Omega

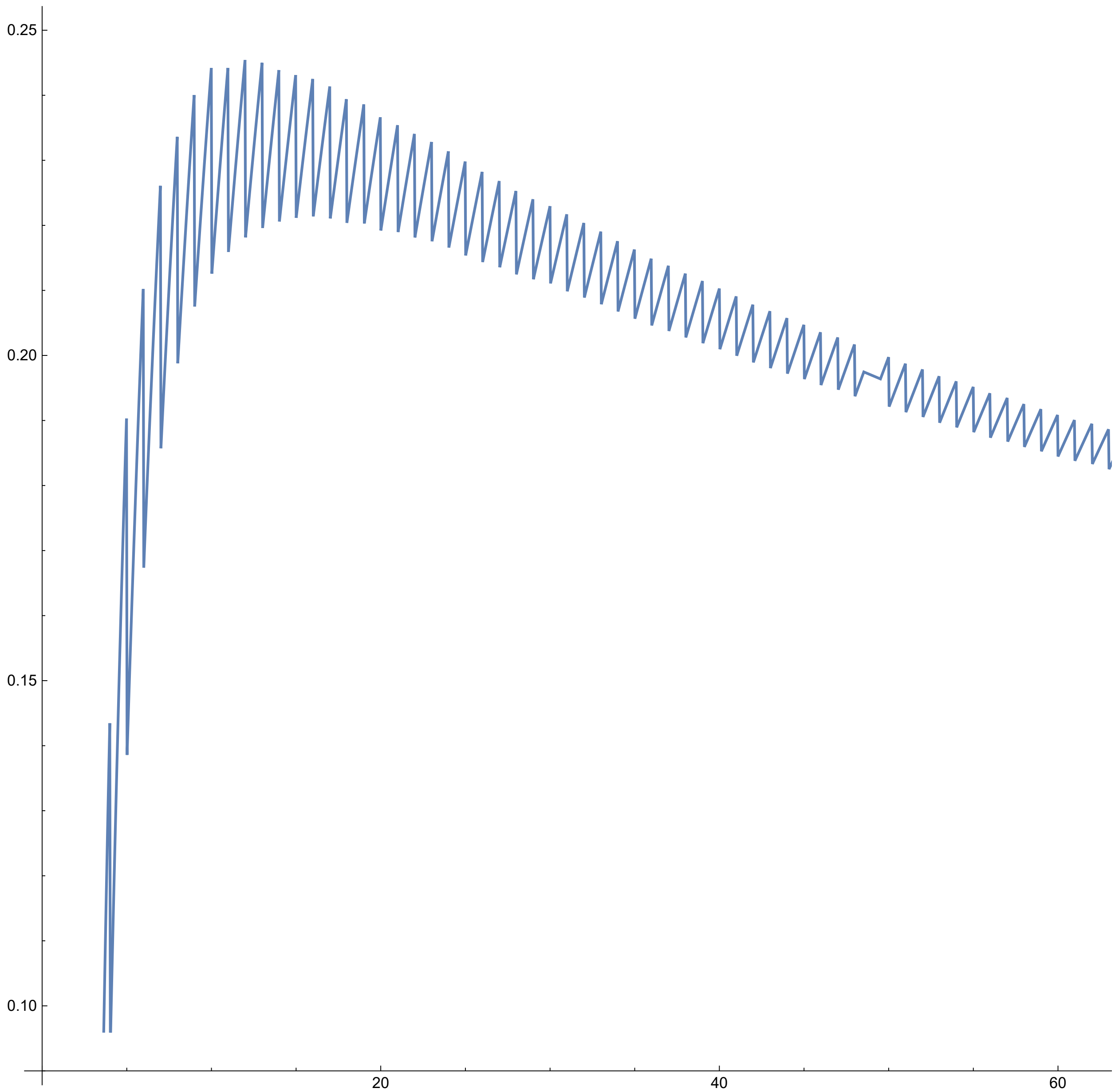
```
Ω[x_] := Sum[1/i - 1/Prime[i], {i, 1, x}]
```

```
Plot[{Log[PrimePi[x]] - Sum[1/i - 1/Prime[i], {i, 1, x}],
  Log[LogIntegral[x]] - Log[PrimePi[x]]}, {x, 1, 1000}]
```

```
x = 40 000;
1/2 (Log[LogIntegral[x]] - Ω[x]) // N
```

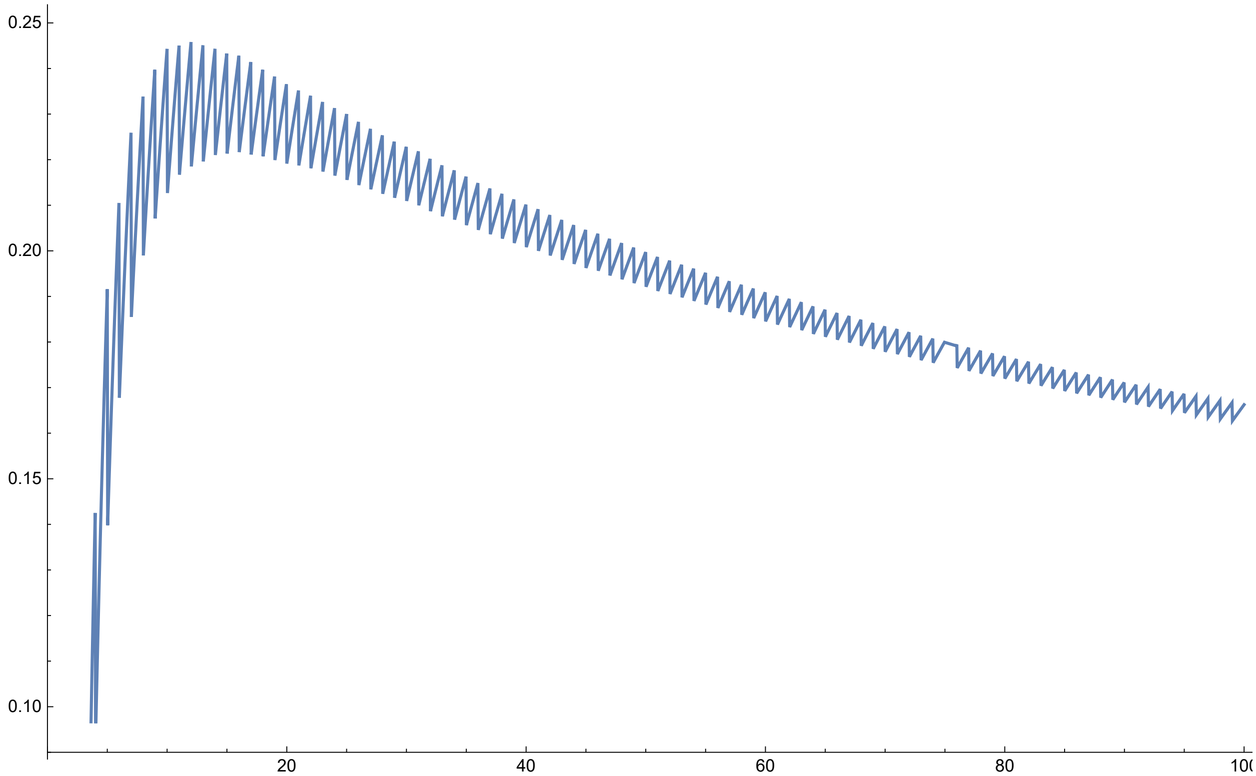
```
0.00479942
```

```
Plot[{1/2 (Log[LogIntegral[x]] - Ω[x])}, {x, 1, 100}]
```

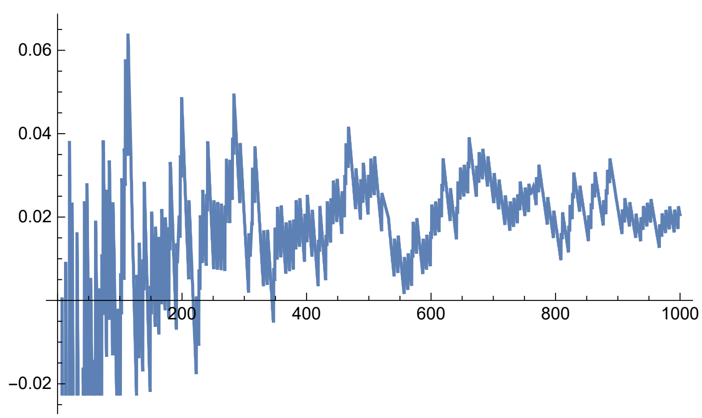



```
A[x_] := 1/2 (Log[LogIntegral[x]] - Ω[x])
B[x_] := (Log[PrimePi[x]]) - (1/2 (Log[LogIntegral[x]] + Ω[x]))
SumofA[xmax_] := Sum[1/2 (Log[LogIntegral[x]] - Ω[x]), {x, 2, xmax}]
A[14]
1.74391
```

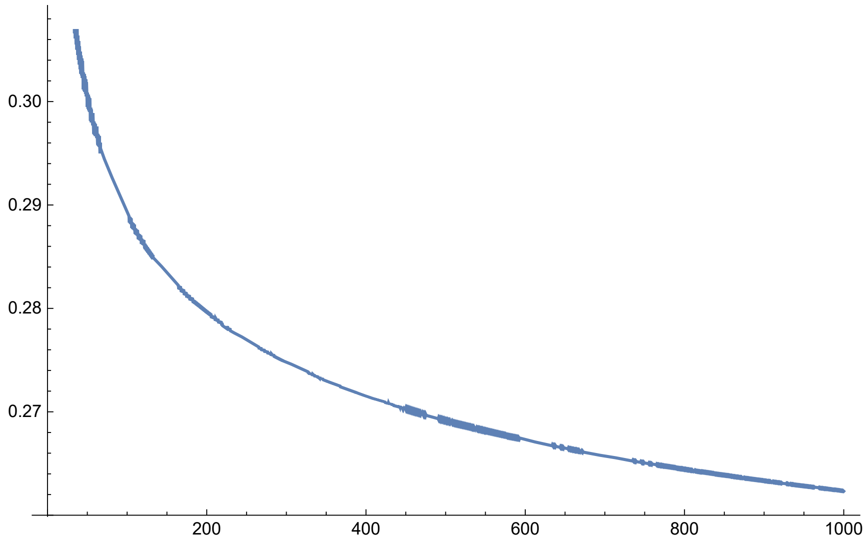
```
Plot[{A[x]}, {x, 2, 100}]
```



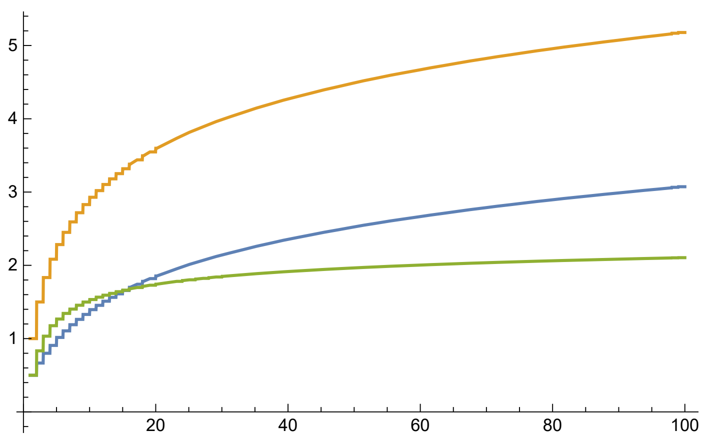

```
Plot[{B[x]}, {x, 2, 1000}]
```



```
Plot[{1/2 (Sum[1/Prime[i], {i, 1, x}] - Log[Log[x]])}, {x, 1, 1000}]
```




```
Plot[{Sum[1/i - 1/Prime[i], {i, 1, x}],
  Sum[1/i, {i, 1, x}], Sum[1/Prime[i], {i, 1, x}]}, {x, 1, 100}]
```



Linearity relation and estimate of large Mersenne primes

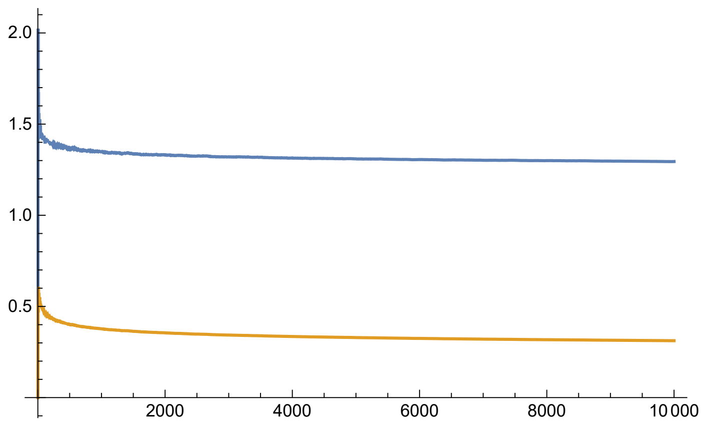
Analytics

From Dusert ' s inequality we have that
Prime[x] < x Log[x] + x Log[Log[x]], for x > 6
so that

$$\frac{\text{Prime}[x]}{\text{Log}[\text{PrimePi}[x]]} < \frac{x\,\text{Log}[x]}{\text{Log}[\text{PrimePi}[x]]} + \frac{x\,\text{Log}[\text{Log}[x]]}{\text{Log}[\text{PrimePi}[x]]} = x\left(\frac{\text{Log}[x]}{\text{Log}[\text{PrimePi}[x]]} + \frac{\text{Log}[\text{Log}[x]]}{\text{Log}[\text{PrimePi}[x]]}\right)$$

So b $= \left(\frac{\text{Log}[x]}{\text{Log}[\text{PrimePi}[x]]} + \frac{\text{Log}[\text{Log}[x]]}{\text{Log}[\text{PrimePi}[x]]}\right)$ precisely, but

```
Plot[{Log[x]/Log[PrimePi[x]], Log[Log[x]]/Log[PrimePi[x]]}, {x, 1, 10000}]
```




```
firstterm[x_] := Log[x]/Log[PrimePi[x]]
secondterm[x_] := Log[Log[x]]/Log[PrimePi[x]]
```

```
x0 = 500 000 000 000;
firstterm[x0] // N
secondterm[x0] // N

firstterm[x0] / secondterm[x0]  // N
```

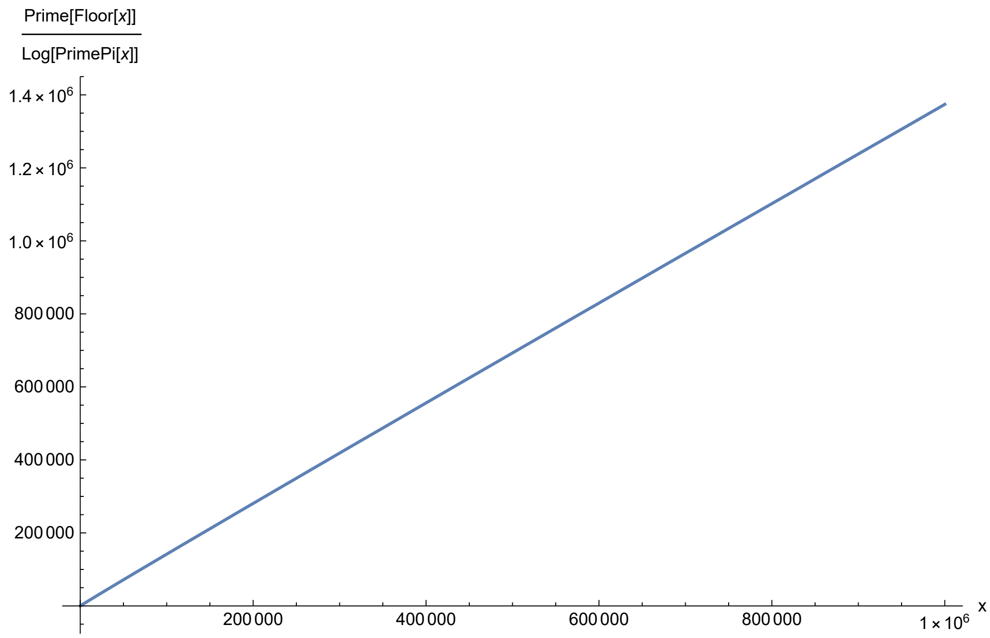
1.1374

0.139063

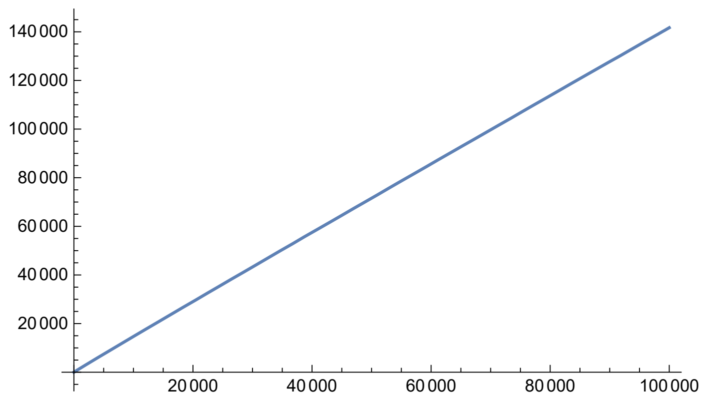
8.17902

Plots and numerics

$$\texttt{Plot}\left[\left\{\frac{\texttt{Prime[Floor[x]]}}{\texttt{Log[PrimePi[x]]}}\right\}, \{\texttt{x, 1, 1000000}\}, \texttt{AxesLabel} \to \left\{\texttt{"x"}, \texttt{"}\frac{\texttt{Prime[Floor[x]]}}{\texttt{Log[PrimePi[x]]}}\texttt{"}\right\}\right]$$



$$\texttt{Plot}\left[\left\{\frac{\texttt{Prime[Floor[x]]}}{\texttt{Log[LogIntegral[x]]}}\right\}, \{\texttt{x, 1, 100000}\}\right]$$



$$\texttt{Do}\left[\texttt{Print}\left[\texttt{" \& "}, \frac{\left(\frac{1}{10^n}\left(\frac{\texttt{Prime}[\texttt{Floor}[10^n]]}{\texttt{Log}[\texttt{PrimePi}[10^n]]}\right)\right)}{\left(\frac{\texttt{Log}[10^n]}{\texttt{Log}[\texttt{PrimePi}[10^n]]} + \left(\frac{\texttt{Log}[\texttt{Log}[10^n]]}{\texttt{Log}[\texttt{PrimePi}[10^n]]}\right)\right)} \texttt{ // N}\right], \{\texttt{n, 1, 12}\}\right] \texttt{ // MatrixForm}$$

```
 & 0.924563
 & 0.882207
 & 0.895774
 & 0.916211
 & 0.931264
 & 0.941888
 & 0.949435
 & 0.955311
 & 0.959891
 & 0.963586
 & 0.966629
 & 0.969185
Null
```

Table[{StringJoin["n = ", ToString[n]], " & ", $\frac{1}{10^n}\left(\frac{\text{Prime[Floor[}10^n\text{]]}}{\text{Log[PrimePi[}10^n\text{]]}}\right)$, " & ", $\left(\frac{\text{Log[}10^n\text{]}}{\text{Log[PrimePi[}10^n\text{]]}}\right)$, " & ", $\left(\frac{\text{Log[Log[}10^n\text{]]}}{\text{Log[PrimePi[}10^n\text{]]}}\right)$}, {n, 1, 12}] // N // MatrixForm

n = 1	&	2.09191	&	1.66096	&	0.601627
n = 2	&	1.68071	&	1.43068	&	0.474445
n = 3	&	1.54548	&	1.34813	&	0.377178
n = 4	&	1.47216	&	1.29469	&	0.312109
n = 5	&	1.41755	&	1.25568	&	0.266502
n = 6	&	1.37398	&	1.22578	&	0.232972
n = 7	&	1.3383	&	1.20222	&	0.207351
n = 8	&	1.30925	&	1.18334	&	0.187161
n = 9	&	1.28502	&	1.16788	&	0.170829
n = 10	&	1.26454	&	1.15499	&	0.157335
n = 11	&	1.24702	&	1.14408	&	0.145986
n = 12	&	1.23185	&	1.13472	&	0.136299

Table[$\frac{1}{10^n}\left(\frac{\text{Prime[Floor[}10^n\text{]]}}{\text{Log[PrimePi[}10^n\text{]]}}\right)$, {n, 1, 12}] // N

```
{2.09191, 1.68071, 1.54548, 1.47216, 1.41755,
 1.37398, 1.3383, 1.30925, 1.28502, 1.26454, 1.24702, 1.23185}
```

N[$\frac{1}{10^7}\left(\frac{\text{Prime[Floor[}10^7\text{]]}}{\text{Log[PrimePi[}10^7\text{]]}}\right)$, 18]

```
1.33830006985348365
```

```
constlist = Table[1/10^n (Prime[Floor[10^n]]/Log[LogIntegral[10^n]]), {n, 1, 12}] // N
```

{1.5943, 1.58866, 1.52889, 1.4693, 1.41694,
1.37378, 1.33825, 1.30924, 1.28501, 1.26454, 1.24702, 1.23185}

```
constlist[[9]] - constlist[[10]]
constlist[[10]] - constlist[[11]]
constlist[[11]] - constlist[[12]]
(* constlist[[9]] = constlist[[n]], n=9 *)
```

0.0204756

0.0175232

0.0151637

Might estimate that constlist[[13]] − constlist[[14]] ≈ 0.013,
so that for $n = 13$ we have a linear constant of appx **b ≈ 1.218**

$$\frac{1}{10^n}\left(\frac{\text{Prime}[10^n]}{\text{Log}[\text{LogIntegral}[10^n]]}\right) \approx b$$

$$\Rightarrow \text{Prime}[10^n] \approx b\,(10^n)\,\text{Log}[\text{LogIntegral}[10^n]]$$

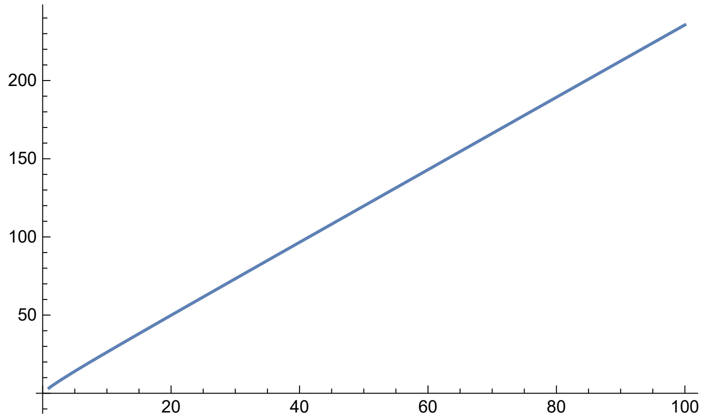
the, for $n = 13$, we have that

$$\text{Prime}\left[10^{13}\right] \approx 1.218\,\left(10^{13}\right)\,\text{Log}\left[\text{LogIntegral}\left[10^{13}\right]\right]$$

using a fit $b[10^x] \approx \frac{1}{1.08703 - 0.495126\, e^{-0.0512058\, x}}$, we have that

$$\text{Prime}[10^n] \approx \frac{1}{1.08703 - 0.495126\, e^{-0.0512058\, n}}\,(10^n)\,\text{Log}[\text{LogIntegral}[10^n]]$$

```
Plot[Log[ (1 / (1.087026743375239` - 0.49512587313630557` ⅇ^(-0.051205804449252136` n)))
   (10^n) Log[LogIntegral[10^n]]], {n, 1, 100}]
```



```
n = 10;
(1 / (1.087026743375239` - 0.49512587313630557` ⅇ^(-0.051205804449252136` n)))
 (10^n) Log[LogIntegral[10^n]]
```

10 000 000 000

$Aborted

```
Prime[10^13]
```

Prime[10 000 000 000 000]

```
Log[LogIntegral[10^13]] // N
1.218 (10^13) Log[LogIntegral[10^13]] (*appx for n=13*)
```

26.5699

3.23621×10^{14}

```
fit2 /. x → 30
```

1.01991

```
Log[LogIntegral[10^30]] // N
(1.0199146114912094`) (10^30) Log[LogIntegral[10^30]] (*appx for n=30*)
```

64.8571

6.61487×10^{31}

```
fit2 /. x → 35
```

0.995478

```
Log[LogIntegral[10^35]] // N
(0.9954781699824219`) (10^35) Log[LogIntegral[10^35]] (*appx for n=13*)
```

76.2137

7.58691×10^{36}

```
fit2 /. x → 100
```

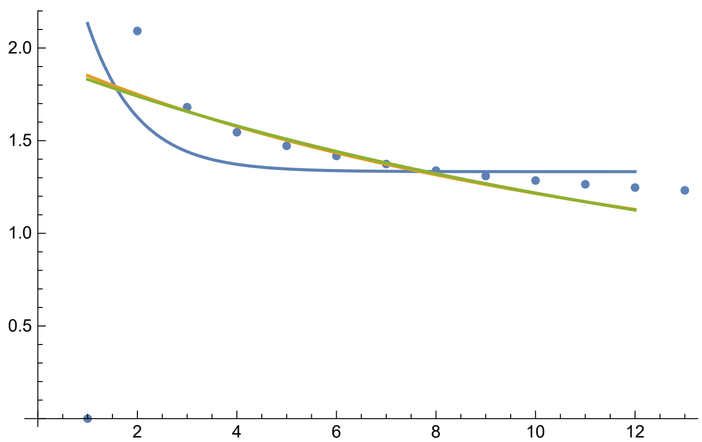
0.92245

```
Log[LogIntegral[10^100]] // N
(0.9224500174233891`) (10^100) Log[LogIntegral[10^100]] (*appx for n=100,
 much rougher, probably have more like b ≈1, but use fit in this appx*)
```

224.824

2.07389×10^{102}

```
Show[
 ListPlot[{0.`, 2.0919078092889967`, 1.6807109979837502`, 1.545483151683329`,
   1.4721625827740545`, 1.4175522912973006`, 1.3739773524220535`,
   1.3383000698534837`, 1.309252869136104`, 1.2850159928687523`,
   1.2645403768352896`, 1.2470171880593464`, 1.2318534959925638`}],
 Plot[{constfit, fit2, ansatzfit}, {x, 1, 12}]
]
```




```
ansatztestfit = 1/(a Exp[b x] - 1.045); (*Since LogIntegral[~1.045] =0 *)
testfitfn = 1/(a Exp[b x] + c);

fit2 = testfitfn /. FindFit[constlist, testfitfn, {a, b, c}, x]
ansatzfit = ansatztestfit /. FindFit[constlist, ansatztestfit, {a, b, c}, x]
```

$$\frac{1}{1.08703 - 0.495126\, e^{-0.0512058\, x}}$$

$$\frac{1}{-1.045 + 1.65214\, e^{0.0106448\, x}}$$

```
fit2 /. x → 10
fit2 /. x → 100
fit2 /. x → 1000
fit2 /. x → 1 000 000
fit2 /. x → 1 000 000 000

ansatzfit /. x → 12
ansatzfit /. x → 100
ansatzfit /. x → 10 000
```

1.26531

0.92245

0.919941

0.919941

0.919941

1.20156

0.267015

3.56509×10^{-47}

```
modifiedconstlist = {1.6807109979837502`, 1.545483151683329`, 1.4721625827740545`,
   1.4175522912973006`, 1.3739773524220535`, 1.3383000698534837`, 1.309252869136104`,
   1.2850159928687523`, 1.2645403768352896`, 1.2470171880593464`, 1.2318534959925638`};
modifiedfit2 = testfitfn /. FindFit[modifiedconstlist, testfitfn, {a, b, c}, x]
```

$$\frac{1}{0.842523 - 0.296995\, e^{-0.194691\, x}}$$

```
modifiedfit2 /. x → 10
modifiedfit2 /. x → 100
modifiedfit2 /. x → 1000
modifiedfit2 /. x → 1 000 000
modifiedfit2 /. x → 1 000 000 000
```

1.24978

1.18691

1.18691

1.18691

1.18691

Mersenne Primes

For very very large primes, have that

$$\left(\frac{\text{Prime}[10^n]}{\text{Log}[\text{LogIntegral}[10^n]]}\right) \approx b * 10^n$$

$$\Rightarrow \text{Prime}[10^n] \approx b\,(10^n)\,\text{Log}[\text{LogIntegral}[10^n]]$$

Imagine there is some large Mersenne prime such that

$\text{Prime}[\sim 10^n] \approx b\,(10^n)\,\text{Log}[\text{LogIntegral}[10^n]] \approx M_k = 2^k - 1 \approx 2^k$,
k some prime since it ' s a Mersenne prime

$\Rightarrow b\,(10^n)\,\text{Log}[\text{LogIntegral}[10^n]] \approx 2^k$, k some prime since it ' s a Mersenne prime

$\Rightarrow \text{Log}[2, b\,(10^n)\,\text{Log}[\text{LogIntegral}[10^n]]] \approx k$,
k some prime since it ' s a Mersenne prime

$k \approx \text{Log}[2, b\,(10^n)\,\text{Log}[\text{LogIntegral}[10^n]]] =$
$\text{Log}[2, b] + \text{Log}[2, (10^n)] + \text{Log}[2, \text{Log}[\text{LogIntegral}[10^n]]]$

$k \approx \text{Log}[2, b] + n\,\text{Log}[2, 10] + \text{Log}[2, \text{Log}[\text{LogIntegral}[10^n]]]$

$\text{Log}[\text{LogIntegral}[10^{50}]] = 110.392 \quad (n = 5 * 10^1)$
$\text{Log}[\text{LogIntegral}[10^{100}]] = 224.824 \quad (n = 10^2)$
$\text{Log}[\text{LogIntegral}[10^{1\,000\,000}]] = 2.30257 \times 10^6 \quad (n = 10^6)$
$\text{Log}[\text{LogIntegral}[10^{10\,000\,000}]] = 2.30258 \times 10^7 \quad (n = 10^7)$
$\text{Log}[\text{LogIntegral}[10^{100\,000\,000}]] = 2.30258 \times 10^8 \quad (n = 10^8)$

$\text{Log}[\text{LogIntegral}[10^n]] \approx 2.30258\,n$ for large n

giving us the very large n appx

$k \approx \text{Log}[2, b] + n\,\text{Log}[2, 10] + \text{Log}[2, 2.30258\,n] =$
$\text{Log}[2, b] + n\,\text{Log}[2, 10] + \text{Log}[2, 2.30258] + \text{Log}[2, n]$

$k \approx \text{Log}[2, 10]\,\mathbf{n} + \text{Log}[2, \mathbf{n}] + (\text{Log}[2, b] + \text{Log}[2, 2.30258])$

b is of order 1 , even if b is not close to 1 exactly is is
negligible since k is very large (not as large as 10^n but still large)

Now want to find n for which k is prime

For such prime k, $2^k - 1$ is possibly a very large n Mersenne prime

and also $\text{Log}[2, \mathbf{n}] = \dfrac{\text{Log}[E, \mathbf{n}]}{\text{Log}[E, \mathbf{2}]} = \dfrac{\text{Log}[\mathbf{n}]}{\text{Log}[\mathbf{2}]}$

$k \approx \text{Log}[2, 10]\, \mathbf{n} + \dfrac{\text{Log}[\mathbf{n}]}{\text{Log}[\mathbf{2}]} + (\text{Log}[2, b] + \text{Log}[2, 2.30258])$

k[n_] := Log[2, 10] **n** + Log[2, **n**] + (Log[2, 1.2] + Log[2, 2.30258])

k[10^8] = 3.32193×10^8
k[10^9] = 3.32193×10^9

so look for a prime around 3.32193×10^n for some n

```
k[n_] := Log[2, 10] n  + Log[2, n]  + (Log[2, 1.0] +  Log[2, 2.30258])
knumappx[npow_] := N[Log[2, 10] (10^npow)  + Log[2, (10^npow)], (npow + 1)]
```

```
(Log[2, 0.91] +  Log[2, 2.30258])
```

1.06719

```
N[k[10^8], 9]
k[10^9]
knumappx[5]
knumappx[7]
knumappx[8]
knumappx[9]
```

```
N[Log[2, 10] (10^8)  + Log[2, (10^8)], 9]
```

3.32193×10^8

3.32193×10^9

332 209.

3.3219304×10^7

3.32192836×10^8

3.321928125×10^9

3.32192836×10^8

```
knumappx[10]
knumappx[11]
knumappx[12]
```

$3.3219280982 \times 10^{10}$

$3.32192809525 \times 10^{11}$

$3.321928094927 \times 10^{12}$

```
knumappx[8] + 1
```

3.32192837×10^{8}

So one such appx is

k ~ 332, 192, 837

Note that there are actually primes around this value. Some of the nearest are
{332 192 779, **332 192 831, 332 192 857**, 332 192 863, 332 192 873, 332 192 879}

Which are actually very close

Moreover we obtained a value within 6 places of an actual prime. While this may very well be coincidental, it is somewhat ecouraging

(data from https : // primes.utm.edu / lists / small / millions /)

So perhaps one of these is a Mersenne Prime

Furthermore these primes are in the 18 th to 19 th millionth prime range

time complexity ~ O (p log p log log p)

```
332 192 831 * Log[332 192 831] * Log[Log[332 192 831]] // N
74 207 281 * Log[74 207 281] * Log[Log[74 207 281]] // N
```

1.94016×10^{10}

3.89612×10^{9}

```
Log[LogIntegral[10^10000000]] // N
Log[2, Log[LogIntegral[10^1000000]]] // N
```

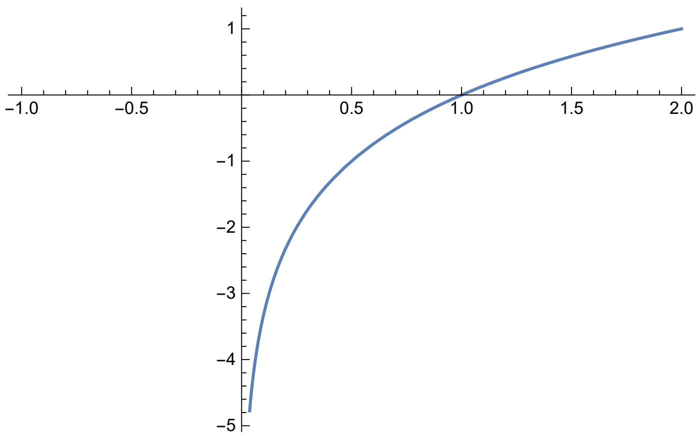
2.30258×10^{7}

21.1348

```
Log[LogIntegral[10^1000000000]] // N
```

2.30259×10^{9}

```
Plot[Log[2, x], {x, -1, 2}]
```




```
knumappx[8]
knumappx[9]
knumappx[10]
knumappx[11]
knumappx[12]
```

3.32192836×10^{8}

$3.321928125 \times 10^{9}$

$3.3219280982 \times 10^{10}$

$3.32192809525 \times 10^{11}$

$3.321928094927 \times 10^{12}$

```
knumappx[9]
```

$3.321928125 \times 10^{9}$

```
NextPrime[3 321 928 125] (* next prime after this number *)
NextPrime[3 321 928 125, 2] (* 2nd next prime after this number *)
NextPrime[3 321 928 125, -1] (* previous prime *)
NextPrime[3 321 928 125, -2] (* 2nd previous prime *)
```

3 321 928 171

3 321 928 189

3 321 928 121

3 321 928 109

$$\frac{\text{Prime}[159\,500\,000]}{(10\text{^}9)} \text{ // N}$$

3.32923

```
knumappx[10]
```

$3.3219280982 \times 10^{10}$

```
NextPrime[33 219 280 982]
NextPrime[33 219 280 982, 2]
NextPrime[33 219 280 982, -1]
NextPrime[33 219 280 982, -2]
```

33 219 281 003

33 219 281 023

33 219 280 951

33 219 280 937

```
knumappx[11]
```

$3.32192809525 \times 10^{11}$

```
NextPrime[332 192 809 525]
NextPrime[332 192 809 525, 2]
NextPrime[332 192 809 525, -1]
NextPrime[332 192 809 525, -2]
```

332 192 809 589

332 192 809 603

332 192 809 477

332 192 809 471

```
knumappx[12]
```

$3.321928094927 \times 10^{12}$

```
NextPrime[3 321 928 094 927]
NextPrime[3 321 928 094 927, 2]
NextPrime[3 321 928 094 927, -1]
NextPrime[3 321 928 094 927, -2]
```

3 321 928 094 941

3 321 928 094 977

3 321 928 094 861

3 321 928 094 851

$\frac{33\,219\,304}{32\,582\,657}$ // N

$\frac{32\,582\,657 - 33\,219\,304}{32\,582\,657}$ // N

1.01954

-0.0195394

```
NextPrime[33 219 304, -101]
```

33 217 333

$\frac{1}{\text{Log}[10\text{^}12]}$ // N

0.0361912

"for large enough N,the probability that a random integer not greater than N is prime is very close to 1/log(N)."
https://en.wikipedia.org/wiki/Prime_number_theorem

Wagstaff's Conjecture

If q_a is the a − th prime such that M_{q_a} is a Mersenne Prime, then
q_a ~ (2^(Exp[−EulerGamma]))^a

```
Exp[-EulerGamma] // N
2^(Exp[-EulerGamma]) // N
E^(Exp[-EulerGamma]) // N
```

0.561459

1.47576

1.75323

For very very large primes, have that

$$\left(\frac{\text{Prime}[10^n]}{\text{Log}[\text{LogIntegral}[10^n]]}\right) \approx b * 10^n$$

$$\Rightarrow \text{Prime}[10^n] \approx b\,(10^n)\,\text{Log}[\text{LogIntegral}[10^n]]$$

Imagine there is some large Mersenne prime such that

$\text{Prime}[\sim 10^n] \approx b\,(10^n)\,\text{Log}[\text{LogIntegral}[10^n]] \approx M_k = 2^k - 1 \approx 2^k$,
k some prime since it ' s a Mersenne prime

$$k \approx \text{Log}[2, b] + n\,\text{Log}[2, 10] + \text{Log}[2, \text{Log}[\text{LogIntegral}[10^n]]]$$

For Large n appx :

$$k \approx \text{Log}[2, 10]\,\mathbf{n} + \frac{\text{Log}[\mathbf{n}]}{\text{Log}[\mathbf{2}]} + C$$

and also $\text{Log}[2, \mathbf{n}] = \dfrac{\text{Log}[E, \mathbf{n}]}{\text{Log}[E, \mathbf{2}]} = \dfrac{\text{Log}[\mathbf{n}]}{\text{Log}[\mathbf{2}]}$

$$k = q_a \sim (2^\wedge(\text{Exp}[-\text{EulerGamma}]))^\wedge a$$

$$\Rightarrow (2^\wedge(\text{Exp}[-\text{EulerGamma}]))^\wedge \mathbf{a} \sim \text{Log}[2, 10]\,\mathbf{n} + \frac{\text{Log}[\mathbf{n}]}{\text{Log}[\mathbf{2}]} + C$$

Let $W = (2^\wedge(\text{Exp}[-\text{EulerGamma}])) = 1.47576\ldots$

$$\Rightarrow \mathbf{a} \sim \text{Log}[W, \text{Log}[2, 10]\,\mathbf{n}] + \text{Log}\left[W, \frac{\text{Log}[\mathbf{n}]}{\text{Log}[\mathbf{2}]}\right] + \text{Log}[W, C] =$$

$$\frac{1}{\text{Log}[\mathbf{W}]}\left(\text{Log}[\text{Log}[2, 10]\,\mathbf{n}] + \text{Log}\left[\frac{\text{Log}[\mathbf{n}]}{\text{Log}[\mathbf{2}]}\right] + \text{Log}[C]\right)$$

$$\Rightarrow \text{Log}[\mathbf{W}]\,\mathbf{a} \sim \text{Log}[\mathbf{n}] + \text{Log}[\text{Log}[2, 10]] + \text{Log}[\text{Log}[\mathbf{n}]] - \text{Log}[\text{Log}[\mathbf{2}]] + \text{Log}[C]$$

$$\Rightarrow \text{Log}[\mathbf{W}]\,\mathbf{a} - \text{Log}[\mathbf{n}] - \text{Log}[\text{Log}[\mathbf{n}]] \sim \text{Log}\left[\frac{\text{Log}[10]}{\text{Log}[2]}\right] - \text{Log}[\text{Log}[\mathbf{2}]] + \text{Log}[C] =$$

$$\text{Log}\left[C\,\frac{\text{Log}[10]}{(\text{Log}[2])^2}\right]$$

$\Rightarrow$ Log[W] a - Log[n] - Log[Log[n]] ~ $\text{Log}\left[C \frac{\text{Log}[10]}{(\text{Log}[2])^2}\right]$

q_a is the a – th prime such that M_{q_a} is a Mersenne Prime,

so take a ~10 * n,

since we can see that q_{50} lands us at n ~ 7 or 8

```
q[a_] := ((2^(Exp[-EulerGamma])))^a
Table[q[a], {a, 1, 50}] // N
```

{1.47576, 2.17787, 3.21402, 4.74313, 6.99972, 10.3299, 15.2445, 22.4972, 33.2006, 48.9961, 72.3065, 106.707, 157.474, 232.395, 342.959, 506.126, 746.921, 1102.28, 1626.7, 2400.62, 3542.74, 5228.24, 7715.63, 11 386.4, 16 803.7, 24 798.2, 36 596.2, 54 007.3, 79 701.8, 117 621., 173 580., 256 163., 378 036., 557 890., 823 313., 1.21501×10^6, 1.79307×10^6, 2.64614×10^6, 3.90508×10^6, 5.76296×10^6, 8.50476×10^6, 1.2551×10^7, 1.85223×10^7, 2.73345×10^7, 4.03391×10^7, 5.95309×10^7, 8.78535×10^7, 1.29651×10^8, 1.91334×10^8, 2.82363×10^8}

$\text{Log}\left[C \frac{\text{Log}[10]}{(\text{Log}[2])^2}\right]$ // N

$\text{Log}\left[\frac{\text{Log}[10]}{(\text{Log}[2])^2}\right]$ // N

Log[4.79253 C]

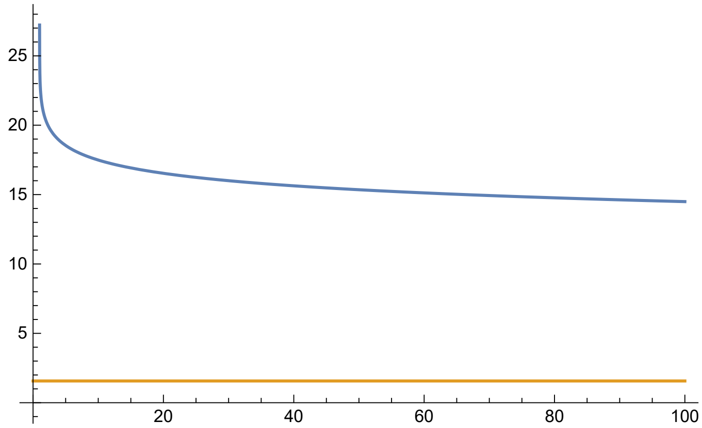
1.56706

```
1.5670582864112845`^2
```

2.45567

Log[(2^(Exp[-EulerGamma]))] (10 * n) - Log[n] - Log[Log[n]] ~ $\text{Log}\left[C \frac{\text{Log}[10]}{(\text{Log}[2])^2}\right]$

```
Plot[{Log[(2^(Exp[-EulerGamma]))] (53) - Log[n] - Log[Log[n]], Log[Log[10]/(Log[2])^2]},
 {n, 0, 100}]
```



```
Log[2, 1]
```

```
0
```

Euler product and mod forms

```
Product[(1 - (1/Prime[n])^-1), {n, 1, 70}]
```

```
24 870 664 731 984 424 921 712 795 598 268 731 795 116 429 648 130 797 708 653 967 166 904 109 759 177 699
 910 661 000 809 149 663 592 236 226 909 906 665 472 000 000 000 000 000 000 000
```

```
epdenomprodfn[nmax_] := Product[(1 - (1/Prime[n])^-1), {n, 1, nmax}]
```

```
Log[epdenomprodfn[nmax_]]
```

```
Table[epdenomprodfn[k], {k, 1, 20}]
Table[Log[epdenomprodfn[k]], {k, 1, 20}]
Table[Abs[Log[epdenomprodfn[k]]] // N, {k, 1, 20}]
```

```
{-1, 2, -8, 48, -480, 5760, -92 160, 1 658 880, -36 495 360, 1 021 870 080, -30 656 102 400,
 1 103 619 686 400, -44 144 787 456 000, 1 854 081 073 152 000, -85 287 729 364 992 000,
 4 434 961 926 979 584 000, -257 227 791 764 815 872 000, 15 433 667 505 888 952 320 000,
 -1 018 622 055 388 670 853 120 000, 71 303 543 877 206 959 718 400 000}
```

```
Table[Abs[Log[epdenomprodfn[k]]] // N, {k, 1, 50}]
```

```
{3.14159, 0.693147, 3.76745, 3.8712, 6.92714, 8.65869, 11.8551, 14.3217, 17.6938, 20.7449,
 24.3496, 27.7296, 31.5752, 35.1562, 39.1112, 42.9361, 47.1014, 51.0908, 55.3697, 59.529,
 63.8829, 68.1624, 72.6371, 77.0464, 81.6712, 86.2159, 90.8952, 95.5043, 100.236, 104.905,
 109.786, 114.609, 119.563, 124.449, 129.484, 134.457, 139.542, 144.594, 149.739, 154.853,
 160.066, 165.228, 170.504, 175.733, 181.038, 186.299, 191.672, 197.049, 202.494, 207.899}
```

```
Table[(Abs[Log[epdenomprodfn[k + 1]]] - Abs[Log[epdenomprodfn[k]]]) // N, {k, 1, 50}]
Table[(Abs[Log[epdenomprodfn[k + 1]] - Log[epdenomprodfn[k]]]) // N, {k, 1, 50}]
Table[Re[(Log[epdenomprodfn[k + 1]] - Log[epdenomprodfn[k]])] // N, {k, 1, 50}]
```

```
{-2.44845, 3.0743, 0.103751, 3.05594, 1.73155, 3.19642, 2.46654, 3.37218, 3.05107, 3.60471,
 3.38, 3.84556, 3.58099, 3.95502, 3.82487, 4.16533, 3.98946, 4.27885, 4.1593, 4.35396,
 4.27941, 4.47469, 4.40937, 4.62479, 4.54473, 4.67928, 4.60913, 4.73138, 4.66925, 4.88124,
 4.82258, 4.95394, 4.88597, 5.03533, 4.97252, 5.08522, 5.05223, 5.14495, 5.11453, 5.21262,
 5.16212, 5.27597, 5.22855, 5.30538, 5.26101, 5.37286, 5.37693, 5.44491, 5.40497, 5.46987}
```

```
{3.21715, 3.43386, 3.61663, 3.89506, 4.00554, 4.19009, 4.26894, 4.40728, 4.57965, 4.63009,
 4.76563, 4.84535, 4.8826, 4.95259, 5.04796, 5.13389, 5.16074, 5.23668, 5.28387, 5.30655,
 5.37127, 5.41191, 5.46957, 5.54102, 5.57469, 5.59106, 5.62292, 5.63844, 5.66867, 5.76708,
 5.79331, 5.83128, 5.84358, 5.90269, 5.91406, 5.94732, 5.9794, 6.00017, 6.03045, 6.05974,
 6.0693, 6.11562, 6.12461, 6.14232, 6.15105, 6.20171, 6.24968, 6.26513, 6.27275, 6.28781}
```

```
{0.693147, 1.38629, 1.79176, 2.30259, 2.48491, 2.77259, 2.89037, 3.09104, 3.3322, 3.4012,
 3.58352, 3.68888, 3.73767, 3.82864, 3.95124, 4.06044, 4.09434, 4.18965, 4.2485, 4.27667,
 4.35671, 4.40672, 4.47734, 4.56435, 4.60517, 4.62497, 4.66344, 4.68213, 4.7185, 4.83628,
 4.86753, 4.91265, 4.92725, 4.99721, 5.01064, 5.04986, 5.0876, 5.11199, 5.14749, 5.18178,
 5.19296, 5.24702, 5.2575, 5.27811, 5.28827, 5.34711, 5.40268, 5.42053, 5.42935, 5.44674}
```

$$\left(1-\left(\frac{1}{\mathrm{Prime}[n]}\right)^{-1}\right)\left(1-\left(\frac{1}{\mathrm{Prime}[n+1]}\right)^{-1}\right)\left(1-\left(\frac{1}{\mathrm{Prime}[n+2]}\right)^{-1}\right)\ldots$$

$$\to \mathrm{Log}\left[\left(1-\left(\frac{1}{\mathrm{Prime}[n]}\right)^{-1}\right)\left(1-\left(\frac{1}{\mathrm{Prime}[n+1]}\right)^{-1}\right)\left(1-\left(\frac{1}{\mathrm{Prime}[n+2]}\right)^{-1}\right)\ldots\right]$$

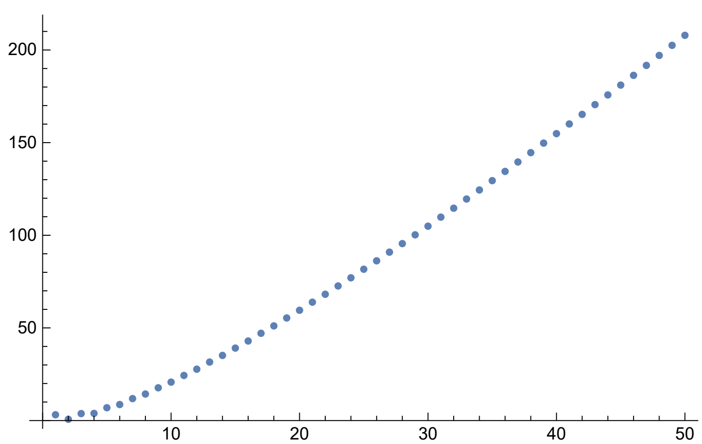
$$= \mathrm{Log}\left[\left(1-\left(\frac{1}{\mathrm{Prime}[n]}\right)^{-1}\right)\right]+$$

$$\mathrm{Log}\left[\left(1-\left(\frac{1}{\mathrm{Prime}[n+1]}\right)^{-1}\right)\right]+\mathrm{Log}\left[\left(1-\left(\frac{1}{\mathrm{Prime}[n+2]}\right)^{-1}\right)\right]+\ldots$$

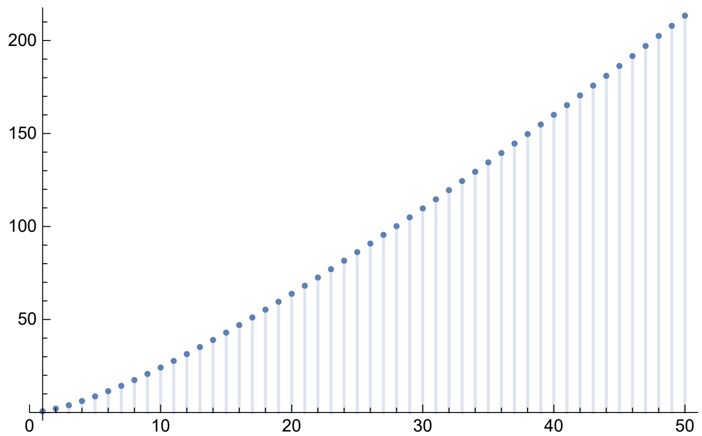
```
Sum[Re[(Log[epdenomprodfn[k + 1]] - Log[epdenomprodfn[k]])] // N, {k, 1, 500}]
```

3504.85

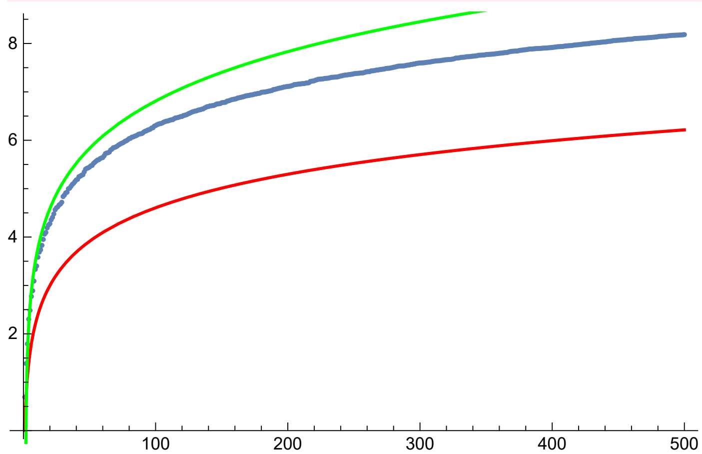
```
(*ListPlot[Table[epdenomprodfn[k],{k,1,5}]]*)
ListPlot[Table[Abs[Log[epdenomprodfn[k]]] // N, {k, 1, 50}]]
```



```
DiscretePlot[Sum[
  Re[(Log[epdenomprodfn[k + 1]] - Log[epdenomprodfn[k]])] // N, {k, 1, kmax}], {kmax, 1, 50}]
```




```
kmax = 500;
Show[ListPlot[
  Table[Re[(Log[epdenomprodfn[k + 1]] - Log[epdenomprodfn[k]])] // N, {k, 1, kmax}]],
 Plot[{Log[x], 2 Log[LogIntegral[x]]}, {x, 1, kmax}, PlotStyle → {Red, Green, Black}]
]
```



```
(1 - 2^-s) (1 - 3^-s) (1 - 5^-s) (1 - 7^-s) (1 - 11^-s) (1 - 13^-s) (1 - 17^-s)
  (1 - 19^-s) (1 - 23^-s) (1 - 29^-s) (1 - 31^-s) (1 - 37^-s) (1 - 41^-s) (1 - 43^-s) (1 - 47^-s)
  (1 - 53^-s) (1 - 59^-s) (1 - 61^-s) (1 - 67^-s) (1 - 71^-s) /. s → (-1) // Expand
```

```
(1 - 2^-s) (1 - 3^-s) (1 - 5^-s) (1 - 7^-s) (1 - 11^-s) (1 - 13^-s) (1 - 17^-s)
  (1 - 19^-s) (1 - 23^-s) (1 - 29^-s) (1 - 31^-s) (1 - 37^-s) (1 - 41^-s) (1 - 43^-s)
  (1 - 47^-s) (1 - 53^-s) (1 - 59^-s) (1 - 61^-s) (1 - 67^-s)  /. s → (-1) // Expand
```

```
(1 - 2^-s) (1 - 3^-s) (1 - 5^-s) (1 - 7^-s) (1 - 11^-s) (1 - 13^-s)
  (1 - 17^-s) (1 - 19^-s) (1 - 23^-s) (1 - 29^-s) (1 - 31^-s) (1 - 37^-s) (1 - 41^-s)
  (1 - 43^-s) (1 - 47^-s) (1 - 53^-s) (1 - 59^-s) (1 - 61^-s) /. s → (-1) // Expand
```

```
(1 - 2^-s) (1 - 3^-s) (1 - 5^-s) (1 - 7^-s) (1 - 11^-s) (1 - 13^-s) (1 - 17^-s) (1 - 19^-s) (1 - 23^-s) (1 - 29^-s)
  (1 - 31^-s) (1 - 37^-s) (1 - 41^-s) (1 - 43^-s) (1 - 47^-s) (1 - 53^-s) (1 - 59^-s)  /. s → (-1) // Expand
```

```
(1 - 2^-s) (1 - 3^-s) (1 - 5^-s) (1 - 7^-s) (1 - 11^-s) (1 - 13^-s) (1 - 17^-s) (1 - 19^-s) (1 - 23^-s) (1 - 29^-s)
  (1 - 31^-s) (1 - 37^-s) (1 - 41^-s) (1 - 43^-s) (1 - 47^-s) (1 - 53^-s)  /. s → (-1) // Expand
```

71 303 543 877 206 959 718 400 000

-1 018 622 055 388 670 853 120 000

15 433 667 505 888 952 320 000

-257 227 791 764 815 872 000

4 434 961 926 979 584 000

Why do they all end in sequences of 0’s !?

```
4 434 961 926 979 584 000 + -257 227 791 764 815 872 000 + 15 433 667 505 888 952 320 000 +
 -1 018 622 055 388 670 853 120 000 + 71 303 543 877 206 959 718 400 000
```

70 300 102 696 494 339 981 312 000