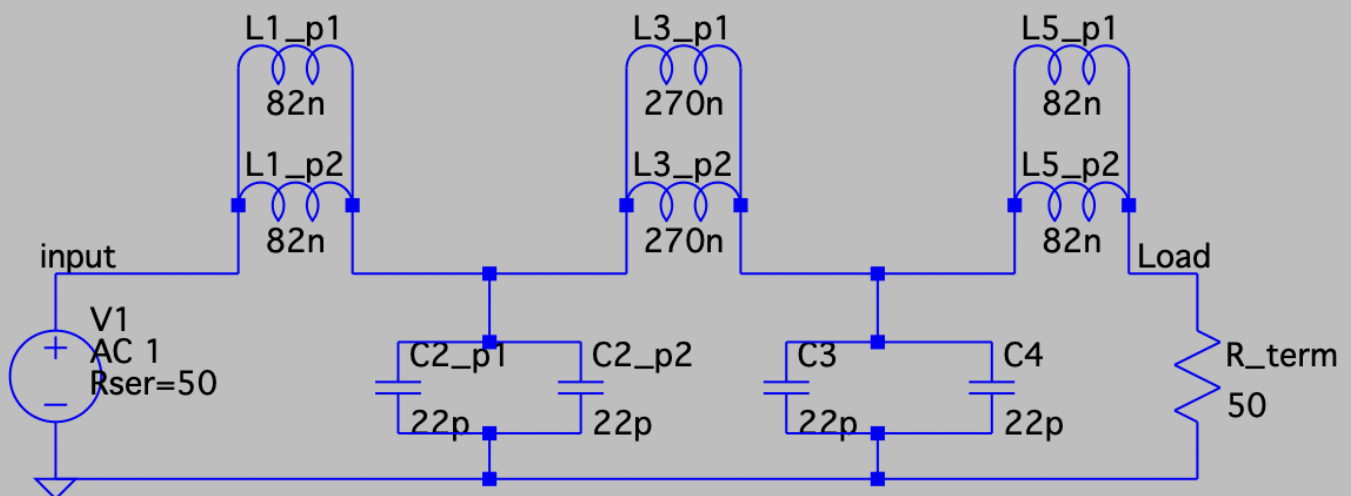


## 1) Table

Parameter	Analytical	Simulated w/ ideal components	Simulated w/ real components	Measured
Filter type (Butterworth, Chebyshev I, etc.)	<i>Butterworth</i>	NA	NA	NA
Filter order	<i>5</i>	NA	NA	NA
Pass Band Edge (defined as exceeding 1dB ripple)	<i>100MHz</i>	<i>100MHz</i>	<i>101.6MHz</i>	<i>63MHz</i>
Stop Band Start (defined @20dB of rejection)	<i>181.2</i>	<i>181.1MHz</i>	<i>185.4MHz</i>	<i>200MHz</i>
Insertion Loss	<i>0dB</i>	<i>0dB</i>	<i>0dB</i>	<i>1.2dB</i>
In-Band Ripple	<i>-1dB</i>	<i>-1dB</i>	<i>-1dB</i>	<i>-1dB</i>

## 2) Pictures and Schematics

### LTSpice Simulations - Real Components

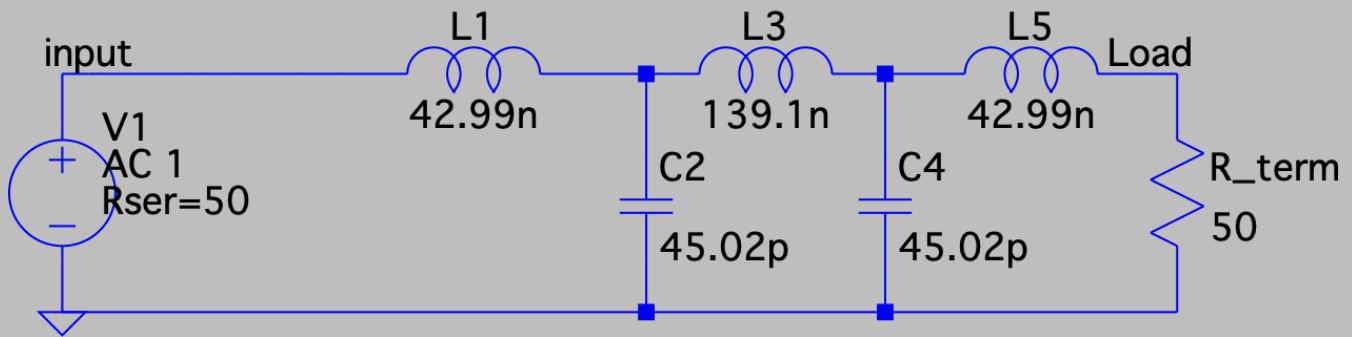


```
.ac lin 1000 70Meg 200Meg
```

```
.net I(R_term) V1
```

# LTSpice Simulations - Ideal Components

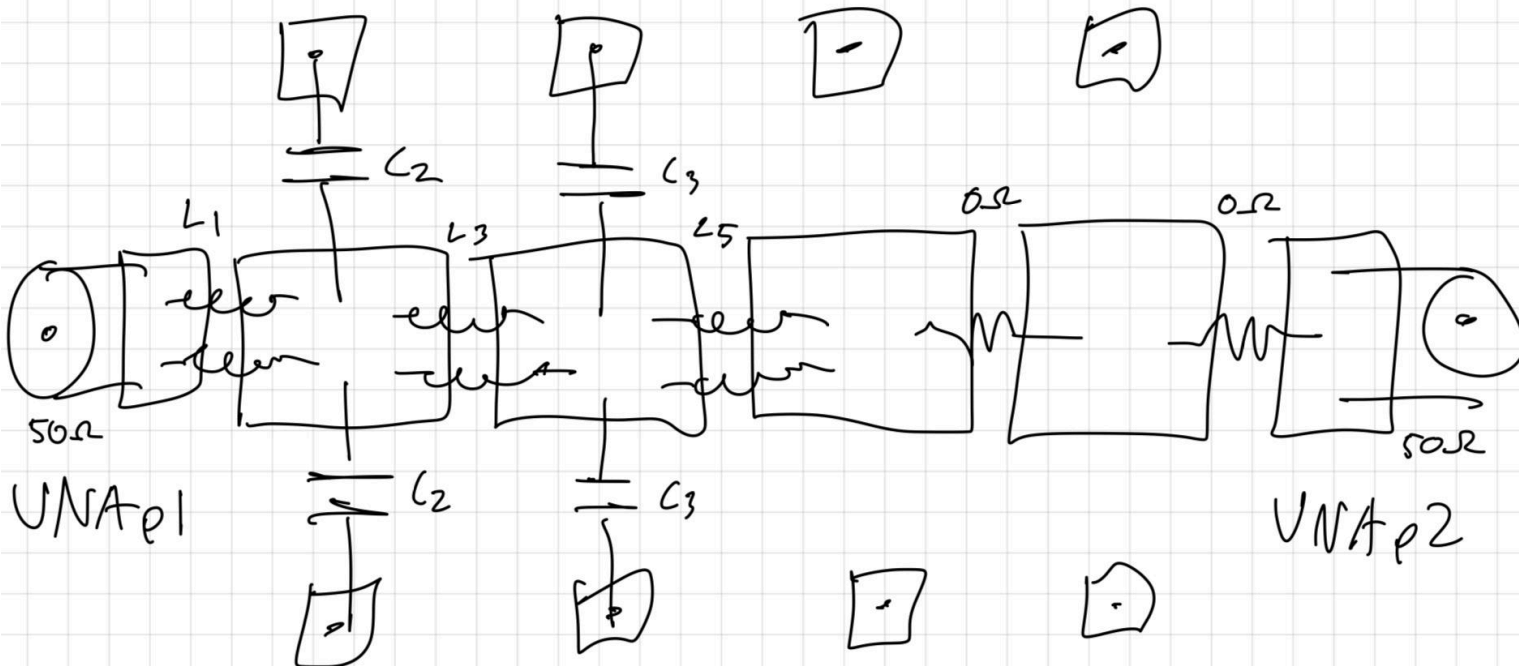
## Ideal Components



```
.ac lin 1000 70Meg 200Meg
```

```
.net I(R_term) V1
```

## Plan for PCB



## Assembled Filter Board



### 3) Hand Calculations

Solve for order of Butterworth Low-pass filter

$$|H(j\omega)|^2 = \frac{1}{1 + \varepsilon^2 F_n^2(\frac{\omega}{\omega_p})}$$

$$F_n = \left(\frac{\omega}{\omega_p}\right)^n$$

$$\varepsilon = \sqrt{10^{0.18} - 1}, \quad \delta = 1 \text{ dB}$$

$$\varepsilon = \sqrt{10^{0.1} - 1} \approx 0.50847 \text{ dB}$$

$$n = \frac{\ln\left(\frac{A_s}{\varepsilon}\right)}{\ln\left(\frac{\omega_s}{\omega_p}\right)}, \quad \begin{aligned} A_s &\Rightarrow | -20 | = 20 \log(A_s) \quad A_s = 10 \\ \omega_s &= 200 \text{ MHz} \cdot 2\pi \\ \omega_p &= 100 \text{ MHz} \cdot 2\pi \end{aligned}$$

$$n = \ln(10/0.50847) / \ln(2) \approx 4.2977$$

n must be an integer so round 4.2977 up to 5

$$|H(j\omega)|^2 = \frac{1}{1 + \varepsilon^2 F_n^2(\frac{\omega}{\omega_p})}$$

$$|H(j\omega)|^2 = \frac{1}{1 + (10^{0.1} - 1) \left(\frac{\omega}{100 \text{ MHz}}\right)^{2 \cdot 5}}$$

$$10 \log(|H(j\omega)|^2) = -3 \text{ dB}$$

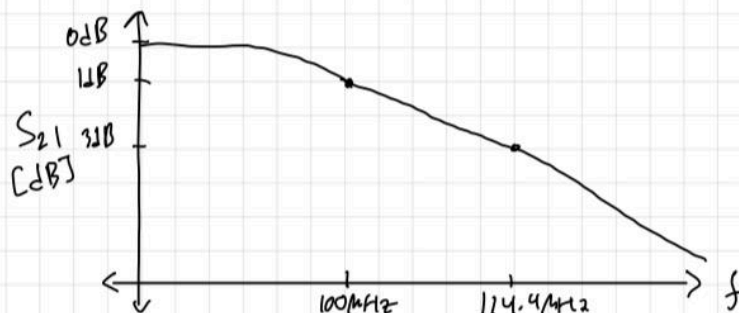
$$10^{-0.3} = \frac{1}{1 + (10^{0.1} - 1) \left(\frac{\omega}{100 \text{ MHz}}\right)^{10}}$$

$$1 + (10^{0.1} - 1) \left(\frac{\omega}{10^8}\right)^{10} = 10^{0.3}$$

$$\frac{\omega}{10^8} = \left(10^{0.3} - 1\right)^{1/10} / (10^{0.1} - 1)$$

$$\omega = \left(\frac{10^{0.3} - 1}{10^{0.1} - 1}\right)^{1/10} \cdot 10^8$$

$$\omega_c = 1.144 \times 10^8 \text{ of } 114.4 \text{ MHz}$$

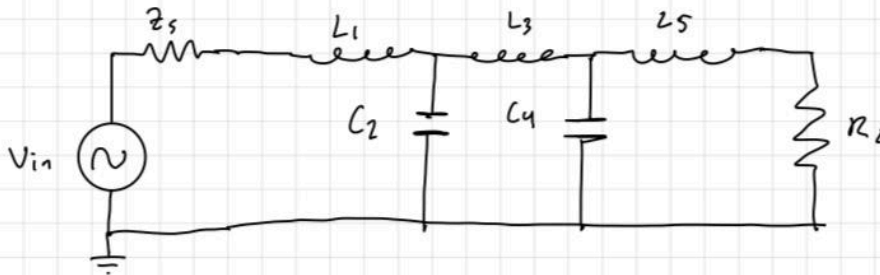




low pass butterworth filter table found on rfcafe.com

Capacitor Input, $R_0=R_L=1\ \Omega$ , $\omega=1\ \text{rad/sec}$										
Order	C1	L2	C3	L4	C5	L6	C7	L8	C9	L10
1	2.000									
2	1.41421	1.41421								
3	1.00000	2.00000	1.00000							
4	0.76537	1.84776	1.84776	0.76537						
5	0.61803	1.61803	2.00000	1.61803	0.61803					
6	0.51764	1.41421	1.93185	1.93185	1.41421	0.51764				
7	0.44504	1.24698	1.80194	2.00000	1.80194	1.24698	0.44504			
8	0.39018	1.11114	1.66294	1.96157	1.96157	1.66294	1.11114	0.39018		
9	0.34730	1.00000	1.53209	1.87938	2.00000	1.87938	1.53209	1.00000	0.34730	
10	0.31287	0.90798	1.41421	1.78201	1.97538	1.97538	1.78201	1.41421	0.90798	0.31287
Inductor Input, $R_0=R_L=1\ \Omega$ , $\omega=1\ \text{rad/sec}$										
	L1	C2	L3	C4	L5	C6	L7	C8	L9	C10

- I will use a series first filter because it matches the layout of the filter board PCBs. This way we will not need to use an extra 0 ohm resistor in our final design.



$$\Rightarrow Z_s = Z_0 = Z_L = 50\ \Omega$$

$$V_{in} = 1\text{V} \quad 70\text{MHz} \rightarrow 130\text{MHz}$$

$$\omega_p L = Z_0 [1\text{rad/sec}] \cdot L_{norm}$$

$$L = \frac{Z_0}{\omega_c} \cdot L_{norm}$$

$$\frac{1}{\omega_p C} = \frac{Z_0}{[1\text{rad/sec}] \cdot C_{norm}}$$

$$C = C_{norm} / \omega_c Z_0$$

$$\omega_c = 2\pi \cdot 114.4 \times 10^6$$

$L_{1,norm} = 0.61803$	$L_1 = \frac{50}{\omega_c} \cdot 0.61803 = 4.29906 \times 10^{-8}$ or <b>42.99 nH</b>
$C_{2,norm} = 1.61803$	$C_2 = \frac{1}{50 \cdot \omega_c} \cdot 1.61803 = 4.50205 \times 10^{-11}$ or <b>45.02 pF</b>
$L_{3,norm} = 2.00000$	$L_3 = \frac{50}{\omega_c} \cdot 2.00000 = 1.39121 \times 10^{-7}$ or <b>139.121 nH</b>
$C_{4,norm} = 1.61803$	$C_4 = \frac{1}{50 \cdot \omega_c} \cdot 1.61803 = 4.50205 \times 10^{-11}$ or <b>45.02 pF</b>
$L_{5,norm} = 0.61803$	$L_5 = \frac{50}{\omega_c} \cdot 0.61803 = 4.29906 \times 10^{-8}$ or <b>42.99 nH</b>

Table source: <https://www.rfcafe.com/references/electrical/butter-proto-values.htm>

Solve for stop band edge:

$$10 \log(|H(j\omega)|^2) = -20 \text{ dB}$$

$$10^{-2} = \frac{1}{1 + (10^{0.1} - 1) \left(\frac{\omega}{100 \text{ MHz}}\right)^{10}}$$

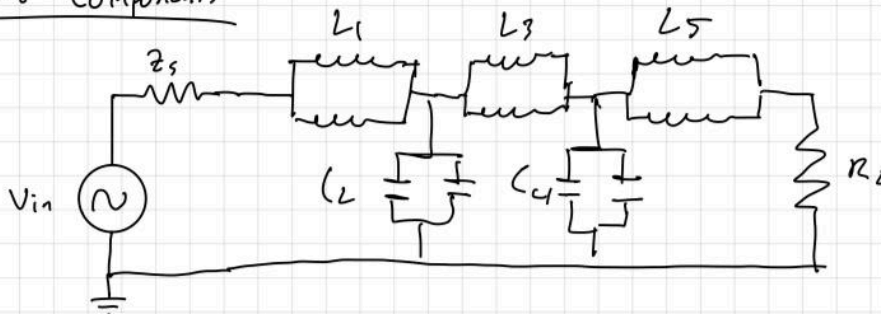
$$1 + (10^{0.1} - 1) \left(\frac{\omega}{10^8}\right)^{10} = 10^2$$

$$\frac{\omega}{10^8} = (10^2 - 1)^{1/10} / (10^{0.1} - 1)$$

$$\omega = \left(\frac{10^2 - 1}{10^{0.1} - 1}\right)^{1/10} \cdot 10^8$$

$$\omega_s = 181.2 \text{ MHz}$$

Real Components



• Some components are not available in lab

$$L_1 = 82 \text{ nH} \parallel 82 \text{ nH} = 41 \text{ nH}$$

$$C_2 = 22 \text{ pF} \parallel 22 \text{ pF} = 44 \text{ pF}$$

$$L_3 = 270 \text{ nH} \parallel 270 \text{ nH} = 140 \text{ nH}$$

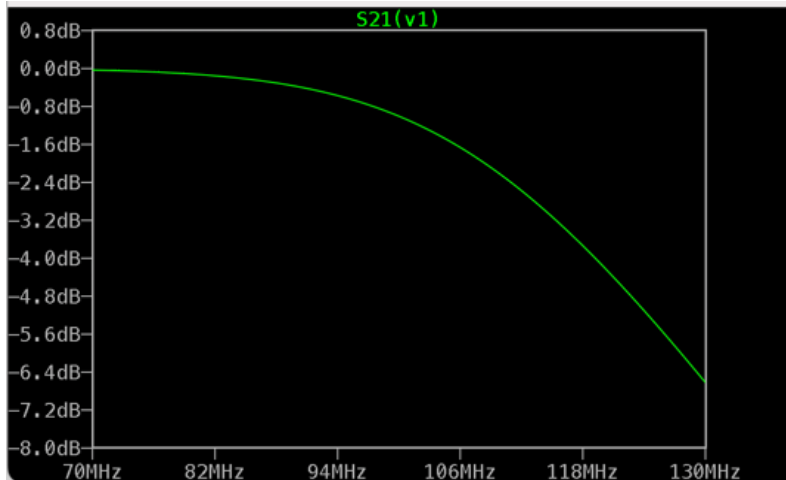
$$C_4 = 22 \text{ pF} \parallel 22 \text{ pF} = 44 \text{ pF}$$

$$L_5 = 82 \text{ nH} \parallel 82 \text{ nH} = 41 \text{ nH}$$

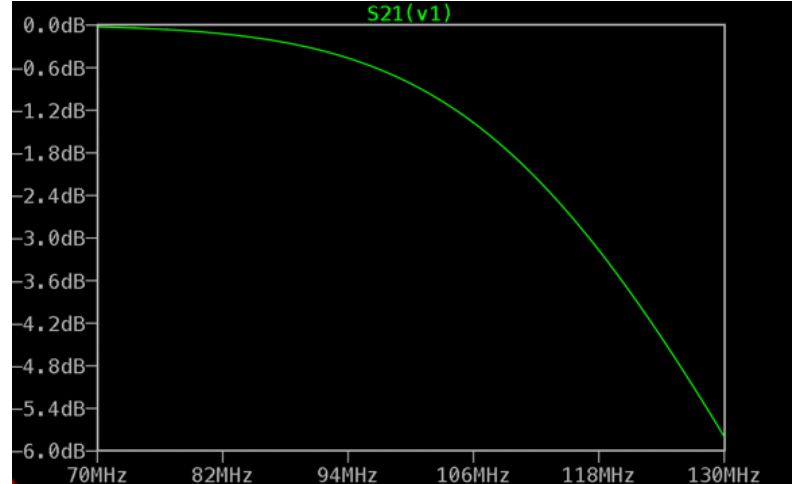
## 4) Magnitude of $S_{21}$

(70MHz - 130MHz)

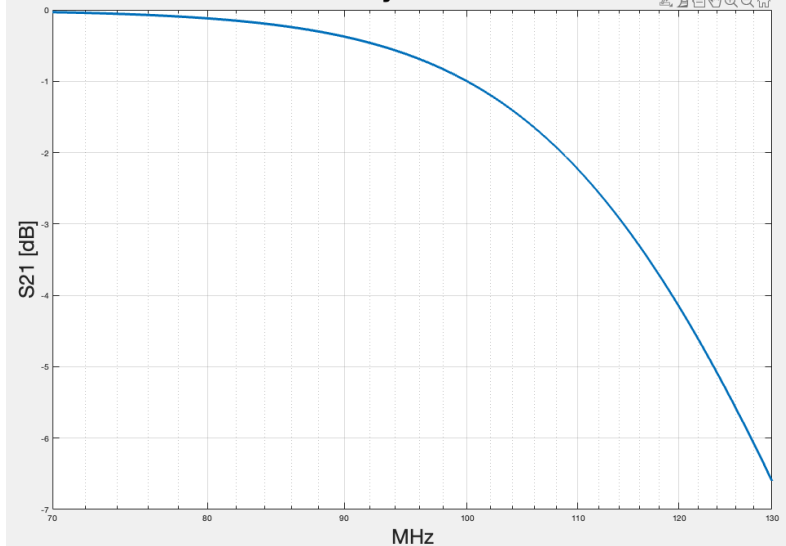
Simulated with Ideal Components



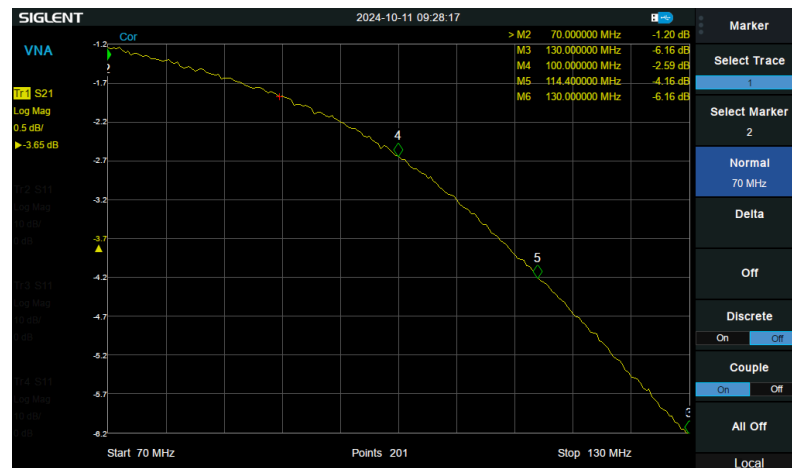
Simulated with Real Components



Analytical  $S_{21}$

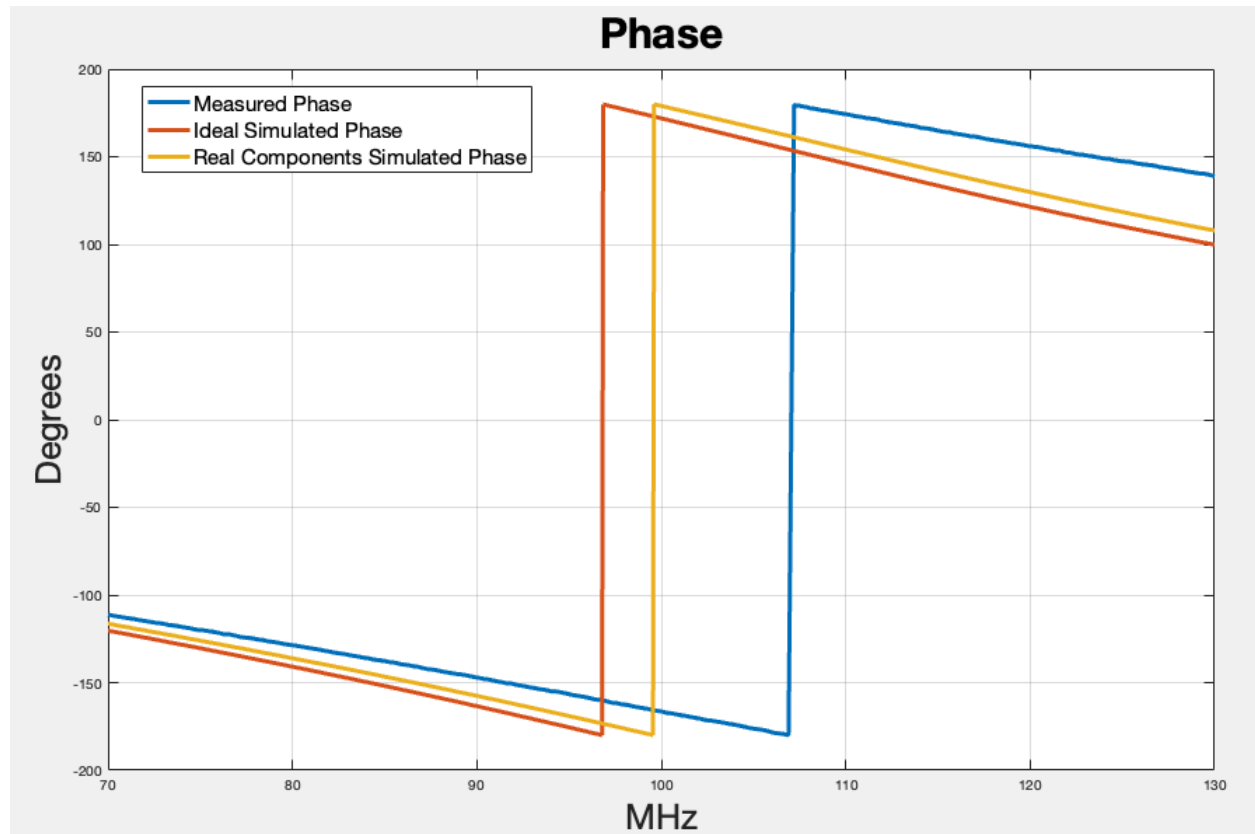


Measured Lab Results



## 5) Phase of $S_{21}$ (70MHz - 130MHz)

(70MHz - 130MHz)

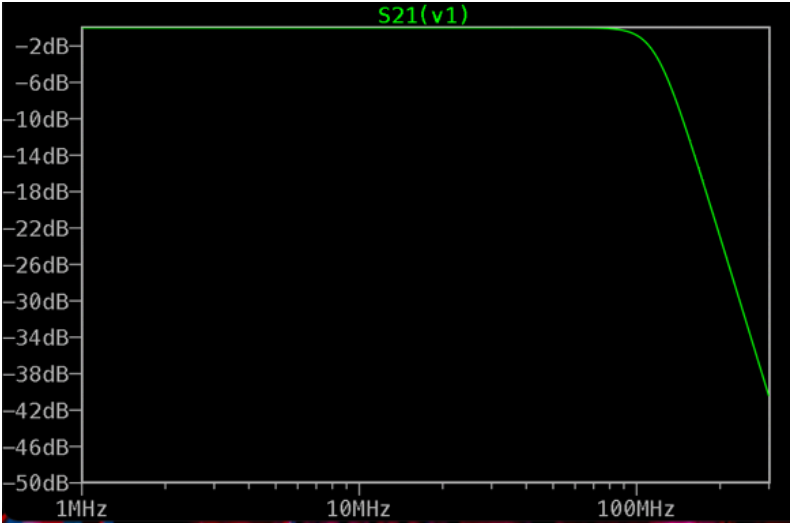




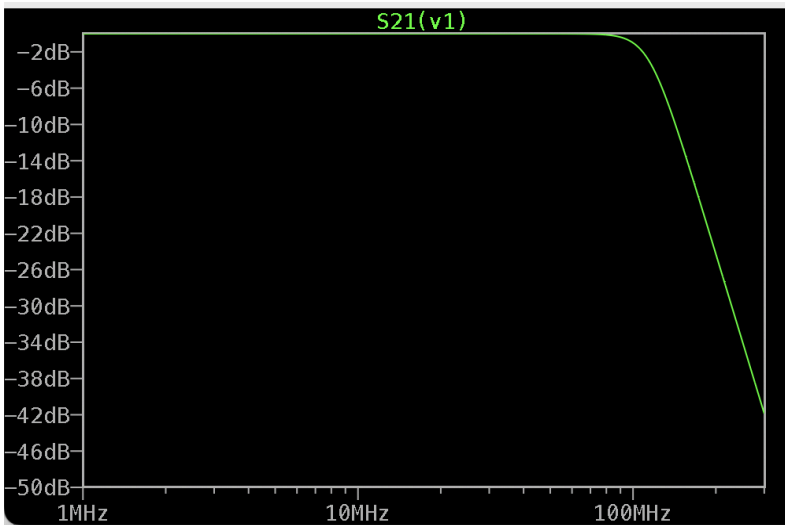
# 6) Magnitude of S<sub>21</sub> (0MHz - 300MHz)

(0MHz - 300MHz)

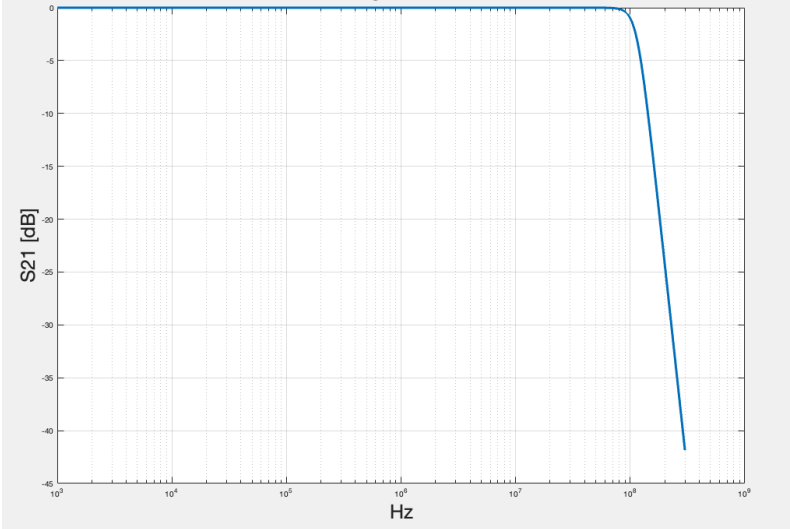
Simulated - Ideal Components



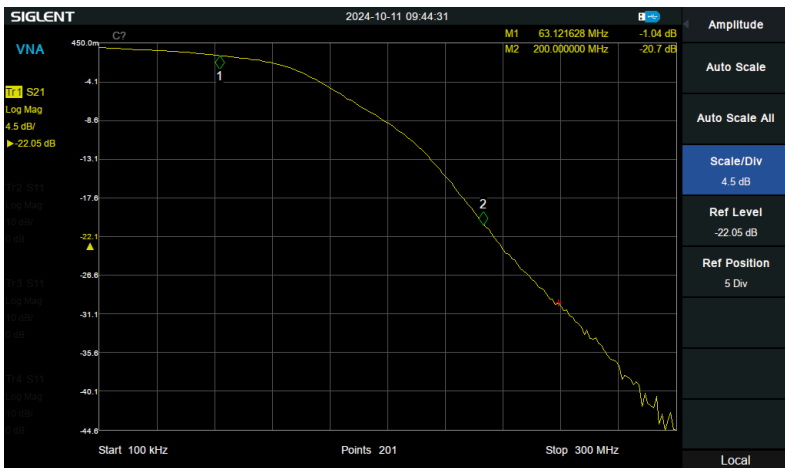
Simulated - Real Components



Analytical S21



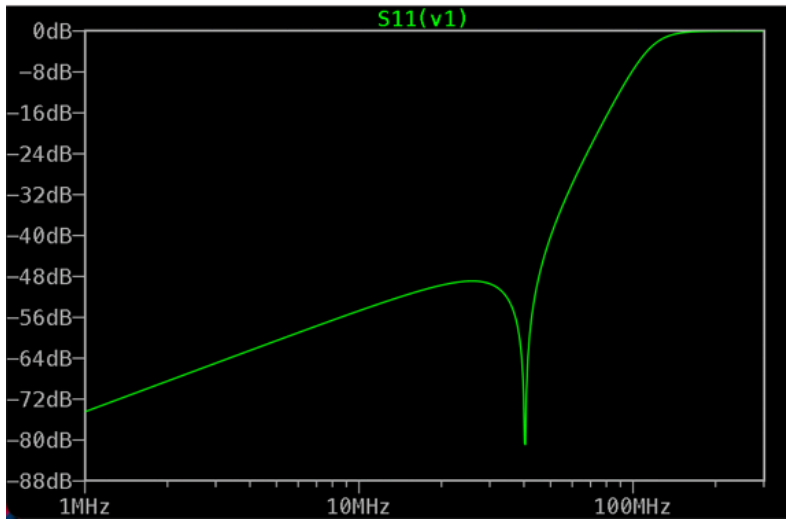
Measured Lab Results



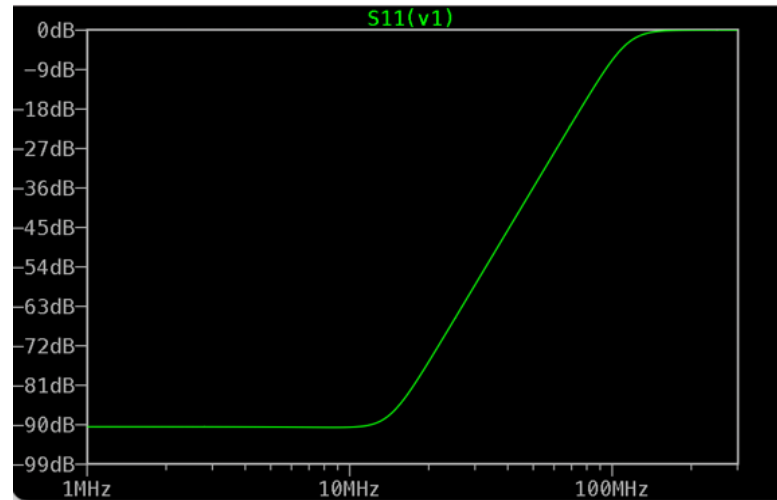
# 7) Magnitude of $S_{11}$

(0MHz - 300MHz)

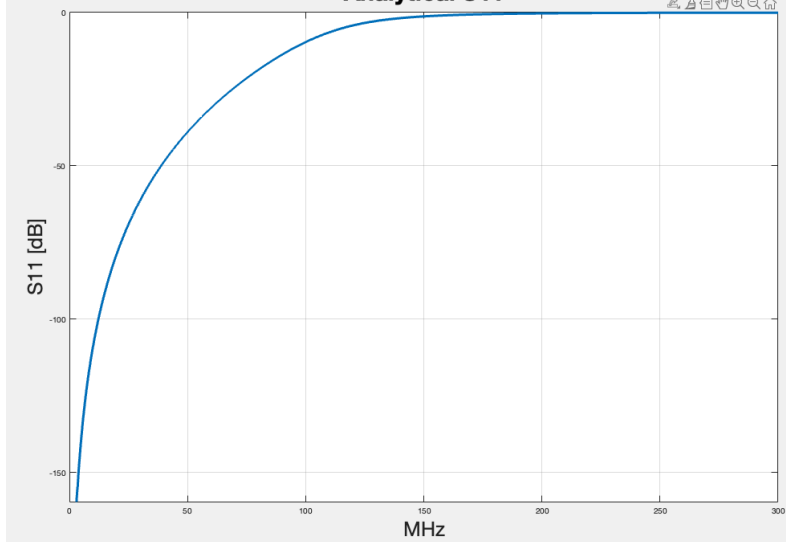
Simulated - Real Component Values



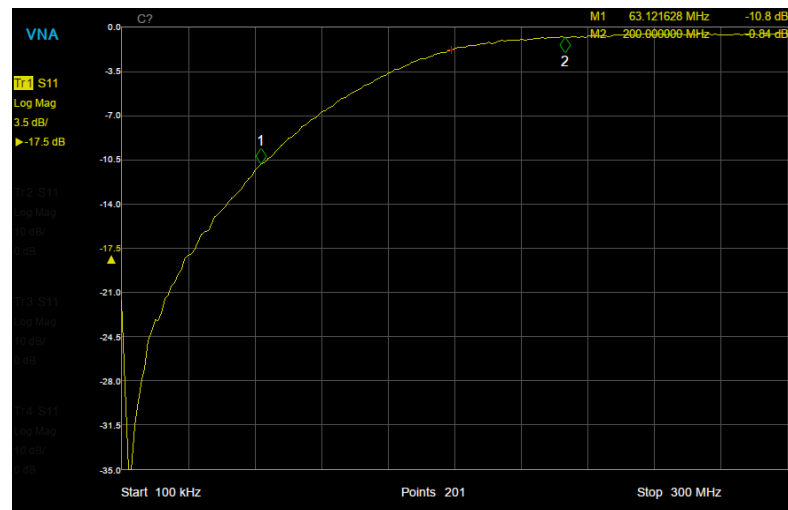
Simulated - Ideal Component Values



Analytical  $S_{11}$



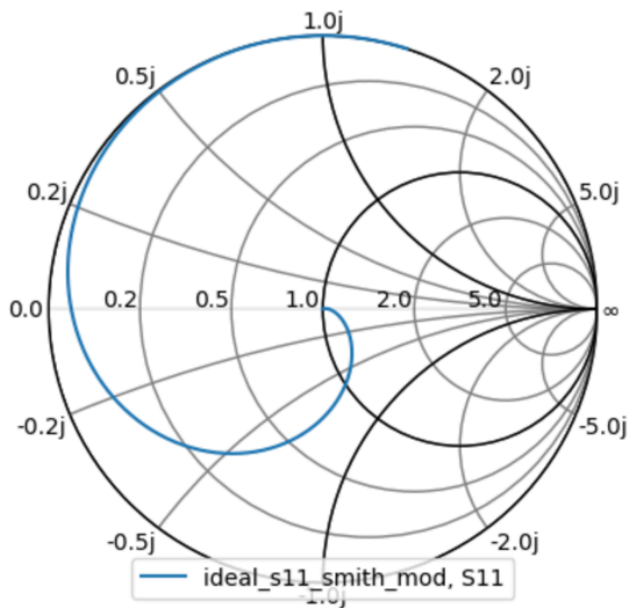
Measured Lab Results



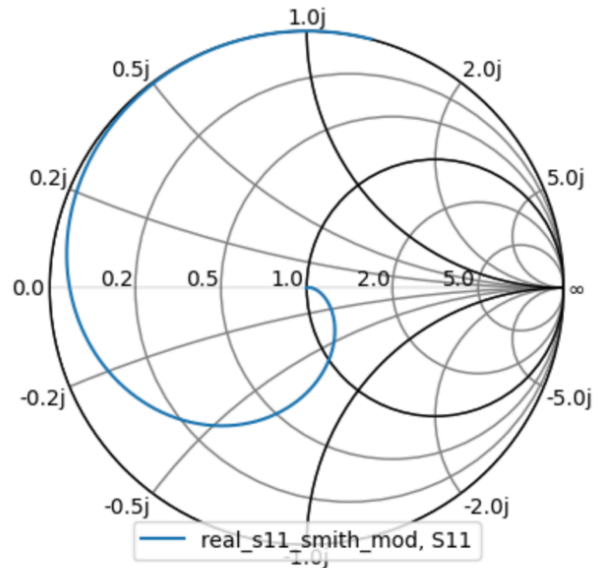
## 8) Smith Charts of $S_{11}$ and $S_{21}$

(0MHz - 300MHz)

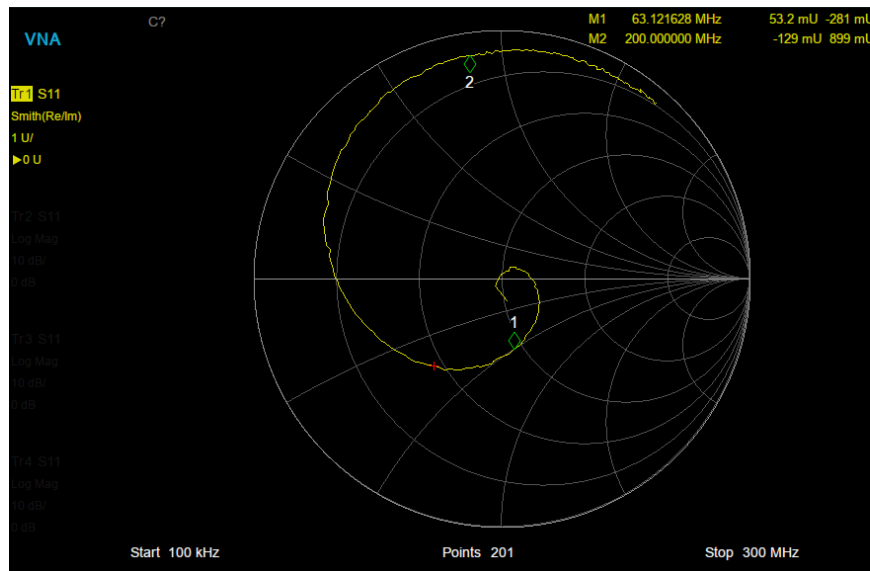
Simulated Ideal Components



Simulated Real Components



Measured Lab Data



## 9) Discussion

The Butterworth filter that we designed operated as it filtered out frequencies near the stop band edge and began attenuating around 100 MHz. The four separate designs that were a part of this lab matched qualitatively. The main difference between predicted (analytical and simulated) and measured results was the pass band edge being lower than anticipated. We designed the circuit to hit -1dB at 100MHz, indicating the pass band's end. The many complicating factors of the real-life circuit implementation were the cause of this difference. These complicating factors include the parasitics of the board, the +/- 5% error in the components, and the power loss over the transmission lines between the gen out and our network. Parasitic components are a result of the geometry of the circuit used. In general, the amount of parasitic inductance and capacitance is 2pF per ground pad connection and 1 nH per vias both in shunt. Secondly, the power loss over the transmission line would lead to a perceived increase in impedance by port 2. Therefore due to parasitics and insertion loss, the filter loses some accuracy. Additionally, the simulated magnitude of the S11 using real components yields a very different result from what we saw in the other three designs. There appears to be resonance at ~40 Hhz. We attribute this to the fact that we set up parallel inductors and capacitors to get a parallel equivalent component value that would match our ideal values closer based on what was available to us in the lab. These larger components would ring out at different frequencies resulting in the resonant dip seen. This occurs at very large negative dB values so it does not have much of an effect on our filter.

## 10) Learning

This Design Project hit home the intuition behind the S network and how it applies to a DUT but more specifically a filter. What made this click for me was looking at the S11 plots that we made. I saw that as frequencies were attenuated by our filter the S11 magnitude increased. This made sense as I knew the energy must be conserved and it is via reflections back off of the s network. Additionally, I learned how to design and build a two-port network. While our circuit in this lab was rather simple I now understand the thought process that goes into order number and component value decisions.