

Program Description - Midterm Project

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There are 4 distinct files for this project. There are a few things they all have in common:

1. They all only request two inputs from the user, s and q . The purpose of these inputs is shown in the paper. For the purposes of this project s and q are assumed to be real numbers, and the zeta function is only convergent for $\text{Re}(s) > 1, \text{Re}(q) > 0$, so only inputs meeting these criteria will get useful outputs.
2. Note that for all the files except HurwitzZeta.cpp, infinities will appear at integer values. This is due to the nature of the functions, not an error in the code. nan outputs will also appear, these are due to the nature of the functions as well.
3. All the functions have set tolerance 1e-16. All functions sum up to NMAX and/or up from -NMAX. NMAX = 1e6 and -NMAX = -1e-6. All these summations are intended to be approximations of infinite sums, so NMAX is fixed to reflect that.

Note: all of the Zeta files below can be run simultaneously by running the AllZetas.cpp file, and the output name of each will correspond to its file of origin. The files are:

- GenZeta.cpp - This file is the most general of the programs. It calculates the sum $\zeta_\beta(s, q) = \sum_{a=-\infty}^{\infty} \sum_{b=-\infty}^{\infty} (a + bi + q)^{-s}$ and thus makes use of complex numbers via the <complex> environment. This file also outputs terms called “PosGenZeta” which gives the summation from $a = 0$ to $a = \infty$ and $b = 0$ to $b = \infty$; and “NegGenZeta” which gives the summation from $a = -\infty$ to $a = -1$ and $b = -\infty$ to $b = -1$ so that $\text{PosGenZeta}(s, q) + \text{NegGenZeta}(s, q) = \text{GenZeta}(s, q)$.
- RealZeta.cpp - This is the second most general program, approximating the equation $\sum_{a=-\infty}^{\infty} (a + q)^{-s}$ summing over all real integers. This program is basically the sum of the sums given by NegZeta.cpp and HurwitzZeta.cpp.
- NegZeta.cpp - This approximates the sum $\sum_{a=-\infty}^{-1} (a + q)^{-s}$, so it's basically the Hurwitz Zeta function but over negative integers. This is of note because it's the most easily calculated by other mathematics software (besides the Hurwitz Zeta function). For instance, for $s = 2$ and $q = 0.5$, the sum is $\sum_{b=-1000000}^{-1} (b + 0.5)^{-2}$, which can be verified by software such as mathematica.
- HurwitzZeta.cpp - This calculates the Hurwitz Zeta function $\zeta(s, q) = \sum_{a=0}^{\infty} \frac{1}{(a+q)^s}$ and is the only one of the functions that I didn't define myself. In mathematica it is calculable by inputting $\zeta[s, q]$.
- AllZetas.cpp - Runs all the above simultaneously