EEEE2055: Modelling Methods and Tools - Coursework1 (Fourier Transforms) 22-23

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Questions

The following questions are assessed and you must write a report describing your results. The questions get you to analysis the Fourier Transform of a signal (Question 1), the frequency response of a filter circuit (Question 2), and the output of the filer circuit when the signal of Question 2 is applied as an input. You will validate analytical calculations using LTSpice.

1 Fourier Transform of signal

1.1 Fourier Transform derivation

Derive the Fourier Transform of the $S_2(t)$ (see Figure 1 *[Report: derivation of Fourier Transform including some comments to explain each step]*

1.2 How can an FFT differ from theory?

Use LTSpice to verify your answer and to look for differences between an FFT and the theoretical Fourier Transform.

As a hint, your LTSpice and calculated plots should be very similar at lower frequencies but there will be some small differences at higher frequencies. If your plot looks completely different then something isn't correct.

You should try to compare your result from the derivation and the LTSpice result on the same plot (export data from LTSpice to Excel, etc). Also think carefully about:

- The frequency range used - you need to make this is wide enough to see any/all differences (i.e. starts at a low enough frequency and finishes at a high enough frequency)
- The axis scales used, see tutorial notes and example spreadsheet.
- The simulation settings used - does changing the simulation stop time, time-step and FFT sample settings have any influence?

[Report: Figure comparing your computed FT with an FFT from LTSpice, some comments on how well they agree and an explanation of what can cause any differences.]

1.3 Exploring effect of falling edge approximation

How does the falling edge time used in the simulation (which should in theory be zero) affect the results? *[Report: Figure showing how approximation used for falling edge in LTSpice affects the Fourier Transform, some comments explaining why.]*

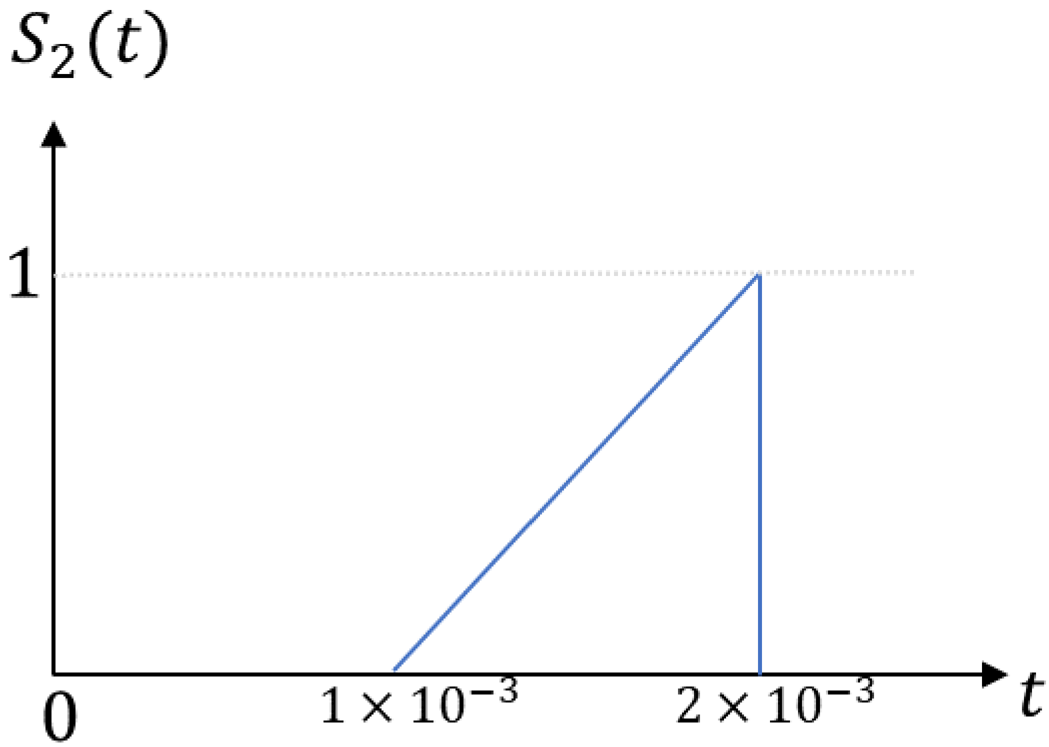

Figure 1: Time domain plot of S2. This will be used as the input to a filter circuit in Question 2.

2 Impulse and Frequency Response of a circuit

2.1 Computing the Frequency Response

Find the Frequency Response of the circuit in Figure 2 by developing an appropriate differential equation relating V_o to V_i and taking a Fourier Transform. *[Report: Derivation of Frequency Response with a short comment explaining each step.]*

2.2 Computing the Impulse Response

Determine an expression for the Impulse Response of the circuit using the Frequency Response from Question 2.1 as a starting point. *[Report: Derivation of Impulse Response with a short comment explaining each step.]*

2.3 Approximating the Impulse Response using LTSpice

Obtain an approximation for the impulse response of the circuit using a time-domain simulation in LTSpice. You need to drive the filter circuit with a Dirac delta function to obtain an output voltage waveform that represents the Impulse Response. You cannot use an actual Dirac delta function in LTSpice so you must use an approximation.

Justify the delta approximation you have used and modify the simulation model to test if your approximation is influencing the results you obtain - try a few variations on the Delta approximation to see what happens.

[Report: Figure(s) comparing computed Impulse Response to response obtained using LTSpice and effect of different delta approximations, some comments to explain how/why certain delta functions give the correct answer but some don't.]

2.4 Checking your Frequency Response using LTSpice

Generate Frequency Response plots from your Impulse Response using an FFT. Compare the LTSpice Frequency Response with your answer to 2.1, how does the Impulse Approximation affect the frequency response that you see?

Include an answer to this question: The frequency response of a circuit is not usually computed using LT-Spice by taking an FFT of the Impulse Response, there is another method. What is it? (You don't need to do any further simulations) *[Report: Figure(s) comparing computed Frequency Response (2.1) to response obtained using LTSpice and comments on: how IR approximation affects results, how FR is usually calculated in software such as LTSpice.]*

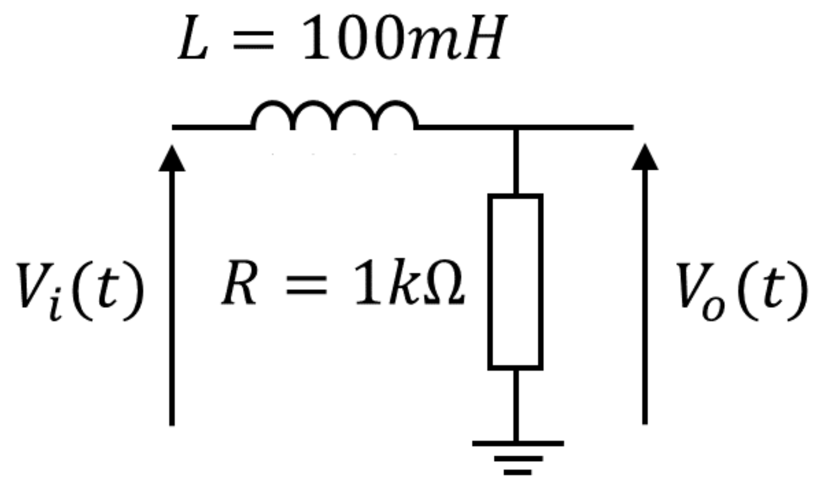

Figure 2: Filter circuit

3 Modelling the filter output using convolution

3.1 Deriving the filter output waveform using convolution

Using convolution, compute an expression for the time-domain output waveform of the filter when $V_i(t) = S_2(t)$.

This is the final question for the coursework and is more difficult problem to solve. The process you need to follow is exactly the same as in the lecture slides.
What are the limits of the convolution integral (i.e. where is the product of $F(\tau)$ and $S_2(t-\tau)$ non zero)? You will need to consider two different sets of limits for different ranges of t. You can start by sketching $F(\tau)$ $S_2(t-\tau)$ on a plot with τ on the horizontal axis to understand these limits.

As a hint, the only real difference to the lecture example, is that you have a triangle pulse rather than a step input. Other than the obvious difference in function shape / different equation for V_in, the limits will be different (a step goes on to infinity, and also this pulse doesn't start at $t = 0$ like the step in the lecture example).

[Report: Derivation of time-domain output waveform. Even if you can't quite get the correct answer for the final part, there are marks for attempting this.]

3.2 Comparing the convolution answer to LTSpice

Use LTSpice to find V_o when $V_i = S_2(t)$. Compare your answer to 3.1 with this.

[Report: Figure comparing your answer to the time-domain waveform obtained as part of solution to 3.1.]

4 Report guidance

Structure the report into subsections as indicated in the question descriptions. (1.1, 1.2, etc). For each subsection just include the minimum that is required to answer the question - typically this is a derivation and/or a figure, and some comments. Bullet points are ok for the comments, there is no need to write an essay!