```
FactorInteger[504]
FactorInteger[504][[3, 1]]
```

```
{{2, 3}, {3, 2}, {7, 1}}
```

```
7
```

```
Length[FactorInteger[504]]
Table[FactorInteger[504][[i, 1]], {i, 1, Length[FactorInteger[504]]}]
```

```
3
```

```
{2, 3, 7}
```

## Definitional fns for unique (non- repeating) prime factors

Unique, so for instance 10 and 100 have the same exact unique prime factorization {2,5}

```
PrimeFactors[n_] := Table[FactorInteger[n][[i, 1]], {i, 1, Length[FactorInteger[n]]}]
```

```
PrimeFactors[10]
PrimeFactors[100]
```

```
{2, 5}
```

```
{2, 5}
```

Defintiional funtions:

```
pfprod[n_] := Product[i, {i, PrimeFactors[n]}]
pfsum[n_] := Total[PrimeFactors[n]]
pfmean[n_] := Mean[PrimeFactors[n]]
pfmax[n_] := Max[PrimeFactors[n]]
pfmin[n_] := Min[PrimeFactors[n]]
```

```
n = 10;
{pfprod[n], pfsum[n], pfmean[n], pfmax[n], pfmin[n]}
```

$\left\{10, 7, \frac{7}{2}, 5, 2\right\}$

Example:

```
n = 10;
PrimeFactors[n]
Product[i, {i, PrimeFactors[n]}]
Total[PrimeFactors[n]]
Mean[PrimeFactors[n]]
Max[PrimeFactors[n]]
Min[PrimeFactors[n]]
```

{2, 5}

10

7

$\frac{7}{2}$

5

2

## Definitional fns for all (including repeat) prime factors

```
AllPrimeFactors[n_] := Flatten[
  Table[
   Table[
    FactorInteger[n][[i, 1]]
    , {j, 1, FactorInteger[n][[i, 2]]}
   ]
   , {i, 1, Length[FactorInteger[n]]}
  ]
 ]
```

```
AllPrimeFactors[504]
```

{2, 2, 2, 3, 3, 7}

```
AllPrimeFactors[10]
AllPrimeFactors[100]
```

{2, 5}

{2, 2, 5, 5}

```
apfprod[n_] := Product[i, {i, AllPrimeFactors[n]}]
apfsum[n_] := Total[AllPrimeFactors[n]]
apfmean[n_] := Mean[AllPrimeFactors[n]]
apfmax[n_] := Max[AllPrimeFactors[n]]
apfmin[n_] := Min[AllPrimeFactors[n]]
```

```
n = 700;
{apfprod[n], apfsum[n], apfmean[n], apfmax[n], apfmin[n]}
```

$\left\{700, 21, \frac{21}{5}, 7, 2\right\}$

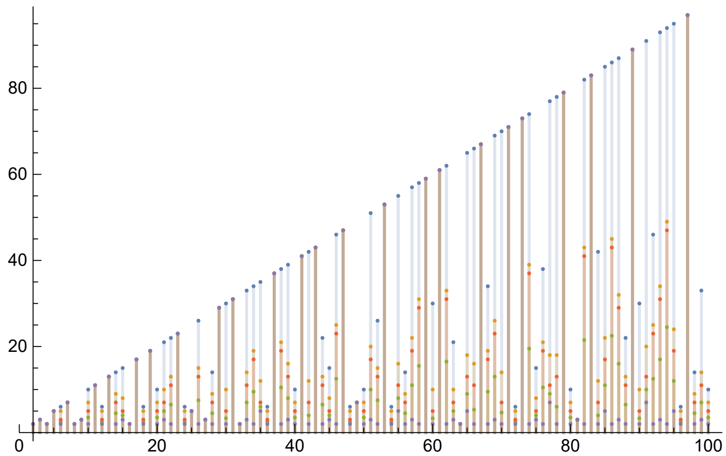
```
{apfprod[x], apfsum[x], apfmean[x], apfmax[x], apfmin[x]}
```

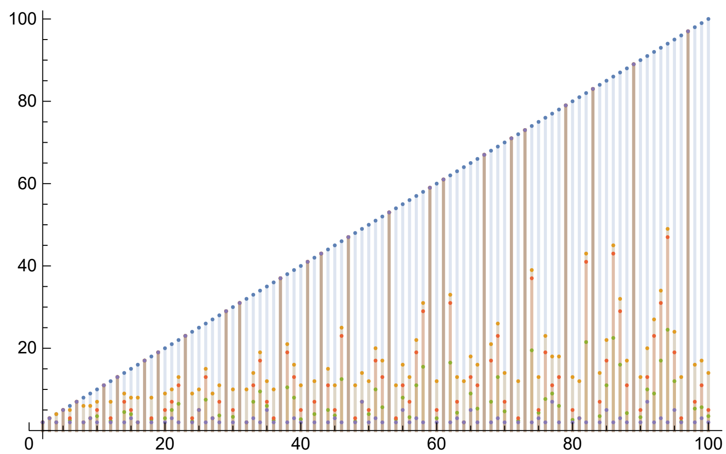
## Plots

Have the functions {pfprod[x],pfsum[x],pfmean[x],pfmax[x],pfmin[x]}

For a given x th

```
DiscretePlot[{pfprod[x], pfsum[x], pfmean[x], pfmax[x], pfmin[x]}, {x, 2, 100}]
DiscretePlot[{apfprod[x], apfsum[x], apfmean[x], apfmax[x], apfmin[x]}, {x, 2, 100}]
```



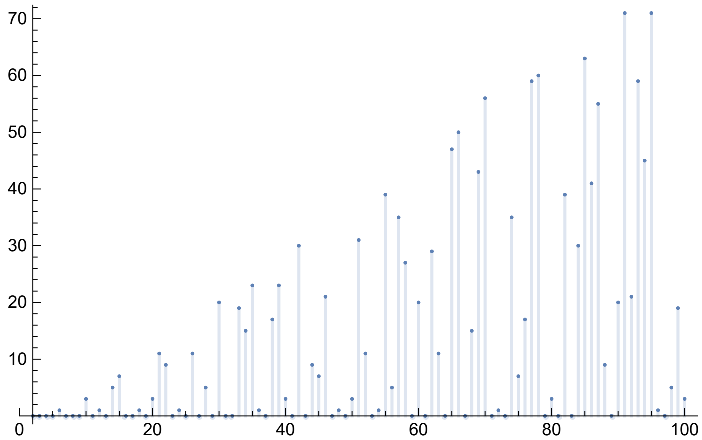

$$ab - (a + b) = ab\left(1 - \left(\frac{1}{b} + \frac{1}{a}\right)\right)$$

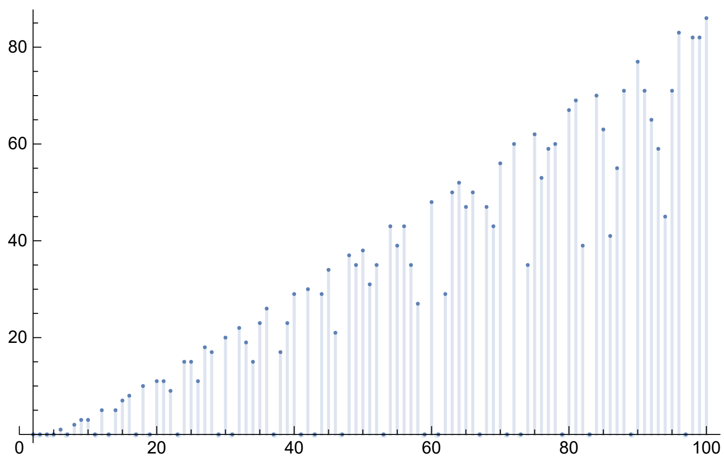
Product

```
Product[i, {i, 1, 10}] - Sum[i, {i, 1, 10}]
```

3 628 745

```
DiscretePlot[{pfprod[x] - pfsum[x]}, {x, 2, 100}]
DiscretePlot[{apfprod[x] - apfsum[x]}, {x, 2, 100}]
```

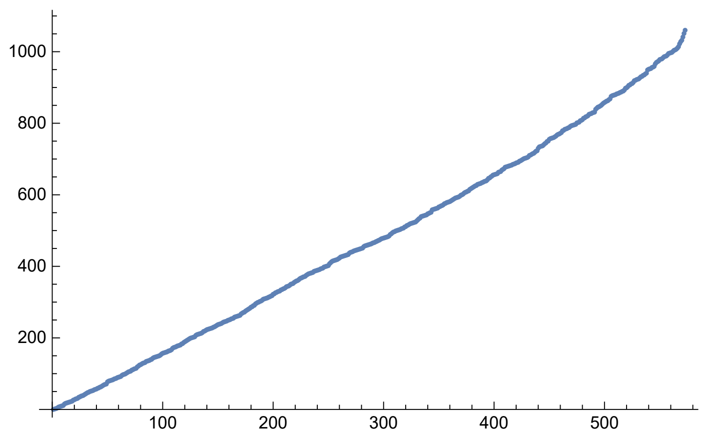


```
xmax = 1080;
Table[apfprod[x] - apfsum[x], {x, 2, xmax}];
Union[Table[apfprod[x] - apfsum[x], {x, 2, xmax}]];
Max[Union[Table[apfprod[x] - apfsum[x], {x, 2, xmax}]]]
Length[Union[Table[apfprod[x] - apfsum[x], {x, 2, xmax}]]]
ListPlot[Union[Table[apfprod[x] - apfsum[x], {x, 2, xmax}]]]
```

1060

573




Seems that,
letting A be the list / set Union[Table[apfprod[x] − apfsum[x], {x, 2, 100}]]

let N ≤ xmax (since the union fn removes repeated elements)
be the number of elements of the list

max element of the list seems to be ~ 2 N

Moreover we seem to have that the N − th element of A, $A_N$, is ~ 2 N

i.e. $A_N \sim 2N$ very nearly for the N − th element of list A

seems to hold for larger N too, tested up to N ~ 10, 000 so far

```
xmax = 1000;
Max[Union[Table[apfprod[x] - apfsum[x], {x, 2, xmax}]]]
Length[Union[Table[apfprod[x] - apfsum[x], {x, 2, xmax}]]]
DiscretePlot[{Max[Union[Table[apfprod[x] - apfsum[x], {x, 2, xmax}]]]/2 -
   Length[Union[Table[apfprod[x] - apfsum[x], {x, 2, xmax}]]],
  (-1/2) (Log[xmax])^2}, {xmax, 1, 400}]
```

979

537