  $H_\alpha$ , excluding  $H_1, H_2$ ", Black, Bold, 12]]
ListPlot[Table[{i, Log[2, PCV1KP3373491RenyiEntropyofEigenvalues[[i]]]},
  {i, 4, Length[PCV1KP3373491RenyiEntropyofEigenvalues]}],
  AxesLabel → {Style[" $\alpha$ ", Large, Bold], Style[" $\text{Log}_2[H_\alpha]$ ", Large, Bold]},
  PlotLabel → Style["Logplot of  $H_\alpha$ , excluding  $H_1, H_2, H_3$ ", Black, Bold, 12]]

(*ListPlot[Table[{Log[2,i],Log[2,PCV1KP3373491RenyiEntropyofEigenvalues[[i]]]},
  {i,1,Length[PCV1KP3373491RenyiEntropyofEigenvalues]}]]*)

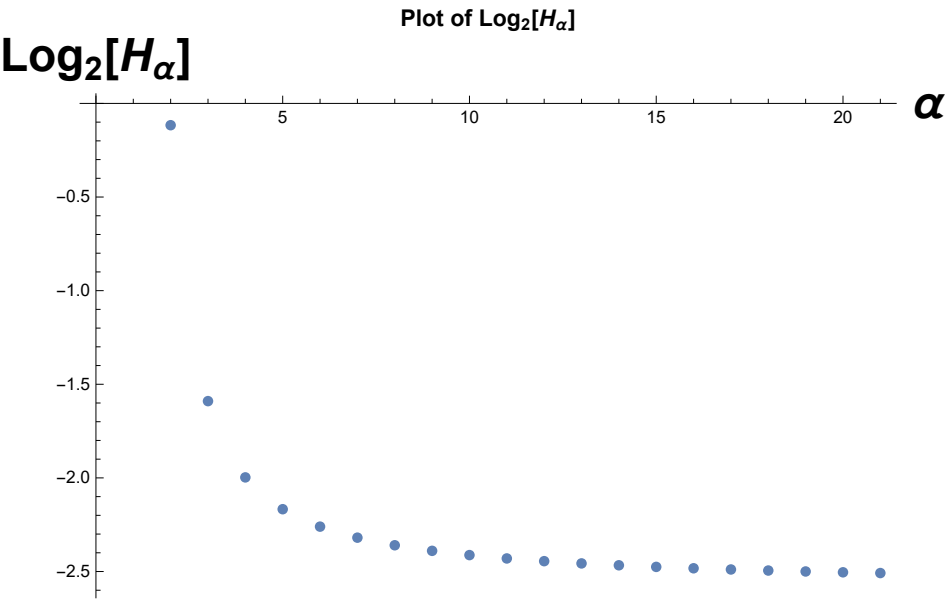
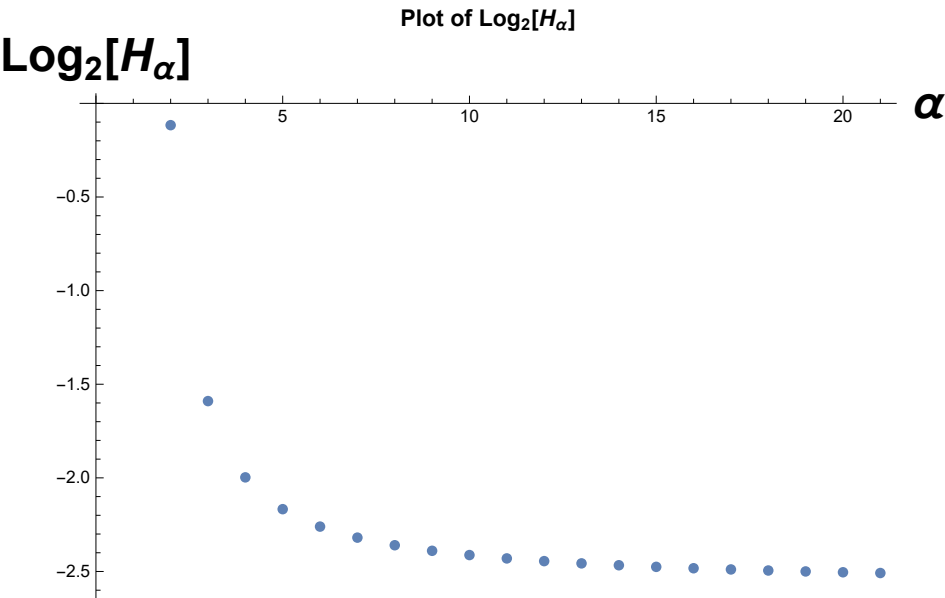
For PCV1 Genbank:KP337349.1

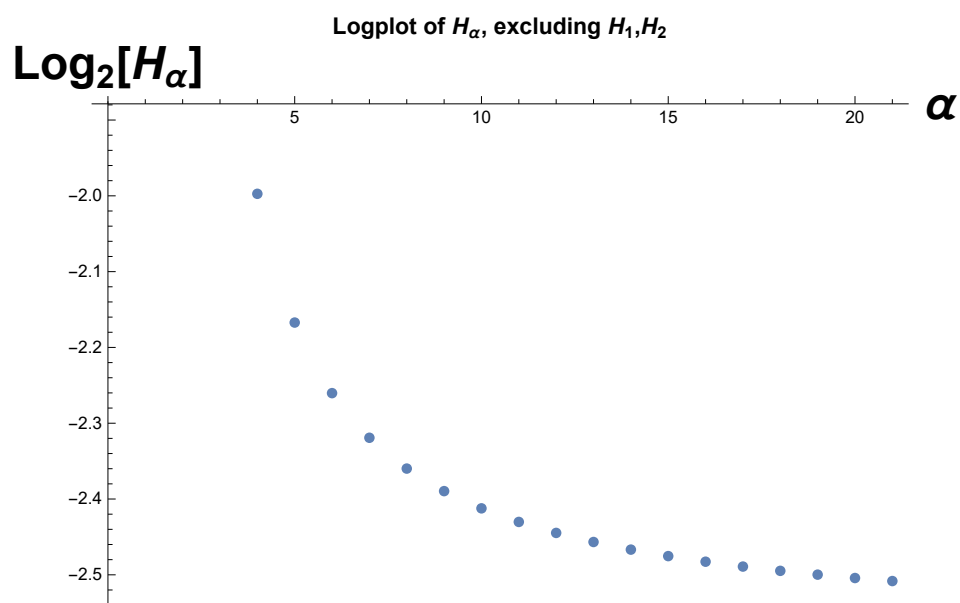
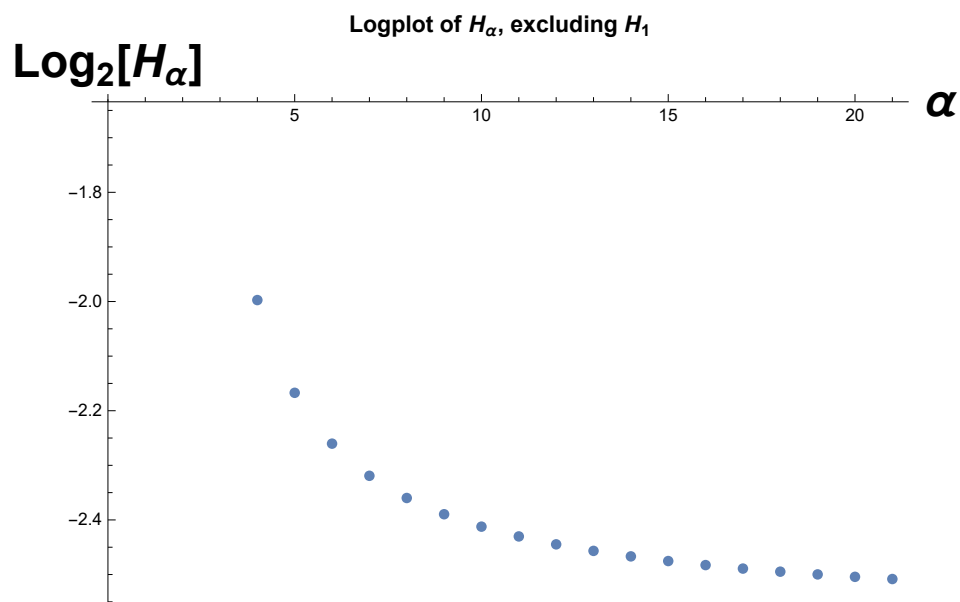
Values of  $H_\alpha$  {3.13549, 0.922488, 0.332164, 0.250462, 0.222643,
  0.208728, 0.200379, 0.194813, 0.190837, 0.187855, 0.185536, 0.183681, 0.182163,
  0.180898, 0.179827, 0.17891, 0.178115, 0.177419, 0.176805, 0.176259, 0.175771}

```

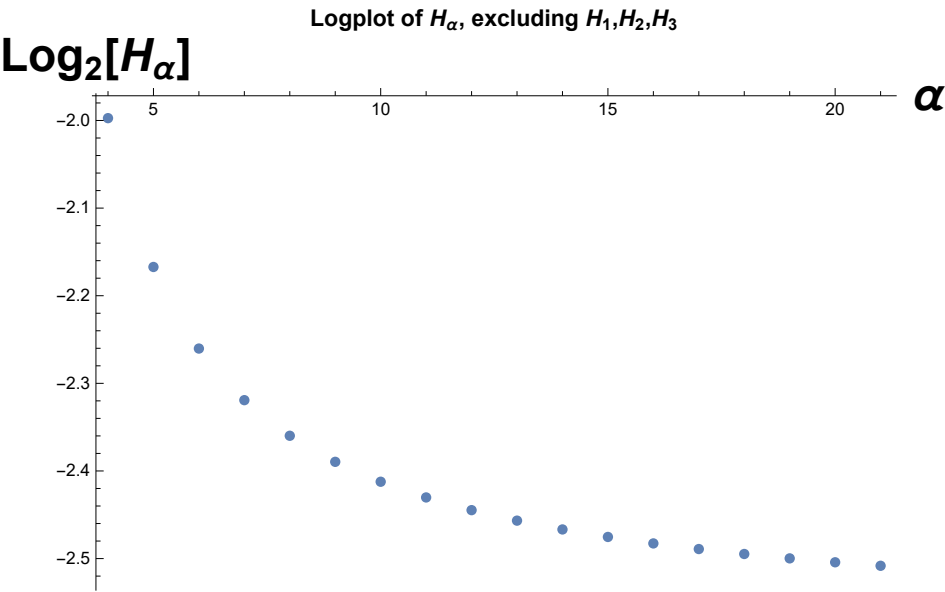


Values of  $\text{Log}_2[H_\alpha]$  {1.14279, -0.0806813, -1.10213, -1.38445, -1.50218,  
 -1.56672, -1.60754, -1.63572, -1.65633, -1.67208, -1.68451, -1.69456, -1.70285,  
 -1.70982, -1.71576, -1.72087, -1.72533, -1.72924, -1.73271, -1.7358, -1.73857}









```

HumanMitoChonDQ1129612RenyiEntropyofEigenvalues =
  {4.812184355372417`, 3.028884662199708`, 1.1916259910182405`, 0.8985929912796556`,
   0.7987837436451032`, 0.7488601080689298`, 0.7189057081568627`,
   0.6989361052172268`, 0.6846721030709843`, 0.6739741014605183`,
   0.6656534335412528`, 0.6589968992058404`, 0.6535506438405028`, 0.6490120977027216`,
   0.6451717894322913`, 0.6418800966290653`, 0.6390272961996027`, 0.6365310958238231`,
   0.634328566080488`, 0.6323707618641902`, 0.6306190423022395`};
Print["For ", "HumanMitoChondrial Genbank:DQ112961.2"]

Table[{i, HumanMitoChonDQ1129612RenyiEntropyofEigenvalues[[i]]},
  {i, 2, Length[HumanMitoChonDQ1129612RenyiEntropyofEigenvalues]}};
Table[{i, Log[HumanMitoChonDQ1129612RenyiEntropyofEigenvalues[[i]]]},
  {i, 2, Length[HumanMitoChonDQ1129612RenyiEntropyofEigenvalues]}};
Table[{Log[i], Log[HumanMitoChonDQ1129612RenyiEntropyofEigenvalues[[i]]]},
  {i, 2, Length[HumanMitoChonDQ1129612RenyiEntropyofEigenvalues]}};

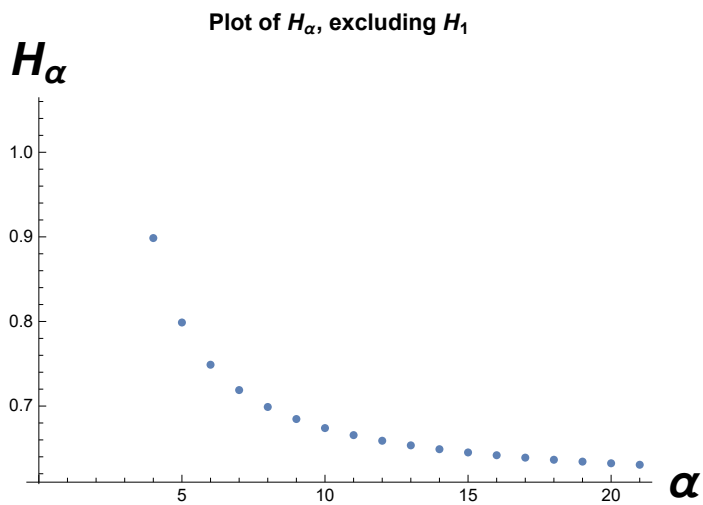
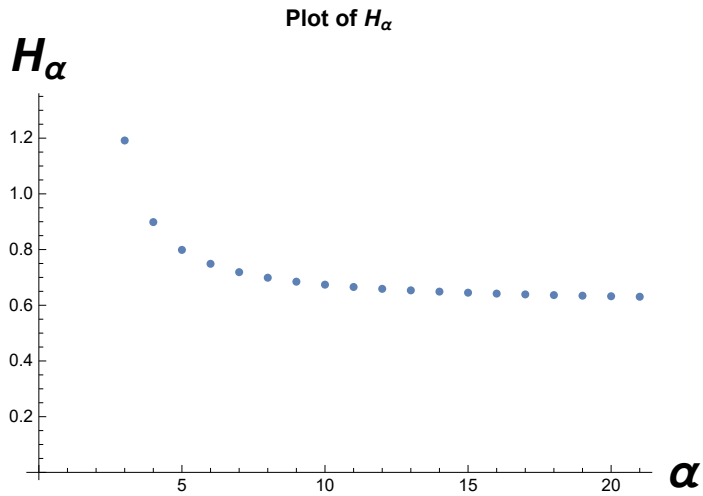
Print["Values of  $H_\alpha$  ", HumanMitoChonDQ1129612RenyiEntropyofEigenvalues]
ListPlot[Table[{i, HumanMitoChonDQ1129612RenyiEntropyofEigenvalues[[i]]},
  {i, 1, Length[HumanMitoChonDQ1129612RenyiEntropyofEigenvalues]}}],
  AxesLabel → {Style[" $\alpha$ ", Large, Bold], Style[" $H_\alpha$ ", Large, Bold]},
  PlotLabel → Style["Plot of  $H_\alpha$ ", Black, Bold, 12]]
ListPlot[Table[{i, HumanMitoChonDQ1129612RenyiEntropyofEigenvalues[[i]]},
  {i, 2, Length[HumanMitoChonDQ1129612RenyiEntropyofEigenvalues]}}],
  AxesLabel → {Style[" $\alpha$ ", Large, Bold], Style[" $H_\alpha$ ", Large, Bold]},
  PlotLabel → Style["Plot of  $H_\alpha$ , excluding  $H_1$ ", Black, Bold, 12]]

Print["Values of  $\text{Log}_2[H_\alpha]$  ", Log[HumanMitoChonDQ1129612RenyiEntropyofEigenvalues]]
ListPlot[Table[{i, Log[2, HumanMitoChonDQ1129612RenyiEntropyofEigenvalues[[i]]]},
  {i, 1, Length[HumanMitoChonDQ1129612RenyiEntropyofEigenvalues]}}],
  AxesLabel → {Style[" $\alpha$ ", Large, Bold], Style[" $\text{Log}_2[H_\alpha]$ ", Large, Bold]},
  PlotLabel → Style["Plot of  $\text{Log}_2[H_\alpha]$ ", Black, Bold, 12]]

ListPlot[Table[{i, Log[2, HumanMitoChonDQ1129612RenyiEntropyofEigenvalues[[i]]]},
  {i, 2, Length[HumanMitoChonDQ1129612RenyiEntropyofEigenvalues]}}],
  AxesLabel → {Style[" $\alpha$ ", Large, Bold], Style[" $\text{Log}_2[H_\alpha]$ ", Large, Bold]},
  PlotLabel → Style["Logplot of  $H_\alpha$ , excluding  $H_1$ ", Black, Bold, 12]]
ListPlot[Table[{i, Log[2, HumanMitoChonDQ1129612RenyiEntropyofEigenvalues[[i]]]},
  {i, 3, Length[HumanMitoChonDQ1129612RenyiEntropyofEigenvalues]}}],
  AxesLabel → {Style[" $\alpha$ ", Large, Bold], Style[" $\text{Log}_2[H_\alpha]$ ", Large, Bold]},
  PlotLabel → Style["Logplot of  $H_\alpha$ , excluding  $H_1, H_2$ ", Black, Bold, 12]]
ListPlot[Table[{i, Log[2, HumanMitoChonDQ1129612RenyiEntropyofEigenvalues[[i]]]},
  {i, 4, Length[HumanMitoChonDQ1129612RenyiEntropyofEigenvalues]}}],
  AxesLabel → {Style[" $\alpha$ ", Large, Bold], Style[" $\text{Log}_2[H_\alpha]$ ", Large, Bold]},
  PlotLabel → Style["Logplot of  $H_\alpha$ , excluding  $H_1, H_2, H_3$ ", Black, Bold, 12]]

For HumanMitoChondrial Genbank:DQ112961.2
Values of  $H_\alpha$  {4.81218, 3.02888, 1.19163, 0.898593, 0.798784,
  0.74886, 0.718906, 0.698936, 0.684672, 0.673974, 0.665653, 0.658997, 0.653551,
  0.649012, 0.645172, 0.64188, 0.639027, 0.636531, 0.634329, 0.632371, 0.630619}

```



Values of  $\text{Log}_2[H_\alpha]$  {1.57115, 1.10819, 0.175319, -0.106925, -0.224665, -0.289203, -0.330025, -0.358196, -0.378815, -0.394564, -0.406986, -0.417036, -0.425335, -0.432304, -0.438239, -0.443354, -0.447808, -0.451722, -0.455188, -0.458279, -0.461053}

