

$$\sum_{n=1}^N \left( \frac{n}{2n+1} \right) // \text{FullSimplify}$$

$$\frac{1}{4} \left( 2N + \text{PolyGamma} \left[ 0, \frac{3}{2} \right] - \text{PolyGamma} \left[ 0, \frac{3}{2} + N \right] \right)$$

$$\sum_{n=1}^N \left( \frac{n+1}{2n+1} \right)$$

$$\frac{1}{4} \left( 2N - \text{PolyGamma} \left[ 0, \frac{3}{2} \right] + \text{PolyGamma} \left[ 0, \frac{3}{2} + N \right] \right) // \text{FullSimplify}$$

$$\frac{1}{4} \left( -2 + 2N + \text{HarmonicNumber} \left[ \frac{1}{2} + N \right] + \text{Log}[4] \right)$$

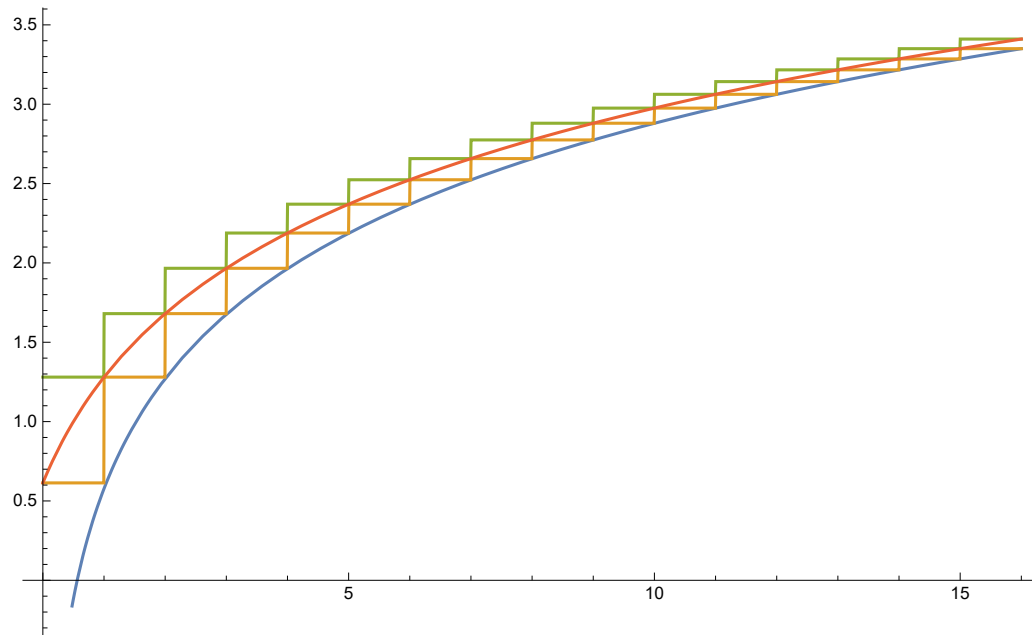
$$N \left[ \text{HarmonicNumber} \left[ \frac{1}{2} + N \right] \right]$$

$$\text{HarmonicNumber}[0.5 + N]$$

$$\text{FunctionExpand}[\text{HarmonicNumber}[0.5 + N]]$$

$$\text{EulerGamma} + \text{PolyGamma}[0, 1.5 + N]$$

**Plot** [{EulerGamma + Log[N], HarmonicNumber[0.5` + Floor[N]],  
HarmonicNumber[0.5` + Ceiling[N]], HarmonicNumber[0.5` + N]}, {N, 0, 16}]



**Solve[HarmonicNumber[0.5` + N] == 2.5]**

**Solve[HarmonicNumber[0.5` + N] == 3.5]**

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

{ {N → 5.8339} }

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

{ {N → 17.5907} }

**HarmonicNumber[0.5` + Floor[1]]**

1.28037

**PolyGamma[0, 1.5` + N]**

PolyGamma[0, 1.5 + N]

$$\sum_{n=1}^N ((n^{-p}) + q)^{-s}$$

$$\sum_{n=1}^N (n^{-p} + q)^{-s}$$

$$\sum_{n=1}^N (n^{-p} + q)^{-s}$$

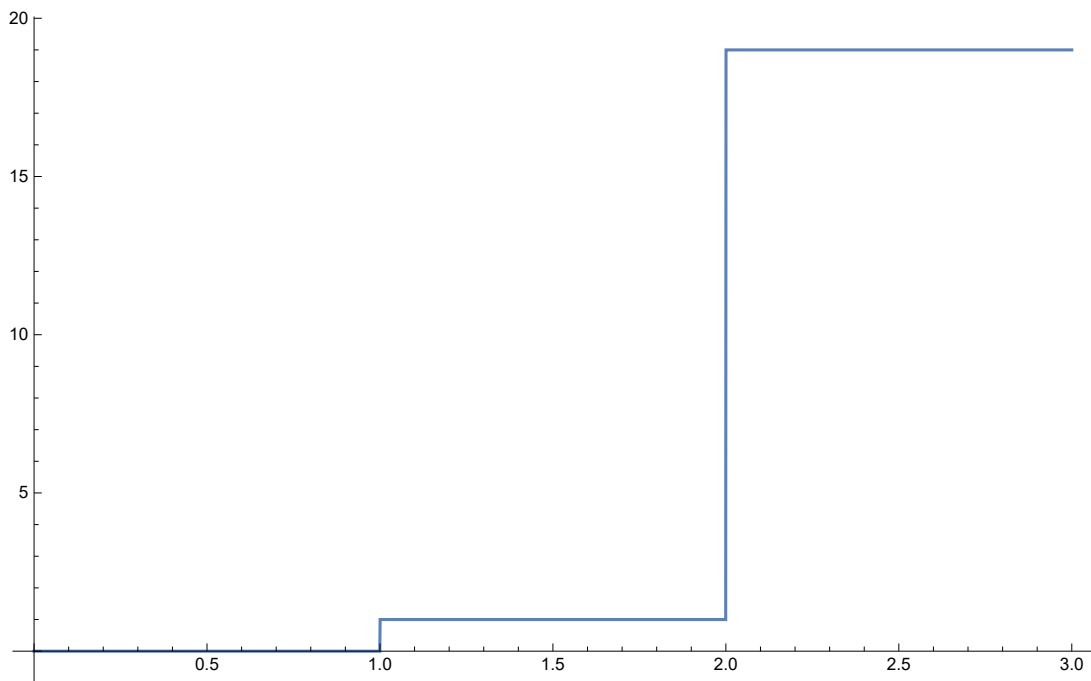
$$\prod_{n=1}^N (((n^{-1})) + q)^{-s}$$

$$\left( \frac{q^N \text{Pochhammer}\left[1 + \frac{1}{q}, N\right]}{N!} \right)^{-s}$$

$$\left( \sum_{n=1}^N (((n^{-1})) + .5)^{(1)} \right) / \left( \prod_{n=1}^N (((n^{-1})) + .5)^{(1)} \right)$$

$$\frac{1. \, e^{0.693147 N} (N + 2. \, \text{HarmonicNumber}[N])}{(1. + N) (2. + N)}$$

`Plot[ $\left(\sum_{n=1}^N ((n^1) + .5)^{-3}\right) / \left(\prod_{n=1}^N ((n^1) + .5)^{-3}\right)$ , {N, 0, 3}]`



$$\prod_{n=1}^N \left( \frac{n}{2n+1} \right)$$

$$\prod_{n=1}^N \left( \frac{n+1}{2n+1} \right)$$

$$\frac{2^{-1-N} \sqrt{\pi} \Gamma[1+N]}{\Gamma\left[\frac{3}{2}+N\right]}$$

$$\frac{2^{-1-N} \sqrt{\pi} \Gamma[2+N]}{\Gamma\left[\frac{3}{2}+N\right]}$$

`Solve[Zeta[s] == 2.5, s]`

`Solve[Zeta[s] == 3.5, s]`

`{{s -> 0.37681 - 17.8479 i}}`

`{{s -> -1.76932 - 10.3593 i}}`

$$\left( \sum_{n=1}^N (n^{-s}) \right) / \left( \prod_{n=1}^N (n^{-s}) \right) // \text{FullSimplify}$$

`(N!)^s HarmonicNumber[N, s]`