Theory & Implementation of Grover's Algorithm in IBM's QISkit OpenQASM 2.0 Representation

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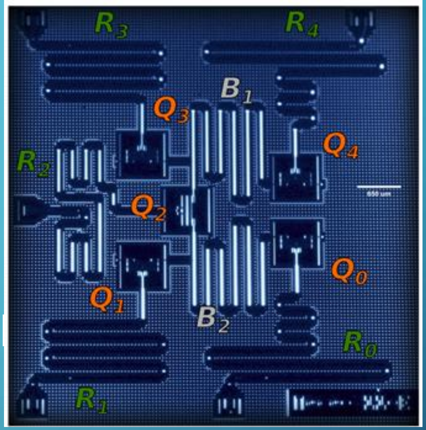
OUTLINE

- Basics of Software/Hardware
  - Backend
  - Basic examples of circuits on IBMQX4
- Grover's Algorithm:
  - The Problem
  - Conceptual basics
- Grover's Algorithm:
  - Implementation on IBMQX
  - Results

IBMQX4 CHIP



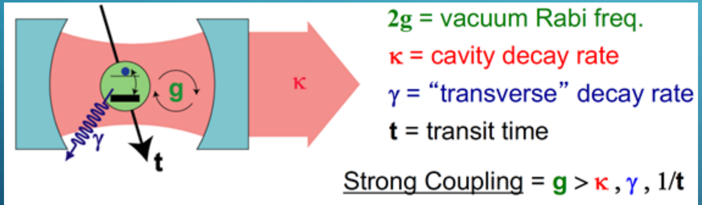
Parameters of the chip

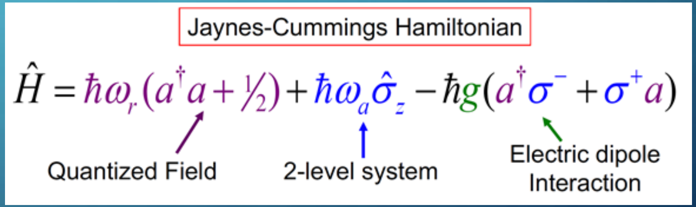
- 5 qubit chip
- ωR_i = Resonant frequency of readout resonator
- ω_i = qubit frequency
- δ_i = anharmonicity, difference b/w freq of state 1→ 2 transmission and freq of state 0→1 transition
- χ = qubit-cavity coupling strength

Qubit	$\omega^R_i/2\pi$ (GHz)	$\omega_i/2\pi$ (GHz)	$\delta_i/2\pi$ (MHz)	$\chi/2\pi$ (kHz)
Q0	6.52396	5.2461	-330.1	410
Q1	6.48078	5.3025	-329.7	512
Q2	6.43875	5.3562	-323.0	408
Q3	6.58036	5.4317	-327.9	434
Q4	6.52698	5.1824	-332.5	458

PHYSICS OF QUBIT-CAVITY SYSTEM

- Described by cavity quantum electrodynamics (cQED)
- Vacuum particles disturb the qubit → don't have perfect coherence
- Constitutes a *Hybrid Quantum System* → Dissipates into environment



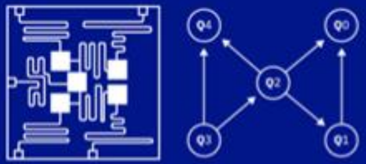


Qubit Cavity

PHYSICS OF QUBIT-CAVITY SYSTEM

- Keep at low temperature T = 0.021 K to help preserve coherence

Backend: ibmqx4 (5 Qubits)



Last Calibration: 2018-04-17 07:05:15 Fridge Temperature: 0.021 K

	Q0	Q1	Q2	Q3	Q4
Gate Error (10^{-3})	0.94	0.69	1.97	3.69	1.97
Readout Error (10^{-2})	4.80	3.50	5.50	32.60	8.30
MultiQubit Gate Error (10^{-2})		CX1_0 2.09	CX2_0 3.09	CX3_2 7.90	
			CX2_1 3.11	CX3_4 8.38	

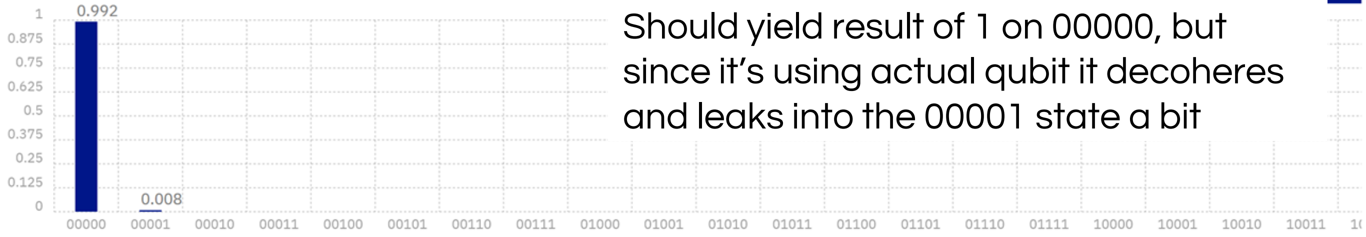
(Retrived April 7th appx 15:00)

SIMPLE EXAMPLES OF IMPLEMENTATION

Device: ibmqx4

Quantum State: Computation Basis

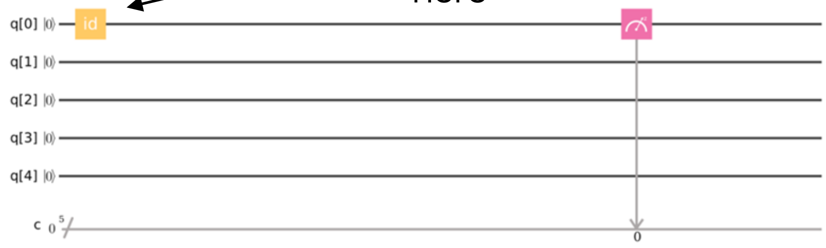
Download CSV



Should yield result of 1 on 00000, but since it's using actual qubit it decoheres and leaks into the 00001 state a bit

Quantum Circuit

No entanglement here

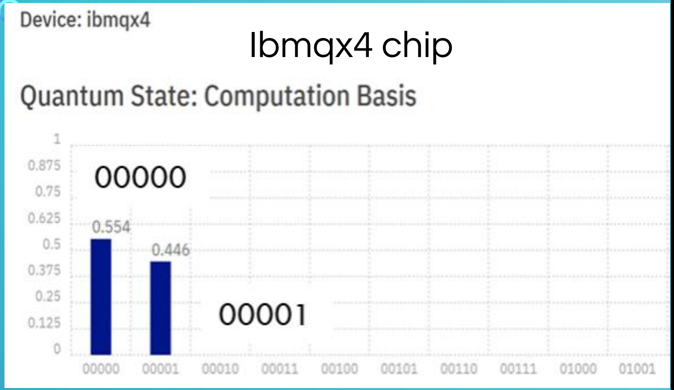


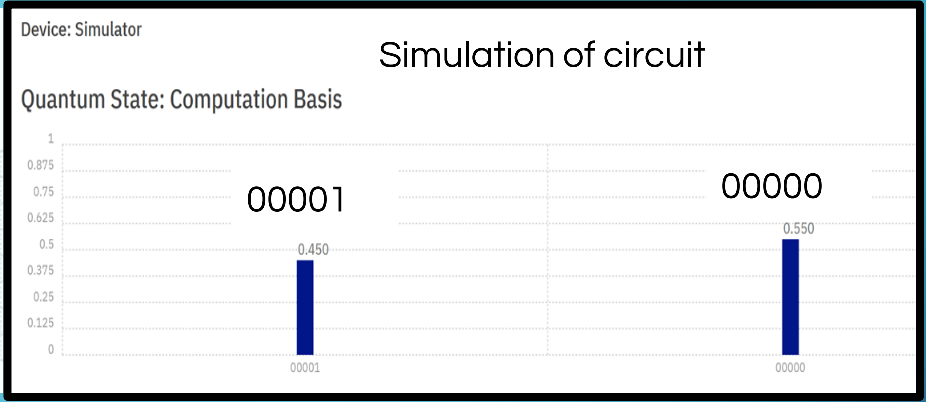
OPENQASM 2.0

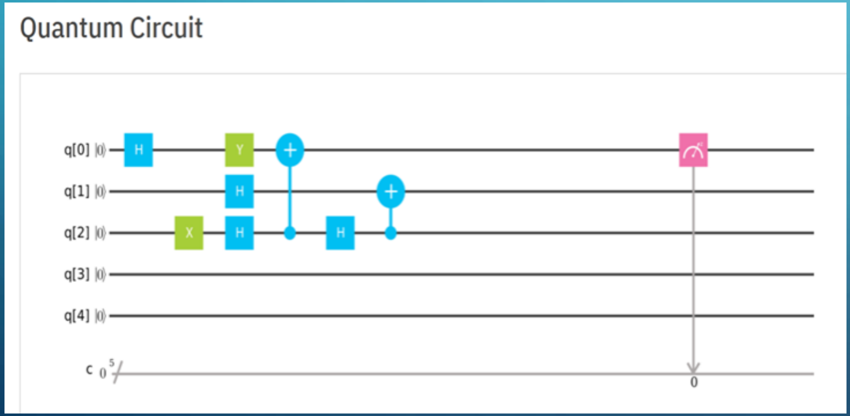
```
include "qelib1.inc";

qreg q[5];
creg c[5];

id q[0];
measure q[0] -> c[0];

```

Open in Composer

Edit in QASM Editor







OPENQASM 2.0

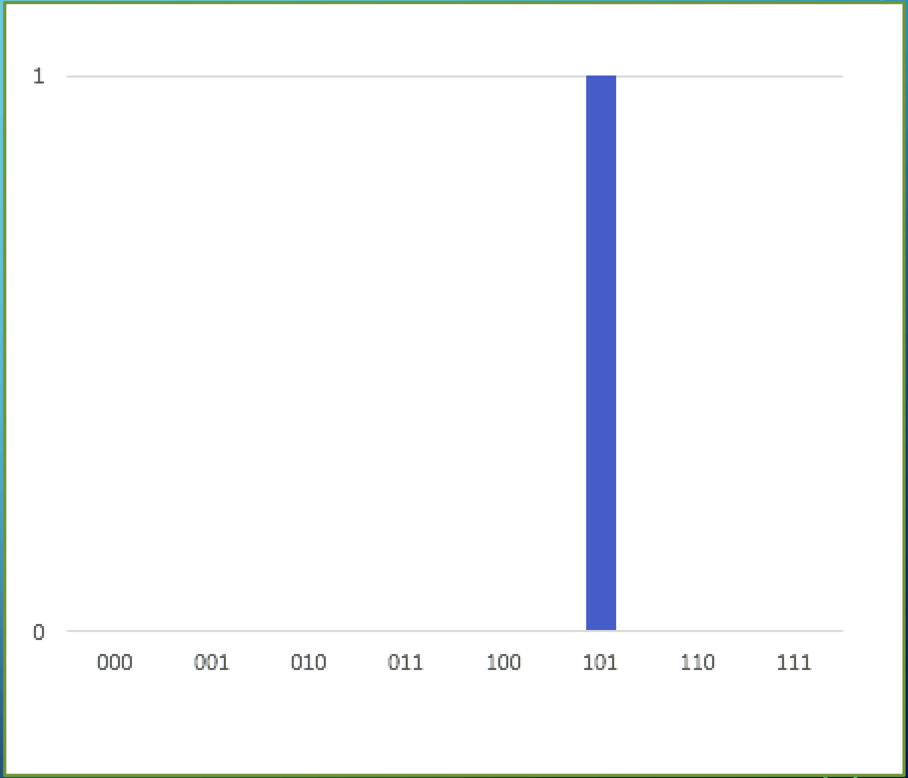
```
include "qelib1.inc";

qreg q[5];
creg c[5];

h q[0];
x q[2];
y q[0];
h q[1];
h q[2];
cx q[2],q[0];
h q[2];
cx q[2],q[1];
measure q[0] -> c[0];

```

GROVER'S ALGORITHM QISKIT OPENQASM 2.0 REPRESENTATION

THE PROBLEM

Given a Boolean black box $f : \{0, 1, \ldots, 2^n - 1\} \rightarrow \{0, 1\}$ that indicates whether a given item is marked.

The problem is to find the marked item given that one exists.



GROVER'S ALGORITHM

Starts by preparing the superposition of all bases using Hadamard gates.

$$|s\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle$$

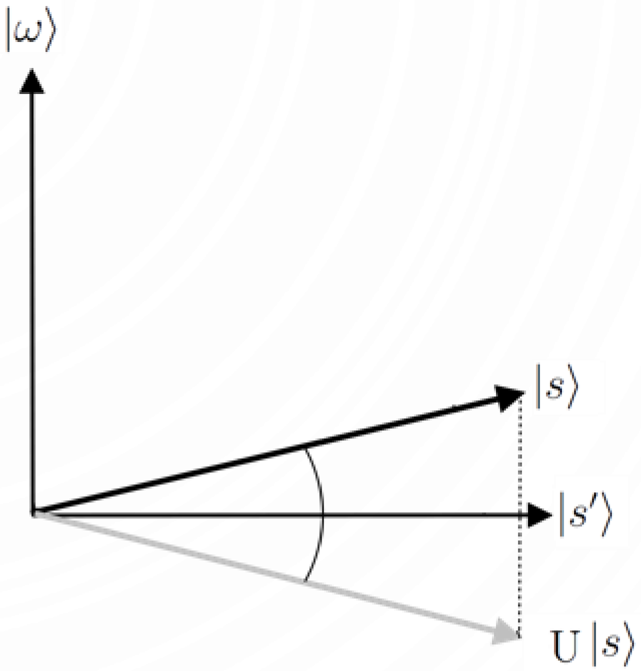
Repeat sqrt(N) times:

Apply, f, the black box operation.

Apply reflection across the average.

GEOMETRICALLY

The black box (phase oracle) reflects the current state around the uniform superposition of all unmarked bases.

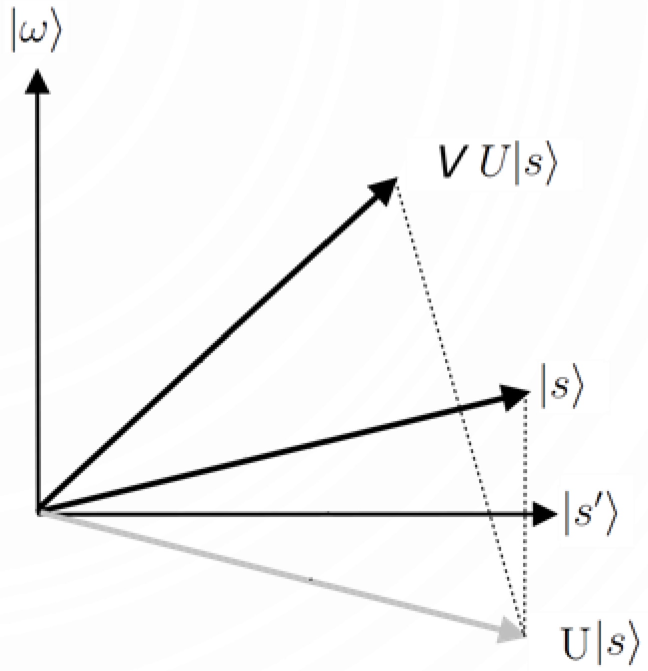


GROVER'S DIFFUSION OPERATION

- After application of the oracle, the state can be pushed towards ω by a reflection around $|s\rangle$.

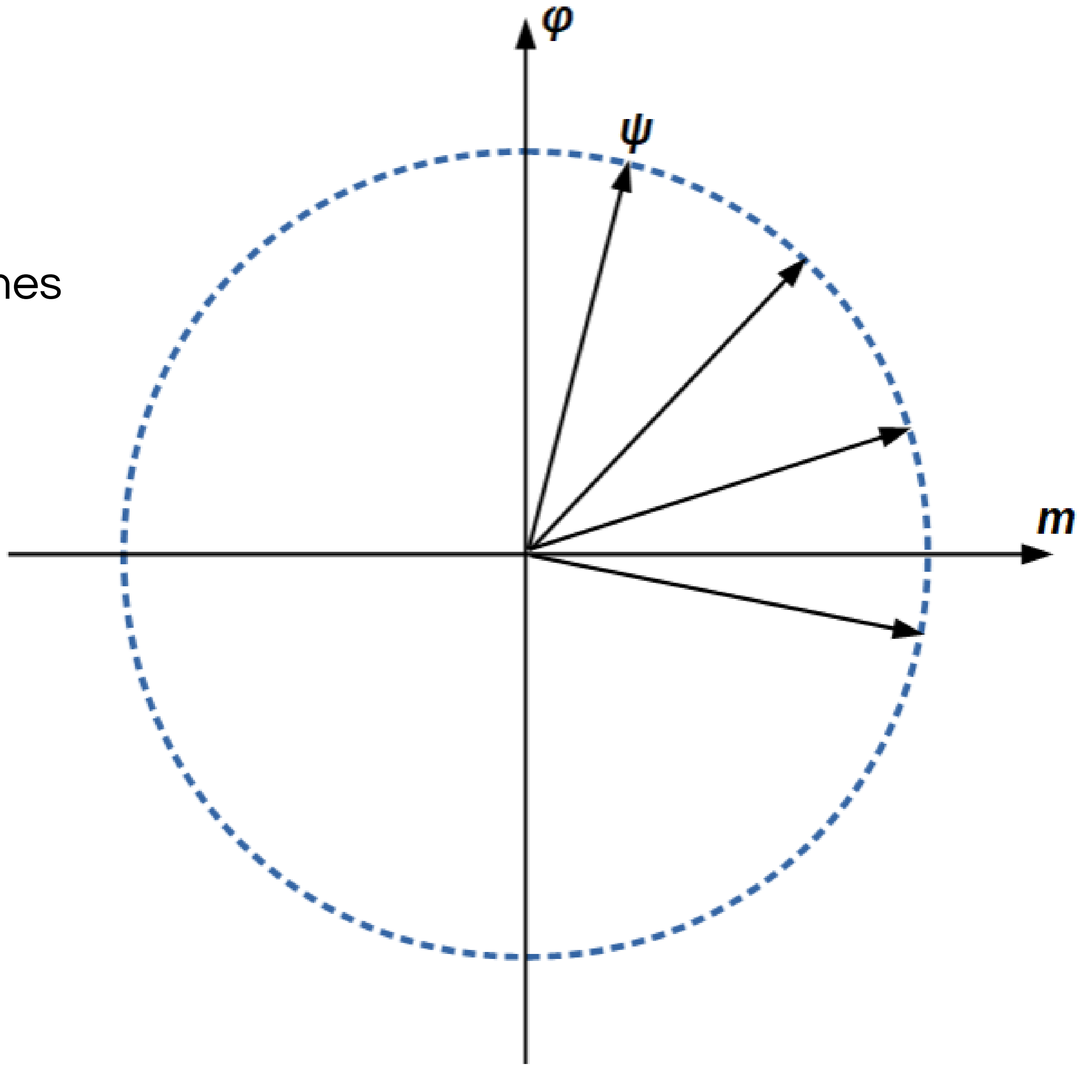
- The operation for this reflection is:

$$V = I - 2|s\rangle\langle s|$$
$$= H\,(I - 2|0\rangle\langle 0|)\,H$$

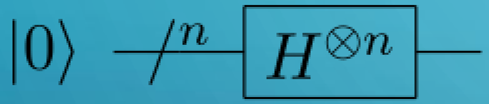


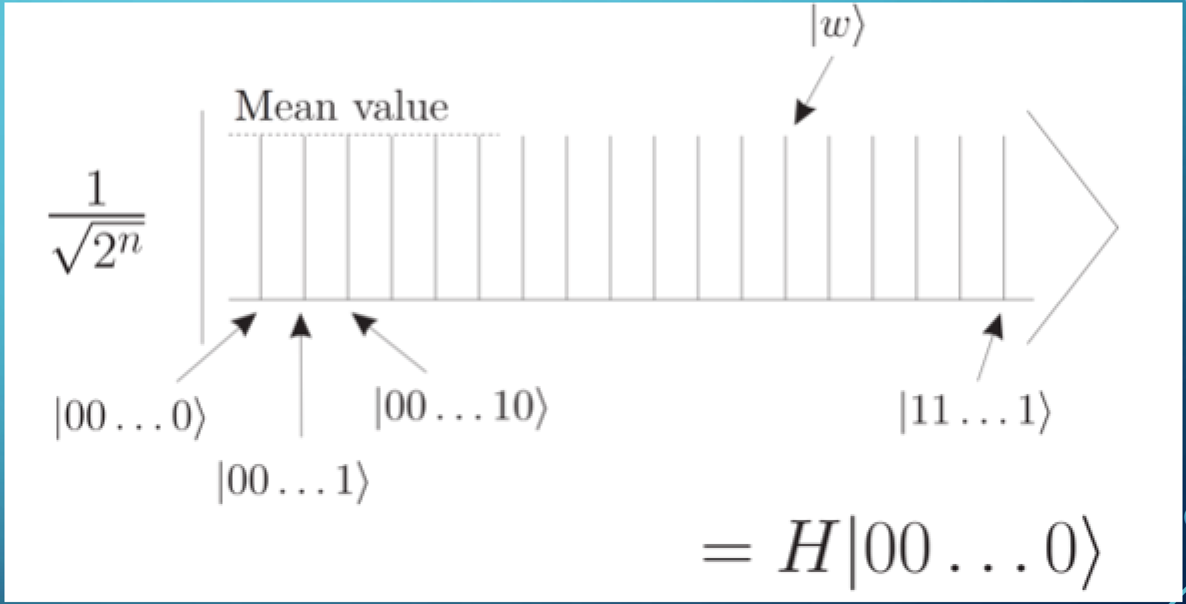
N = 16

- The process can be repeated 3 times



SUPERPOSITION OF ALL BASES





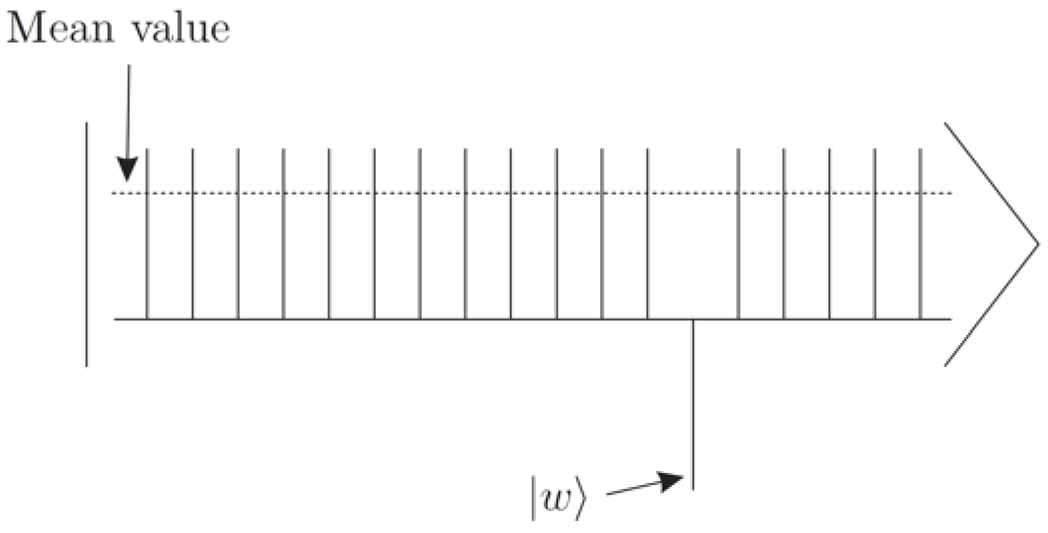
PHASE ORACLE

f can be written as a reversible operation as:

$$U|x\rangle = (-1)^{f(x)}|x\rangle = \begin{cases} |x\rangle & x \text{ is not marked} \\ -|x\rangle & x \text{ is marked} \end{cases}$$

FIRST OPERATION / REFLECTION

$$|x\rangle \longmapsto (-1)^{f(x)}|x\rangle$$

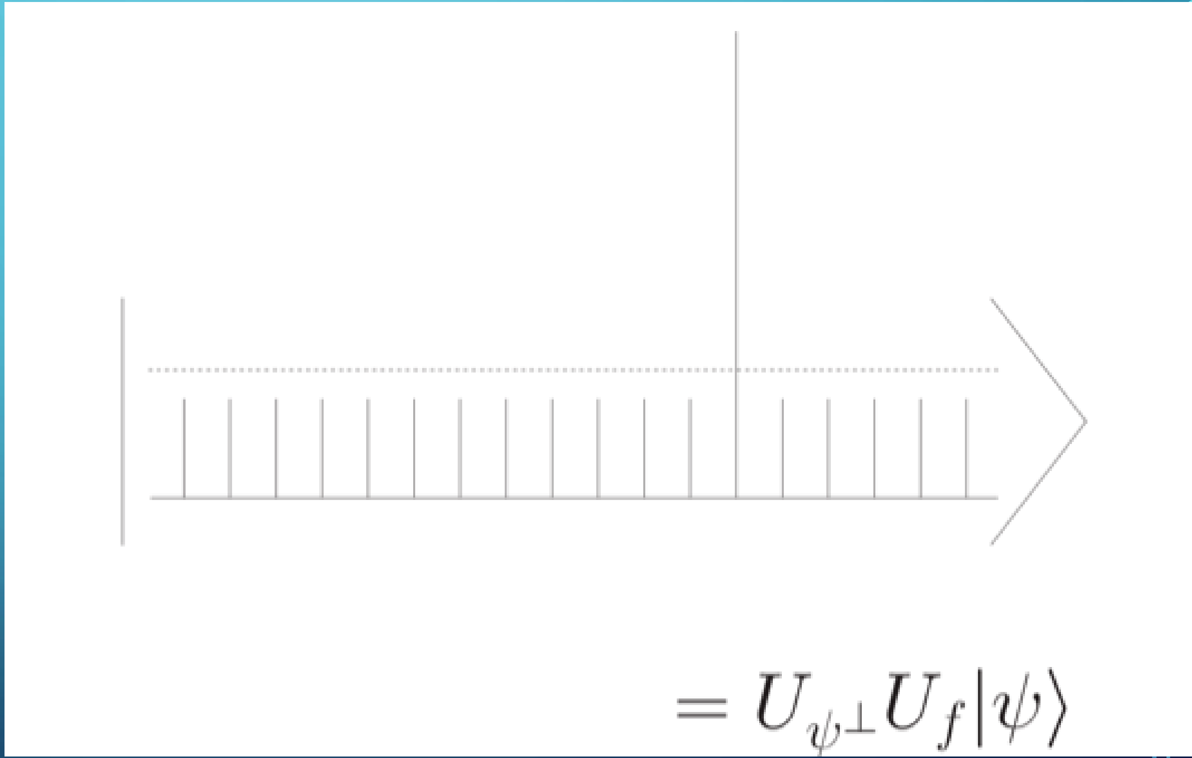


$$= U_f|\psi\rangle$$

REFLECTION ACROSS THE AVERAGE

After 2nd reflection:

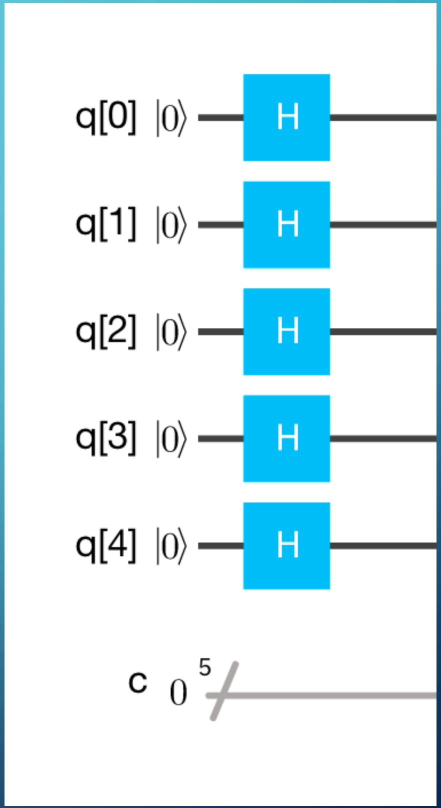
$V = I - 2|s\rangle\langle s|$



IMPLEMENTATION

Prepare the state of superposition of all bases using Hadamard gates.

```python
for i in range(N):
    circuit.h(q_register[i])
```



IMPLEMENTATION / REFLECTION ABOUT $|000\ldots0\rangle$

We need operator:

$$I - 2|0\rangle\langle 0|$$

that maps:

$$|0\rangle \longmapsto -|0\rangle$$
$$|x\rangle \longmapsto |x\rangle$$

Pauli Z gate:

$$|0\rangle \longmapsto |0\rangle$$
$$|1\rangle \longmapsto -|1\rangle$$

Controlled Z:

$$|00\rangle \longmapsto |00\rangle$$
$$|01\rangle \longmapsto |01\rangle$$
$$|10\rangle \longmapsto |10\rangle$$
$$|11\rangle \longmapsto -|11\rangle$$

IMPLEMENTATION / REFLECTION USING CONTROLLED Z

NOT Controlled Z NOT:

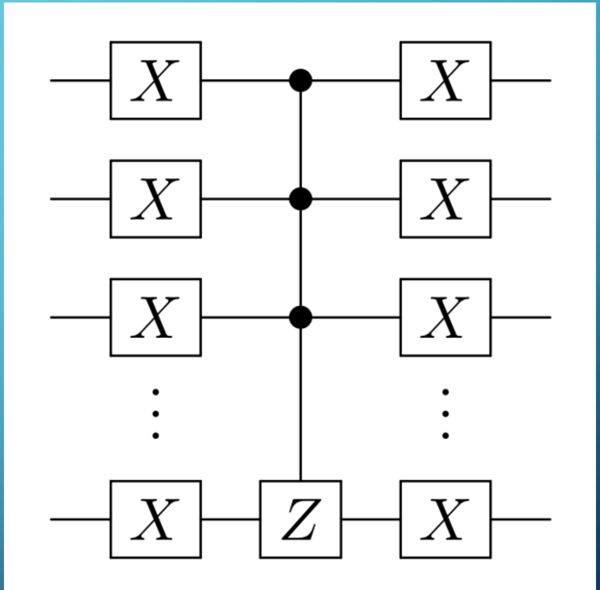
$|00\rangle \longmapsto -|00\rangle$

$|01\rangle \longmapsto |01\rangle$

$|10\rangle \longmapsto |10\rangle$

$|11\rangle \longmapsto |11\rangle$

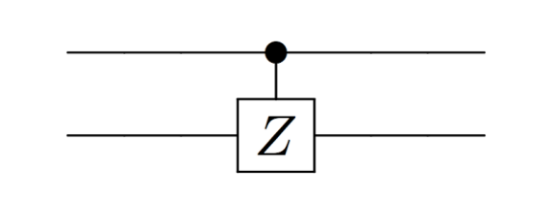
$\mathrm{NOT}^{\otimes n} \quad C^{n-1}Z \quad \mathrm{NOT}^{\otimes n}$



Giacomo Nannicini, “An Introduction to Quantum Computing, Without the Physics”, 2018

IMPLEMENTATION / CONTROLLED Z USING CNOT

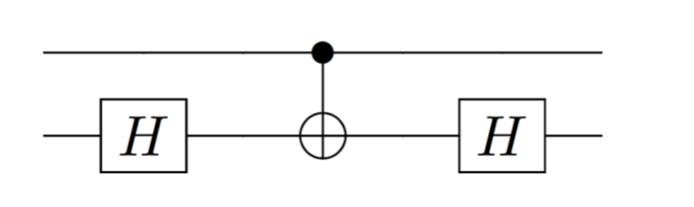
There is no Controlled Z operator in QISKit.



But Z operator is equal to:

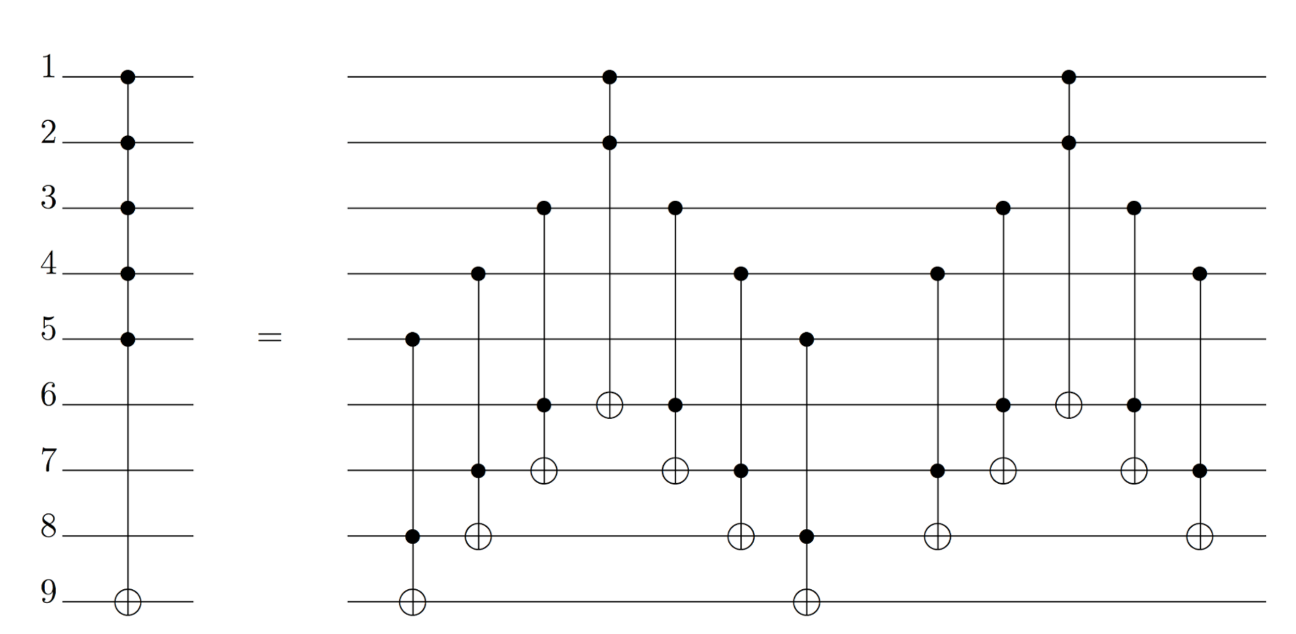
$Z = H \quad NOT \quad H$

Therefore Controlled Z can be written as:



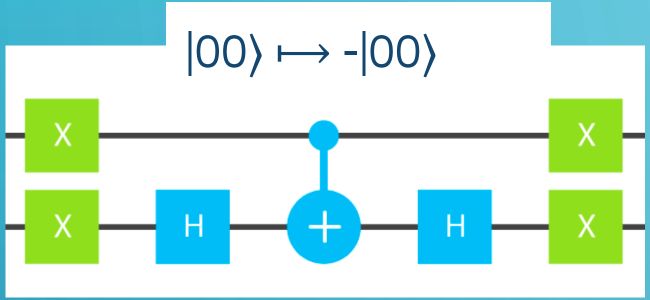
$CZ = (I \otimes H) \quad CNOT \quad (I \otimes H)$

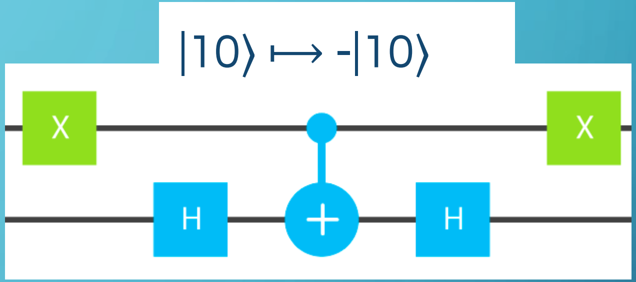
IMPLEMENTATION / N CONTROLLED NOT

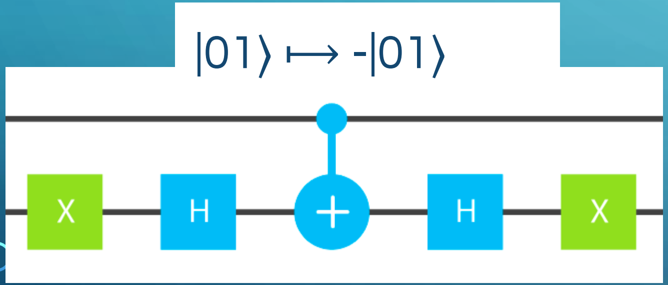


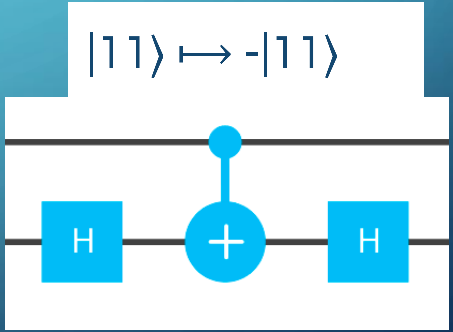
Adriano Barenco et. al, “Elementary gates for quantum computation", Quantum Physics, 1995, Lemma 7.2.

IMPLEMENTATION / PHASE ORACLE









IMPLEMENTATION / REFLECTION ACROSS AVERAGE

So:

$I - 2|s\rangle\langle s|$

can be implemented as:

$H^{\otimes n}\ NOT^{\otimes n}\ CZ\ \ NOT^{\otimes n}\ H^{\otimes n}$

For 2 qbits: