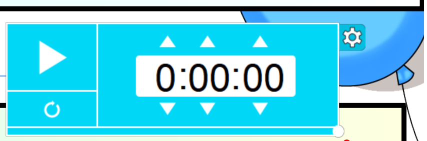
(e)	the equation $pV = \frac{1}{3}Nm\overline{c^2}$, where N is the number of particles (atoms or molecules) and $\overline{c^2}$ is the mean square speed	Derivation of this equation is not required. HSW2
(f)	root mean square (r.m.s.) speed; mean square speed	Learners should know about the general characteristics of the Maxwell-Boltzmann distribution.
(g)	the Boltzmann constant; $k = \frac{R}{N_A}$	
(h)	$pV = NkT$; $\frac{1}{2}m\overline{c^2} = \frac{3}{2}kT$	Learners will also be expected to know the derivation of the equation $\frac{1}{2}m\overline{c^2} = \frac{3}{2}kT$ from $pV = \frac{1}{3}Nm\overline{c^2}$ and $pV = NkT$. HSW2

(6) M - Define and calculate 'root mean square speed'
(7) S - Apply the equation for root mean square speed
(8) C - Derive an equation for root mean square speed.

Lesson 6. Root mean square speed



STARTER:

V m M

A rigid cylinder of volume 0.030 m^3 holds 4.0 g of air. The molar mass of air is about 29 g.

a Calculate the pressure exerted by the air when its temperature is 34 °C. T

b What is the temperature of the gas in degrees Celsius when the pressure is *twice* your value from part **a**?

$V = 0.03$
$p = ?$
$n = 0.14$
$R = 8.31$
$T = (34 + 273) = 307k$

a $PV = nRT$ [1]

$n = \frac{4.0}{29} = 0.138$ moles [1]

$P = \frac{nRT}{V} = \frac{0.138 \times 8.31 \times (273 + 34)}{0.030}$ [1]

$P = 1.17 \times 10^4$ Pa $\approx 1.2 \times 10^4$ Pa (12 kPa) [1]

b $\frac{P}{T}$ is constant when the volume of the gas is constant. [1]

The pressure is doubled, hence the absolute temperature of the gas is also doubled. [1]

Therefore:

temperature $= 2 \times (273 + 34) = 614$ K [1]

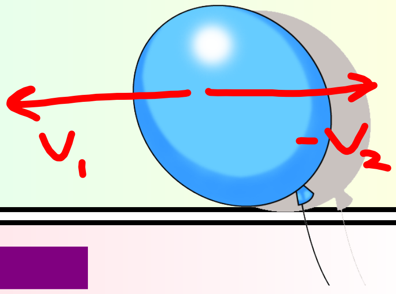
temperature in °C $= 614 - 273 = 341$ °C ≈ 340 °C [1]

(6) M - Define and calculate 'root mean square speed'
(7) S - Apply the equation for root mean square speed
(8) C - Derive an equation for root mean square speed.

Lesson 6. Root mean square speed

0:00:00

Discuss: What do you think is the average velocity of gas particles in this balloon?



Answer - 0m/s   Can you explain why?

Kilo 10^3
Mega 10^6 Justify your answer...
Giga 10^9

Key point

r.m.s. speed can be determined instead which is a more useful measure.

c - Speed

c^2 - Square speed

$\overline{c^2}$ - mean square speed.

$\sqrt{\overline{c^2}}$ - root mean square speed.

(6) M - Define and calculate 'root mean square speed'
(7) S - Apply the equation for root mean square speed
(8) C - Derive an equation for root mean square speed.

Lesson 6. Root mean square speed

Question 1: Imagine a small sample of 4 gas molecules moving in a line. Their velocites are: -450, -50, 100 and 400 m/s. Find:

a) the mean velocity.

b) mean speed

c) root, mean, square speed.

Worked example: Average speeds

A very small sample of gas contains just four molecules moving in one line. Their velocities in $m\,s^{-1}$ are: −450, −50, 100, 400. Calculate the mean velocity, the mean speed $\bar{c}$, and the r.m.s. speed.

Step 1: For the mean velocity, you must take account of the signs of the velocities, because they are vectors.

$$\text{mean velocity} = \frac{(-450 - 50 + 100 + 400)}{4} = 0\,m\,s^{-1}$$

Step 2: Speed is a scalar, so mean speed $\bar{c}$ is calculated by ignoring the negative signs.

$$\bar{c} = \frac{(450 + 50 + 100 + 400)}{4} = 250\,m\,s^{-1}.$$

Step 3: To determine the r.m.s. speed, first square the speeds, then determine the mean.

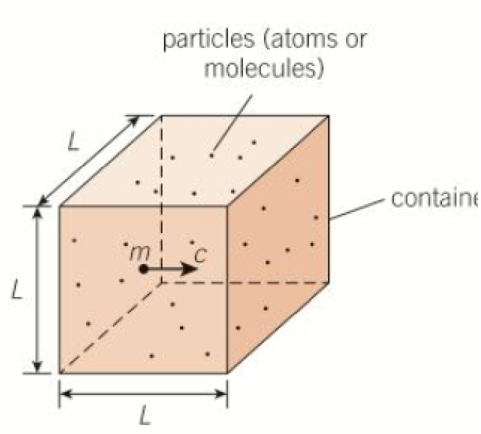
mean square speed =

$$\frac{(202\,500 + 2500 + 100\,000 + 160\,000)}{4} = 116\,250\,m^2\,s^{-2}$$

$$c_{r.m.s.} = \sqrt{116\,250} = \cancel{340}\,m\,s^{-1}\ (2\ s.f.).$$ 306 ms^{-1}

The average speed $\bar{c}$ is not the same as the r.m.s. speed.

(6) M - Define and calculate 'root mean square speed'
(7) S - Apply the equation for root mean square speed
(8) C - Derive an equation for root mean square speed.



▲ Figure 1 *Gas particles (atoms or molecules) in a container*

Mean square speed equation:

Activity:
Identify each of the quantities and their units.

$pV = 1/3Nm\overline{c^2}$

Kilo 10^3
Mega 10^6
Giga 10^9

Extension: Can you use the diagram of an atom in a box to derive the equation above? Get help on page 293

0:00:00

Key point

$pV = 1/3Nm\overline{c^2}$

N -

m -

$\overline{c^2}$ -

(6) M - Define and calculate 'root mean square speed'
(7) S - Apply the equation for root mean square speed
(8) C - Derive an equation for root mean square speed.

Mean square speed equation:

Activity:
Complete the worksheet on mean square speed.

dont do worksheet - errors and too much content into next lesson

Kilo 10^3
Mega 10^6
Giga 10^9

Extension: Can you use the diagram of an atom in a box to derive the equation above? Get help on page 293


Key point

Pressure of a gas

Specification references

- *5.1.4 a) b) c) d) (i) e) f)*
- *HSW1, 2*

Answers

1 **a)** $\frac{(-150+100+200+250)}{4} = 100\ \text{ms}^{-1}$

(1 mark)

b) $\frac{150+100+200+250}{4} = \frac{700}{4} = 180\ \text{ms}^{-1}$ (2 s.f.)

(1 mark)

c) $\frac{22\,500+10\,000+40\,000+62\,500}{4} = 31\,250\ \text{m}^2\,\text{s}^{-2}$

(1 mark)

r.m.s. speed = $180\ \text{ms}^{-1}$ (2 s.f.)

(1 mark)
(4 marks)

2 Substituting the values into the equation $pV = \frac{1}{3}N m\overline{c^2}$ we obtain:

$p = \frac{1}{3} \times \frac{2\times10^{24}\times7\times10^{-27}\times400^2}{4.8}$

(1 mark)

which gives us
p = 160 Pa (2 s.f.)

(1 mark)
(2 marks)

3 $p = \frac{1}{3} \times \frac{2\times10^{24}\times7\times10^{-27}\times360^2}{4.8}$

(1 mark)

p = 130 Pa (2 s.f.)

(1 mark)
(2 marks)

4 $pV = \frac{1}{3}N m\overline{c^2}$ and $pV = nRT$. Hence,

$\frac{1}{3}N m\overline{c^2} = nRT$

(1 mark)

Rearranging we obtain $\frac{1}{3}m\overline{c^2} = \frac{nRT}{N}$

(1 mark)

Since $\frac{nR}{N} = k$, we can write $\frac{1}{3}m\overline{c^2} = kT$

(1 mark)

Multiplying by $\frac{3}{2}$ gives $\frac{1}{2}m\overline{c^{-2}} = \frac{3}{2}kT$

(1 mark)
(4 marks)

5 Substituting the values into $pV = nRT$ we obtain $p(2 \times 10^{-2}) = 2.4 \times 8.31 \times 313$

(1 mark)

Rearranging to make p the subject gives us $p = \frac{2.4\times8.31\times313}{2\times10^{-2}}$

(1 mark)

Hence, p = 312 124 Pa or 310 kPa (2 s.f.).

(1 mark)
(3 marks)

6 The equations can be described as energy equations because pV, nRT, and $\frac{1}{3}N m\overline{c^2}$ all have units equivalent to joules when the units are substituted.

(1 mark)

7 The kinetic energy of a ball of mass 5 kg moving at $12\ \text{ms}^{-1}$ is given by $\frac{1}{2}mv^2$, giving a value of 360 J.

(1 mark)

If this kinetic energy equates to the temperature of 1 mole of helium gas, then the kinetic energy of 1 helium atom, $\frac{1}{2}mc^2$, is equal to $\frac{360}{6.02\times10^{23}}$ or 6.0×10^{-22} J

(1 mark)

Hence, from $\frac{1}{2}m\overline{c^2} = \frac{3}{2}kT$

(1 mark)

we get $T = 6.0 \times 10^{-22} \times \frac{2}{3} \times 1.38 \times 10^{-23} = 29$ K

(1 mark)
(4 marks)

8

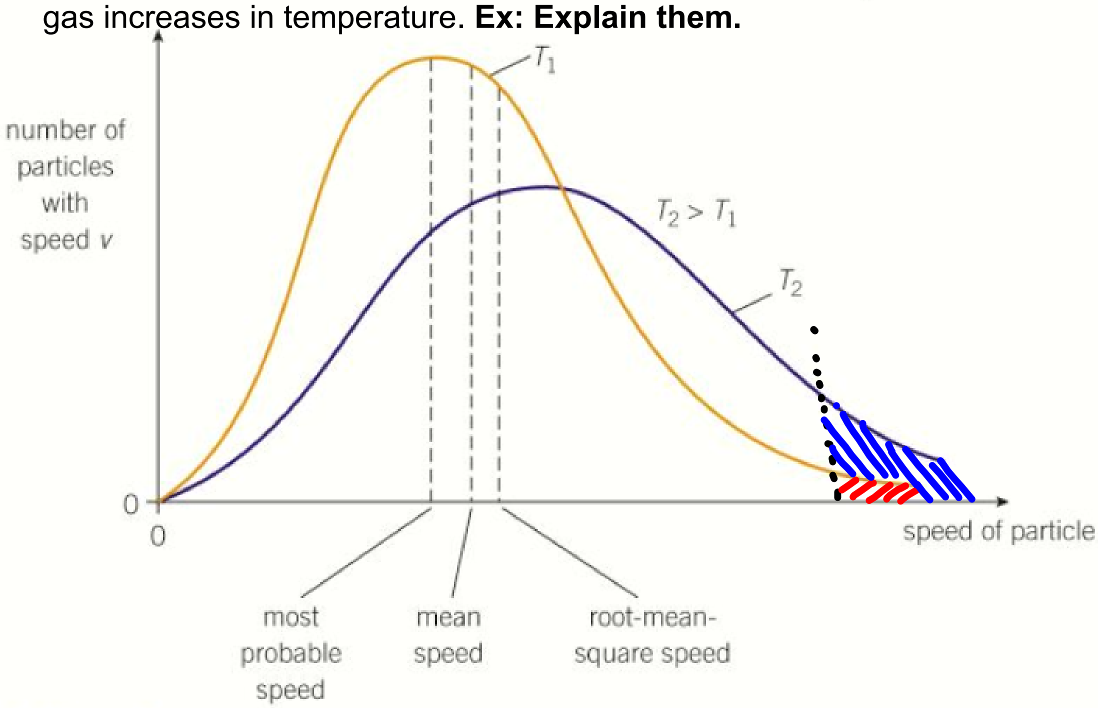
p	V	n	N	m	$\overline{c^2}$	T
4×10^5 Pa	**$6.2 \times 10^{-5}\ \text{m}^3$**	1×10^{-2}	**6.02×10^{21}**	**6.6×10^{-27} kg**	**$1.9 \times 10^6\ \text{m}^2\text{s}^{-2}$**	300 K
2.4×10^5 Pa	$3.2\ \text{m}^3$	**218.5**	**1.3×10^{26}**	**6.6×10^{-27} kg**	**$2.7 \times 10^6\ \text{m}^2\,\text{s}^{-2}$**	150 °C

(1 mark for each cell in bold = 8 marks)

(6) M - Define and calculate 'root mean square speed'
(7) S - Apply the equation for root mean square speed
(8) C - Derive an equation for root mean square speed.

Maxwell-Boltzmann distribution

Activity: Describe the changes in the distribution as the gas increases in temperature. **Ex: Explain them.**



▲ **Figure 2** *The spread of speeds of particles in a gas is called the Maxwell–Boltzmann distribution, and is broader at the high temperature T_2 than at the low temperature T_1*

(6) M - Define and calculate 'root mean square speed'
(7) S - Apply the equation for root mean square speed
(8) C - Derive an equation for root mean square speed.

Plenary

a A small dust particle in the upper atmosphere is struck by five molecules in succession. The speeds of the molecules are 300 ms^{-1}, 500 ms^{-1}, 700 ms^{-1}, 400 ms^{-1}, and 600 ms^{-1}.

Calculate:

i the mean speed of the molecules, $\bar{c}$

........500 ✓........ (*1 mark*)

ii the root mean square speed of the molecules, $\sqrt{\overline{c^2}}$.

........520 ✓........

.... (*2 marks*)

b Show that for an ideal gas the r.m.s speed of its molecules is given by the formula

$$\sqrt{\overline{c^2}} = \sqrt{\frac{3RT}{M}}$$

Question number	Answer	Marks	Guidance
4 a i	$\bar{c} = \frac{(300+500+700+400+600)}{5} = 500\text{ ms}^{-1}$	A1	
4 a ii	$\overline{c^2} = \frac{(300^2+500^2+700^2+400^2+600^2)}{5}$ $\overline{c^2} = 270\,000$ r.m.s. speed $= \sqrt{\overline{c^2}} = 520\text{ ms}^{-1}$	C1 A1	
4 b	$\frac{1}{2}m\overline{c^2} = \frac{3}{2}kT$ $N \times m\overline{c^2} = 3NkT$ but $Nk = nR$ $\overline{c^2} = \frac{3nRT}{Nm}$ but $Nm = M$ when $n = 1$ $\sqrt{\overline{c^2}} = \sqrt{\frac{3RT}{M}}$	M1 M1 M1 A0	From data book Using $pV = NkT = nRT$ in data book **ALLOW** alternative method if all steps are shown and are logical
4 c	$\frac{\text{r.m.s speed at } 250\,°\text{C}}{\text{r.m.s speed at } 25\,°\text{C}} = \frac{\sqrt{(273+250)}}{\sqrt{(273+25)}}$ $\frac{\text{r.m.s speed at } 250\,°\text{C}}{\text{r.m.s speed at } 25\,°\text{C}} = 1.3(2)$	C1 A1	Mark can be deducted if a unit is provided with the answer
4 d	$\sqrt{\overline{c^2}} = \sqrt{\frac{3RT}{M}}$ $11 \times 10^3 = \sqrt{\frac{3 \times 8.31 \times T}{2.0 \times 10^{-3}}}$ $T = 9700\text{ K}$ (or 9434 °C)	C1 A1	