UNITED KINGDOM · CHINA · MALAYSIA

Department of Electrical and Electronic Engineering
Faculty of Engineering

Modelling: Methods and Tools

(EEEE2055 UNUK) (FYR1 22-23)

Coursework 1 (Fourier Transforms)

Author:
George Downing

Student Number:
20273662

December 20, 2022

Contents

1 Fourier Transform of a Signal

1.1 Fourier Transform derivation

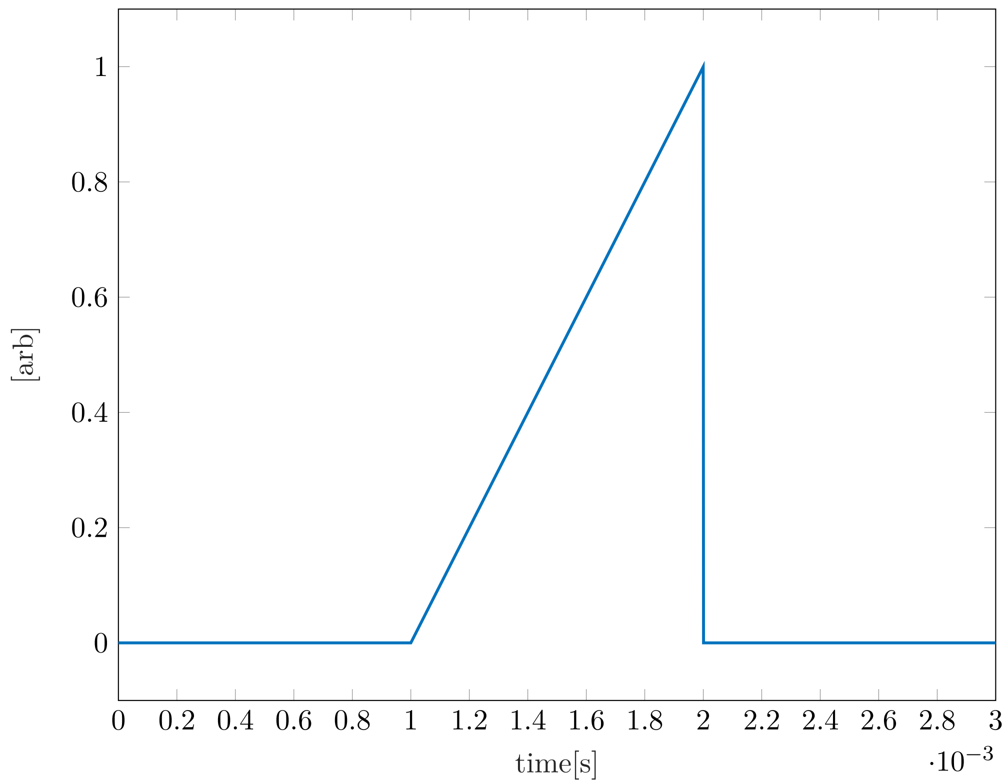

Figure 1: $S_2(t)$

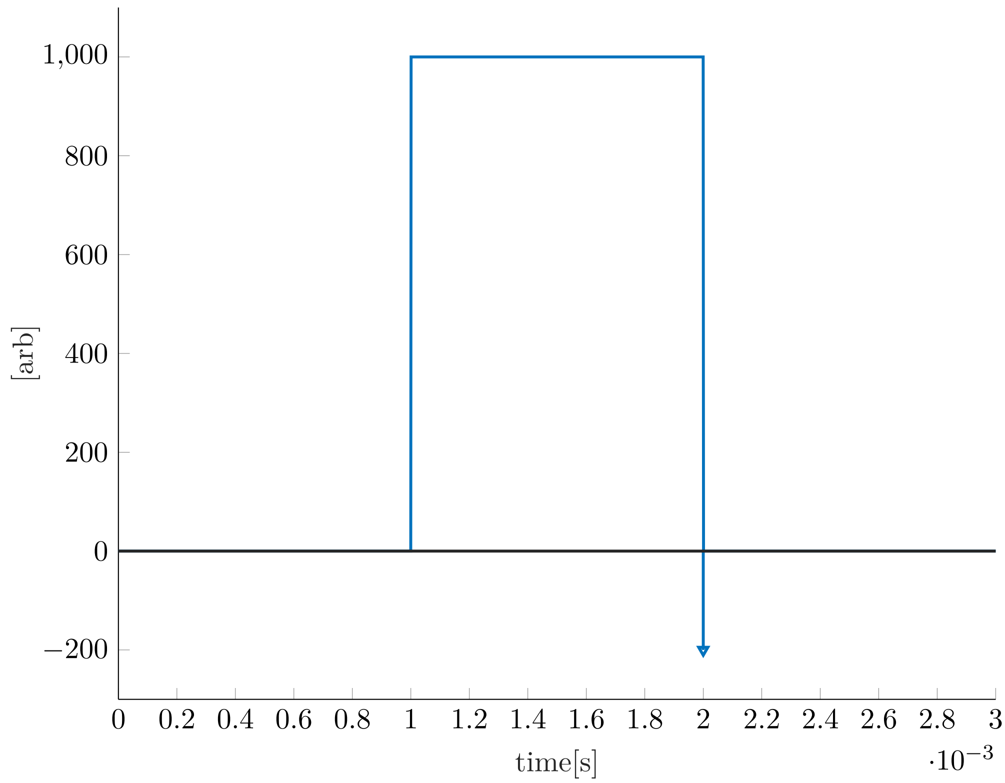

Figure 2: $\frac{dS_2(t)}{dt}$

Figure 1 shows the triangular pulse $S_2(t)$ and Figure 2 shows the derivative of $S_2(t)$. where the gradient of the triangle represents a rise of 1 in 1ms.

To calculate the fourier transform of the signal we can split the signal allowing us to differentiate the the different components to create delta functions. The first derivative of $S_2(t)$ is shown in (**??**). This signal consists of a rectangular pulse with a width of 1ms and a height of 1000 can be differentiated again to form two unit impulses. Also there is unit impulse at 0.002ms.

The second derivative of $S_2(t)$ as a summation of delta functions is shown in (1).

$$
\begin{aligned}
\frac{d^2 S_2(t)}{dt^2} &= \frac{d}{dt}\left[\frac{dS_2(t)}{dt}\right] \\
&= \left(\frac{1}{m}\delta(t-m) - \frac{1}{m}\delta(t-2m)\right) + \frac{d}{dt}\left[\delta(t-2m)\right] \\
&= 1000\delta(t-0.001) - 1000\delta(t-0.002) + \frac{d}{dt}\left[\delta(t-0.002)\right]
\end{aligned} \tag{1}
$$

In order to calculate the fourier transform of the Signal $S_2(t)$ we can use the derivative theorem. The derivative theorem states that the fourier transform of the derivative of a signal is the fourier transform of the signal divided by the frequency. Application of derivative theorem is shown in (2). Where the first line is the derivative theorem and the second line is the application of the derivative theorem to the second derivative of $S_2(t)$.

$$\begin{aligned}(j\omega)^2 \mathcal{F}\{S(t)\} =& \mathcal{F}\left\{\frac{d^2 S_2(t)}{dt^2}\right\}\\ \therefore \widetilde{S}\{\omega\} =& \frac{1}{(j\omega)^2} \cdot \mathcal{F}\{1000\delta(t-0.001) - 1000\delta(t-0.002)\}\\ &+ \frac{1}{(j\omega)} \cdot \mathcal{F}\delta(t-0.002)\end{aligned} \tag{2}$$

The Fourier transform is evaluated using integration as shown in (3).

$$\begin{aligned}\widetilde{S}\{\omega\} =& \frac{1000}{(j\omega)^2} \cdot \int_{-\infty}^{\infty} \delta(t-0.001)\, e^{-j\omega t} dt\\ &+ \frac{-1000}{(j\omega)^2} \cdot \int_{-\infty}^{\infty} \delta(t-0.002)\, e^{-j\omega t} dt\\ &+ \frac{-1}{j\omega} \cdot \int_{-\infty}^{\infty} \delta(t-0.002)\, e^{-j\omega t} dt\end{aligned} \tag{3}$$

The equation given in (3) is then evaluated, using the properties of the delta function, to give the following equation (4).

$$\begin{aligned}\widetilde{S}\{\omega\} =& \frac{-1000}{\omega^2} \cdot e^{-j\omega 0.001}\\ &+ \frac{1000}{\omega^2} \cdot e^{-j\omega 0.002m}\\ &+ \frac{-1}{j\omega} \cdot e^{-j\omega 0.002m}\\ =& \frac{1}{(j\omega)^2 m}\left[e^{-j\omega 0.001m} - e^{-j\omega 0.002m}\right] - \frac{1}{(j\omega)}\left[e^{-j\omega 0.001m} - e^{-j\omega 0.002m}\right]\end{aligned} \tag{4}$$

The equation given in (4) is then simplified to give the following equation (5).

$$\widetilde{S}\{\omega\} = \frac{je^{-\frac{j\omega}{500}}}{\omega} + \frac{1000e^{-\frac{j\omega}{500}} - 1000e^{-\frac{j\omega}{1000}}}{\omega^2} \tag{5}$$

Converting the equation into polar form to avoid imaginary exponentials gives the following equation (6).

$$\widetilde{S}\left\{\omega\right\} = \frac{t\sin\left(\frac{\omega}{500}\right) - 2000\sin\left(\frac{\omega}{2000}\right)\sin\left(\frac{3t}{2000}\right)}{t^2} + j\frac{t\cos\left(\frac{\omega}{500}\right) - 2000\sin\left(\frac{\omega}{2000}\right)\cos\left(\frac{\omega}{2000} - \frac{\omega}{500}\right)}{t^2} \quad (6)$$

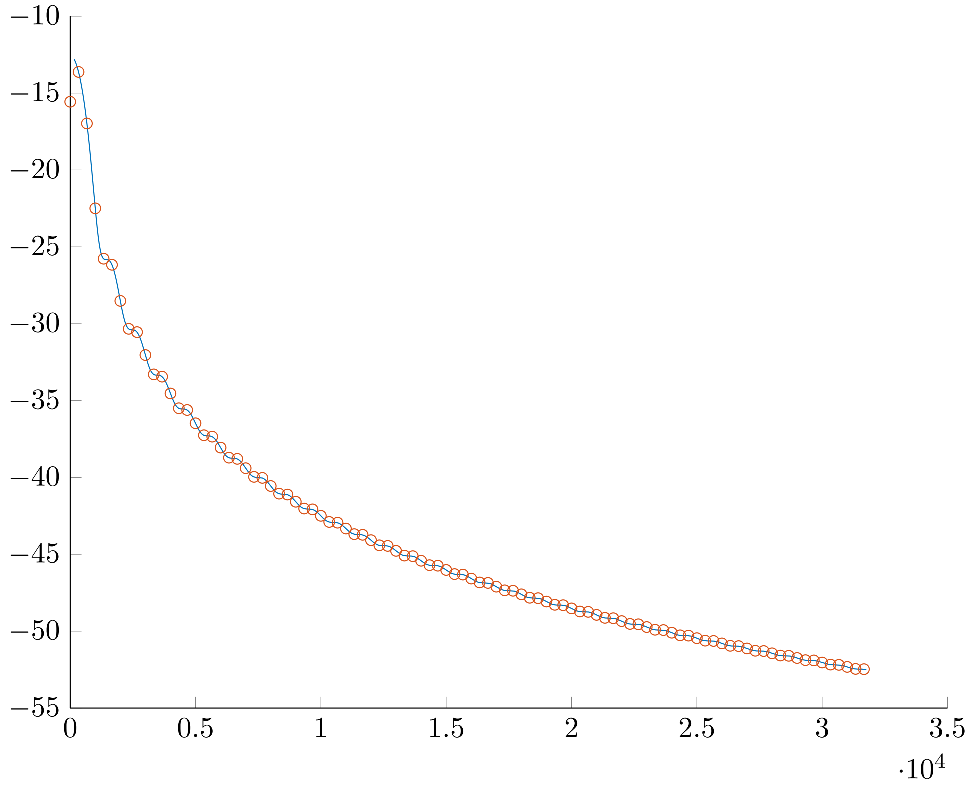

Figure 3: $S_2\left(f\right)$

1.2 How can an FFT differ from theory

1.3 Exploring effect of falling edge approximation

2 Impulse and Frequency Response of a Circuit

2.1 Computing the Frequency Response

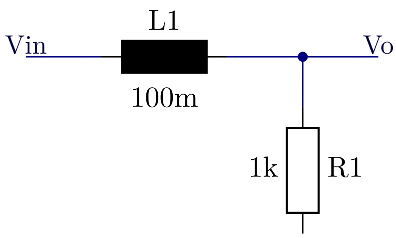

Figure 4: Figure 2: LR filter circuit

Figure 4 Shows the circuit is comprised of two impedances in series, $R1$ and $L1$ in a potential divider configuration.

The equation relating the impedance of resistor $R1$ to the frequency is derived as shown by eq. 7

$$
\begin{aligned}
V &= IR \\
Z &= R = \frac{V}{I} \\
\therefore Z &= R
\end{aligned}
\tag{7}
$$

The equation relating the impedance of inductor $L1$ to the frequency is derived as shown by eq. 8

$$
\begin{aligned}
V_L &= \frac{di}{dt}L \\
Z &= \frac{V_L}{I} = \frac{\frac{d}{dt}L}{I} = \frac{d}{dt}L \\
\therefore Z &= j\omega L
\end{aligned}
\tag{8}
$$

where: $V = \text{IR}$
$\frac{d}{dt} = \text{j}\omega$

The equation that relates V_o to V_i with respect to such impedances is given by eq. 9

$$
\begin{aligned}
\frac{V_o}{V_i} &= \frac{Z_2}{Z_1 + Z_2} \\
&= \frac{R_1}{\frac{d}{dt}L_1 + R_1}
\end{aligned}
\tag{9}
$$

The fourier frequency response of the circuit is obtained by inserting $j\omega$ in place of $\frac{d}{dt}$ in eq. 10

$$\begin{aligned}\widetilde{\mathcal{F}}\left\{\omega\right\} &= \frac{\widetilde{V_o}\left(\omega\right)}{\widetilde{V_{in}}\left(\omega\right)} \\ &= \frac{R_1}{j\omega L_1 + R_1} \\ &= \frac{1k\Omega}{j\omega 100mH + 1k\Omega} \\ &= \frac{1000}{j\omega 0.1 + 1000}\end{aligned} \tag{10}$$

$\widetilde{\mathcal{F}}\left(\omega\right)$ can be writen as a transfer function as shown by eq. 11

$$H\left(s\right) = \frac{1000}{0.1\left(s\right) + 1000} \tag{11}$$

Plotting the transfer function in matlabs bode plot function shows the frequency response of the circuit as shown in figure 5

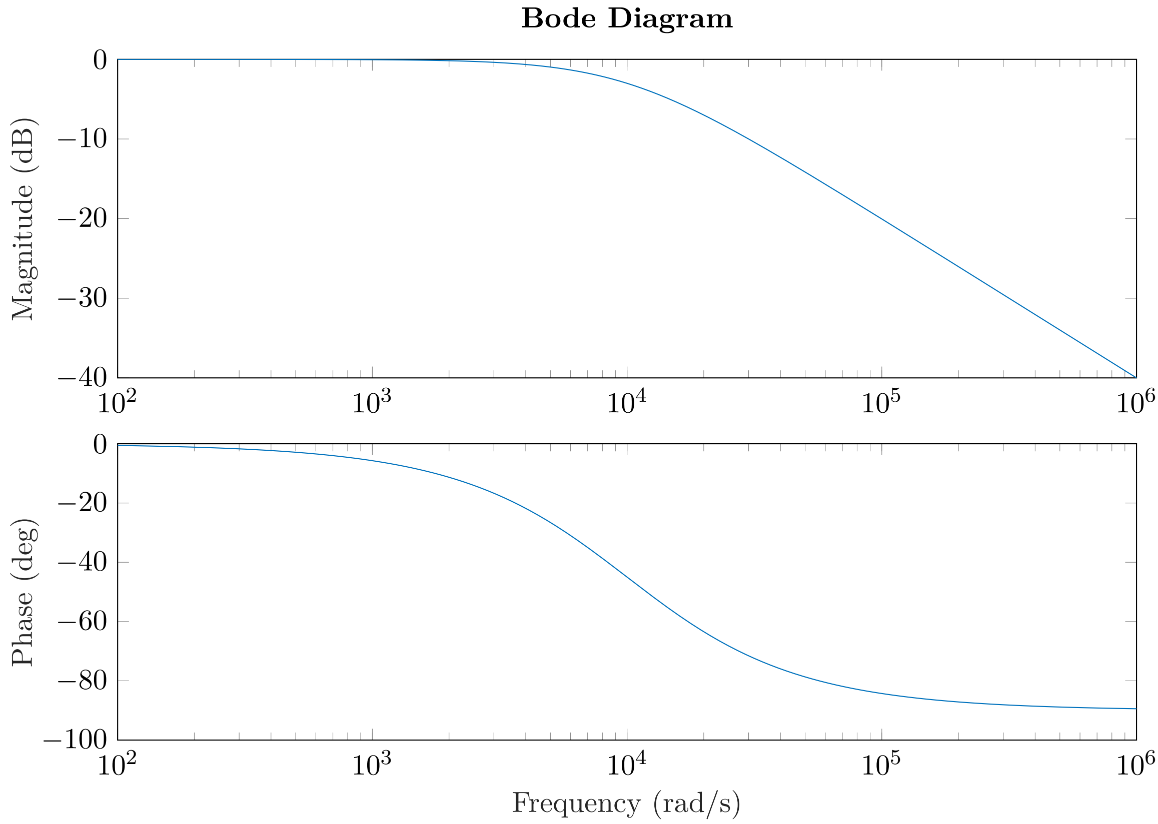

Figure 5: Figure 3: Frequency Response of the circuit

2.2 Computing the Impulse Response

2.3 Approximating the Impulse Response using LTSpice

2.4 Checking the Frequency Response using LTSpice

3 Modelling the filter output using convolution

3.1 Deriving the filter output waveform using convolution

3.2 Comparing the convolution answer to LTSpice

References