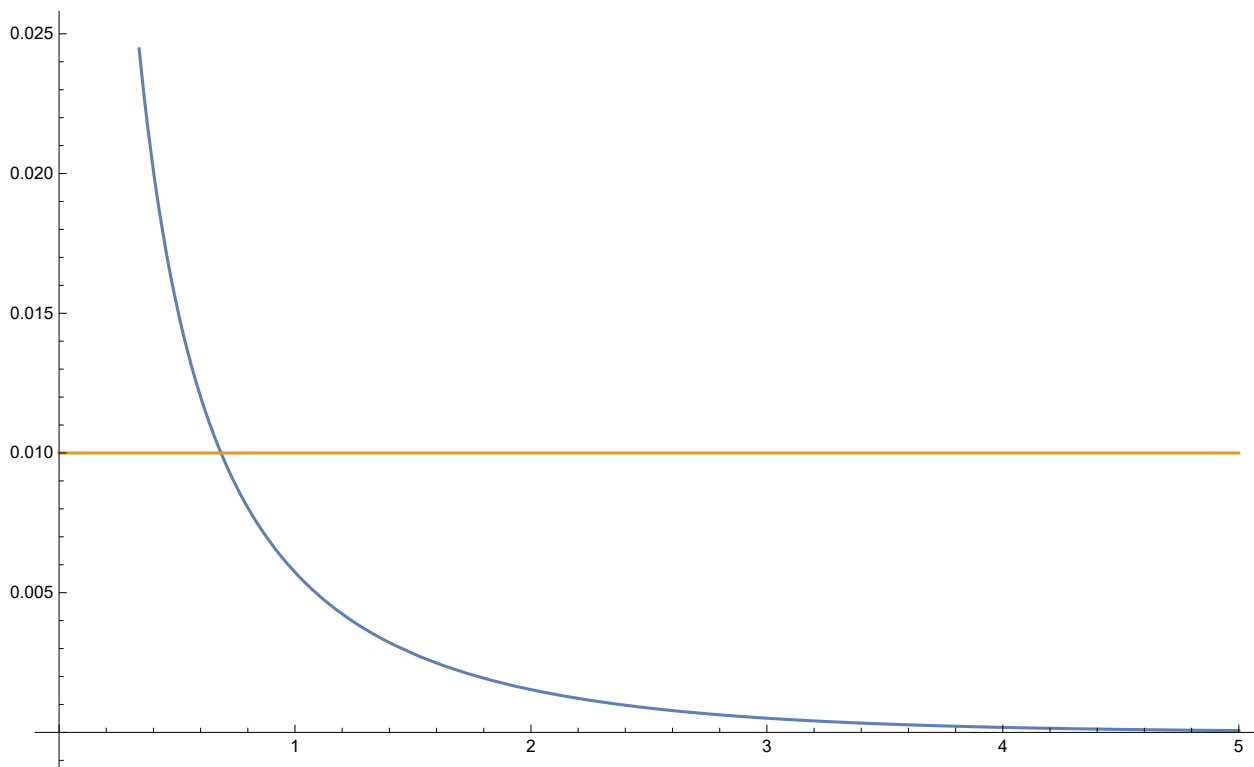```
Assuming[Re[s] > 1, 1/Gamma[s] Integrate[x^(s-1) (Exp[x] - 1)^(-1), {x, 0, ∞}]]
```

```
PolyLog[s, 1]
```

```
a = 100;
Plot[{(1/a) ((Exp[(x + a x)/a] - 1)^(-1)), (1/a)}, {x, 0, 5}]
```



```
F[A_, s_, x_] :=
  1/A ((1 + 1/A)^s) Exp[(1 + 1/A) x] ((Exp[(1 + 1/A) x] - 1)^(-s-1)) ((-1)^s) Product[n, {n, 1, s}]
```
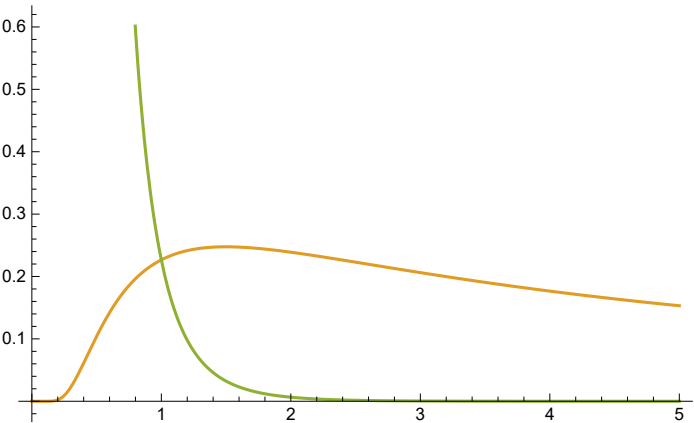
```
Table[F[i, 2, 1], {i, 1, 10}] // N
```

```
{0.226657, 0.238921, 0.206215, 0.176555,
 0.153125, 0.134748, 0.120122, 0.108268, 0.0984942, 0.0903105}
```

```
F[5, 2, 1] // N
```

```
0.153125
```

```
Plot[{F[100 000, x, x], F[x, 2, 1], F[x, 2, x]}, {x, 0, 5}]
```
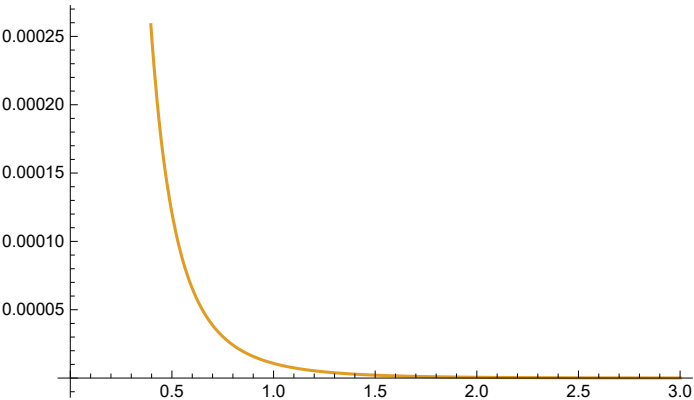


```
Zeta[2]
Zeta[4] // N
```

$$\frac{\pi^2}{6}$$

```
1.08232
```

```
N[ π²/6 ]
```

$$N\left[\frac{\pi^2}{6}\right]$$

```
1.64493
```

```
G[s_, x_] := F[100 000, s, x]
```

```
Limit[F[A, s, x], A → Infinity]
```

```
0
```

```
Plot[{G[x, x], G[2, x], G[x, 2]}, {x, 0, 3}]
```

```
n = 4;
Table[Solve[G[i, x] == Zeta[i], x] // N // MatrixForm, {i, 2, 10}]
```

$$
\left\{
\begin{pmatrix}
x \rightarrow \text{ConditionalExpression}\left[0.99999 \left(0.0229067 + \left(0. + 6.28319\,\dot{\imath}\right) C[1]\right), C[1] \in \text{Integer} \right. \\
x \rightarrow \text{ConditionalExpression}\left[0.99999 \left(\left(-0.0114533 + 0.0199904\,\dot{\imath}\right) + \left(0. + 6.28319\,\dot{\imath}\right) C[1]\right), C[1] \in \right. \\
x \rightarrow \text{ConditionalExpression}\left[0.99999 \left(\left(-0.0114533 - 0.0199904\,\dot{\imath}\right) + \left(0. + 6.28319\,\dot{\imath}\right) C[1]\right), C[1] \in \right.
\end{pmatrix}
\right.
$$