For this code make imax an even power of 2:

```
imax = 2^(2 * (6));
imax
Log[2, imax]
```

4096

12

Prime[i]

```
Wgenesample = "Primes";
numgenesample = Table[Prime[i], {i, 1, imax}];
SpecialNote = " ";
```

```
lengthofgeneitself = Length[Flatten[numgenesample]];
M = numgenesample;

For[npow = 1, npow < 1000, npow++, If[Length[M] < (2^(npow)), Break[]];
  FilledSize = 2^(npow + 1)];
Filler[vecvar1_] := Table[4, {i, 1, FilledSize - lengthofgeneitself}]
FilledVec[vecvar2_] := Join[Flatten[vecvar2], Filler[vecvar2]]

Filler[vecvar4_] := Table[4, {i, 1, FilledSize - lengthofgeneitself}]
FilledVec[vecvar5_] := Join[Flatten[vecvar5], Filler[vecvar5]]
For[npow = 1, npow < 1000, npow++, If[lengthofgeneitself ≤ (2^npow), Break[]]];
(* gives npow such that  2^npow > lengthofgeneitself > 2^(npow -1) *)
FilledSize = 2^npow;
FilledM = FilledVec[M];
numrowsW = √Length[FilledM] ;
W = Table[Table[FilledM[[i]],
    {i, (((j - 1) * (numrowsW)) + 1), (j * (numrowsW))}], {j, 1, numrowsW}];

lengthofgeneitself
Length[FilledM]
```

16 384

16 384

Run this for output and pdf

```
Wgenesample
ρ = (W.Transpose[W]); (* ρ as inner product *)
rhoEigens = Sort[DeleteCases[Eigenvalues[ρ] // N, 0.], Greater];
(*DeleteCases Removes 0's from the set of Eigenvalues,
Sort puts the list in order of greatest to least *)
rhoEigensinczeroes = Sort[Eigenvalues[ρ] // N, Greater];
(*DeleteCases Removes 0's from the set of Eigenvalues,
Sort puts the list in order of greatest to least *)
set = rhoEigens / Total[rhoEigens];
(* This is the set of nonzero normalized eigenvalues in order of greatest to least *)
setinczeroes = rhoEigensinczeroes / Total[rhoEigensinczeroes];
n = Length[set];
H[α_] := 1/(1 - α) Log[2, Sum[(set[[i]])^α, {i, 1, n}]] // N
H0 = Log[n] // N; (* H0 = Hartley Entropy*)
H1 = -Sum[((set[[i]]) (Log[2, set[[i]]])), {i, 1, n}] // N;
(* H1 = Shannon Entropy*)
H2onward = Table[H[a], {a, 2, 20}] // N; (* H2 onward *)
RenyiEntropyofEigenvalues = Join[{H0}, {H1}, H2onward];
```

```
button =
  Button["Click here for output and pdf", Print[Style[Wgenesample, Black, Bold, 28]] ×
    Print[Style["The ", Blue, Italic, 18], Style[Wgenesample, Black, Italic, 18],
     Style["  has  ", Blue, Italic, 18], Style[lengthofgeneitself, Black, Italic, 18],
     Style["   base pairs   ", Blue, Italic, 18]] ×
    If[StringLength[SpecialNote] > 3, Print[Style["(Special Note): ", Black, Bold, 16],
      Style[SpecialNote, Black, Italic, 12]], Print["  "]] ×
    Print[Style["W is a   ", Blue, Italic, 18], Style[Length[W], Black, Italic, 18],
     Style["  by  ", Blue, Italic, 18], Style[Length[W[[1]]], Black, Italic, 18],
     Style["   matrix with   ", Blue, Italic, 18],
     Style[Length[W] * Length[W[[1]]], Black, Italic, 18],
     Style[" = 2^b elements", Blue, Italic, 18], Style["  for b = ", Blue, Italic, 18],
     Style[Log[2, Length[W] * Length[W[[1]]]], Black, Italic, 18]] ×
    If[(Length[W] * Length[W[[1]]]) == (Length[W])^2,
     Print[Style["(If statement safecheck):  ", Black, Bold, 12],
      Style[Length[W], Black, Italic, 12], Style["  times  ", Red, Italic, 12],
      Style[Length[W[[1]]], Black, Italic, 12],
      Style["   equals   ", Red, Italic, 12], Style[(Length[W]^2), Black, Italic, 12],
      Style["  W is of the right size, you may proceed ", Red, Italic, 12]],
     Print[Style["(If statement safecheck):  ", Black, Bold, 12],
      Style["Warning!!!", Red, Italic, 28],
      Style["   W is of wrong size, STOP and check W ", Red, Italic, 12]]] ×
```

```
Print["The number of nonzero eigenvalues is  =   ", Length[rhoEigens]]
Do[Print["The i-th Eigenvalue " λi, " is = ", (rhoEigens)[[i]]],
 {i, 1, Length[rhoEigens]}]
Print[Graphics[ListPlot[rhoEigens // N, AxesLabel → {Style["i", Medium, Bold],
     Style["λi", Medium, Bold]}, PlotLabel → "Eigenvalue PLOT"]]]
Print[Graphics[ListLogPlot[rhoEigens // N, AxesLabel → {Style["i", Medium, Bold],
     Style["Log[λi]", Medium, Bold]}, PlotLabel → "Eigenvalue Log PLOT"]]]
Print["Zooming in on the Log Plot so as to Exclude the first
   eigenvalue gives the following plot:"]
Print[Graphics[ListLogPlot[Table[{i, rhoEigens[[i]]}, {i, 2, Length[rhoEigens]}],
   AxesLabel → {Style["i", Medium, Bold], Style["Log[λi]", Medium, Bold]},
   PlotRange → {{10, 2 * rhoEigens[[2]]}}, PlotStyle → Red,
   PlotLabel → Style["Logplot of Eigenvalues, excluding λ1", Red, Bold, 16]] ]]
Print["The approximate linearity of the above plot tells us
   that the eigenvalues decrease exponentially. If it's
   nowhere near linear try adjusting the plot range. "]

Print[Graphics[ListLogPlot[
   Table[{i, rhoEigensinczeroes[[i]]}, {i, 2, Length[rhoEigensinczeroes]}],
   AxesLabel → {Style["i", Medium, Bold], Style["Log[λi]", Medium, Bold]},
   PlotRange → {{10, 2 * rhoEigensinczeroes[[2]]}}, PlotStyle → Red, PlotLabel →
    Style["Logplot of Eigenvalues, including zero-valued ones", Red, Bold, 16]] ]]

Print["  "]
Print[Style[
  "████████████████████████████████████████████████████████████████", 18]]
Print["  "]

Print[Style["The First normalized eigenvector is:   ", Blue, Italic, 18],
 Style[set[[1]], Blue, Italic, 18]]
Print[Style["The Second normalized eigenvector is:   ", Blue, Italic, 18],
 Style[set[[2]], Blue, Italic, 18]]
Print[Style["The Last (n-th) normalized (nonzero) eigenvector is:   ",
  Blue, Italic, 18], Style[set[[n]], Blue, Italic, 16]]
If[Total[set] == 1, Print[Style["(If statement safecheck):  ", Black, Bold, 12],
  Style["Total[set] = ", Red, Italic, 12], Style[Total[set], Black, Italic, 12],
  Style[" = 1, so the Eigenvalue set is properly normalized", Red, Italic, 12]],
 Print[Style["(If statement safecheck):  ", Black, Bold, 12],
  Style["Warning!!!", Red, Italic, 28], Style["  Total[set] = ", Red, Italic, 12],
  Style[Total[set], Black, Italic, 12], Style[" ≠ 1, ", Red, Italic, 12],
  Style[" so the Eigenvalue set is NOT properly normalized.", Red, Italic, 12],
  Style[" This will render the entropies invalid. Fix it. ", Red, Italic, 12]]]

Print["  "]
Print[Style[
  "████████████████████████████████████████████████████████████████", 18]]
Print["  "]

Do[Print["The α-th Renyi Entropy Hα ->  "Hi-1, " is = ",
  RenyiEntropyofEigenvalues[[i]]], {i, 1, Length[RenyiEntropyofEigenvalues]}]
Print[Graphics[Show[
```

```
        ListPlot[RenyiEntropyofEigenvalues, PlotRange → All,
         AxesLabel → {Style["α", Large, Bold], Style["Hα", Large, Bold]}],
        ListLinePlot[RenyiEntropyofEigenvalues, PlotStyle → {Red, Thin}]
       ]]]
     Export["PrimePiEigenEntropies.pdf", EvaluationNotebook[]]
     NotebookSave[EvaluationNotebook[], "PrimePirhoCalcOutput"];
    SystemOpen["PrimePiEigenEntropies.pdf"]
    , Background → Green];
nb = CreateDocument[];
Paste[nb, button]
NotebookEvaluate[nb];
```

PrimePi[i]

```
Wgenesample = "PrimePi";
numgenesample = Table[PrimePi[i], {i, 1, imax}];
SpecialNote = " ";
```

```
lengthofgeneitself = Length[Flatten[numgenesample]];
M = numgenesample;

For[npow = 1, npow < 1000, npow++, If[Length[M] < (2^(npow)), Break[]];
  FilledSize = 2^(npow + 1)];
Filler[vecvar1_] := Table[4, {i, 1, FilledSize - lengthofgeneitself}]
FilledVec[vecvar2_] := Join[Flatten[vecvar2], Filler[vecvar2]]

Filler[vecvar4_] := Table[4, {i, 1, FilledSize - lengthofgeneitself}]
FilledVec[vecvar5_] := Join[Flatten[vecvar5], Filler[vecvar5]]
For[npow = 1, npow < 1000, npow++, If[lengthofgeneitself ≤ (2^npow), Break[]]];
(* gives npow such that  2^npow > lengthofgeneitself > 2^(npow -1)  *)
FilledSize = 2^npow;
FilledM = FilledVec[M];
numrowsW = √Length[FilledM] ;
W = Table[Table[FilledM[[i]],
    {i, (((j - 1) * (numrowsW)) + 1), (j * (numrowsW))}], {j, 1, numrowsW}];

lengthofgeneitself
Length[FilledM]
```

4096

4096

Run this for output and pdf

```
Wgenesample
ρ = (W.Transpose[W]); (* ρ as inner product *)
rhoEigens = Sort[DeleteCases[Eigenvalues[ρ] // N, 0.], Greater];
(*DeleteCases Removes 0's from the set of Eigenvalues,
Sort puts the list in order of greatest to least *)
rhoEigensinczeroes = Sort[Eigenvalues[ρ] // N, Greater];
(*DeleteCases Removes 0's from the set of Eigenvalues,
Sort puts the list in order of greatest to least *)
set = rhoEigens / Total[rhoEigens];
(* This is the set of nonzero normalized eigenvalues in order of greatest to least *)
setinczeroes = rhoEigensinczeroes / Total[rhoEigensinczeroes];
n = Length[set];
H[α_] := 1/(1 - α) Log[2, Sum[(set[[i]])^α, {i, 1, n}]] // N
H0 = Log[n] // N;(* H0 = Hartley Entropy*)
H1 = -Sum[((set[[i]]) (Log[2, set[[i]]])), {i, 1, n}] // N;
(* H1 = Shannon Entropy*)
H2onward = Table[H[a], {a, 2, 20}] // N;(* H2 onward *)
RenyiEntropyofEigenvalues = Join[{H0}, {H1}, H2onward];
```

PrimePi

```
button =
  Button["Click here for output and pdf", Print[Style[Wgenesample, Black, Bold, 28]] ×
    Print[Style["The ", Blue, Italic, 18], Style[Wgenesample, Black, Italic, 18],
     Style["  has  ", Blue, Italic, 18], Style[lengthofgeneitself, Black, Italic, 18],
     Style["   base pairs   ", Blue, Italic, 18]] ×
    If[StringLength[SpecialNote] > 3, Print[Style["(Special Note): ", Black, Bold, 16],
      Style[SpecialNote, Black, Italic, 12]], Print["  "]] ×
    Print[Style["W is a   ", Blue, Italic, 18], Style[Length[W], Black, Italic, 18],
     Style["  by  ", Blue, Italic, 18], Style[Length[W[[1]]], Black, Italic, 18],
     Style["   matrix with   ", Blue, Italic, 18],
     Style[Length[W] * Length[W[[1]]], Black, Italic, 18],
     Style[" = 2^b elements", Blue, Italic, 18], Style["  for b = ", Blue, Italic, 18],
     Style[Log[2, Length[W] * Length[W[[1]]]], Black, Italic, 18]] ×
    If[(Length[W] * Length[W[[1]]]) == (Length[W])^2,
     Print[Style["(If statement safecheck):  ", Black, Bold, 12],
      Style[Length[W], Black, Italic, 12], Style["  times  ", Red, Italic, 12],
      Style[Length[W[[1]]], Black, Italic, 12],
      Style["   equals   ", Red, Italic, 12], Style[(Length[W]^2), Black, Italic, 12],
      Style["  W is of the right size, you may proceed ", Red, Italic, 12]],
     Print[Style["(If statement safecheck):  ", Black, Bold, 12],
      Style["Warning!!!", Red, Italic, 28],
```

```
  Style["   𝕎 is of wrong size, STOP and check 𝕎 ", Red, Italic, 12]]]
Print["The number of nonzero eigenvalues is  =   ", Length[rhoEigens]]
Do[Print["The i-th Eigenvalue "λi, " is = ", (rhoEigens)[[i]]],
 {i, 1, Length[rhoEigens]}]
Print[Graphics[ListPlot[rhoEigens // N, AxesLabel → {Style["i", Medium, Bold],
     Style["λi", Medium, Bold]}, PlotLabel → "Eigenvalue PLOT"]]]
Print[Graphics[ListLogPlot[rhoEigens // N, AxesLabel → {Style["i", Medium, Bold],
     Style["Log[λi]", Medium, Bold]}, PlotLabel → "Eigenvalue Log PLOT"]]]
Print["Zooming in on the Log Plot so as to Exclude the first
   eigenvalue gives the following plot:"]
Print[Graphics[ListLogPlot[Table[{i, rhoEigens[[i]]}, {i, 2, Length[rhoEigens]}],
   AxesLabel → {Style["i", Medium, Bold], Style["Log[λi]", Medium, Bold]},
   PlotRange → {{10, 2 * rhoEigens[[2]]}}, PlotStyle → Red,
   PlotLabel → Style["Logplot of Eigenvalues, excluding λ1", Red, Bold, 16]] ]]
Print["The approximate linearity of the above plot tells us
   that the eigenvalues decrease exponentially. If it's
   nowhere near linear try adjusting the plot range. "]

Print[Graphics[ListLogPlot[
   Table[{i, rhoEigensinczeroes[[i]]}, {i, 2, Length[rhoEigensinczeroes]}],
   AxesLabel → {Style["i", Medium, Bold], Style["Log[λi]", Medium, Bold]},
   PlotRange → {{10, 2 * rhoEigensinczeroes[[2]]}}, PlotStyle → Red, PlotLabel →
    Style["Logplot of Eigenvalues, including zero-valued ones", Red, Bold, 16]] ]]

Print["  "]
Print[Style[
  "■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■", 18]]
Print["  "]

Print[Style["The First normalized eigenvector is:   ", Blue, Italic, 18],
 Style[set[[1]], Blue, Italic, 18]]
Print[Style["The Second normalized eigenvector is:   ", Blue, Italic, 18],
 Style[set[[2]], Blue, Italic, 18]]
Print[Style["The Last (n-th) normalized (nonzero) eigenvector is:   ",
  Blue, Italic, 18], Style[set[[n]], Blue, Italic, 16]]
If[Total[set] == 1, Print[Style["(If statement safecheck):  ", Black, Bold, 12],
  Style["Total[set] = ", Red, Italic, 12], Style[Total[set], Black, Italic, 12],
  Style[" = 1, so the Eigenvalue set is properly normalized", Red, Italic, 12]],
 Print[Style["(If statement safecheck):  ", Black, Bold, 12],
  Style["Warning!!!", Red, Italic, 28], Style["  Total[set] = ", Red, Italic, 12],
  Style[Total[set], Black, Italic, 12], Style[" ≠ 1, ", Red, Italic, 12],
  Style[" so the Eigenvalue set is NOT properly normalized.", Red, Italic, 12],
  Style[" This will render the entropies invalid. Fix it. ", Red, Italic, 12]]]

Print["  "]
Print[Style[
  "■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■", 18]]
Print["  "]

Do[Print["The α-th Renyi Entropy Hα ->  "Hi-1, " is = ",
  RenyiEntropyofEigenvalues[[i]]], {i, 1, Length[RenyiEntropyofEigenvalues]}]
```

```
     Print[Graphics[Show[
        ListPlot[RenyiEntropyofEigenvalues, PlotRange → All,
         AxesLabel → {Style["α", Large, Bold], Style["Hα", Large, Bold]}],
        ListLinePlot[RenyiEntropyofEigenvalues, PlotStyle → {Red, Thin}]
       ]]]
     Export["PrimePiEigenEntropies.pdf", EvaluationNotebook[]]
     NotebookSave[EvaluationNotebook[], "PrimePirhoCalcOutput"];
    SystemOpen["PrimePiEigenEntropies.pdf"]
    , Background → Green];
nb = CreateDocument[];
Paste[nb, button]
NotebookEvaluate[nb];
```

Int[i]

```
Wgenesample = "Int";
numgenesample = Table[i, {i, 1, imax}];
SpecialNote = " ";
```

```
lengthofgeneitself = Length[Flatten[numgenesample]];
M = numgenesample;

For[npow = 1, npow < 1000, npow++, If[Length[M] < (2^(npow)), Break[]];
  FilledSize = 2^(npow + 1)];
Filler[vecvar1_] := Table[4, {i, 1, FilledSize - lengthofgeneitself}]
FilledVec[vecvar2_] := Join[Flatten[vecvar2], Filler[vecvar2]]

Filler[vecvar4_] := Table[4, {i, 1, FilledSize - lengthofgeneitself}]
FilledVec[vecvar5_] := Join[Flatten[vecvar5], Filler[vecvar5]]
For[npow = 1, npow < 1000, npow++, If[lengthofgeneitself ≤ (2^npow), Break[]]];
(* gives npow such that  2^npow > lengthofgeneitself > 2^(npow -1) *)
FilledSize = 2^npow;
FilledM = FilledVec[M];
numrowsW = √(Length[FilledM]) ;
W = Table[Table[FilledM[[i]],
    {i, (((j - 1) * (numrowsW)) + 1), (j * (numrowsW))}], {j, 1, numrowsW}];

lengthofgeneitself
Length[FilledM]
```

65 536

65 536

Run this for output and pdf

```
Wgenesample
ρ = (W.Transpose[W]); (* ρ as inner product *)
rhoEigens = Sort[DeleteCases[Eigenvalues[ρ] // N, 0.], Greater];
(*DeleteCases Removes 0's from the set of Eigenvalues,
Sort puts the list in order of greatest to least *)
rhoEigensinczeroes = Sort[Eigenvalues[ρ] // N, Greater];
(*DeleteCases Removes 0's from the set of Eigenvalues,
Sort puts the list in order of greatest to least *)
set = rhoEigens / Total[rhoEigens];
(* This is the set of nonzero normalized eigenvalues in order of greatest to least *)
setinczeroes = rhoEigensinczeroes / Total[rhoEigensinczeroes];
n = Length[set];
H[α_] := 1/(1 - α) Log[2, Sum[(set[[i]])^α, {i, 1, n}]] // N
H0 = Log[n] // N; (* H0 = Hartley Entropy*)
H1 = -Sum[((set[[i]]) (Log[2, set[[i]]])), {i, 1, n}] // N;
(* H1 = Shannon Entropy*)
H2onward = Table[H[a], {a, 2, 20}] // N; (* H2 onward *)
RenyiEntropyofEigenvalues = Join[{H0}, {H1}, H2onward];
```

```
Int
```

```
$Aborted
```

```
$Aborted
```

```
button =
  Button["Click here for output and pdf", Print[Style[Wgenesample, Black, Bold, 28]] ×
    Print[Style["The ", Blue, Italic, 18], Style[Wgenesample, Black, Italic, 18],
     Style["  has  ", Blue, Italic, 18], Style[lengthofgeneitself, Black, Italic, 18],
     Style["   base pairs    ", Blue, Italic, 18]] ×
    If[StringLength[SpecialNote] > 3, Print[Style["(Special Note): ", Black, Bold, 16],
      Style[SpecialNote, Black, Italic, 12]], Print["  "]] ×
    Print[Style["W is a   ", Blue, Italic, 18], Style[Length[W], Black, Italic, 18],
     Style["  by  ", Blue, Italic, 18], Style[Length[W[[1]]], Black, Italic, 18],
     Style["   matrix with   ", Blue, Italic, 18],
     Style[Length[W] * Length[W[[1]]], Black, Italic, 18],
     Style[" = 2^b elements", Blue, Italic, 18], Style["  for b = ", Blue, Italic, 18],
     Style[Log[2, Length[W] * Length[W[[1]]]], Black, Italic, 18]] ×
    If[(Length[W] * Length[W[[1]]]) == (Length[W])^2,
     Print[Style["(If statement safecheck):  ", Black, Bold, 12],
      Style[Length[W], Black, Italic, 12], Style["  times  ", Red, Italic, 12],
      Style[Length[W[[1]]], Black, Italic, 12],
      Style["   equals   ", Red, Italic, 12], Style[(Length[W]^2), Black, Italic, 12],
```

```
    Style["  W is of the right size, you may proceed ", Red, Italic, 12]],
   Print[Style["(If statement safecheck):  ", Black, Bold, 12],
    Style["Warning!!!", Red, Italic, 28],
    Style["   W is of wrong size, STOP and check W ", Red, Italic, 12]]]
Print["The number of nonzero eigenvalues is  =   ", Length[rhoEigens]]
Do[Print["The i-th Eigenvalue "λi, " is = ", (rhoEigens)[[i]]],
  {i, 1, Length[rhoEigens]}]
Print[Graphics[ListPlot[rhoEigens // N, AxesLabel → {Style["i", Medium, Bold],
     Style["λi", Medium, Bold]}, PlotLabel → "Eigenvalue PLOT"]]]
Print[Graphics[ListLogPlot[rhoEigens // N, AxesLabel → {Style["i", Medium, Bold],
     Style["Log[λi]", Medium, Bold]}, PlotLabel → "Eigenvalue Log PLOT"]]]
Print["Zooming in on the Log Plot so as to Exclude the first
   eigenvalue gives the following plot:"]
Print[Graphics[ListLogPlot[Table[{i, rhoEigens[[i]]}, {i, 2, Length[rhoEigens]}],
   AxesLabel → {Style["i", Medium, Bold], Style["Log[λi]", Medium, Bold]},
   PlotRange → {{10, 2 * rhoEigens[[2]]}}, PlotStyle → Red,
   PlotLabel → Style["Logplot of Eigenvalues, excluding λ1", Red, Bold, 16]] ]]
Print["The approximate linearity of the above plot tells us
   that the eigenvalues decrease exponentially. If it's
   nowhere near linear try adjusting the plot range. "]

Print[Graphics[ListLogPlot[
   Table[{i, rhoEigensinczeroes[[i]]}, {i, 2, Length[rhoEigensinczeroes]}],
   AxesLabel → {Style["i", Medium, Bold], Style["Log[λi]", Medium, Bold]},
   PlotRange → {{10, 2 * rhoEigensinczeroes[[2]]}}, PlotStyle → Red, PlotLabel →
    Style["Logplot of Eigenvalues, including zero-valued ones", Red, Bold, 16]] ]]

Print["  "]
Print[Style[
  "▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬", 18]]
Print["  "]

Print[Style["The First normalized eigenvector is:   ", Blue, Italic, 18],
  Style[set[[1]], Blue, Italic, 18]]
Print[Style["The Second normalized eigenvector is:   ", Blue, Italic, 18],
  Style[set[[2]], Blue, Italic, 18]]
Print[Style["The Last (n-th) normalized (nonzero) eigenvector is:   ",
   Blue, Italic, 18], Style[set[[n]], Blue, Italic, 16]]
If[Total[set] == 1, Print[Style["(If statement safecheck):  ", Black, Bold, 12],
   Style["Total[set] = ", Red, Italic, 12], Style[Total[set], Black, Italic, 12],
   Style[" = 1, so the Eigenvalue set is properly normalized", Red, Italic, 12]],
  Print[Style["(If statement safecheck):  ", Black, Bold, 12],
   Style["Warning!!!", Red, Italic, 28], Style["  Total[set] = ", Red, Italic, 12],
   Style[Total[set], Black, Italic, 12], Style[" ≠ 1, ", Red, Italic, 12],
   Style[" so the Eigenvalue set is NOT properly normalized.", Red, Italic, 12],
   Style[" This will render the entropies invalid. Fix it. ", Red, Italic, 12]]]

Print["  "]
Print[Style[
  "▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬▬", 18]]
Print["  "]
```

```
    Do[Print["The α-th Renyi Entropy Hα ->  " Hi-1, " is = ",
      RenyiEntropyofEigenvalues[[i]]], {i, 1, Length[RenyiEntropyofEigenvalues]}]
    Print[Graphics[Show[
       ListPlot[RenyiEntropyofEigenvalues, PlotRange → All,
        AxesLabel → {Style["α", Large, Bold], Style["Hα", Large, Bold]}],
       ListLinePlot[RenyiEntropyofEigenvalues, PlotStyle → {Red, Thin}]
      ]]]
    Export["PrimePiEigenEntropies.pdf", EvaluationNotebook[]]
    NotebookSave[EvaluationNotebook[], "PrimePirhoCalcOutput"];
   SystemOpen["PrimePiEigenEntropies.pdf"]
   , Background → Green];
nb = CreateDocument[];
Paste[nb, button]
NotebookEvaluate[nb];
```

Zeta[i]

```
Zeta[1024] // N
```

```
0.999999999999971
```

```
Wgenesample = "Zeta";
numgenesample = Table[Zeta[i] // N, {i, 2, imax}];
SpecialNote = " ";
```

```
lengthofgeneitself = Length[Flatten[numgenesample]];
M = numgenesample;

For[npow = 1, npow < 1000, npow++, If[Length[M] < (2^(npow)), Break[]];
  FilledSize = 2^(npow + 1)];
Filler[vecvar1_] := Table[4, {i, 1, FilledSize - lengthofgeneitself}]
FilledVec[vecvar2_] := Join[Flatten[vecvar2], Filler[vecvar2]]

Filler[vecvar4_] := Table[4, {i, 1, FilledSize - lengthofgeneitself}]
FilledVec[vecvar5_] := Join[Flatten[vecvar5], Filler[vecvar5]]
For[npow = 1, npow < 1000, npow++, If[lengthofgeneitself ≤ (2^npow), Break[]]];
(* gives npow such that  2^npow > lengthofgeneitself > 2^(npow -1) *)
FilledSize = 2^npow;
FilledM = FilledVec[M];
numrowsW = √Length[FilledM] ;
W = Table[Table[FilledM[[i]],
    {i, (((j - 1) * (numrowsW)) + 1), (j * (numrowsW))}], {j, 1, numrowsW}];

lengthofgeneitself
Length[FilledM]
```

255

256

```
Do[rhoEigens1[[i]] = If[(Eigenvalues[ρ][[i]] // N) < 0, 0. * Eigenvalues[ρ][[i]] // N,
  Eigenvalues[ρ][[i]] // N], {i, 1, Length[Eigenvalues[ρ]]}]
```

```
Log[rhoEigens]
```

{13.5917, 6.68389, 1.75364, 1.12502, 0.937959, 0.625578, -0.67433, -1.51898}

Run this for output and pdf

```
Wgenesample
ρ = (W.Transpose[W]); (* ρ as inner product *)
rhoEigens1 = Table[0, {i, 1, Length[Eigenvalues[ρ]]}];

Do[rhoEigens1[[i]] = If[(Eigenvalues[ρ][[i]] // N) < 0, 0. * Eigenvalues[ρ][[i]] // N,
   Eigenvalues[ρ][[i]] // N], {i, 1, Length[Eigenvalues[ρ]]}]
rhoEigens = Sort[DeleteCases[rhoEigens1 // N, 0.], Greater];

rhoEigensinczeroes = Sort[Eigenvalues[ρ] // N, Greater];
(*DeleteCases Removes 0's from the set of Eigenvalues,
Sort puts the list in order of greatest to least *)
set = rhoEigens / Total[rhoEigens];
(* This is the set of nonzero normalized eigenvalues in order of greatest to least *)
setinczeroes = rhoEigensinczeroes / Total[rhoEigensinczeroes];
n = Length[set];
H[α_] := 1/(1 - α) Log[2, Sum[(set[[i]])^α, {i, 1, n}]] // N
H0 = Log[n] // N; (* H0 = Hartley Entropy*)
H1 = -Sum[((set[[i]]) (Log[2, set[[i]]])), {i, 1, n}] // N;
(* H1 = Shannon Entropy*)
H2onward = Table[H[a], {a, 2, 20}] // N; (* H2 onward *)
RenyiEntropyofEigenvalues = Join[{H0}, {H1}, H2onward];
```

Zeta

```
button =
  Button["Click here for output and pdf", Print[Style[Wgenesample, Black, Bold, 28]] ×
    Print[Style["The ", Blue, Italic, 18], Style[Wgenesample, Black, Italic, 18],
     Style["  has  ", Blue, Italic, 18], Style[lengthofgeneitself, Black, Italic, 18],
     Style["   base pairs   ", Blue, Italic, 18]] ×
    If[StringLength[SpecialNote] > 3, Print[Style["(Special Note): ", Black, Bold, 16],
      Style[SpecialNote, Black, Italic, 12]], Print["  "]] ×
    Print[Style["W is a   ", Blue, Italic, 18], Style[Length[W], Black, Italic, 18],
     Style["  by  ", Blue, Italic, 18], Style[Length[W[[1]]], Black, Italic, 18],
     Style["   matrix with   ", Blue, Italic, 18],
     Style[Length[W] * Length[W[[1]]], Black, Italic, 18],
     Style[" = 2^b elements", Blue, Italic, 18], Style["  for b = ", Blue, Italic, 18],
     Style[Log[2, Length[W] * Length[W[[1]]]], Black, Italic, 18]] ×
    If[(Length[W] * Length[W[[1]]]) == (Length[W])^2,
     Print[Style["(If statement safecheck):  ", Black, Bold, 12],
      Style[Length[W], Black, Italic, 12], Style["  times  ", Red, Italic, 12],
      Style[Length[W[[1]]], Black, Italic, 12],
      Style["   equals   ", Red, Italic, 12], Style[(Length[W]^2), Black, Italic, 12],
```

```
    Style["  𝕎 is of the right size, you may proceed ", Red, Italic, 12]],
   Print[Style["(If statement safecheck):  ", Black, Bold, 12],
    Style["Warning!!!", Red, Italic, 28],
    Style["   𝕎 is of wrong size, STOP and check 𝕎 ", Red, Italic, 12]]]
Print["The number of nonzero eigenvalues is  =   ", Length[rhoEigens]]
Do[Print["The i-th Eigenvalue "λi, " is = ", (rhoEigens)[[i]]],
 {i, 1, Length[rhoEigens]}]
Print[Graphics[ListPlot[rhoEigens // N, AxesLabel → {Style["i", Medium, Bold],
     Style["λi", Medium, Bold]}, PlotLabel → "Eigenvalue PLOT"]]]
Print[Graphics[ListLogPlot[rhoEigens // N, AxesLabel → {Style["i", Medium, Bold],
     Style["Log[λi]", Medium, Bold]}, PlotLabel → "Eigenvalue Log PLOT"]]]
Print["Zooming in on the Log Plot so as to Exclude the first
   eigenvalue gives the following plot:"]
Print[Graphics[ListLogPlot[Table[{i, rhoEigens[[i]]}, {i, 2, Length[rhoEigens]}],
   AxesLabel → {Style["i", Medium, Bold], Style["Log[λi]", Medium, Bold]},
   PlotStyle → Red,
   PlotLabel → Style["Logplot of Eigenvalues, excluding λ1", Red, Bold, 16]] ]]
Print["The approximate linearity of the above plot tells us
   that the eigenvalues decrease exponentially. If it's
   nowhere near linear try adjusting the plot range. "]

Print[Graphics[ListLogPlot[
   Table[{i, rhoEigensinczeroes[[i]]}, {i, 2, Length[rhoEigensinczeroes]}],
   AxesLabel → {Style["i", Medium, Bold], Style["Log[λi]", Medium, Bold]},
   PlotStyle → Red, PlotLabel →
    Style["Logplot of Eigenvalues, including zero-valued ones", Red, Bold, 16]] ]]

Print["  "]
Print[Style[
  "■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■", 18]]
Print["  "]

Print[Style["The First normalized eigenvector is:   ", Blue, Italic, 18],
 Style[set[[1]], Blue, Italic, 18]]
Print[Style["The Second normalized eigenvector is:   ", Blue, Italic, 18],
 Style[set[[2]], Blue, Italic, 18]]
Print[Style["The Last (n-th) normalized (nonzero) eigenvector is:   ",
  Blue, Italic, 18], Style[set[[n]], Blue, Italic, 16]]
If[Total[set] == 1, Print[Style["(If statement safecheck):  ", Black, Bold, 12],
  Style["Total[set] = ", Red, Italic, 12], Style[Total[set], Black, Italic, 12],
  Style[" = 1, so the Eigenvalue set is properly normalized", Red, Italic, 12]],
 Print[Style["(If statement safecheck):  ", Black, Bold, 12],
  Style["Warning!!!", Red, Italic, 28], Style["  Total[set] = ", Red, Italic, 12],
  Style[Total[set], Black, Italic, 12], Style[" ≠ 1, ", Red, Italic, 12],
  Style[" so the Eigenvalue set is NOT properly normalized.", Red, Italic, 12],
  Style[" This will render the entropies invalid. Fix it. ", Red, Italic, 12]]]

Print["  "]
Print[Style[
  "■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■", 18]]
Print["  "]
```

```
     Do[Print["The α-th Renyi Entropy Hα ->  "Hi-1, " is = ",
       RenyiEntropyofEigenvalues[[i]]], {i, 1, Length[RenyiEntropyofEigenvalues]}]
     Print[Graphics[Show[
        ListPlot[RenyiEntropyofEigenvalues, PlotRange → All,
         AxesLabel → {Style["α", Large, Bold], Style["Hα", Large, Bold]}],
        ListLinePlot[RenyiEntropyofEigenvalues, PlotStyle → {Red, Thin}]
       ]]]
     Export["PrimePiEigenEntropies.pdf", EvaluationNotebook[]]
     NotebookSave[EvaluationNotebook[], "PrimePirhoCalcOutput"];
    SystemOpen["PrimePiEigenEntropies.pdf"]
    , Background → Green];
nb = CreateDocument[];
Paste[nb, button]
NotebookEvaluate[nb];
```

DenomEvenZeta[i]

```
Table[Denominator[Zeta[2 (i)]], {i, 1, 4}]
```

```
{6, 90, 945, 9450}
```

```
Zeta[1024] // N
```

```
0.999999999999971
```

```
Denominator[Zeta[2]]
Denominator[Zeta[2 * 2]]
```

```
6
```

```
90
```

```
Wgenesample = "DenomEvenZeta";
numgenesample = Table[Denominator[Zeta[2 (i)]], {i, 1, imax}];
SpecialNote = " ";
```

```
lengthofgeneitself = Length[Flatten[numgenesample]];
M = numgenesample;

For[npow = 1, npow < 1000, npow++, If[Length[M] < (2^(npow)), Break[]];
  FilledSize = 2^(npow + 1)];
Filler[vecvar1_] := Table[4, {i, 1, FilledSize - lengthofgeneitself}]
FilledVec[vecvar2_] := Join[Flatten[vecvar2], Filler[vecvar2]]

Filler[vecvar4_] := Table[4, {i, 1, FilledSize - lengthofgeneitself}]
FilledVec[vecvar5_] := Join[Flatten[vecvar5], Filler[vecvar5]]
For[npow = 1, npow < 1000, npow++, If[lengthofgeneitself ≤ (2^npow), Break[]]];
(* gives npow such that  2^npow > lengthofgeneitself > 2^(npow -1) *)
FilledSize = 2^npow;
FilledM = FilledVec[M];
numrowsW = √Length[FilledM] ;
W = Table[Table[FilledM[[i]],
     {i, (((j - 1) * (numrowsW)) + 1), (j * (numrowsW))}], {j, 1, numrowsW}];

lengthofgeneitself
Length[FilledM]
```

16

16

Run this for output and pdf

```
Wgenesample
ρ = (W.Transpose[W]); (* ρ as inner product *)
rhoEigens = Sort[DeleteCases[Eigenvalues[ρ] // N, 0.], Greater];
(*DeleteCases Removes 0's from the set of Eigenvalues,
Sort puts the list in order of greatest to least *)
rhoEigensinczeroes = Sort[Eigenvalues[ρ] // N, Greater];
(*DeleteCases Removes 0's from the set of Eigenvalues,
Sort puts the list in order of greatest to least *)
set = rhoEigens / Total[rhoEigens];
(* This is the set of nonzero normalized eigenvalues in order of greatest to least *)
setinczeroes = rhoEigensinczeroes / Total[rhoEigensinczeroes];
n = Length[set];
H[α_] := 1/(1 - α) Log[2, Sum[(set[[i]])^α, {i, 1, n}]] // N
H0 = Log[n] // N;(* H0 = Hartley Entropy*)
H1 = -Sum[((set[[i]]) (Log[2, set[[i]]])), {i, 1, n}] // N;
(* H1 = Shannon Entropy*)
H2onward = Table[H[a], {a, 2, 20}] // N;(* H2 onward *)
RenyiEntropyofEigenvalues = Join[{H0}, {H1}, H2onward];
```

DenomEvenZeta

```
button =
  Button["Click here for output and pdf", Print[Style[Wgenesample, Black, Bold, 28]] ×
    Print[Style["The ", Blue, Italic, 18], Style[Wgenesample, Black, Italic, 18],
     Style["  has  ", Blue, Italic, 18], Style[lengthofgeneitself, Black, Italic, 18],
     Style["   base pairs   ", Blue, Italic, 18]] ×
    If[StringLength[SpecialNote] > 3, Print[Style["(Special Note): ", Black, Bold, 16],
      Style[SpecialNote, Black, Italic, 12]], Print["  "]] ×
    Print[Style["W is a   ", Blue, Italic, 18], Style[Length[W], Black, Italic, 18],
     Style["  by  ", Blue, Italic, 18], Style[Length[W[[1]]], Black, Italic, 18],
     Style["   matrix with   ", Blue, Italic, 18],
     Style[Length[W] * Length[W[[1]]], Black, Italic, 18],
     Style[" = 2^b elements", Blue, Italic, 18], Style["  for b = ", Blue, Italic, 18],
     Style[Log[2, Length[W] * Length[W[[1]]]], Black, Italic, 18]] ×
    If[(Length[W] * Length[W[[1]]]) == (Length[W])^2,
     Print[Style["(If statement safecheck):  ", Black, Bold, 12],
      Style[Length[W], Black, Italic, 12], Style["  times  ", Red, Italic, 12],
      Style[Length[W[[1]]], Black, Italic, 12],
      Style["   equals   ", Red, Italic, 12], Style[(Length[W]^2), Black, Italic, 12],
      Style["  W is of the right size, you may proceed ", Red, Italic, 12]],
     Print[Style["(If statement safecheck):  ", Black, Bold, 12],
      Style["Warning!!!", Red, Italic, 28],
```

```
  Style["   W is of wrong size, STOP and check W ", Red, Italic, 12]]]
Print["The number of nonzero eigenvalues is  =   ", Length[rhoEigens]]
Do[Print["The i-th Eigenvalue " λi, " is = ", (rhoEigens)[[i]]],
 {i, 1, Length[rhoEigens]}]
Print[Graphics[ListPlot[rhoEigens // N, AxesLabel → {Style["i", Medium, Bold],
     Style["λi", Medium, Bold]}, PlotLabel → "Eigenvalue PLOT"]]]
Print[Graphics[ListLogPlot[rhoEigens // N, AxesLabel → {Style["i", Medium, Bold],
     Style["Log[λi]", Medium, Bold]}, PlotLabel → "Eigenvalue Log PLOT"]]]
Print["Zooming in on the Log Plot so as to Exclude the first
   eigenvalue gives the following plot:"]
Print[Graphics[ListLogPlot[Table[{i, rhoEigens[[i]]}, {i, 2, Length[rhoEigens]}],
   AxesLabel → {Style["i", Medium, Bold], Style["Log[λi]", Medium, Bold]},
   PlotRange → {{10, 2 * rhoEigens[[2]]}}, PlotStyle → Red,
   PlotLabel → Style["Logplot of Eigenvalues, excluding λ1", Red, Bold, 16]] ]]
Print["The approximate linearity of the above plot tells us
   that the eigenvalues decrease exponentially. If it's
   nowhere near linear try adjusting the plot range. "]

Print[Graphics[ListLogPlot[
   Table[{i, rhoEigensinczeroes[[i]]}, {i, 2, Length[rhoEigensinczeroes]}],
   AxesLabel → {Style["i", Medium, Bold], Style["Log[λi]", Medium, Bold]},
   PlotRange → {{10, 2 * rhoEigensinczeroes[[2]]}}, PlotStyle → Red, PlotLabel →
    Style["Logplot of Eigenvalues, including zero-valued ones", Red, Bold, 16]] ]]

Print["  "]
Print[Style[
  "■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■", 18]]
Print["  "]

Print[Style["The First normalized eigenvector is:   ", Blue, Italic, 18],
 Style[set[[1]], Blue, Italic, 18]]
Print[Style["The Second normalized eigenvector is:   ", Blue, Italic, 18],
 Style[set[[2]], Blue, Italic, 18]]
Print[Style["The Last (n-th) normalized (nonzero) eigenvector is:   ",
  Blue, Italic, 18], Style[set[[n]], Blue, Italic, 16]]
If[Total[set] == 1, Print[Style["(If statement safecheck):  ", Black, Bold, 12],
  Style["Total[set] = ", Red, Italic, 12], Style[Total[set], Black, Italic, 12],
  Style[" = 1, so the Eigenvalue set is properly normalized", Red, Italic, 12]],
 Print[Style["(If statement safecheck):  ", Black, Bold, 12],
  Style["Warning!!!", Red, Italic, 28], Style["  Total[set] = ", Red, Italic, 12],
  Style[Total[set], Black, Italic, 12], Style[" ≠ 1, ", Red, Italic, 12],
  Style[" so the Eigenvalue set is NOT properly normalized.", Red, Italic, 12],
  Style[" This will render the entropies invalid. Fix it. ", Red, Italic, 12]]]

Print["  "]
Print[Style[
  "■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■", 18]]
Print["  "]

Do[Print["The α-th Renyi Entropy Hα ->  "Hi-1, " is = ",
  RenyiEntropyofEigenvalues[[i]]], {i, 1, Length[RenyiEntropyofEigenvalues]}]
```

```
     Print[Graphics[Show[
        ListPlot[RenyiEntropyofEigenvalues, PlotRange → All,
         AxesLabel → {Style["α", Large, Bold], Style["Hα", Large, Bold]}],
        ListLinePlot[RenyiEntropyofEigenvalues, PlotStyle → {Red, Thin}]
       ]]]
     Export["PrimePiEigenEntropies.pdf", EvaluationNotebook[]]
     NotebookSave[EvaluationNotebook[], "PrimePirhoCalcOutput"];
    SystemOpen["PrimePiEigenEntropies.pdf"]
    , Background → Green];
nb = CreateDocument[];
Paste[nb, button]
NotebookEvaluate[nb];
```

ZetaZero[i]

```
Zeta[1024] // N
```

0.999999999999971

```
Im[ZetaZero[1] // N]
Im[ZetaZero[2] // N]
Im[ZetaZero[1023] // N]
Im[ZetaZero[1024] // N]
```

14.1347

21.022

1445.83

1447.23

```
Wgenesample = "ZetaZero";
numgenesample = Table[Im[ZetaZero[i] // N], {i, 1, imax}];
SpecialNote = " ";
```

```
lengthofgeneitself = Length[Flatten[numgenesample]];
M = numgenesample;

For[npow = 1, npow < 1000, npow++, If[Length[M] < (2^(npow)), Break[]];
  FilledSize = 2^(npow + 1)];
Filler[vecvar1_] := Table[4, {i, 1, FilledSize - lengthofgeneitself}]
FilledVec[vecvar2_] := Join[Flatten[vecvar2], Filler[vecvar2]]

Filler[vecvar4_] := Table[4, {i, 1, FilledSize - lengthofgeneitself}]
FilledVec[vecvar5_] := Join[Flatten[vecvar5], Filler[vecvar5]]
For[npow = 1, npow < 1000, npow++, If[lengthofgeneitself ≤ (2^npow), Break[]]];
(* gives npow such that  2^npow > lengthofgeneitself > 2^(npow -1) *)
FilledSize = 2^npow;
FilledM = FilledVec[M];
numrowsW = √Length[FilledM] ;
W = Table[Table[FilledM[[i]],
     {i, (((j - 1) * (numrowsW)) + 1), (j * (numrowsW))}], {j, 1, numrowsW}];

lengthofgeneitself
Length[FilledM]
```

64

64

```
(5) // Negative
(-5) // Negative
```

False

True

Run this for output and pdf

```
Wgenesample
ρ = (W.Transpose[W]); (* ρ as inner product *)

Do[rhoEigens1[[i]] = If[(Eigenvalues[ρ][[i]] // N) < 0, 0. * Eigenvalues[ρ][[i]] // N,
   Eigenvalues[ρ][[i]] // N], {i, 1, Length[Eigenvalues[ρ]]}]
rhoEigens = Sort[DeleteCases[rhoEigens1 // N, 0.], Greater];

rhoEigensinczeroes = Sort[Eigenvalues[ρ] // N, Greater];
(*DeleteCases Removes 0's from the set of Eigenvalues,
Sort puts the list in order of greatest to least *)
set = rhoEigens / Total[rhoEigens];
(* This is the set of nonzero normalized eigenvalues in order of greatest to least *)
setinczeroes = rhoEigensinczeroes / Total[rhoEigensinczeroes];
n = Length[set];
H[α_] := 1/(1 - α) Log[2, Sum[(set[[i]])^α, {i, 1, n}]] // N
H0 = Log[n] // N ; (* H₀ = Hartley Entropy*)
H1 = -Sum[((set[[i]]) (Log[2, set[[i]]])), {i, 1, n}] // N;
(* H₁ = Shannon Entropy*)
H2onward = Table[H[a], {a, 2, 20}] // N ; (* H₂ onward *)
RenyiEntropyofEigenvalues = Join[{H0}, {H1}, H2onward];
```

ZetaZero

```
button =
  Button["Click here for output and pdf", Print[Style[Wgenesample, Black, Bold, 28]] ×
    Print[Style["The ", Blue, Italic, 18], Style[Wgenesample, Black, Italic, 18],
     Style["  has  ", Blue, Italic, 18], Style[lengthofgeneitself, Black, Italic, 18],
     Style["   base pairs   ", Blue, Italic, 18]] ×
    If[StringLength[SpecialNote] > 3, Print[Style["(Special Note): ", Black, Bold, 16],
      Style[SpecialNote, Black, Italic, 12]], Print["  "]] ×
    Print[Style["W is a   ", Blue, Italic, 18], Style[Length[W], Black, Italic, 18],
     Style["  by  ", Blue, Italic, 18], Style[Length[W[[1]]], Black, Italic, 18],
     Style["   matrix with   ", Blue, Italic, 18],
     Style[Length[W] * Length[W[[1]]], Black, Italic, 18],
     Style[" = 2^b elements", Blue, Italic, 18], Style["  for b = ", Blue, Italic, 18],
     Style[Log[2, Length[W] * Length[W[[1]]]], Black, Italic, 18]] ×
    If[(Length[W] * Length[W[[1]]]) == (Length[W])^2,
     Print[Style["(If statement safecheck):  ", Black, Bold, 12],
      Style[Length[W], Black, Italic, 12], Style["  times  ", Red, Italic, 12],
      Style[Length[W[[1]]], Black, Italic, 12],
      Style["   equals   ", Red, Italic, 12], Style[(Length[W]^2), Black, Italic, 12],
      Style["  W is of the right size, you may proceed ", Red, Italic, 12]],
```

```
 Print[Style["(If statement safecheck):  ", Black, Bold, 12],
  Style["Warning!!!", Red, Italic, 28],
  Style["   W is of wrong size, STOP and check W ", Red, Italic, 12]]]
Print["The number of nonzero eigenvalues is  =   ", Length[rhoEigens]]
Do[Print["The i-th Eigenvalue " λi, " is = ", (rhoEigens)[[i]]],
 {i, 1, Length[rhoEigens]}]
Print[Graphics[ListPlot[rhoEigens // N, AxesLabel → {Style["i", Medium, Bold],
     Style["λi", Medium, Bold]}, PlotLabel → "Eigenvalue PLOT"]]]
Print[Graphics[ListLogPlot[rhoEigens // N, AxesLabel → {Style["i", Medium, Bold],
     Style["Log[λi]", Medium, Bold]}, PlotLabel → "Eigenvalue Log PLOT"]]]
Print["Zooming in on the Log Plot so as to Exclude the first
   eigenvalue gives the following plot:"]
Print[Graphics[ListLogPlot[Table[{i, rhoEigens[[i]]}, {i, 2, Length[rhoEigens]}],
   AxesLabel → {Style["i", Medium, Bold], Style["Log[λi]", Medium, Bold]},
   PlotStyle → Red,
   PlotLabel → Style["Logplot of Eigenvalues, excluding λ1", Red, Bold, 16]] ]]
Print["The approximate linearity of the above plot tells us
   that the eigenvalues decrease exponentially. If it's
   nowhere near linear try adjusting the plot range. "]

Print[Graphics[ListLogPlot[
   Table[{i, rhoEigensinczeroes[[i]]}, {i, 2, Length[rhoEigensinczeroes]}],
   AxesLabel → {Style["i", Medium, Bold], Style["Log[λi]", Medium, Bold]},
   PlotStyle → Red, PlotLabel →
    Style["Logplot of Eigenvalues, including zero-valued ones", Red, Bold, 16]] ]]

Print["  "]
Print[Style[
  "████████████████████████████████████████████████████████████", 18]]
Print["  "]

Print[Style["The First normalized eigenvector is:   ", Blue, Italic, 18],
 Style[set[[1]], Blue, Italic, 18]]
Print[Style["The Second normalized eigenvector is:   ", Blue, Italic, 18],
 Style[set[[2]], Blue, Italic, 18]]
Print[Style["The Last (n-th) normalized (nonzero) eigenvector is:   ",
  Blue, Italic, 18], Style[set[[n]], Blue, Italic, 16]]
If[Total[set] == 1, Print[Style["(If statement safecheck):  ", Black, Bold, 12],
  Style["Total[set] = ", Red, Italic, 12], Style[Total[set], Black, Italic, 12],
  Style[" = 1, so the Eigenvalue set is properly normalized", Red, Italic, 12]],
 Print[Style["(If statement safecheck):  ", Black, Bold, 12],
  Style["Warning!!!", Red, Italic, 28], Style["  Total[set] = ", Red, Italic, 12],
  Style[Total[set], Black, Italic, 12], Style[" ≠ 1, ", Red, Italic, 12],
  Style[" so the Eigenvalue set is NOT properly normalized.", Red, Italic, 12],
  Style[" This will render the entropies invalid. Fix it. ", Red, Italic, 12]]]

Print["  "]
Print[Style[
  "████████████████████████████████████████████████████████████", 18]]
Print["  "]
```

```
     Do[Print["The α-th Renyi Entropy Hα ->  " Hi-1, " is = ",
       RenyiEntropyofEigenvalues[[i]]], {i, 1, Length[RenyiEntropyofEigenvalues]}]
     Print[Graphics[Show[
        ListPlot[RenyiEntropyofEigenvalues, PlotRange → All,
         AxesLabel → {Style["α", Large, Bold], Style["Hα", Large, Bold]}],
        ListLinePlot[RenyiEntropyofEigenvalues, PlotStyle → {Red, Thin}]
       ]]]
     Export["PrimePiEigenEntropies.pdf", EvaluationNotebook[]]
     NotebookSave[EvaluationNotebook[], "PrimePirhoCalcOutput"];
    SystemOpen["PrimePiEigenEntropies.pdf"]
    , Background → Green];
nb = CreateDocument[];
Paste[nb, button]
NotebookEvaluate[nb];
```