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RANDOM SIGNAL PROCESSING: FINAL MATLAB ASSIGNMENT

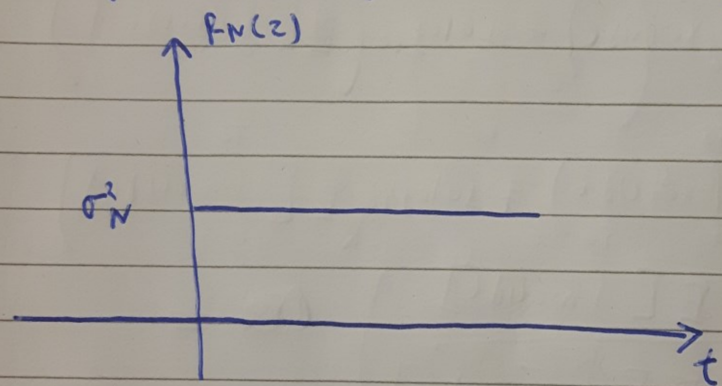
EXERCISE 1

Exercise 01

We are given that $E[N(t)] = 0$ and $Var[N(t)] = \sigma_N^2$ and also it is assumed that the noise $N(t)$ is a stochastic stationary and uncorrelated signal. This implies that:

$$R_N(t,\tau) = R_N(\tau) = E[N(\tau) \cdot N(t+\tau)] \overset{t=0}{=} E[N^2(\tau)] = \\ = \sigma_N^2 .$$

So, the sketch of the autocorrelation function of the noise process $N(t)$ is given by the next figure:



EXERCISE 2

Exercise 02

We should compute the quantity $SNR_e - SNR$.

So, $SNR_e - SNR = 10\log_{10} \dfrac{\int_{t=0}^{T_0} x_0^2(t)\,dt}{E\left[\int_0^{T_0} \tilde{n}^2(t)\,dt\right]} -$

$$- 10\log_{10}\left(\frac{\int_0^{T_0} x_0^2(t)\,dt}{E\int_0^{T_0} n^2(t)\,dt}\right)$$

$$= 10\log_{10}\left(\int_{t=0}^{T_0} x_0^2(t)\,dt\right) - 10\log_{10}\left(E\left[\int_0^{T_0} \tilde{n}^2(t)\,dt\right]\right)$$

$$- 10\log_{10}\left(\int_0^{T_0} x_0^2(t)\,dt\right) + 10\log_{10}\left(E\left[\int_0^{T_0} n^2(t)\,dt\right]\right)$$

$$= 10\log_{10}\left(\frac{E\left[\int_0^{T_0} n^2(t)\,dt\right]}{E\left[\int_0^{T_0} \tilde{n}^2(t)\,dt\right]}\right) =$$

$$= 10\log_{10} \frac{\int_0^{T_0} E[n^2(t)]\,dt}{\int_0^{T_0} E[\tilde{n}^2(t)]\,dt} \quad (*)$$

Let compute the average times that are inside the integrals seperately.

$E[n^2(t)] = Var[N(t)] + (E[\cancel{N(t)}])^2 = Var[N(t)]$
$= \sigma_N^2$ (1)

$E[\bar{N}^2(t)] = Var[\bar{N}(t)] + E[\bar{N}(t)]$ (2)

We have that:

$E[\bar{N}(t)] = E\left[\frac{1}{10}(n(t) + n(t+T_0) + \dots + n(t+9T_0))\right] =$

$= \frac{1}{10}\left\{E[\cancel{n(t)}] + E[\cancel{n(t+T_0)}] + \dots + E[\cancel{n(t+9T_0)}]\right\}$

(3)

$= 0$, as we have that $E[n(t)] = E[n(t+T_0)] = \dots = E[n(t+9T_0)]$
$= 0$

$Var[\bar{N}(t)] = Var\left[\frac{1}{10}[n(t) + \dots + n(t+9T_0)]\right] =$

n(t) uncorrelat

$= \frac{1}{100}\left[Var[N(t)] + \dots + Var[n(t+9T_0)]\right] =$

$= \frac{1}{100}(\underbrace{\sigma^2_N + \dots + \sigma^2_N}_{10 \text{ times}}) = \frac{10\sigma^2_N}{100} = \frac{\sigma^2_N}{10}$ (4)

So, (2) $\underset{(4)}{\overset{(3)}{\Rightarrow}}$ $E[\bar{N}^2(t)] = \frac{\sigma^2_N}{10}$ (5)

Thus, (*) $\underset{(5)}{\overset{(1)}{\Rightarrow}}$ $\boxed{SNR_e - SNR} = 10\log_{10}\frac{\int_0^{T_0}\sigma^2_N dt}{\int_0^{T_0}\frac{\sigma^2_N}{10}dt}$

$= 10\log_{10}\left(\frac{T_0\sigma^2_N}{T_0\frac{\sigma^2_N}{10}}\right) = 10\log_{10}10$

$= \boxed{10}$

EXERCISE 3

$\boxed{\text{Exercise 03}}$

In order to obtain an SNR increase of 90dB using the epoch folding, we should calculate the value of the variable K for given $T_0 = 0.5$.

So, $$SNR_e - SNR = 10\log_{10}\left(\frac{\int_0^{0.5} \sigma^2 N \, dt}{\int_0^{0.5} \frac{\sigma^2 N}{K} \, dt}\right) = 90$$

$$\Leftrightarrow 10\log_{10}\left(\frac{\frac{0.5\sigma^2 N}{1}}{\frac{0.5\sigma^2 N}{K}}\right) = 90$$

$$\Leftrightarrow 10\log_{10} K = 90 \Leftrightarrow \log_{10} K = 9$$

$$\Leftrightarrow 10^{\log_{10} K} = 10^9$$

$$\Leftrightarrow \boxed{K = 10^9}$$

EXERCISE 4

Exercise 04

These signals that are given are sampled at 10 kHz. So, we can obtain that the frequency $fs = 10 \cdot 10^3 = 10^4$ Hz.

Thus, the periodicity To in samples by observing signal x is given by: $\boxed{T_0} = \frac{1}{fs} = \frac{1}{10^4} = \boxed{10^{-4}\ s}$

Also, we computed the SNR of the original signal using signals x and n by using the MATLAB command $\boxed{SNR(x,n)}$

The result is $\boxed{-34.9972}$

EXERCISE 5

The epoch_folding gives as an output the epoch folded signal z by taking as an input a signal y, the period T_o and the number of frames K.

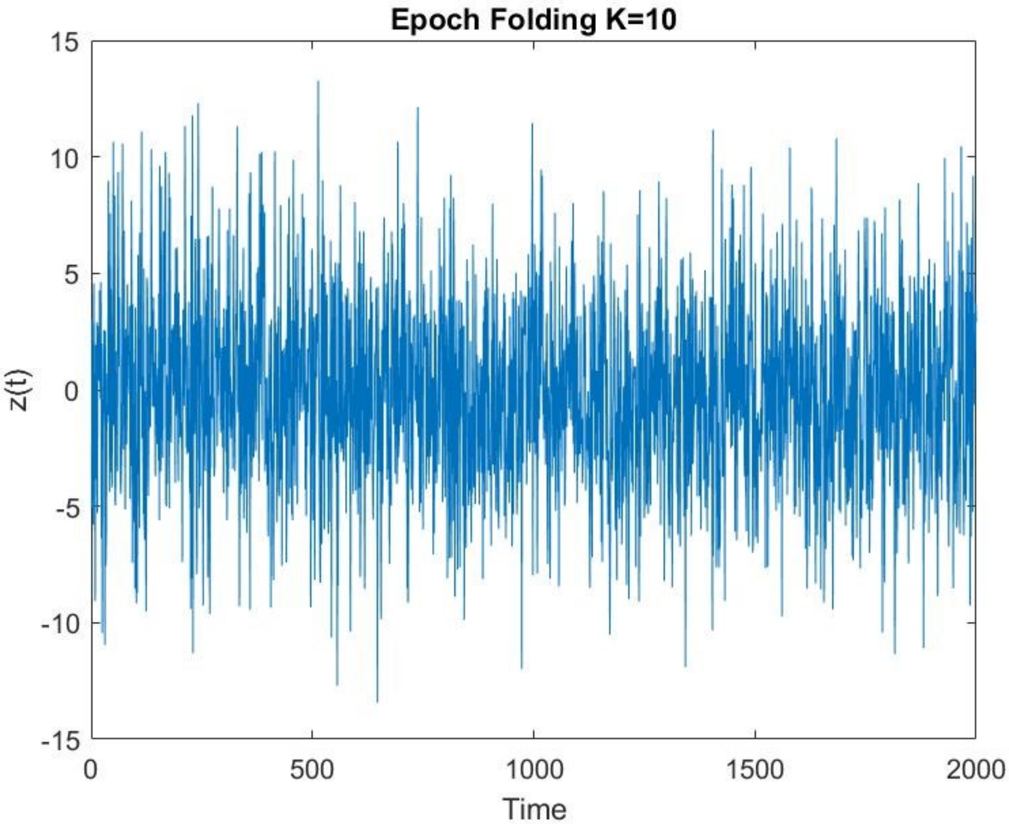
The accompanying code follows:

```
function [z] = Epoch_folding (y, To, k)

z=zeros (length(y), 1)
for i= 1:length(y)-(k-1)*To
	for j=0:k-1
		z(i)=(1/k)*y(i+(To*j))+z(i);

	end
end

for i=1:length(z)
	time(i)=i;
end

for i=1:length(z)
	a(i)=z(i);
end

plot(time, a);
xlabel ('Time');
ylabel ('z(t)');
end
```

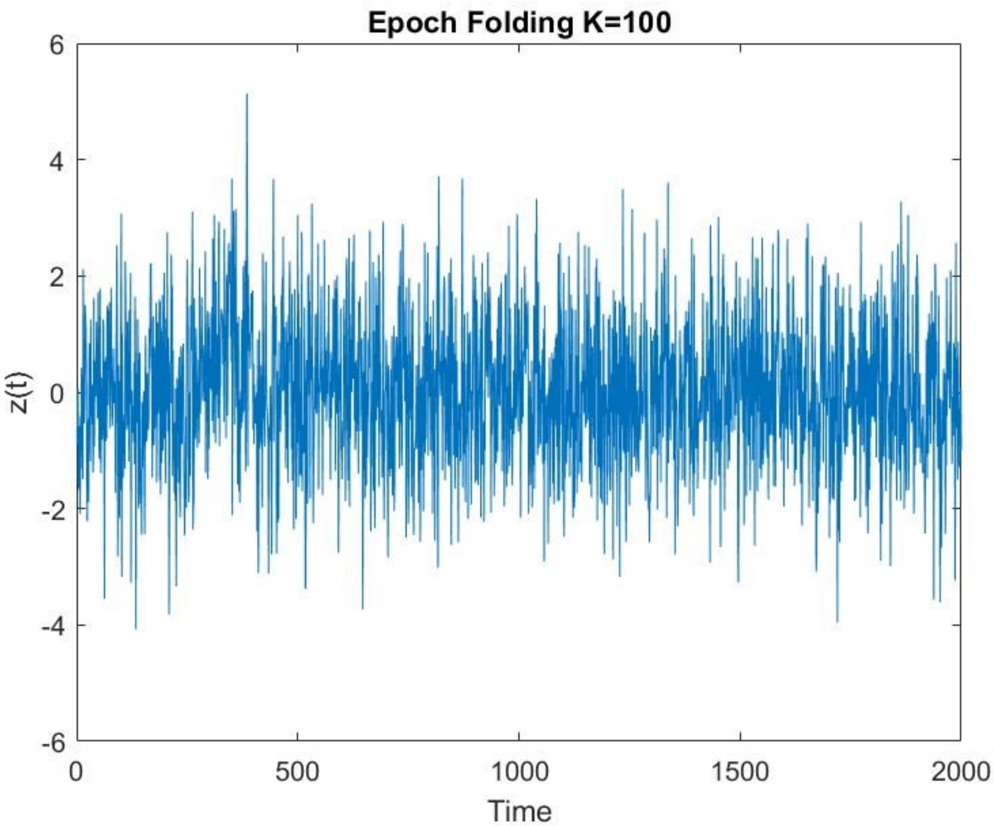
EXERCISE 6

We use the above function epoch_folding in order to detect the pulse in the pulsar signal. An increment of SNR of 10, 20, and 30 dB implies that the K will be 10 ,100, 1000 respectively.
Hence, for the corresponding values of k:

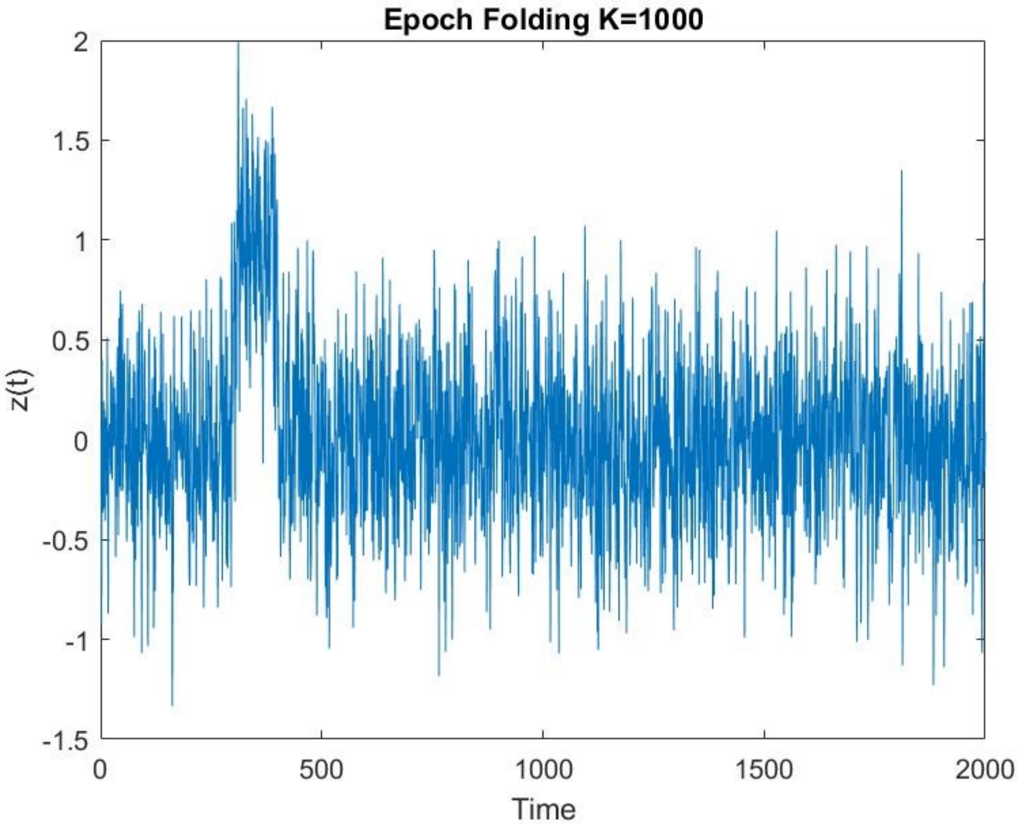
k=10



k=100



k=1000


Epoch Folding K=1000
z(t)
Time
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
500
1000
1500
2000

EXERCISE 7

Exercise 07

It is given that $z_m(t) = \int_0^{T_0} x_0(s)\,x(-t+s)\,ds + \int_0^{T_0} \tilde{n}(s)\,x(-t+s)\,ds$

By assuming that $x_0(t)$ consists of a perfect pulse and taking into account that for $K \to +\infty$ we have that

$$\lim_{K \to +\infty} \frac{1}{K} \sum_{k=0}^{K-1} n(t+kT_0) = \lim_{K \to +\infty} \tilde{n}(t) \to 0$$

So, as the noise is assumed to be uncorrelated, it is going to cancel out as $K \to +\infty$.

Thus our sketch consists of the graph of our initial signal $y(t)$. We notice that it contains a lot of noise. However later on, as we use techniques such as epoch folding and matched filtering and we are able to have better results.

EXERCISE 8

EXERCISE 8

Exercise 08

We need to prove that $E[z_m(t)] = T_0 R_{x_0 x}(-t)$

We have that $z_m(t) = \int_0^{T_0} x_0(s)x(-t+s)\,ds + \int_0^{T_0} \tilde{n}(s)x(-t+s)\,ds$

So, $E[z_m(t)] = E\left[\int_0^{T_0} x_0(s)x(-t+s)\,ds + \int_0^{T_0} \tilde{n}(s)x(-t+s)\,ds\right]$

$= \int_0^{T_0} E[x_0(s)x(-t+s)\,ds] + \int_0^{T_0} E[\tilde{n}(s)x(-t+s)\,ds]$ (*)

We notice that:

$$E[\tilde{n}(s)\cdot x(-t+s)] \overset{\tilde{n},\,x \text{ independent}}{=} E[\tilde{n}(s)]\cdot E[x(-t+s)]$$

Also, we proved in exercise 2 that $E[\tilde{n}(s)] = 0$.

Thus, $E[\tilde{n}(s)\cdot x(-t+s)] = 0$ (1)

(*) $\overset{(1)}{\Longrightarrow} E[z_m(t)] = \int_0^{T_0} E[x_0(s)x(-t+s)]\,ds$ (2)

By definition, $E[x_0(s)x(-t+s)] = R_{x_0 x}(s,-t) \overset{(3)}{=} R_{x_0 x}(-t)$ (4)

x_0, x jointly WSS

In general we have that:

$$R_{XY}(t,\tau) = E[X(t)\cdot Y(t+\tau)]$$

Let $\boxed{t=s}$, then $t+\tau = -t+s$

and $s+\tau = -t+s \Rightarrow \boxed{\tau=-t}$

So, $\boxed{\begin{array}{c} t=s \\ \tau=-t \end{array}}$ (3)

So, (2) $\Rightarrow$ $E[\tilde{n}(t)] = \int_0^{T_0} R_{X_0}(-t)\, ds =$ (4)

$$= R_{X_0X}(-t)\,[s]_0^{T_0} = \boxed{T_0 R_{X_0X}(-t)}$$

Hence, we made the assumptions that the variables $\tilde{n}$, X are intependontent and the variables X_0 and X are wide sense stationary jointly.

EXERCISE 9

Exercise 09

It is given in equation (18) that:

$$zm(L) = \frac{T_0}{N} \sum_{m=0}^{N-1} ze(m)\, x(-L+m) \quad (1)$$

we know that $ze(m) = \frac{1}{K} \sum_{k=0}^{K-1} x(m+kT_0) + \frac{1}{K} \sum_{k=0}^{K-1} n(m+kT_0)$ (2)

Thus, $(1) \overset{(2)}{\Longrightarrow} zm(L) = \frac{T_0}{N} \sum_{k=0}^{K-1} x_0(m) \cdot x(-L+m) + \frac{T_0}{N} \sum_{k=0}^{K-1} \tilde{n}(m) \cdot x(-L+m)$

$$= \frac{T_0}{N} R_{x_0 x}(-L) + \frac{T_0}{N} R_{\tilde{N} x}(-L)$$

So, for large N, $N \to +\infty$, $zm(L)$ converges to:

$$\lim_{N \to +\infty} \frac{1}{N} \cdot T_0 R_{x_0 x}(-L) + \lim_{N \to +\infty} \frac{1}{N} T_0 R_{\tilde{n} x}(-L) =$$

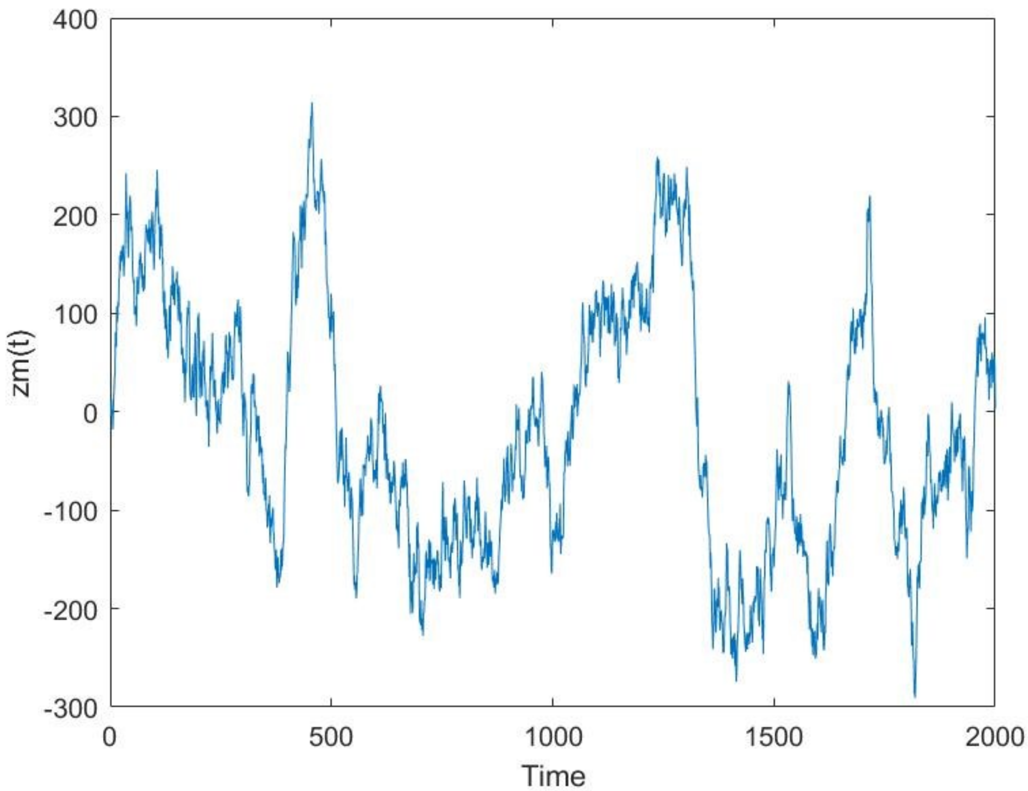
$$= T_0 R_{x_0 X}(-L) + T_0 R_{\tilde{N} X}(-L).$$

Taking into consideration (the property that $\tilde{N}$, x are indepedent related to estimation of autocorrelation functions that the underlying processes shall have), we have that $T_0 R_{\tilde{N} x}(-L) = 0$.

Thus $zm(L) \simeq T_0 R_{x_0 x}(-L)$

EXERCISE 10

The **plot** of the results:



Accompanying code:

```
function [zm] = Matched_Filter (ze,y)

zm=zeros(length(x),1);
a(1)=x(1);
for i=0:length(x)-2
        a(i+2)=x(length(x)-i);
end
a=a';
Tp = toeplitz(a,x);
zm=Ta*ze ;
for i=1:length(x)
        t(i)=i;
end
for i=1:length(x)
        s(i)=zm(i);
end
plot(t,s);
xlabel('Time');
```

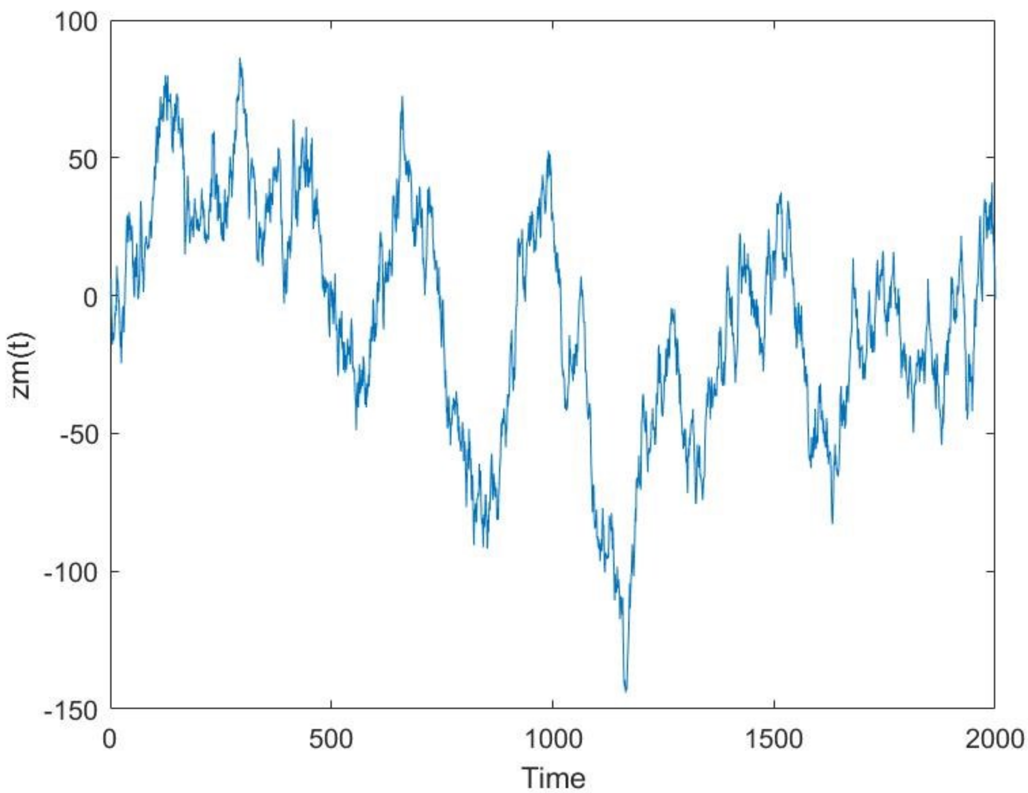
```
ylabel('zm(t)');
end
```

We assumed that the ze will be the y variable and the x equals to template in order to compute the output of the matched filter.

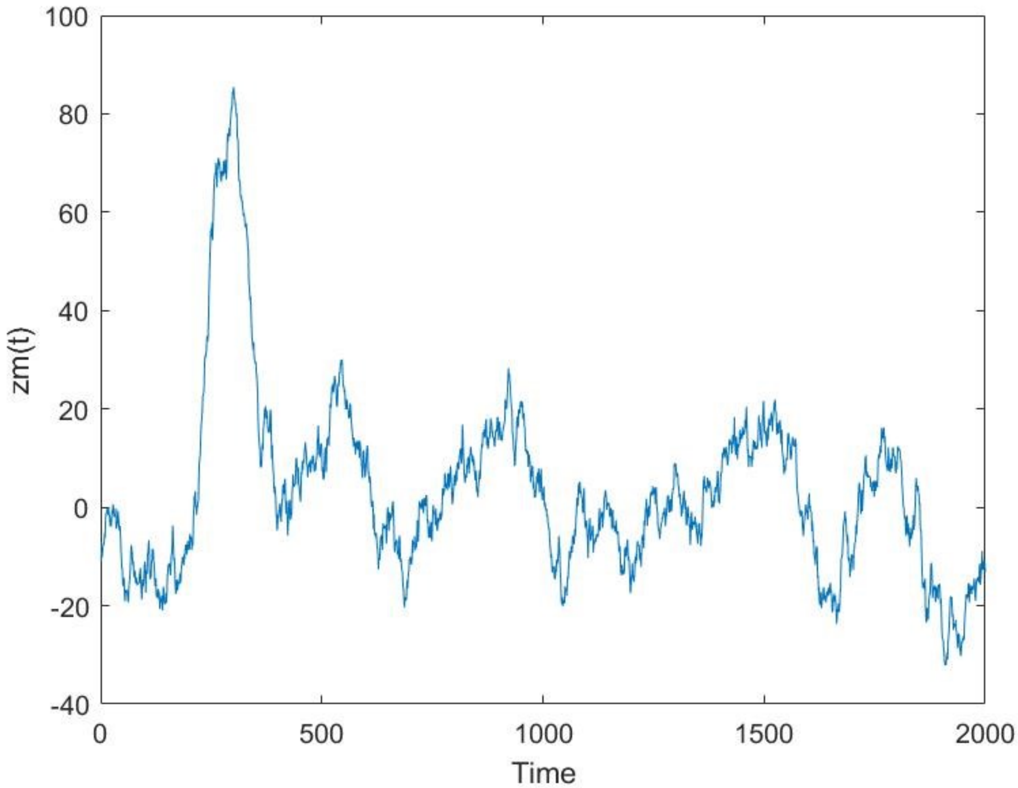
EXERCISE 11

First we generate epoch folded signals using the function epoch folding on the noisy signal y for the K-values which we computed in exercise 6 in order to determine the pulse location of the pulsar signal. Then we use each of these epoch folded signals as input ze in the function matched filter. Hence, for the corresponding values of k we attach the results:

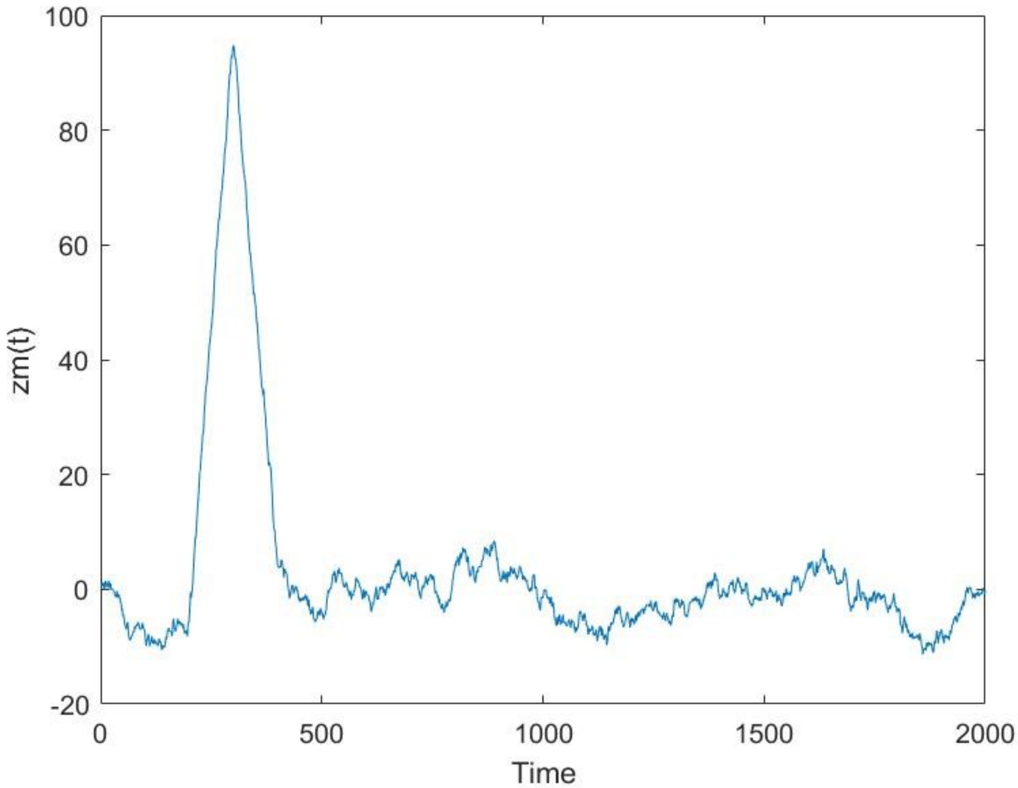
k=10



k=100



k=1000


100
80
60
40
20
0
-20
zm(t)
0
500
1000
1500
2000
Time