Functions

Functions

```
Dag[A_] := Transpose[Conjugate[A]];
(*Dagger and bar ONLY same in Dirac Basis, otherwise Bar[A_] = Dag[A_].γ0*)
```

Dirac + Pauli Matrices

$$\gamma0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}; \ \gamma1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}; \ \gamma2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix};$$

$$\gamma3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}; \ (*\text{Dirac Basis}*)$$

```
γ5 = i γ0.γ1.γ2.γ3;
γ5 // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\sigma1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \ \sigma2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \ \sigma3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix};$$

Guage Covariant Derivative & 4-Derivative

(-+++) convention used

$$\text{QFTMetric} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

```
COOD = {x0, x1, x2, x3};

Bar[A_] := Transpose[Conjugate[A]].γ0

GDer[A_] := i (γ0.(D[A, x0]) + γ1.(D[A, x1]) + γ2.(D[A, x2]) + γ3.(D[A, x3]));
(*If EM is included need to put in EM potential terms*)

FourDer[A_] := D[A, x0] + D[A, x1] + D[A, x2] + D[A, x3];

FourDerSQ[A_] :=
 Sum[Sum[QFTMetric[[i, i]] ((D[A, COOD[[i]]])^2), {i, 1, 4}][[j]], {j, 1, 4}][[1]]
```

```
FourLap[A_] := Sum[1/Sqrt[Abs[Det[QFTMetric]]] D[Sqrt[Abs[Det[QFTMetric]]]
    QFTMetric[[i, i]] D[A[[i]], COOD[[i]]], COOD[[i]]], {i, 1, 4}][[1]];

FourLap2[A_] := Sum[1/Sqrt[Abs[Det[QFTMetric]]] D[
   Sqrt[Abs[Det[QFTMetric]]] QFTMetric[[i, i]] D[A[[i]], COOD[[i]]], COOD[[i]]], {i, 1, 4}];

(* Metric = metric in 4D Spacetime
   FourLapGen[A_]:=
  Sum[1/Sqrt[Abs[Det[Metric]]] D[Sqrt[Abs[Det[Metric]]] Metric[[i,i]] D[A[[i]],COOD[[i]]],COOD[[i]]],
   {i,1,4}][[1]];*)

LaplaceBeltrami[A_, Metric_] :=
  Sum[1/Sqrt[Abs[Det[Metric]]] D[Sqrt[Abs[Det[Metric]]] Metric[[i, i]] D[A[[i]], COOD[[i]]],
    COOD[[i]]], {i, 1, Dimensions[Metric][[1]]}];
```

This LaplaceBeltrami function applies the Lapace Beltrami operator (tensor version of Laplacian) to some vector A which is in a spacetime described geometrically by a given metric
For instance, for the laplacian of a vector ϕ in 4 Dimensional AdS space do LaplaceBeltrami[ϕ, AdSMetric4]
This fn is only for vector A though

For matrix M , it should do something like

```
Sum[
 D[Sqrt[-Det[metric]] metric.D[M, COOD[[i]]], COOD[[i]]] / (Sqrt[-Det[metric]]), {i, 1, 4}]
```

AdS Math Stuff

AdS Metrics

```
AdSMetric5 = ( -L^2 (1 + (r^2))   0              0                         0                         
               0                  (L^2) (r^2)    0                         0                         
               0                  0              (L^2) (r^2) (Sin[α]^2)    0                         
               0                  0              0                         (L^2) (r^2) (Sin[α]^2     
               0                  0              0                         0                         

COOD5 = {x0, x1, x2, x3, r};
```

```
AdSMetric4 =
  ( -L^2 (1 + (r^2))    0              0                         0
    0                   (L^2) (r^2)    0                         0
    0                   0              (L^2) (r^2) (Sin[α]^2)    0
    0                   0              0                         (L^2) (1 + (r^2))^(-1) );
(* D = 4, grr = g33 3rd row *)
COOD4 = {x0, x1, x2, r};

AdSMetric3 = ( -L^2 (1 + (r^2))    0              0
               0                   (L^2) (r^2)    0
               0                   0              (L^2) (1 + (r^2))^(-1) );

COOD3 = {x0, x1, r};

AdSMetric2 = ( -L^2 (1 + (r^2))    0
               0                   (L^2) (1 + (r^2))^(-1) );
(* D = 1, grr = g22 2nd row *)
COOD2 = {x0, r};

AdSMetric1 = (L^2) (1 + (r^2))^(-1);
(* D = 0, grr = g11 *)
COOD1 = {r};
```

AdS Laplacian Operators

Use Laplace–Beltrami operator:

```
FourLapAdS1[A_] := Sum[1/Sqrt[Abs[Det[AdSMetric1]]]
    D[Sqrt[Abs[Det[AdSMetric1]]] AdSMetric1 D[A, COOD1[[i]]], COOD1[[i]]], {i, 1, 1}];
FourLapAdS2[A_] := Sum[1/Sqrt[Abs[Det[AdSMetric2]]] D[Sqrt[Abs[Det[AdSMetric2]]]
      AdSMetric2[[i, i]] D[A[[i]], COOD2[[i]]], COOD2[[i]]], {i, 1, 2}][[1]];
FourLapAdS3[A_] := Sum[1/Sqrt[Abs[Det[AdSMetric3]]] D[Sqrt[Abs[Det[AdSMetric3]]]
      AdSMetric3[[i, i]] D[A[[i]], COOD3[[i]]], COOD3[[i]]], {i, 1, 3}][[1]];
FourLapAdS4[A_] := Sum[1/Sqrt[Abs[Det[AdSMetric4]]] D[Sqrt[Abs[Det[AdSMetric4]]]
      AdSMetric4[[i, i]] D[A[[i]], COOD4[[i]]], COOD4[[i]]], {i, 1, 4}][[1]];
FourLapAdS5[A_] := Sum[1/Sqrt[Abs[Det[AdSMetric5]]] D[Sqrt[Abs[Det[AdSMetric5]]]
      AdSMetric5[[i, i]] D[A[[i]], COOD5[[i]]], COOD5[[i]]], {i, 1, 5}][[1]];
FourLapAdS5Scalar[A_] := Sum[1/Sqrt[Abs[Det[AdSMetric5]]] D[Sqrt[Abs[Det[AdSMetric5]]]
      AdSMetric5[[i, i]] D[A, COOD5[[i]]], COOD5[[i]]], {i, 1, 5}][[1]];
```

```
√Abs[Det[AdSMetric1]] // FullSimplify
√Abs[Det[AdSMetric2]] // FullSimplify
√Abs[Det[AdSMetric3]] // FullSimplify
√Abs[Det[AdSMetric4]] // FullSimplify
√Abs[Det[AdSMetric5]] // FullSimplify
```

$$\sqrt{\mathrm{Abs}\left[\mathrm{Det}\left[\frac{L^2}{1+r^2}\right]\right]}$$

$\mathrm{Abs}[L]^2$

$\mathrm{Abs}\left[L^3\, r\right]$

$\mathrm{Abs}\left[L^4\, r^2\, \mathrm{Sin}[\alpha]\right]$

$\mathrm{Abs}\left[L^5\, r^3\, \mathrm{Sin}[\alpha]^2\, \mathrm{Sin}[\beta]\right]$

Lagrangian

```
LScalar[ϕ_, ψ_] :=
  (1/2) FourDerSQ[ϕ] + Bar[ψ].GDer[ψ] + Bar[ψ].ψ.ϕ - (a/2 (ϕ^2) + b/3! (ϕ^3) + c/4! (ϕ^4))

ϕ = {{ϕ0[x0, x1, x2, x3]}, {ϕ1[x0, x1, x2, x3]}, {ϕ2[x0, x1, x2, x3]}, {ϕ3[x0, x1, x2, x3]}};
ϕ5 = {{ϕ0[x0, x1, x2, x3, r]}, {ϕ1[x0, x1, x2, x3, r]},
   {ϕ2[x0, x1, x2, x3, r]}, {ϕ3[x0, x1, x2, x3, r]}, {ϕr[x0, x1, x2, x3, r]}};
ψ = {{ψ0[x0, x1, x2, x3]}, {ψ1[x0, x1, x2, x3]}, {ψ2[x0, x1, x2, x3]}, {ψ3[x0, x1, x2, x3]}};

FourDerSQ[ϕ]
FourLap[ϕ] (*To check the code *)
```

$$\phi0^{(0,0,0,1)}[x0, x1, x2, x3]^2 + \phi1^{(0,0,0,1)}[x0, x1, x2, x3]^2 + \phi2^{(0,0,0,1)}[x0, x1, x2, x3]^2 + \phi3^{(0,0,0,1)}[x0, x1, x2, x3]^2 + \phi0^{(0,0,1,0)}[x0, x1, x2, x3]^2 + \phi1^{(0,0,1,0)}[x0, x1, x2, x3]^2 + \phi2^{(0,0,1,0)}[x0, x1, x2, x3]^2 + \phi3^{(0,0,1,0)}[x0, x1, x2, x3]^2 + \phi0^{(0,1,0,0)}[x0, x1, x2, x3]^2 + \phi1^{(0,1,0,0)}[x0, x1, x2, x3]^2 + \phi2^{(0,1,0,0)}[x0, x1, x2, x3]^2 + \phi3^{(0,1,0,0)}[x0, x1, x2, x3]^2 - \phi0^{(1,0,0,0)}[x0, x1, x2, x3]^2 - \phi1^{(1,0,0,0)}[x0, x1, x2, x3]^2 - \phi2^{(1,0,0,0)}[x0, x1, x2, x3]^2 - \phi3^{(1,0,0,0)}[x0, x1, x2, x3]^2$$

$$\phi3^{(0,0,0,2)}[x0, x1, x2, x3] + \phi2^{(0,0,2,0)}[x0, x1, x2, x3] + \phi1^{(0,2,0,0)}[x0, x1, x2, x3] - \phi0^{(2,0,0,0)}[x0, x1, x2, x3]$$

After applying Euler-Lagrange Eqn it becomes

$\mathrm{FourLap}[\phi c] - \mathrm{Bar}[\psi].\psi + \left(a\ (\phi c) + \frac{b}{2}\ (\phi c\verb|^|2) + \frac{c}{6}\ (\phi c\verb|^|3)\right) = 0$

$\mathrm{FourLap}[\phi c] - \mathrm{Bar}[\psi].\psi - ((m\verb|^|2)\ (\phi \mathrm{AdS}) + \beta\ (\phi \mathrm{AdS}\verb|^|2) + (2\,\lambda)\ (\phi \mathrm{AdS}\verb|^|3)) = 0$

Potential U min occurs at ϕ=0 and at ϕc

```
ϕcplus = -(3 b)/(2 c) (1 - Sqrt[1 - (8 a c)/(3 b^2)]);

ϕcminus = -(3 b)/(2 c) (1 + Sqrt[1 - (8 a c)/(3 b^2)]);

FourLap2[ϕcplus] - Bar[ψ].ψ + (a (ϕcplus) + b/2 (ϕcplus^2) + c/6 (ϕcplus^3)) // FullSimplify
(* =0 *)
FourLap2[ϕcminus] - Bar[ψ].ψ + (a (ϕcminus) + b/2 (ϕcminus^2) + c/6 (ϕcminus^3)) //
 FullSimplify(* =0 *)
```

$$\{\{-\text{Abs}[\psi0[x0, x1, x2, x3]]^2 - \text{Abs}[\psi1[x0, x1, x2, x3]]^2 + \text{Abs}[\psi2[x0, x1, x2, x3]]^2 + \text{Abs}[\psi3[x0, x1, x2, x3]]^2\}\}$$

$$\{\{-\text{Abs}[\psi0[x0, x1, x2, x3]]^2 - \text{Abs}[\psi1[x0, x1, x2, x3]]^2 + \text{Abs}[\psi2[x0, x1, x2, x3]]^2 + \text{Abs}[\psi3[x0, x1, x2, x3]]^2\}\}$$

As would be expected. So even if a,b,c depend on the xμ, we have the constraint
FourLap[ϕcminus] = Bar[ψ].ψ
Which would govern how they depend on the xμ

```
ϕAdS = Exp[-ⅈ Δ x0] (1 + (r^2))^(-Δ/2);

Δ = (Dim/2) + Sqrt[(Dim^2/4) + (m L)^2];

FourLapAdS5Scalar[ϕAdS] - ((m^2) (ϕAdS)) /. Dim → 4 /. m → 0
FourLapAdS5Scalar[ϕAdS] - ((m^2) (ϕAdS)) /. Dim → 4 /. m → (L^(-1)) // FullSimplify
```

$$\frac{16\, e^{-4 i x0} L^2}{1 + r^2}$$

$$\frac{1}{L^2} e^{-i \left(2+\sqrt{5}\right) x0} \left(1 + r^2\right)^{-1-\frac{\sqrt{5}}{2}} \left(-1 + \left(9 + 4\sqrt{5}\right) L^4 \left(1 + r^2\right)\right)$$

```
FourLapAdS5[ϕ5]
```

$$\left(-\frac{1}{\left(1+r^2\right)^2}2\,L^2\,r\,\sqrt{\mathrm{Abs}\left[-\frac{L^{10}\,r^6\,\mathrm{Sin}[\alpha]^4\,\mathrm{Sin}[\beta]^2}{1+r^2}-\frac{L^{10}\,r^8\,\mathrm{Sin}[\alpha]^4\,\mathrm{Sin}[\beta]^2}{1+r^2}\right]}\right.$$

$$\phi r^{(0,0,0,0,1)}[x0,\,x1,\,x2,\,x3,\,r]+\left(L^2\left(\frac{2\,L^{10}\,r^7\,\mathrm{Sin}[\alpha]^4\,\mathrm{Sin}[\beta]^2}{\left(1+r^2\right)^2}+\right.\right.$$

$$\left.\frac{2\,L^{10}\,r^9\,\mathrm{Sin}[\alpha]^4\,\mathrm{Sin}[\beta]^2}{\left(1+r^2\right)^2}-\frac{6\,L^{10}\,r^5\,\mathrm{Sin}[\alpha]^4\,\mathrm{Sin}[\beta]^2}{1+r^2}-\frac{8\,L^{10}\,r^7\,\mathrm{Sin}[\alpha]^4\,\mathrm{Sin}[\beta]^2}{1+r^2}\right)$$

$$\left.\mathrm{Abs}'\left[-\frac{L^{10}\,r^6\,\mathrm{Sin}[\alpha]^4\,\mathrm{Sin}[\beta]^2}{1+r^2}-\frac{L^{10}\,r^8\,\mathrm{Sin}[\alpha]^4\,\mathrm{Sin}[\beta]^2}{1+r^2}\right]\phi r^{(0,0,0,0,1)}[x0,\,x1,\,x2,\,x3,\,r]\right)\Big/$$

$$\left(2\left(1+r^2\right)\sqrt{\mathrm{Abs}\left[-\frac{L^{10}\,r^6\,\mathrm{Sin}[\alpha]^4\,\mathrm{Sin}[\beta]^2}{1+r^2}-\frac{L^{10}\,r^8\,\mathrm{Sin}[\alpha]^4\,\mathrm{Sin}[\beta]^2}{1+r^2}\right]}\right)+\frac{1}{1+r^2}$$

$$\left.L^2\sqrt{\mathrm{Abs}\left[-\frac{L^{10}\,r^6\,\mathrm{Sin}[\alpha]^4\,\mathrm{Sin}[\beta]^2}{1+r^2}-\frac{L^{10}\,r^8\,\mathrm{Sin}[\alpha]^4\,\mathrm{Sin}[\beta]^2}{1+r^2}\right]}\,\phi r^{(0,0,0,0,2)}[x0,\,x1,\,x2,\,x3,\,r]\right)\Big/$$

$$\left(\sqrt{\mathrm{Abs}\left[-\frac{L^{10}\,r^6\,\mathrm{Sin}[\alpha]^4\,\mathrm{Sin}[\beta]^2}{1+r^2}-\frac{L^{10}\,r^8\,\mathrm{Sin}[\alpha]^4\,\mathrm{Sin}[\beta]^2}{1+r^2}\right]}\right)+L^2$$

$$r^2\,\mathrm{Sin}[\alpha]^2\,\mathrm{Sin}[\beta]^2\,\phi 3^{(0,0,0,2,0)}[x0,\,x1,\,x2,\,x3,\,r]+$$

$$L^2\,r^2\,\mathrm{Sin}[\alpha]^2\,\phi 2^{(0,0,2,0,0)}[x0,\,x1,\,x2,\,x3,\,r]+$$

$$L^2\,r^2\,\phi 1^{(0,2,0,0,0)}[x0,\,x1,\,x2,\,x3,\,r]-$$

$$L^2\left(1+r^2\right)\phi 0^{(2,0,0,0,0)}[x0,\,x1,\,x2,\,x3,\,r]$$

```
B (ϕAdS^2) + (2 λ) (ϕAdS^3) /. Dim → 4 /. m → 0 // FullSimplify
B (ϕAdS^2) + (2 λ) (ϕAdS^3) /. Dim → 4 /. m → (L^(-1)) // FullSimplify
```

$$\frac{e^{-12\,i\,x0}\left(B\,e^{4\,i\,x0}\left(1+r^2\right)^2+2\,\lambda\right)}{\left(1+r^2\right)^6}$$

$$e^{-3\,i\left(2+\sqrt{5}\right)x0}\left(1+r^2\right)^{-\frac{3}{2}\left(2+\sqrt{5}\right)}\left(B\,e^{i\left(2+\sqrt{5}\right)x0}\left(1+r^2\right)^{1+\frac{\sqrt{5}}{2}}+2\,\lambda\right)$$

```
FourLapAdS5Scalar[ϕAdS] - ((m^2) (ϕAdS) + B (ϕAdS^2) + (2 λ) (ϕAdS^3)) /. Dim → 4 /. m → 0 //
 FullSimplify
```

$$\frac{1}{\left(1+r^2\right)^6}e^{-12\,i\,x0}\left(-B\,e^{4\,i\,x0}\left(1+r^2\right)^2+16\,e^{8\,i\,x0}\,L^2\left(1+r^2\right)^5-2\,\lambda\right)$$

```
ϕ50r = {{ϕ0[x0]}, {0}, {0}, {0}, {ϕr[r]}};
```

```
Assuming[{L > 0, r > 0, α ∈ Reals, β ∈ Reals}, FourLapAdS5[ϕ50r] // FullSimplify]
Solve[FourLapAdS5[ϕ50r] - ((m^2) (ϕ50r) + B (ϕ50r^2) + (2 λ) (ϕ50r^3)) == 0, {ϕ0[x0]}] //
  FullSimplify // MatrixForm
```

$$\frac{1}{r\left(1+r^2\right)^2} L^2 \left(\left(-2 r^2+3\left(1+r^2\right) \mathrm{Sign}[\mathrm{Sin}[\alpha]]^4\right) \phi r'[r]+r\left(1+r^2\right)\left(-\left(1+r^2\right)^2 \phi 0''[x0]+\phi r''[r]\right)\right)$$

$$\begin{pmatrix} \phi 0[x0] \to 0 \\ \phi 0[x0] \to -\frac{B+\sqrt{B^2-8 m^2 \lambda}}{4 \lambda} \\ \phi 0[x0] \to \frac{-B+\sqrt{B^2-8 m^2 \lambda}}{4 \lambda} \end{pmatrix}$$

```
Assuming[{L > 0, r > 0, α ∈ Reals, β ∈ Reals}, FourLapAdS5[ϕ5] // FullSimplify]
Solve[FourLapAdS5[ϕ5] - ((m^2) (ϕ5) + B (ϕ5^2) + (2 λ) (ϕ5^3)) == 0, {ϕ0[x0]}] //
  FullSimplify // MatrixForm
```

$$\frac{1}{\left(1+r^2\right)^2} L^2 \left(\frac{1}{r}\left(-2 r^2+3\left(1+r^2\right) \mathrm{Sign}[\mathrm{Sin}[\alpha]]^4\right) \phi r^{(0,0,0,0,1)}[x0, x1, x2, x3, r]+\left(1+r^2\right)\left(\phi r^{(0,0,0,0,2)}[x0, x1, x2, x3, r]+\left(1+r^2\right)\left(r^2\left(\mathrm{Sin}[\alpha]^2\left(\mathrm{Sin}[\beta]^2 \phi 3^{(0,0,0,2,0)}[x0, x1, x2, x3, r]+\phi 2^{(0,0,2,0,0)}[x0, x1, x2, x3, r]\right)+\phi 1^{(0,2,0,0,0)}[x0, x1, x2, x3, r]\right)-\left(1+r^2\right) \phi 0^{(2,0,0,0,0)}[x0, x1, x2, x3, r]\right)\right)\right)$$

$$\left(\{\}\right)$$

```
Solve[FourLapAdS5[ϕ5] - ((m^2) (ϕ5) + B (ϕ5^2) + (2 λ) (ϕ5^3)) == 0,
   {ϕ0[x0, x1, x2, x3, r], ϕ1[x0, x1, x2, x3, r], ϕ2[x0, x1, x2, x3, r],
    ϕ3[x0, x1, x2, x3, r], ϕr[x0, x1, x2, x3, r]}] // FullSimplify // MatrixForm
```

Solve::ivar : {{ϕ0[x0, x1, x2, x3, r]}, {ϕ1[x0, x1, x2, x3, r]}, {ϕ2[x0, x1, x2, x3, r]}, {ϕ3[x0, x1, x2, x3, r]}, {ϕr[x0, x1, x2, x3, r]}} is not a valid variable. >>

```
$Aborted
```