

5.4.4 Gravitational potential and energy

Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

- (a) gravitational potential at a point as the work done in bringing unit mass from infinity to the point; gravitational potential is zero at infinity
- (b) gravitational potential $V_g = \frac{GM}{r}$ at a distance r from a point mass M ; changes in gravitational potential
- (c) force-distance graph for a point or spherical mass; work done is area under graph
- (d) gravitational potential energy $E = mV_g = -\frac{GMm}{r}$ at a distance r from a point mass M
- (e) escape velocity.

Additional guidance

HSW5

HSW1, HSW2 Predicting the escape velocity of atoms from the atmosphere of planets.


(6) M - Define gravitational potential energy
(7) S - Derive and apply the equation for gravitational potential energy
(8) C - Find the escape velocity of an object from the Earth or other planets.

Gravitational potential energy

STARTER: Calculate the **gravitational potential** at a point 1200km above the Earth's surface. Earth's radius is 6370km

$V_g = -\frac{GM}{r} = 5.2 \times 10^7 \text{ J kg}^{-1}$
 -4.6×10^7

Ex: Why can't $E_g = mgh$ be used to calculate the gravitational potential energy of a 1000kg satellite at this point?



Answer: It can only be used in a uniform gravitational field.

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Gravitational potential energy

The **gravitational potential energy** of any object of mass in a gravitational field is defined as the work done to move a mass from infinity to a point in the gravitational field.

$E_g = mV_g$ Or for changes...

$\Delta E_g = m\Delta V_g$

$E_g = mV_g$ Activity: Substitute to find an expression for V_g involving G

$E = -\frac{GMm}{r}$

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Activity: Try Q2 p356 and self assess

Ex: Lowe: Q

Mini plenary:

The mass of the Earth is 5.97×10^{24} kg and it has radius 6370km. Calculate the gravitational potential energy of a 75.8kg satellite at a height of 1200km above the surface of the Earth.

Step 1: Determine the distance of the satellite from the centre of mass of the Earth.

$6370 + 1200 = 7570\text{km}$

Step 2: Calculate the gravitational potential energy.

$E = -\frac{GMm}{r} = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 75.8}{7.570 \times 10^6} = -3.95 \times 10^7 \text{ J (3 s.f.)}$

Note that this is not the energy required to lift the satellite into orbit, since it already had a gravitational potential energy on the surface of the Earth. It will also need some kinetic energy in orbit.

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Activity 1: Try Q2 p356 and self assess

Activity 2: Consider the 27500kg space shuttle. Calculate the gravitational force of attraction at each of these distances: (radius of Earth=6370km)


Height above Earth surface (km)	Gravitational force (N)
0	2.69×10^7
200	2.54×10^6
400	2.39×10^6
600	2.35×10^6
800	2.32×10^6
1000	2.31×10^6

69×10^5

Activity 3: Plot a graph of F against r . Use the graph to find the work done in raising this object to this position in the field. Check this is correct by calculation using an equation from this lesson.

Ex: Why is this not the true value?

$\Delta E_g = \frac{GMm}{r_0} - \frac{GMm}{r_1}$



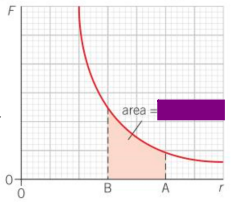

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Starter - Force vs distance graph

What is the significance of the area under the line?

Show algebraically that this is correct, by considering the gravitational force equation.

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Escape velocity

How could an object/projectile totally escape the earth's gravitational field and not be pulled back down to earth?

In order to escape the gravitational field of a mass like our planet, an object must be supplied with the energy = the gain in GPE needed to lift it out of the field.

Consider a projectile. (bullet). The loss of KE must equal the gain in GPE.


$\frac{1}{2}mv^2 = \frac{GMm}{r}$

$V^2 = \frac{2GM}{r}$ $V = \sqrt{\frac{2GM}{r}}$

ACTIVITY 1: Use this idea to find an equation for the minimum escape velocity of a projectile needed to escape the Earth.

ACTIVITY 2: Once found, find the escape velocity at the Earth's surface.

$V_{\text{earth}} = 11.2 \text{ km s}^{-1}$



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Activity:

- Try questions 4 of the previous worksheet.
- Try summary question 3 p356

Kilo 10³	Escape velocity on different planets P356 Lowe: Q9-10 p74
Mega 10⁶	
Giga 10⁹	

Neutron Star
 A neutron star is the collapsed core of a star that has gone supernova. A typical neutron star has about the same mass as our sun, but a diameter of only 20 km or so.
 Determine the release height that would give a 5 g marshmallow the same kinetic energy as a one kiloton nuclear bomb (4.2×10^{12} J) when it struck the surface.

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Plenary	$\frac{1}{2}mv^2$
Define Gravitational PE....	
The Sun has a mass of 2.0×10^{30} kg and a mean radius of 7.0×10^8 m Calculate the escape velocity for the Sun. $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$	
<input type="radio"/> 310 km s^{-1} <input checked="" type="radio"/> 620 km s^{-1} → $\frac{3}{2}kT = E$ <input type="radio"/> $2.0 \times 10^7 \text{ km s}^{-1}$ <input type="radio"/> $3.8 \times 10^6 \text{ km s}^{-1}$	
Ex: What temperature would the surface of the sun need to be to have an average energy hydrogen particles escape?	
Hydrogen particle escape at a much lower temperature. Explain why.	$620,000 \text{ m/s}^2$

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