

(d) solutions to the equation $a = -\omega^2 x$
e.g. $x = A \cos \omega t$ or $x = A \sin \omega t$ M3.9, M3.12

(e) velocity $v = \pm \omega \sqrt{A^2 - x^2}$ hence $v_{\max} = \omega A$ M2.2

(f) the period of a simple harmonic oscillator is independent of its amplitude (isochronous oscillator)

(g) graphical methods to relate the changes in displacement, velocity and acceleration during simple harmonic motion. HSW1

(6) M - Apply the equations of SHM to find the displacement of an oscillator
(7) S - Calculate the max velocity of an oscillator
(8) C - Use a graphical methods to relate changes in displacement, velocity and acceleration during SHM

Lesson 2. Graphing SHM

STARTER: A short pendulum oscillates with s.h.m. such that its acceleration a (m s^{-2}) is related to its displacement x (in m) by the equation $a = -300x$. Determine the frequency of the oscillations.

Answer

2.76 Hz \approx 2.8 Hz

HWK (due next lesson): Complete Summary questions: P

Key point

Sep 29-15:54

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Example SHM question

The stone is displaced 12cm from the equilibrium position and then released. In a time of 45 seconds the stone executes 30 oscillations. Calculate the maximum acceleration of the stone.

Ex: Determine the displacement at time = 2s

The period T is given by:
 $T = \frac{45}{30} = 1.5 \text{ s}$

The acceleration of the stone is given by:
 $a = -(2\pi f)^2 x$ or $a = -\left(\frac{2\pi}{T}\right)^2 x$

The acceleration is a maximum when the displacement x is equal to the amplitude A of the motion, which is 12 cm. Therefore the magnitude of the maximum acceleration is:
 $a = \left(\frac{2\pi}{1.5}\right)^2 \times 12 \times 10^{-2} = 2.11 \text{ m s}^{-2} \approx 2.1 \text{ m s}^{-2}$

The displacement x of the stone after 2.0 s is given by:
 $x = A \cos(2\pi f t)$
 $x = 12 \times 10^{-2} \times \cos\left(2\pi \times \frac{1}{1.5} \times 2.0\right) = -0.06 \text{ m}$

After 2.0 s the stone is displaced a distance of 6.0 cm above the equilibrium position.

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Other equations of SHM

Displacement equation

$x = A \sin(2\pi f t)$ $x = A \sin(\omega t)$

A = amplitude
 ω = angular frequency
 t = time

If displacement starts at the maximum amplitude, then

$x = A \cos(2\pi f t)$ $x = A \cos(\omega t)$

You must be careful when choosing which equation to use: always check where the object's starting point is.

Calculator must be in RADIAN mode.

Velocity equation

$v = \pm \omega \sqrt{A^2 - x^2}$

Max velocity equation

$v_{\max} = \omega A$
 $v_{\max} = 2\pi f A$

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ACTIVITY: Oscillate a 0.5 kg mass on a spring starting from rest (displacement of 10cm). Record the amplitude and find the time period. Calculate the frequency. Complete the table.

Sketch the graphs of x against t , v against t and a against t .

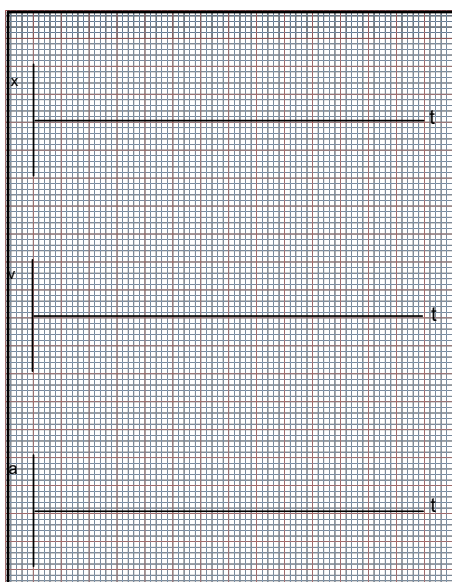
How did you ensure your time period was accurate?

Extension on worksheet

Find v_{\max} from $v = \omega x$

Key point

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Oct 30-20:28

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Plenary

14 The vibration of a component in a machine is represented by the equation:
 $x = 3.0 \times 10^{-4} \sin(240\pi t)$

where the displacement x is in metres. Determine the **a** amplitude, **b** frequency and **c** period of the vibration.

Answer

a $3.0 \times 10^{-4} \text{ m}$ (0.30 mm)
b 120 Hz
c $8.3 \times 10^{-3} \text{ s}$

Key point

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