Part 1

```
ToExpression[
 "\\sum_{a=a_0}^{a_f}(\\sum_{b=b_0}^{b_f}\\frac{1}{(a+ b \\sqrt{-1} +q)^{s}})",
 TeXForm]
```

```
ToExpression["\\sum_{a=a_0}^{a_f}(\\frac{1}{(a+q)^{s}})", TeXForm]
```

$\text{HurwitzZeta}[s, q + a_0] - \text{HurwitzZeta}[s, 1 + q + a_f]$

```
ToExpression["\\sum_{b=b_0}^{b_f}(\\frac{1}{(b+q)^{s}})", TeXForm]
```

$\text{HurwitzZeta}[s, q + b_0] - \text{HurwitzZeta}[s, 1 + q + b_f]$

```
ToExpression["\\sum_{b=b_0}^{b_f}(\\frac{1}{(a+(b+q))^{s}})", TeXForm]
```

$\text{HurwitzZeta}[s, a + q + b_0] - \text{HurwitzZeta}[s, 1 + a + q + b_f]$

```
ToExpression["\\sum_{a=a_0}^{a_f}(\\frac{1}{(b+(a+q))^{s}})", TeXForm]
```

$\text{HurwitzZeta}[s, b + q + a_0] - \text{HurwitzZeta}[s, 1 + b + q + a_f]$

```
ToExpression["\\frac{1}{(a+q)^{s}}", TeXForm]
```

$(a + q)^{-s}$

```
ToExpression["\\zeta(s,q + a) -\\zeta(s,q + a +1)", TeXForm]
```

$\zeta[s, a + q] - \zeta[s, 1 + a + q]$

```
Assuming[{s ∈ Reals, a ∈ Reals, q ∈ Reals, s > 2, a > 2, q > 2},
 Zeta[s, a + q] - Zeta[s, 1 + a + q] === (a + q)^-s]
```

False

```
N[Zeta[2, 2 + 2] - Zeta[2, 1 + 2 + 2]]
```

0.0625

$\text{N}[(2 + 2)^{-2}]$

0.0625

```
N[Zeta[5, 3.14 + 2] - Zeta[5, 1 + 3.14 + 2]]
```

0.00027873

$\text{N}[(3.14 + 2)^{-5}]$

0.00027873

```
N[Zeta[-12, -12 + -12] - Zeta[-12, 1 + -12 + -12]]
```

3.65203×10^{16}

$\text{N}[(-12 + -12)^{12}]$

3.65203×10^{16}

Part 2

```
N[Zeta[2]]
```

1.64493

```
N[1 + Zeta[2, 2]]
```

1.64493

```
Plot[{Zeta[x] - (1 + Zeta[x, 2])}, {x, -3, 5}]
```

```
Zeta[5] - 1 + Zeta[5, 2]
```

-2 + 2 Zeta[5]

```
N[-2 + 2 Zeta[5]]
```

0.0738555

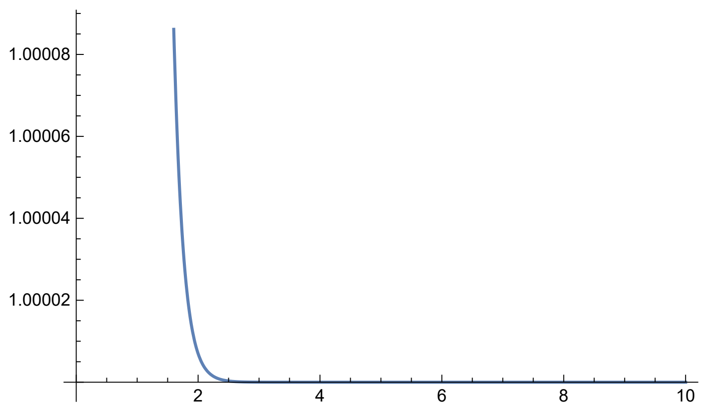
```
Gamma[5]
```

24

$$\sum_{b=1}^{\infty} (1 / (((x)^2) + b^2))$$

$$\frac{-1 + \pi x \operatorname{Coth}[\pi x]}{2 x^2}$$

```
Plot[Coth[π x], {x, 0, 10}]
```



$$\sum_{b=-\infty}^{\infty} (x + b)^{\wedge}(-s)$$

$(-1)^{-s}$ (HurwitzZeta[s, 1 - x] + $(-1)^{s}$ HurwitzZeta[s, x])

Simplify[$(-1)^{-s}$ (HurwitzZeta[s, 1 - x] + $(-1)^{s}$ HurwitzZeta[s, x])]

$(-1)^{-s}$ HurwitzZeta[s, 1 - x] + HurwitzZeta[s, x]

$$\mathbf{N}\left[\sum_{b=1}^{10\,000}\left(\mathbf{Zeta}[0,\,10+b]-\mathbf{Zeta}[0,\,10+b+1]\right)\right]$$

10 000.

$$\sum_{b=b0}^{bf}\left(\left(z+\left(b\,i\right)\right)\right)\wedge(-s)\ \ //\ \mathbf{FullSimplify}$$

$$i^{-s}\left(\mathrm{HurwitzZeta}\left[s,\,b0+\frac{z}{i}\right]-\mathrm{HurwitzZeta}\left[s,\,1+bf+\frac{z}{i}\right]\right)$$

$$\mathbf{Manipulate}\left[i^{s}\,\mathbf{Zeta}[s,\,1+i\,x]+\left(-i\right)^{s}\mathbf{Zeta}[s,\,-i\,x],\,\{x,\,0,\,10\},\,\{s,\,-10,\,10\}\right]$$

$$\left(\left(-1\right)^{-s}\mathbf{Zeta}[s,\,1-z]+\mathbf{Zeta}[s,\,z]\right)+\left(i^{s}\,\mathbf{Zeta}[s,\,1+i\,z]+\left(-i\right)^{s}\mathbf{Zeta}[s,\,-i\,z]\right)\ \ //\ \mathbf{FullSimplify}$$

$$\left(-1\right)^{-s}\mathrm{Zeta}[s,\,1-z]+i^{s}\,\mathrm{Zeta}[s,\,1+i\,z]+\left(-i\right)^{s}\mathrm{Zeta}[s,\,-i\,z]+\mathrm{Zeta}[s,\,z]$$

$$i^{-s}\left(\mathbf{Zeta}\left[s,\,b0+\frac{z}{i}\right]-\mathbf{Zeta}\left[s,\,1+bf+\frac{z}{i}\right]\right)$$

$$\sum_{a=1}^{af}\sum_{b=1}^{bf}\left(\left(\left(q+a+\left(b\,i\right)\right)\right)\wedge(-s)\right)$$

$$\sum_{a=1}^{r}(a)\wedge(s)\ \ //\ \mathbf{FullSimplify}$$

HarmonicNumber[r, -s]

FunctionExpand[HarmonicNumber[r, -s]]

-HurwitzZeta[-s, 1 + r] + Zeta[-s]

Part 3

N[Zeta[2, 2]]

0.644934

$$\sum_{b=-1\,000\,000}^{-1}\left(0.5+b\right)\wedge\left(-2\right)$$

4.9348

$$\texttt{Table}\left[\left\{\texttt{Zeta[s, q]},\ \texttt{Zeta[s]} - \left(\sum_{k=1}^{q-1}\left(\left(k\right)\texttt{\^{}}(-s)\right)\right)\right\},\ \{s, 2, 10\},\ \{q, 1, 10\}\right]$$

$$\left\{\left\{\left\{\frac{\pi^2}{6}, \frac{\pi^2}{6}\right\}, \left\{-1+\frac{\pi^2}{6}, -1+\frac{\pi^2}{6}\right\}, \left\{-\frac{5}{4}+\frac{\pi^2}{6}, -\frac{5}{4}+\frac{\pi^2}{6}\right\},\right.\right.$$
$$\left\{-\frac{49}{36}+\frac{\pi^2}{6}, -\frac{49}{36}+\frac{\pi^2}{6}\right\}, \left\{-\frac{205}{144}+\frac{\pi^2}{6}, -\frac{205}{144}+\frac{\pi^2}{6}\right\}, \left\{-\frac{5269}{3600}+\frac{\pi^2}{6}, -\frac{5269}{3600}+\frac{\pi^2}{6}\right\},$$
$$\left\{-\frac{5369}{3600}+\frac{\pi^2}{6}, -\frac{5369}{3600}+\frac{\pi^2}{6}\right\}, \left\{-\frac{266\,681}{176\,400}+\frac{\pi^2}{6}, -\frac{266\,681}{176\,400}+\frac{\pi^2}{6}\right\},$$
$$\left.\left\{-\frac{1\,077\,749}{705\,600}+\frac{\pi^2}{6}, -\frac{1\,077\,749}{705\,600}+\frac{\pi^2}{6}\right\}, \left\{-\frac{9\,778\,141}{6\,350\,400}+\frac{\pi^2}{6}, -\frac{9\,778\,141}{6\,350\,400}+\frac{\pi^2}{6}\right\}\right\},$$
$$\left\{\{\mathrm{Zeta}[3], \mathrm{Zeta}[3]\}, \{-1+\mathrm{Zeta}[3], -1+\mathrm{Zeta}[3]\}, \left\{\mathrm{Zeta}[3, 3], -\frac{9}{8}+\mathrm{Zeta}[3]\right\},\right.$$
$$\left\{\mathrm{Zeta}[3, 4], -\frac{251}{216}+\mathrm{Zeta}[3]\right\}, \left\{\mathrm{Zeta}[3, 5], -\frac{2035}{1728}+\mathrm{Zeta}[3]\right\},$$
$$\left\{\mathrm{Zeta}[3, 6], -\frac{256\,103}{216\,000}+\mathrm{Zeta}[3]\right\}, \left\{\mathrm{Zeta}[3, 7], -\frac{28\,567}{24\,000}+\mathrm{Zeta}[3]\right\},$$
$$\left\{\mathrm{Zeta}[3, 8], -\frac{9\,822\,481}{8\,232\,000}+\mathrm{Zeta}[3]\right\}, \left\{\mathrm{Zeta}[3, 9], -\frac{78\,708\,473}{65\,856\,000}+\mathrm{Zeta}[3]\right\},$$
$$\left.\left\{\mathrm{Zeta}[3, 10], -\frac{19\,148\,110\,939}{16\,003\,008\,000}+\mathrm{Zeta}[3]\right\}\right\}, \left\{\left\{\frac{\pi^4}{90}, \frac{\pi^4}{90}\right\}, \left\{-1+\frac{\pi^4}{90}, -1+\frac{\pi^4}{90}\right\},\right.$$
$$\left\{-\frac{17}{16}+\frac{\pi^4}{90}, -\frac{17}{16}+\frac{\pi^4}{90}\right\}, \left\{-\frac{1393}{1296}+\frac{\pi^4}{90}, -\frac{1393}{1296}+\frac{\pi^4}{90}\right\}, \left\{-\frac{22\,369}{20\,736}+\frac{\pi^4}{90}, -\frac{22\,369}{20\,736}+\frac{\pi^4}{90}\right\},$$
$$\left\{-\frac{14\,001\,361}{12\,960\,000}+\frac{\pi^4}{90}, -\frac{14\,001\,361}{12\,960\,000}+\frac{\pi^4}{90}\right\}, \left\{-\frac{14\,011\,361}{12\,960\,000}+\frac{\pi^4}{90}, -\frac{14\,011\,361}{12\,960\,000}+\frac{\pi^4}{90}\right\},$$
$$\left\{-\frac{33\,654\,237\,761}{31\,116\,960\,000}+\frac{\pi^4}{90}, -\frac{33\,654\,237\,761}{31\,116\,960\,000}+\frac{\pi^4}{90}\right\}, \left\{-\frac{538\,589\,354\,801}{497\,871\,360\,000}+\frac{\pi^4}{90}, -\frac{538\,589\,354\,801}{497\,871\,360\,000}+\frac{\pi^4}{90}\right\},$$
$$\left.\left\{-\frac{43\,631\,884\,298\,881}{40\,327\,580\,160\,000}+\frac{\pi^4}{90}, -\frac{43\,631\,884\,298\,881}{40\,327\,580\,160\,000}+\frac{\pi^4}{90}\right\}\right\},$$
$$\left\{\{\mathrm{Zeta}[5], \mathrm{Zeta}[5]\}, \{-1+\mathrm{Zeta}[5], -1+\mathrm{Zeta}[5]\}, \left\{\mathrm{Zeta}[5, 3], -\frac{33}{32}+\mathrm{Zeta}[5]\right\},\right.$$
$$\left\{\mathrm{Zeta}[5, 4], -\frac{8051}{7776}+\mathrm{Zeta}[5]\right\}, \left\{\mathrm{Zeta}[5, 5], -\frac{257\,875}{248\,832}+\mathrm{Zeta}[5]\right\},$$
$$\left\{\mathrm{Zeta}[5, 6], -\frac{806\,108\,207}{777\,600\,000}+\mathrm{Zeta}[5]\right\}, \left\{\mathrm{Zeta}[5, 7], -\frac{268\,736\,069}{259\,200\,000}+\mathrm{Zeta}[5]\right\},$$
$$\left\{\mathrm{Zeta}[5, 8], -\frac{4\,516\,906\,311\,683}{4\,356\,374\,400\,000}+\mathrm{Zeta}[5]\right\}, \left\{\mathrm{Zeta}[5, 9], -\frac{144\,545\,256\,245\,731}{139\,403\,980\,800\,000}+\mathrm{Zeta}[5]\right\},$$
$$\left.\left\{\mathrm{Zeta}[5, 10], -\frac{105\,375\,212\,839\,937\,899}{101\,625\,502\,003\,200\,000}+\mathrm{Zeta}[5]\right\}\right\},$$
$$\left\{\left\{\frac{\pi^6}{945}, \frac{\pi^6}{945}\right\}, \left\{-1+\frac{\pi^6}{945}, -1+\frac{\pi^6}{945}\right\}, \left\{-\frac{65}{64}+\frac{\pi^6}{945}, -\frac{65}{64}+\frac{\pi^6}{945}\right\},\right.$$
$$\left\{-\frac{47\,449}{46\,656}+\frac{\pi^6}{945}, -\frac{47\,449}{46\,656}+\frac{\pi^6}{945}\right\}, \left\{-\frac{3\,037\,465}{2\,985\,984}+\frac{\pi^6}{945}, -\frac{3\,037\,465}{2\,985\,984}+\frac{\pi^6}{945}\right\},$$
$$\left\{-\frac{47\,463\,376\,609}{46\,656\,000\,000}+\frac{\pi^6}{945}, -\frac{47\,463\,376\,609}{46\,656\,000\,000}+\frac{\pi^6}{945}\right\}, \left\{-\frac{47\,464\,376\,609}{46\,656\,000\,000}+\frac{\pi^6}{945}, -\frac{47\,464\,376\,609}{46\,656\,000\,000}+\frac{\pi^6}{945}\right\},$$

$$\left\{-\frac{5\,584\,183\,099\,672\,241}{5\,489\,031\,744\,000\,000}+\frac{\pi^6}{945},\ -\frac{5\,584\,183\,099\,672\,241}{5\,489\,031\,744\,000\,000}+\frac{\pi^6}{945}\right\},$$
$$\left\{-\frac{357\,389\,058\,474\,664\,049}{351\,298\,031\,616\,000\,000}+\frac{\pi^6}{945},\ -\frac{357\,389\,058\,474\,664\,049}{351\,298\,031\,616\,000\,000}+\frac{\pi^6}{945}\right\},$$
$$\left\{-\frac{260\,537\,105\,518\,334\,091\,721}{256\,096\,265\,048\,064\,000\,000}+\frac{\pi^6}{945},\ -\frac{260\,537\,105\,518\,334\,091\,721}{256\,096\,265\,048\,064\,000\,000}+\frac{\pi^6}{945}\right\}\right\},$$
$$\left\{\{\text{Zeta}[7],\ \text{Zeta}[7]\},\ \{-1+\text{Zeta}[7],\ -1+\text{Zeta}[7]\},\ \left\{\text{Zeta}[7,3],\ -\frac{129}{128}+\text{Zeta}[7]\right\},\right.$$
$$\left\{\text{Zeta}[7,4],\ -\frac{282\,251}{279\,936}+\text{Zeta}[7]\right\},\ \left\{\text{Zeta}[7,5],\ -\frac{36\,130\,315}{35\,831\,808}+\text{Zeta}[7]\right\},$$
$$\left\{\text{Zeta}[7,6],\ -\frac{2\,822\,716\,691\,183}{2\,799\,360\,000\,000}+\text{Zeta}[7]\right\},\ \left\{\text{Zeta}[7,7],\ -\frac{940\,908\,897\,061}{933\,120\,000\,000}+\text{Zeta}[7]\right\},$$
$$\left\{\text{Zeta}[7,8],\ -\frac{774\,879\,868\,932\,307\,123}{768\,464\,444\,160\,000\,000}+\text{Zeta}[7]\right\},$$
$$\left\{\text{Zeta}[7,9],\ -\frac{99\,184\,670\,126\,682\,733\,619}{98\,363\,448\,852\,480\,000\,000}+\text{Zeta}[7]\right\},$$
$$\left.\left\{\text{Zeta}[7,10],\ -\frac{650\,750\,755\,630\,450\,535\,274\,259}{645\,362\,587\,921\,121\,280\,000\,000}+\text{Zeta}[7]\right\}\right\},$$
$$\left\{\left\{\frac{\pi^8}{9450},\ \frac{\pi^8}{9450}\right\},\ \left\{-1+\frac{\pi^8}{9450},\ -1+\frac{\pi^8}{9450}\right\},\ \left\{-\frac{257}{256}+\frac{\pi^8}{9450},\ -\frac{257}{256}+\frac{\pi^8}{9450}\right\},\right.$$
$$\left\{-\frac{1\,686\,433}{1\,679\,616}+\frac{\pi^8}{9450},\ -\frac{1\,686\,433}{1\,679\,616}+\frac{\pi^8}{9450}\right\},\ \left\{-\frac{431\,733\,409}{429\,981\,696}+\frac{\pi^8}{9450},\ -\frac{431\,733\,409}{429\,981\,696}+\frac{\pi^8}{9450}\right\},$$
$$\left\{-\frac{168\,646\,292\,872\,321}{167\,961\,600\,000\,000}+\frac{\pi^8}{9450},\ -\frac{168\,646\,292\,872\,321}{167\,961\,600\,000\,000}+\frac{\pi^8}{9450}\right\},$$
$$\left\{-\frac{168\,646\,392\,872\,321}{167\,961\,600\,000\,000}+\frac{\pi^8}{9450},\ -\frac{168\,646\,392\,872\,321}{167\,961\,600\,000\,000}+\frac{\pi^8}{9450}\right\},$$
$$\left\{-\frac{972\,213\,062\,238\,348\,973\,121}{968\,265\,199\,641\,600\,000\,000}+\frac{\pi^8}{9450},\ -\frac{972\,213\,062\,238\,348\,973\,121}{968\,265\,199\,641\,600\,000\,000}+\frac{\pi^8}{9450}\right\},$$
$$\left\{-\frac{248\,886\,558\,707\,571\,775\,009\,601}{247\,875\,891\,108\,249\,600\,000\,000}+\frac{\pi^8}{9450},\ -\frac{248\,886\,558\,707\,571\,775\,009\,601}{247\,875\,891\,108\,249\,600\,000\,000}+\frac{\pi^8}{9450}\right\},$$
$$\left.\left\{-\frac{1\,632\,944\,749\,460\,578\,249\,437\,992\,161}{1\,626\,313\,721\,561\,225\,625\,600\,000\,000}+\frac{\pi^8}{9450},\ -\frac{1\,632\,944\,749\,460\,578\,249\,437\,992\,161}{1\,626\,313\,721\,561\,225\,625\,600\,000\,000}+\frac{\pi^8}{9450}\right\}\right\},$$
$$\left\{\{\text{Zeta}[9],\ \text{Zeta}[9]\},\ \{-1+\text{Zeta}[9],\ -1+\text{Zeta}[9]\},\ \left\{\text{Zeta}[9,3],\ -\frac{513}{512}+\text{Zeta}[9]\right\},\right.$$
$$\left\{\text{Zeta}[9,4],\ -\frac{10\,097\,891}{10\,077\,696}+\text{Zeta}[9]\right\},\ \left\{\text{Zeta}[9,5],\ -\frac{5\,170\,139\,875}{5\,159\,780\,352}+\text{Zeta}[9]\right\},$$
$$\left\{\text{Zeta}[9,6],\ -\frac{10\,097\,934\,603\,139\,727}{10\,077\,696\,000\,000\,000}+\text{Zeta}[9]\right\},\ \left\{\text{Zeta}[9,7],\ -\frac{373\,997\,614\,931\,101}{373\,248\,000\,000\,000}+\text{Zeta}[9]\right\},$$
$$\left\{\text{Zeta}[9,8],\ -\frac{15\,092\,153\,145\,114\,981\,831\,307}{15\,061\,903\,105\,536\,000\,000\,000}+\text{Zeta}[9]\right\},$$
$$\left\{\text{Zeta}[9,9],\ -\frac{7\,727\,182\,467\,755\,471\,289\,426\,059}{7\,711\,694\,390\,034\,432\,000\,000\,000}+\text{Zeta}[9]\right\},$$
$$\left.\left\{\text{Zeta}[9,10],\ -\frac{4\,106\,541\,588\,424\,891\,370\,931\,874\,221\,019}{4\,098\,310\,578\,334\,288\,576\,512\,000\,000\,000}+\text{Zeta}[9]\right\}\right\},$$
$$\left\{\left\{\frac{\pi^{10}}{93\,555},\ \frac{\pi^{10}}{93\,555}\right\},\ \left\{-1+\frac{\pi^{10}}{93\,555},\ -1+\frac{\pi^{10}}{93\,555}\right\},\ \left\{-\frac{1025}{1024}+\frac{\pi^{10}}{93\,555},\ -\frac{1025}{1024}+\frac{\pi^{10}}{93\,555}\right\},\right.$$

$$\left\{-\frac{60526249}{60466176}+\frac{\pi^{10}}{93555},\ -\frac{60526249}{60466176}+\frac{\pi^{10}}{93555}\right\},$$
$$\left\{-\frac{61978938025}{61917364224}+\frac{\pi^{10}}{93555},\ -\frac{61978938025}{61917364224}+\frac{\pi^{10}}{93555}\right\},$$
$$\left\{-\frac{605263128567754849}{604661760000000000}+\frac{\pi^{10}}{93555},\ -\frac{605263128567754849}{604661760000000000}+\frac{\pi^{10}}{93555}\right\},$$
$$\left\{-\frac{605263138567754849}{604661760000000000}+\frac{\pi^{10}}{93555},\ -\frac{605263138567754849}{604661760000000000}+\frac{\pi^{10}}{93555}\right\},$$
$$\left\{-\frac{170971856382109814342232401}{170801981216778240000000000}+\frac{\pi^{10}}{93555},\ -\frac{170971856382109814342232401}{170801981216778240000000000}+\frac{\pi^{10}}{93555}\right\},$$
$$\left\{-\frac{175075181098169912564190119249}{174901228765980917760000000000}+\frac{\pi^{10}}{93555},\ -\frac{175075181098169912564190119249}{174901228765980917760000000000}+\frac{\pi^{10}}{93555}\right\},\ \left\{-\frac{10338014371627802833957102351534201}{10327742657402407212810240000000000}+\frac{\pi^{10}}{93555},\ -\frac{10338014371627802833957102351534201}{10327742657402407212810240000000000}+\frac{\pi^{10}}{93555}\right\}\}\}$$