Apply transformation to a and b

$$b' = bp + aq$$
$$a' = bq + aq + ap$$

Apply same transformation again to b

$$b'' = b'p + a'q$$

$$b'' = (bp + aq)p + (bq + aq + ap)q$$
$$b'' = bp^2 + 2aqp + bq^2 + aq^2$$
$$b'' = b(p^2 + q^2) + a(2qp + q^2)$$

Let $b'' = bp' + aq'$

$$bp' + aq' = b(p^2 + q^2) + a(2qp + q^2)$$

$$p' = p^2 + q^2, q' = 2qp + q^2$$