

A.Tastes which mix

`Masses[f_, x_List?VectorQ] := D[f, {x, 2}]`

`msqrdΠ4new = Masses[Wa, Π0vec];`

`msqrdΠ4new // FullSimplify // MatrixForm`

$$\begin{pmatrix} (-1 + \cos A^2) \Delta A - 3 \delta p_{Am} + \cos A^2 (\Delta I + 3 \delta p_{Am} - \Delta V) + \Delta V + 2 \cos A m \mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (-1 + \cos A^2) \Delta A - 3 \delta p_{Am} + \Delta T + \cos A^2 \Delta I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

`msqrdΠ4new[[1, 1]]`

$$\begin{aligned} & 48 (1 - \cos A^2) \left(-\frac{\delta p_{Am}}{32} + \frac{\delta p_{Ap}}{32} \right) - \frac{3}{2} (1 - \cos A^2) (\delta p_{Am} + \delta p_{Ap}) + \\ & 64 \cos A^2 \left(\frac{\Delta A}{64} + \frac{\Delta I}{128} - \frac{\Delta V}{64} \right) - 32 (1 - \cos A^2) \left(\frac{\Delta A}{64} + \frac{\Delta I}{128} - \frac{\Delta V}{64} \right) + \\ & 64 \cos A^2 \left(-\frac{\Delta A}{64} + \frac{\Delta I}{128} + \frac{\Delta V}{64} \right) + 32 (1 - \cos A^2) \left(-\frac{\Delta A}{64} + \frac{\Delta I}{128} + \frac{\Delta V}{64} \right) + 2 \cos A m \mu \\ & 48 \sin A^2 \left(-\frac{\delta p_{Am}}{32} + \frac{\delta p_{Ap}}{32} \right) - \frac{3}{2} \sin A^2 (\delta p_{Am} + \delta p_{Ap}) + \\ & 64 \cos A^2 \left(\frac{\Delta A}{64} + \frac{\Delta I}{128} - \frac{\Delta V}{64} \right) - 32 \sin A^2 \left(\frac{\Delta A}{64} + \frac{\Delta I}{128} - \frac{\Delta V}{64} \right) + 64 \cos A^2 \left(-\frac{\Delta A}{64} + \frac{\Delta I}{128} + \frac{\Delta V}{64} \right) + \\ & 32 \sin A^2 \left(-\frac{\Delta A}{64} + \frac{\Delta I}{128} + \frac{\Delta V}{64} \right) + 2 \cos A m \mu // \text{FullSimplify} \\ & (-1 + \cos A^2) \Delta A - 3 \delta p_{Am} + \cos A^2 (\Delta I + 3 \delta p_{Am} - \Delta V) + \Delta V + 2 \cos A m \mu \end{aligned}$$

Eigenvalues[msqrdπ4new] // FullSimplify // MatrixForm

$$\begin{pmatrix} -3 \delta p_{Am} + \cos^2 A (\Delta A + 3 \delta p_{Am}) + 2 \cos A m \mu \\ -3 \delta p_{Am} + \cos^2 A (\Delta A + 3 \delta p_{Am}) + 2 \cos A m \mu \\ -3 \delta p_{Am} + \cos^2 A (\Delta A + 3 \delta p_{Am}) + 2 \cos A m \mu \\ (-1 + \cos^2 A) (\Delta A + 3 \delta p_{Am}) + 2 \cos A m \mu \\ (-1 + \cos^2 A) \Delta A + 3 (-1 + \cos^2 A) \delta p_{Am} + \Delta T + 2 \cos A m \mu \\ (-1 + \cos^2 A) \Delta A + 3 (-1 + \cos^2 A) \delta p_{Am} + \Delta T + 2 \cos A m \mu \\ (-1 + \cos^2 A) \Delta A + 3 (-1 + \cos^2 A) \delta p_{Am} + \Delta T + 2 \cos A m \mu \\ 3 (-1 + \cos^2 A) \delta p_{Am} + \cos A (\cos A \Delta T + 2 m \mu) \\ 3 (-1 + \cos^2 A) \delta p_{Am} + \cos A (\cos A \Delta T + 2 m \mu) \\ 3 (-1 + \cos^2 A) \delta p_{Am} + \cos A (\cos A \Delta T + 2 m \mu) \\ (-1 + \cos^2 A) \Delta A - 3 \delta p_{Am} + \cos^2 A (\Delta I + 3 \delta p_{Am} - \Delta V) + \Delta V + 2 \cos A m \mu \\ (-1 + \cos^2 A) \Delta A + 3 (-1 + \cos^2 A) \delta p_{Am} + \Delta V + 2 \cos A m \mu \\ (-1 + 2 \cos^2 A) \Delta A + 3 (-1 + \cos^2 A) \delta p_{Am} + 2 \cos A m \mu \\ (-1 + \cos^2 A) \Delta A - 3 \delta p_{Am} + \Delta T + \cos A (\cos A (3 \delta p_{Am} - \Delta T + \Delta V) + 2 m \mu) \\ (-1 + \cos^2 A) \Delta A - 3 \delta p_{Am} + \Delta T + \cos A (\cos A (3 \delta p_{Am} - \Delta T + \Delta V) + 2 m \mu) \\ (-1 + \cos^2 A) \Delta A - 3 \delta p_{Am} + \Delta T + \cos A (\cos A (3 \delta p_{Am} - \Delta T + \Delta V) + 2 m \mu) \end{pmatrix}$$

$$\sin A = \sqrt{1 - \cos^2 A};$$

Eigenvalues[msqrdπ4new] // FullSimplify // MatrixForm;

Assuming[CosA == 1, Eigenvalues[msqrdπ4new] // FullSimplify // MatrixForm]

$$\begin{pmatrix} \Delta A + 2 m \mu \\ \Delta A + 2 m \mu \\ \Delta A + 2 m \mu \\ \Delta A + 2 m \mu \\ 2 m \mu \\ \Delta T + 2 m \mu \\ \Delta T + 2 m \mu \\ \Delta T + 2 m \mu \\ \Delta T + 2 m \mu \\ \Delta T + 2 m \mu \\ \Delta T + 2 m \mu \\ \Delta I + 2 m \mu \\ \Delta V + 2 m \mu \\ \Delta V + 2 m \mu \\ \Delta V + 2 m \mu \\ \Delta V + 2 m \mu \end{pmatrix}$$

Assuming[CosA == 0, Eigenvalues[msqrdΠ4new] // FullSimplify // MatrixForm]

$$\begin{pmatrix} -3 \delta pAm \\ -3 \delta pAm \\ -3 \delta pAm \\ -3 \delta pAm \\ -3 \delta pAm \\ -3 \delta pAm \\ -\Delta A - 3 \delta pAm \\ -\Delta A - 3 \delta pAm \\ -\Delta A - 3 \delta pAm + \Delta T \\ -\Delta A - 3 \delta pAm + \Delta T \\ -\Delta A - 3 \delta pAm + \Delta T \\ -\Delta A - 3 \delta pAm + \Delta T \\ -\Delta A - 3 \delta pAm + \Delta T \\ -\Delta A - 3 \delta pAm + \Delta T \\ -\Delta A - 3 \delta pAm + \Delta V \\ -\Delta A - 3 \delta pAm + \Delta V \end{pmatrix}$$

$$\begin{pmatrix} -3 \delta pAm \\ -3 \delta pAm \\ -3 \delta pAm \\ -3 \delta pAm \\ -3 \delta pAm \\ -3 \delta pAm \\ -\Delta A - 3 \delta pAm \\ -\Delta A - 3 \delta pAm \\ -\Delta A - 3 \delta pAm + \Delta T \\ -\Delta A - 3 \delta pAm + \Delta T \\ -\Delta A - 3 \delta pAm + \Delta T \\ -\Delta A - 3 \delta pAm + \Delta T \\ -\Delta A - 3 \delta pAm + \Delta T \\ -\Delta A - 3 \delta pAm + \Delta T \\ -\Delta A - 3 \delta pAm + \Delta V \\ -\Delta A - 3 \delta pAm + \Delta V \end{pmatrix}$$

{-3 δpAm, -3 δpAm, -3 δpAm, -3 δpAm, -3 δpAm, -3 δpAm, -ΔA - 3 δpAm,
-ΔA - 3 δpAm, -ΔA - 3 δpAm + ΔT, -ΔA - 3 δpAm + ΔT, -ΔA - 3 δpAm + ΔT, -ΔA - 3 δpAm + ΔT,
-ΔA - 3 δpAm + ΔT, -ΔA - 3 δpAm + ΔT, -ΔA - 3 δpAm + ΔV, -ΔA - 3 δpAm + ΔV}

mvecnew = {-3 δpAm, -3 δpAm, -3 δpAm, -3 δpAm, -3 δpAm, -3 δpAm, -ΔA - 3 δpAm,
-ΔA - 3 δpAm, -ΔA - 3 δpAm + ΔT, -ΔA - 3 δpAm + ΔT, -ΔA - 3 δpAm + ΔT, -ΔA - 3 δpAm + ΔT,
-ΔA - 3 δpAm + ΔT, -ΔA - 3 δpAm + ΔT, -ΔA - 3 δpAm + ΔV, -ΔA - 3 δpAm + ΔV};

Disregarding the factors of 4 in the paper, the meson masses are:

msqrdΠ4 = mvecnew[[15]] (*=mvecnew[[16]]*) (*=msqrdetap4?*)

-ΔA - 3 δpAm + ΔV

msqrdΠ5 = mvecnew[[7]] (*=mvecnew[[8]]*)

-ΔA - 3 δpAm

msqrdΠi4 = mvecnew[[9]] (*also 10 to 14*)

-ΔA - 3 δpAm + ΔT

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msqrd $\pi$ i5 = mvecnew[[1]] (*also 2 to 5*)
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- 3  $\delta$ pAm
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msqrdetap5 = msqrd $\pi$ 5;
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```
msqrdetapi4 = msqrd $\pi$ 4;
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Masses[WwA,  $\eta$ pvec] // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Which would give the η 'i5 goldstone bosons

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msqrdetapi5 = 0
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0
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