Explore Lagrangians with Exponentials in them

D[f, {t, 3}] + (D[f, {t, 2}])^2 + 1 gives

```
f[t] = (I / 2) (PolyLog[2, -E^((2 I) (t - Subscript[c, 1]))] +
        (t - Subscript[c, 1]) (t + (2 I) Log[1 + E^((2 I) (t - Subscript[c, 1]))] -
           (2 I) Log[Cos[t - Subscript[c, 1]]] - Subscript[c, 1])) +
    Subscript[c, 2] + t Subscript[c, 3] // FullSimplify
```

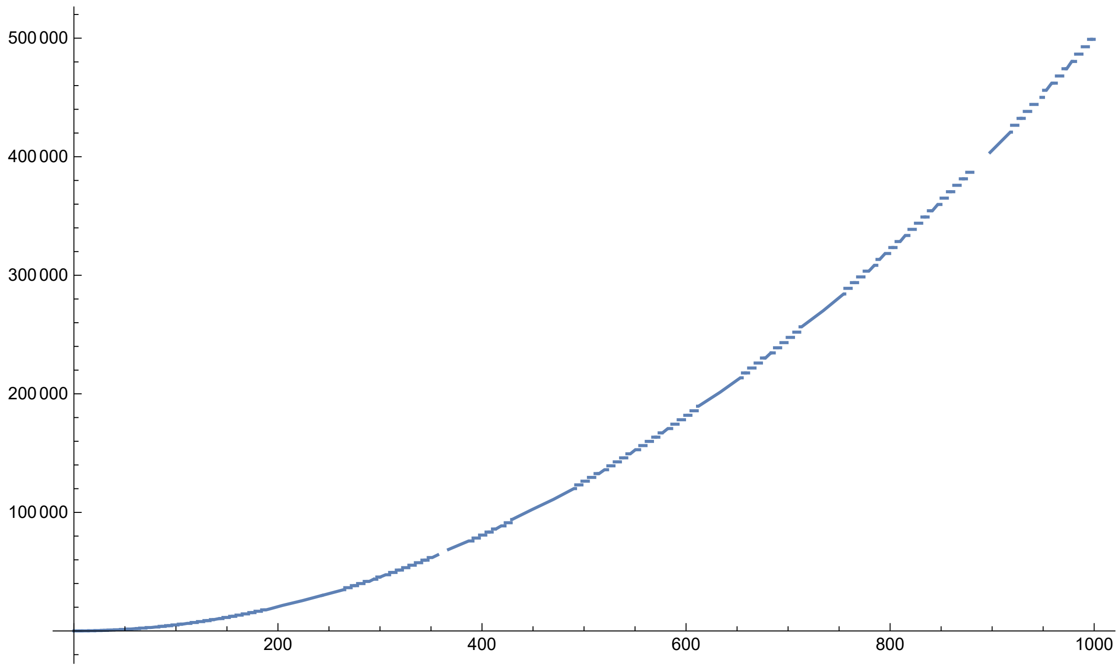
$$\frac{1}{2} i \left(\text{PolyLog}\left[2, -e^{2 i (t-c_1)}\right] + (t - c_1) \left(t + 2 i \text{Log}\left[1 + e^{2 i (t-c_1)}\right] - 2 i \text{Log}[\text{Cos}[t - c_1]] - c_1\right)\right) + c_2 + t\, c_3$$

Second eqn: (D[g, {t, 3}] + (D[g, {t, 2}])^2) Exp[D[g, {t, 1}] - g] + 1

$$\text{Table}\left[\frac{1}{2} i \left(\text{PolyLog}\left[2, -e^{2 i (t)}\right] + (t) \left(t + 2 i \text{Log}\left[1 + e^{2 i (t)}\right] - 2 i \text{Log}[\text{Cos}[t]]\right)\right), \{t, 0, 5\}\right] \text{ // FullSimplify}$$

$$\left\{-\frac{i \pi^2}{24}, -\text{Log}\left[1 + e^{2 i}\right] + \text{Log}[\text{Cos}[1]] + \frac{1}{2} i \left(1 + \text{PolyLog}\left[2, -e^{2 i}\right]\right),\right.$$
$$\frac{1}{2} i \left(-4 + 8\pi + 4 i \text{Log}[2] + \text{PolyLog}\left[2, -e^{4 i}\right]\right),$$
$$\frac{1}{2} i \left(9 + 6\pi + 6 i \text{Log}\left[1 + e^{6 i}\right] - 6 i \text{Log}[-\text{Cos}[3]] + \text{PolyLog}\left[2, -e^{6 i}\right]\right),$$
$$8 i + 2 i \pi + \text{Log}\left[\frac{1}{\left(1 + e^{8 i}\right)^4}\right] + \text{Log}\left[\text{Cos}[4]^4\right] + \frac{1}{2} i\, \text{PolyLog}\left[2, -e^{8 i}\right],$$
$$\left.-5 \text{Log}\left[1 + e^{10 i}\right] + 5 \text{Log}[\text{Cos}[5]] + \frac{1}{2} i \left(25 + \text{PolyLog}\left[2, -e^{10 i}\right]\right)\right\}$$

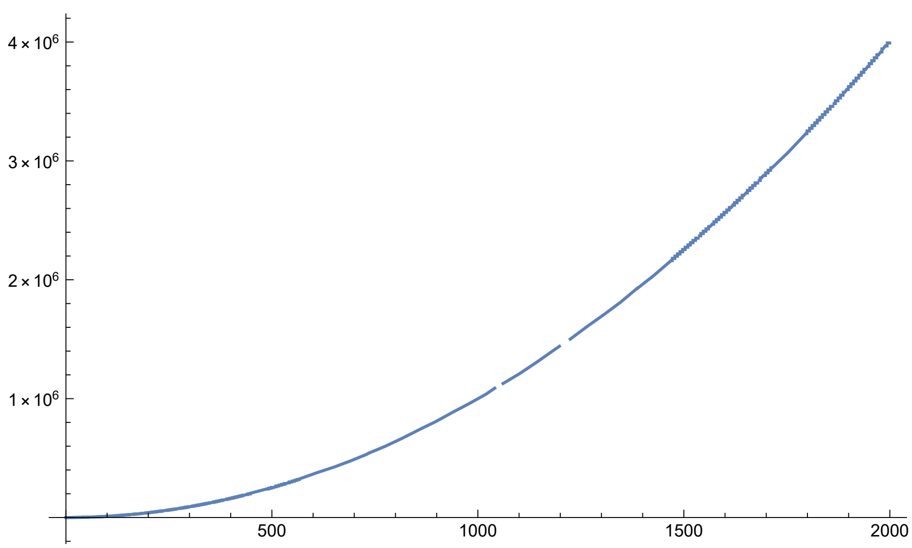
```
Plot[
 Abs[1/2 ⅈ (PolyLog[2, -ℯ^(2 ⅈ (t))] + (t) (t + 2 ⅈ Log[1 + ℯ^(2 ⅈ (t))] - 2 ⅈ Log[Cos[t]]))], {t, 0, 1000}]
```



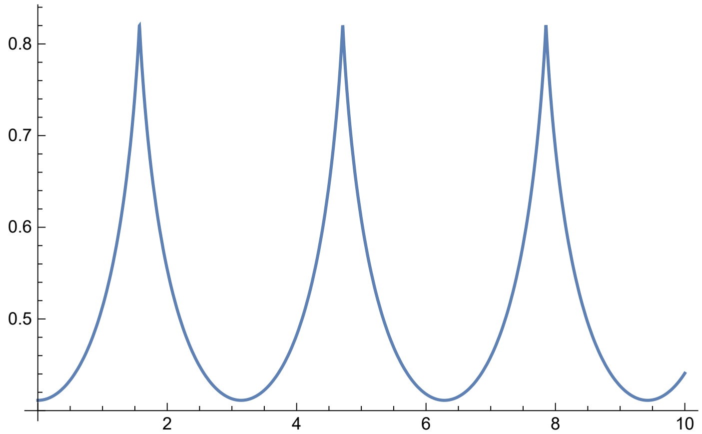

```
E^2 // N
```

```
7.38906
```

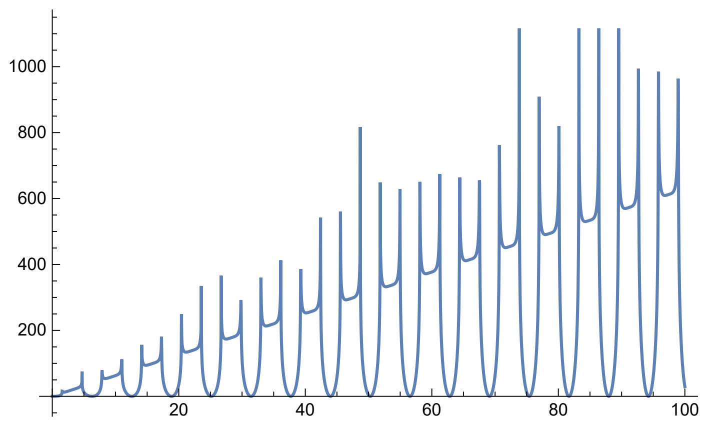
```
Plot[{Abs[(t) (t + 2 ⅈ Log[1 + ℯ^(2 ⅈ (t))] - 2 ⅈ Log[Cos[t]])]}, {t, 0, 2000}]
```



```
Plot[{Abs[1/2 ⅈ (PolyLog[2, -ⅇ^(2 ⅈ (t))])]}, {t, 0, 10}]
```



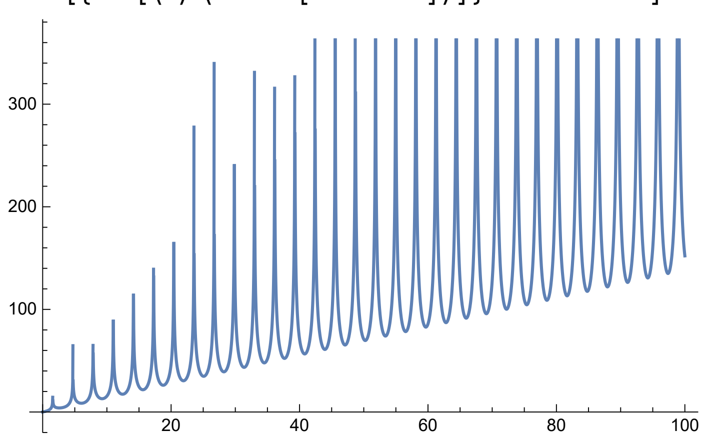

```
Plot[{Abs[(t) (-2 ⅈ Log[Cos[t]])]}, {t, 0, 100}]
```




```
Cos[2 π]
```

1

```
Plot[{Abs[(t) (2 ⅈ Log[1 + ⅇ^(2 ⅈ (t))])]}, {t, 0, 100}]
```




```
EulerEquations[f, u[x], x]
```

```
DSolve[(D[f[t], {t, 3}] + (D[f[t], {t, 2}])^2) + 1 == 0, f[t], t]
```

$$\left\{\left\{f[t] \to C[2] + t\,C[3] + \frac{1}{2}\,i\left((t - C[1])\left(t - C[1] + 2\,i\,\text{Log}\left[1 + e^{2\,i\,(t - C[1])}\right] - 2\,i\,\text{Log}[\text{Cos}[t - C[1]]]\right) + \text{PolyLog}\left[2, -e^{2\,i\,(t - C[1])}\right]\right)\right\}\right\}$$

```
DSolve[(D[g[t], {t, 3}] + (D[g[t], {t, 2}])^2) Exp[D[g[t], {t, 1}] - g[t]] + 1 == 0, g[t], t]
```

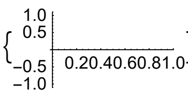
$$\text{DSolve}\left[1 + e^{-g[t] + g'[t]}\left(g''[t]^2 + g^{(3)}[t]\right) == 0, g[t], t\right]$$

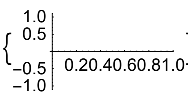
```
NDSolve[{(D[g[t], {t, 3}] + (D[g[t], {t, 2}])^2) Exp[D[g[t], {t, 1}] - g[t]] + 1 == 0,
  g[0] == 1, g'[0] == .1, g''[0] == .2}, g, {t, 0, 3}]
```

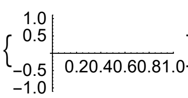
NDSolve::ndsz : At t == 0.9841176613637623`, step size is effectively zero; singularity or stiff system suspected. ≫

{{g → InterpolatingFunction[Domain: {{0., 0.984}} Output: scalar]}}

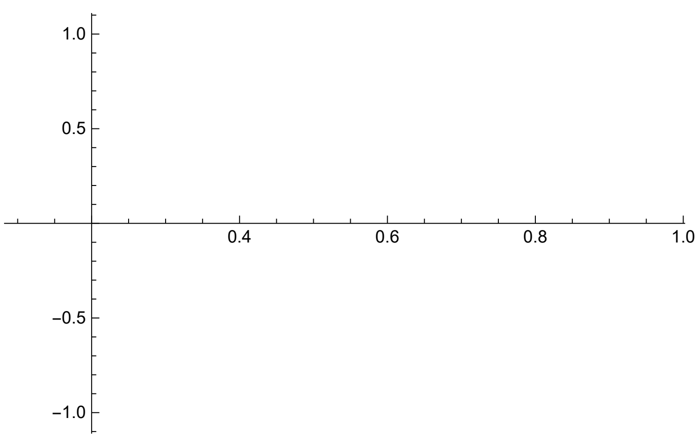
```
Plot[Evaluate[g[t] /. %], {t, 0.1, 0.9841176613637623`}]
```

ReplaceAll::reps : {  } is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing. ≫

ReplaceAll::reps : {  } is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing. ≫

ReplaceAll::reps : {  } is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing. ≫

General::stop : Further output of ReplaceAll::reps will be suppressed during this calculation. ≫



```
NDSolve[{(D[g[t], {t, 3}] + (D[g[t], {t, 2}])^2) Exp[D[g[t], {t, 1}] - g[t]] + 1 == 0,
  g[0] == .001, g[.5] == .5, g[1] == 1}, g, {t, 0, 1}]
```

```
{{g → InterpolatingFunction[ Domain: {{0., 1.}} Output: scalar ]}}
```