Explore fns in particles statistics, mathematical properties and relns to math fns

$$\sum_{i=1}^{r} (\texttt{E}^\wedge(-\beta\, i\, e0)) \;\; \texttt{// FullSimplify}$$

$$-\frac{1 - e^{-e0\, r\, \beta}}{1 - e^{e0\, \beta}}$$

$$-\frac{1 - e^{-e0\, r\, \beta}}{1 - e^{e0\, \beta}} \;\; \texttt{// FullSimplify}$$

$$-\frac{1 - e^{-e0\, r\, \beta}}{1 - e^{e0\, \beta}}$$

Integrals

To see how it might relate to Riemann zeta fn

$$\int -\frac{1 - e^{-x\, r}}{1 - e^{x}}\, dx \;\; \texttt{// FullSimplify}$$

$$-x + e^{-r\, x} \left(e^{x}\right)^{r} \text{Beta}\left[e^{x}, -r, 0\right] + \text{Log}\left[1 - e^{x}\right]$$

$$\int -\frac{1 - e^{-x}}{1 - e^{x}}\, dx \;\; \texttt{(*r=1*)}$$

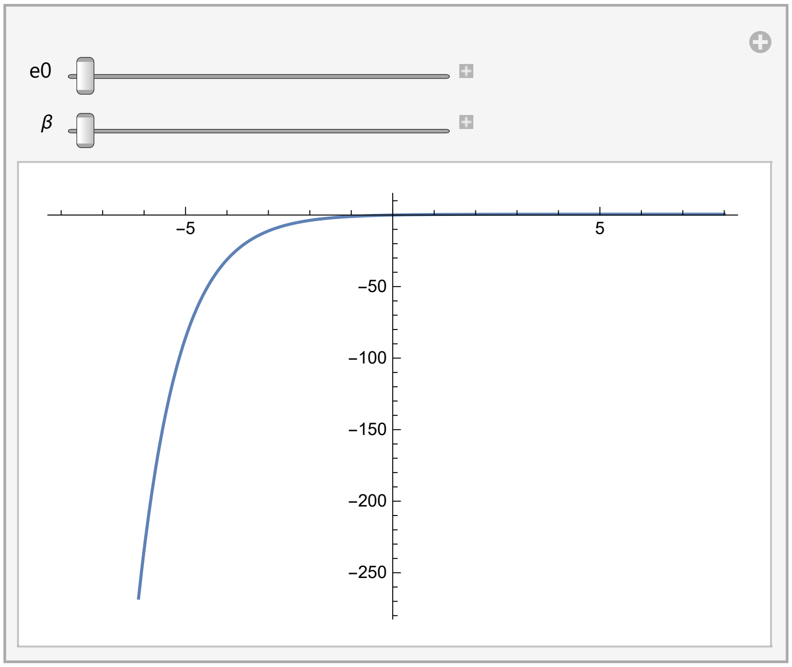
$$-e^{-x}$$

Summation

$$-\frac{1 - e^{-e0\, r\, \beta}}{1 - e^{e0\, \beta}}$$

$$-\frac{1 - e^{-e0\, r\, \beta}}{1 - e^{e0\, \beta}}$$

$$\text{Manipulate}\left[\text{Plot}\left[-\frac{1-e^{-e0\, r\, \beta}}{1-e^{e0\, \beta}}, \{r, -8, 8\}\right], \{e0, 1, 10\}, \{\beta, 1, 2\}\right]$$



GCE BE

$Z = \sum_{i=0}^{r} (E\,\hat{}\,(\beta\, i\, (\mu - e0)))$; b= β (μ-e0) = -β (e0 - μ)

$$\sum_{i=0}^{r} \left(E\,\hat{}\,\left(\beta\, i\, \left(\mu - e0\right)\right)\right) \;\; \text{// FullSimplify}$$

$$\sum_{i=0}^{r} \left(E\,\hat{}\,\left(i\, \left(b\right)\right)\right) \;\; \text{// FullSimplify}$$

$$\frac{e^{e0\, \beta} - e^{\beta\, (-e0\, r + \mu + r\, \mu)}}{e^{e0\, \beta} - e^{\beta\, \mu}}$$

$$\frac{-1 + e^{b + b\, r}}{-1 + e^{b}}$$

$$\frac{1 - e^{\beta\, (-e0\, r + \mu + r\, \mu - e0)}}{1 - e^{\beta\, (\mu - e0)}} \;\; \text{// FullSimplify}$$

$$\frac{-1 + e^{-(1+r)\, \beta\, (e0 - \mu)}}{-1 + e^{\beta\, (-e0 + \mu)}}$$

<N> = (1/β)* (1/Z)* [∂_{μ}(Z)]

$$\left(\frac{1}{\left(\beta\,\frac{-1+e^{-(1+r)\,\beta\,(e0-\mu)}}{-1+e^{-\beta\,(e0-\mu)}}\right)}\right)\left(\partial_\mu\left(\frac{-1+e^{-(1+r)\,\beta\,(e0-\mu)}}{-1+e^{-\beta\,(e0-\mu)}}\right)\right)\ \text{// FullSimplify}$$

$$\frac{1}{-1+e^{\beta\,(e0-\mu)}}-\frac{1+r}{-1+e^{(1+r)\,\beta\,(e0-\mu)}}$$

$$n=\frac{1}{-1+e^{\beta\,(e0-\mu)}}-\frac{1+r}{-1+e^{(1+r)\,\beta\,(e0-\mu)}};$$

$$n\ \text{/.}\ \beta\,(e0-\mu)\to b$$

$$\frac{1}{-1+e^{b}}-\frac{1+r}{-1+e^{b\,(1+r)}}$$

$$-\left(\frac{1}{\left(\left(\sum_{i=0}^{r}\left(E\hat{}\left(-i\,b\right)\right)\right)\right)}\right)\left(\partial_b\left(\left(\sum_{i=0}^{r}\left(E\hat{}\left(-i\,b\right)\right)\right)\right)\right)\ \text{// FullSimplify}$$

$$\frac{1}{-1+e^{b}}+\frac{1+r}{1-e^{b+b\,r}}$$

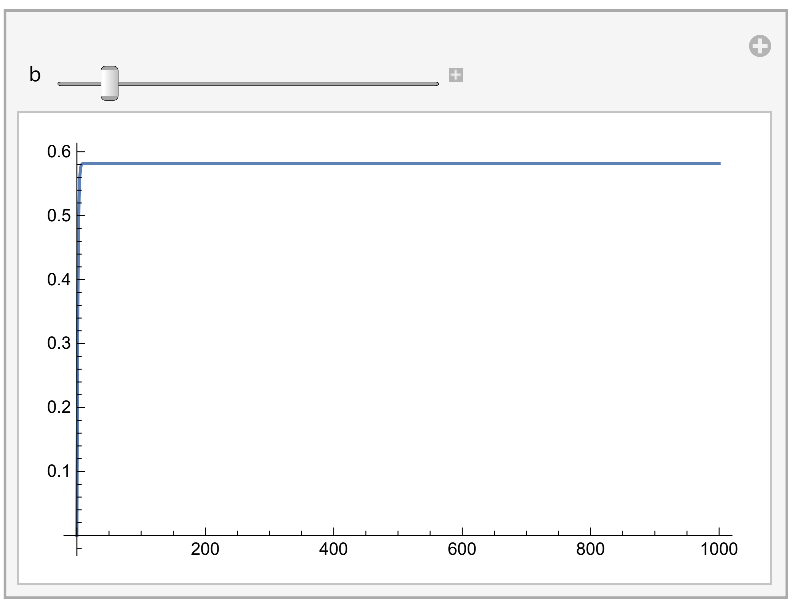
$$\int\left(\frac{1}{-1+e^{b}}-\frac{1+r}{-1+e^{b\,(1+r)}}\right)db\ \text{// FullSimplify}$$

$$b\,r+\text{Log}\left[1-e^{b}\right]-\text{Log}\left[1-e^{b\,(1+r)}\right]$$

$$\int\left(\frac{1}{-1+e^{b}}\right)db\ \text{// FullSimplify (*ordinary BE fn*)}$$

$$-b+\text{Log}\left[1-e^{b}\right]$$

$$\text{Manipulate}\left[\text{Plot}\left[\frac{1}{-1+e^{b}}-\frac{1+r}{-1+e^{b\,(1+r)}},\ \{r,0,1000\}\right],\ \{b,0.001,10\}\right]$$



$$\frac{1}{-1+e^{b}}-\frac{1+1}{-1+e^{b\,(1+1)}}\ \text{// FullSimplify}$$

$$\frac{1}{1+e^{b}}$$

$$\frac{1}{-1+e^{1}}-\frac{1+r}{-1+e^{b\,(1+r)}}\ \text{// FullSimplify}$$

$$\frac{1}{-1+e^{1}}-\frac{1+r}{-1+e^{1\,(1+r)}}\ \text{// FullSimplify}$$

$$\frac{1}{-1+e}+\frac{1+r}{1-e^{b\,(1+r)}}$$

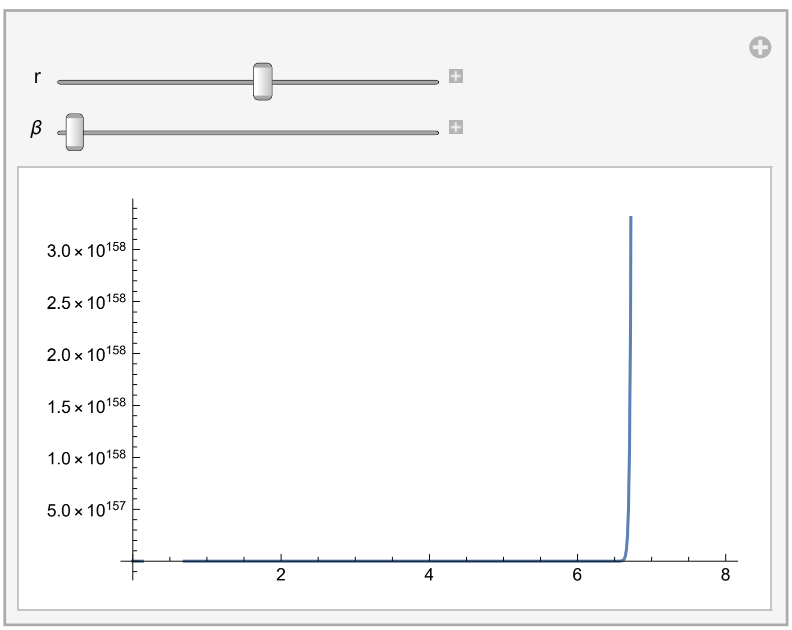
$$\frac{1}{-1+e}+\frac{1+r}{1-e^{1+r}}$$

Integral

$$\int\frac{-1+e^{(1+r)\,\beta\,x}}{-1+e^{\beta\,x}}\,dx\ \text{// FullSimplify}$$

$$-\frac{1}{\beta}\left(-x\,\beta+e^{(1+r)\,x\,\beta}\,\text{Gamma}[1+r]\,\text{Hypergeometric2F1Regularized}\left[1,1+r,2+r,e^{x\,\beta}\right]+\text{Log}\left[1-e^{x\,\beta}\right]\right)$$

$$\text{Manipulate}\Big[\text{Plot}\Big[-\frac{1}{\beta}\left(-x\,\beta+e^{(1+r)\,x\,\beta}\,\text{Gamma}[1+r]\,\text{Hypergeometric2F1Regularized}\left[1,1+r,2+r,e^{x\,\beta}\right]+\text{Log}\left[1-e^{x\,\beta}\right]\right),\{x,0,8\}\Big],\{r,1,100\},\{\beta,1,10\}\Big]$$



log[a + b]

log[a + b]

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Table[{i, N[Zeta[-i, 7], 20]}, {i, 0, 40}] // MatrixForm
```

0	−6.5000000000000000000
1	−21.083333333333333333
2	−91.000000000000000000
3	−440.99166666666666667
4	−2275.0000000000000000
5	−12 201.003968253968254
6	−67 171.000000000000000
7	−376 760.99583333333333
8	$-2.1425950000000000000 \times 10^{6}$
9	$-1.2313161007575757576 \times 10^{7}$
10	$-7.1340451000000000000 \times 10^{7}$
11	$-4.1599868097890720391 \times 10^{8}$
12	$-2.4382357150000000000 \times 10^{9}$
13	$-1.4350108521083333333 \times 10^{10}$
14	$-8.4740914531000000000 \times 10^{10}$
15	$-5.0179068620055674020 \times 10^{11}$
16	$-2.9780358776350000000 \times 10^{12}$
17	$-1.7706908038284053954 \times 10^{13}$
18	$-1.0544376109341100000 \times 10^{14}$
19	$-6.2870926703129454379 \times 10^{14}$
20	$-3.7526288711643550000 \times 10^{15}$
21	$-2.2418196307542722460 \times 10^{16}$
22	$-1.3402351320458109100 \times 10^{17}$
23	$-8.0172161490442243349 \times 10^{17}$
24	$-4.7982677405200318750 \times 10^{18}$
25	$-2.8729438001035715829 \times 10^{19}$
26	$-1.7207635044052328157 \times 10^{20}$
27	$-1.0309589716986332874 \times 10^{21}$
28	$-6.1782671979205334562 \times 10^{21}$
29	$-3.7032206100719074687 \times 10^{22}$
30	$-2.2200639542274564910 \times 10^{23}$
31	$-1.3311047435011710387 \times 10^{24}$
32	$-7.9819626229088860407 \times 10^{24}$
33	$-4.7868455774040716751 \times 10^{25}$
34	$-2.8709417173178748414 \times 10^{26}$
35	$-1.7219823634357478774 \times 10^{27}$
36	$-1.0328981436235479972 \times 10^{28}$
37	$-6.1959327257001275909 \times 10^{28}$
38	$-3.7168316618558302886 \times 10^{29}$
39	$-2.2297350481130085114 \times 10^{30}$
40	$-1.3376590694799440631 \times 10^{31}$

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Zeta[-2, 5]
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−30

BE Partition Function