2.2.1 Measurements and uncertainties

	Learning outcomes	Additional guidance
	Learners should be able to demonstrate and apply their knowledge and understanding of:	
(a)	systematic errors (including zero errors) and random errors in measurements	
(b)	precision and accuracy	As discussed in *The Language of Measurement* (ASE 2010).
(c)	absolute and percentage uncertainties when data are combined by addition, subtraction, multiplication, division and raising to powers	As set out in the ASE publication *Signs, Symbols and Systematics* (*The ASE Companion to 16–19 Science*, 2000). A rigorous statistical treatment is not expected. *M1.5*
(d)	graphical treatment of errors and uncertainties; line of best fit; worst line; absolute and percentage uncertainties; percentage difference.	An elementary knowledge of error bars is expected at A level. HSW5 *M1.5*

(6) M - identify systematic and random errors in measurements.
(7) S - Describe the relationship between pressure and temperature
(8) C - Explain uncertainty and calculate absolute and percentage uncertainty.

Lesson 3. uncertainty

STARTER: A group of engineers are investigating the design of wind turbines.
The maximum input power P from the wind is given by the equation below, where *A* is the area swept out by the rotating blades, *ρ* is the density of air and *v* is the speed of the wind.

Kilo 10^3

Mega 10^6

Giga 10^9

$$P = \frac{1}{2}\rho A v^3$$

How can you make your working as clear as possible?

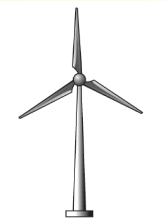


Fig. 18

Key point

$[\frac{1}{2}\rho A v^3] = (kg\,m^{-3})(m^2)(ms^{-1})^3$
$= kg\,m^{-3}\,m^2\,m^3 s^{-3}$
$= kg\,m^2 s^{-3}$

$[P] = J\,s^{-1}$
$= N\,m\,s^{-1}$
$= kg\,m\,s^{-2}\,m\,s^{-1}$
$= kg\,m^2 s^{-3}$

$\therefore\ [\frac{1}{2}\rho A v^3] = [P]$

∴ The equation is homogenous

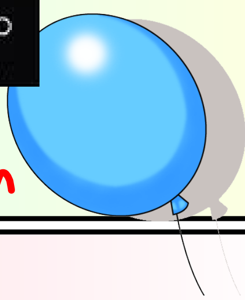
(6) M - identify systematic and random errors in measurements.
(7) S - Describe the relationship between pressure and temperature
(8) C - Explain uncertainty and calculate absolute and percentage uncertainty.

0:00:55

STARTER: How thick is sheet of A4 paper?
Take measurements.

a) In mm 0.106 mm 0.125 mm

b) In m 0.000106 m 0.000125 m



Kilo 10^3

Mega 10^6

Giga 10^9

How could you measure this **accurately.** What does accuracy mean?

Key point

Ex: - A measured value that is close to the true value.

(6) M - identify systematic and random errors in measurements.
(7) S - Describe the relationship between pressure and temperature
(8) C - Explain uncertainty and calculate absolute and percentage uncertainty.

Can be corrected after data is taken.

0:00:00

Discuss: Identify the sources of **systematic** and **random** error in this measurement.

First look up and record these terms

Kilo 10^3
Mega 10^6
Giga 10^9

What is the difference between error and uncertainty?

Key point

RANDOM ERRORS refer to random fluctuations in the measured data due to:

- The readability of the instrument
- The effects of something changing in the surroundings between measurements
- The observer being less than perfect

Random errors can be reduced by **averaging.**
A **precise** experiment has small random error.

SYSTEMATIC ERRORS refer to reproducible fluctuations consistently in the same direction due to:

- An instrument being wrongly calibrated
- An instrument with zero error (it does not read zero when it should – to correct for this, the value should be subtracted from every reading)
- The observer being less than perfect in the same way during each measurement.

(6) M - identify systematic and random errors in measurements.
(7) S - Describe the relationship between pressure and temperature
(8) C - Explain uncertainty and calculate absolute and percentage uncertainty.

What is Uncertainty?

When something is measured there will always be a small difference between the measured value and the true value (error). **Uncertainty is a way of expressing this doubt.**

Key point

r=0.1

In this case, the **uncertainty** is half the resolution.

0.5 ± 0.05

0.55

0.49999

What is the uncertainty in this measurement 0.5kg?

Key point

This is called the **absolute** uncertainty. Defined as: An interval, within which the true value can be expected to lie.

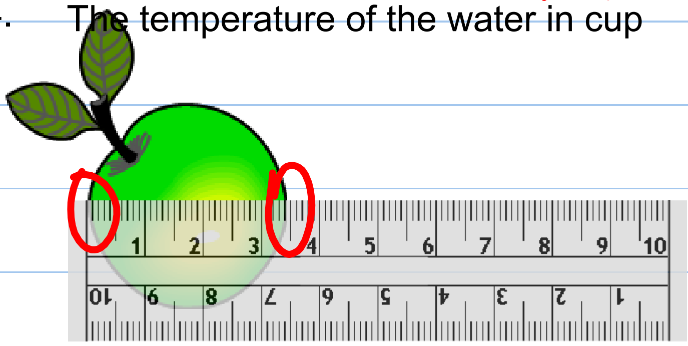
(6) M - identify systematic and random errors in measurements.
(7) S - Describe the relationship between pressure and temperature
(8) C - Explain uncertainty and calculate absolute and percentage uncertainty.

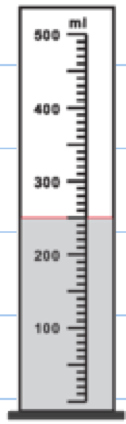
Write the value with the **absolute uncertainty.**

Of the following: 39 ± 0.5 ml

1. The volume of water in a measuring cylinder
2. The weight of your shoe 360 ± 20g
3. The length of your pen. 14.4 ± 0.1cm
4. The temperature of the water in cup

Ex: Which measurement has a slightly different rule?





Key point

(6) M - identify systematic and random errors in measurements.
(7) S - Describe the relationship between pressure and temperature
(8) C - Explain uncertainty and calculate absolute and percentage uncertainty.

Percentage uncertainty

0:00:00

Discuss: How do we calculate percentage uncertainty?

Kilo 10^3
Mega 10^6
Giga 10^9

What is the difference between error and uncertainty?

Key point

$$\text{percentage uncertainty} = \frac{\text{uncertainty}}{\text{measured value}} \times 100\%$$

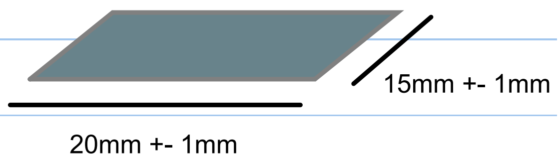
Example measuring cylinder:

Combining uncertainty

In a calculation, if several of the quantities have uncertainties then these will all contribute to the uncertainty in the answer. The following rules will help you calculate the uncertainty in your final answers.

- When quantities are added, the uncertainty is the sum of the *absolute* uncertainties.
- When quantities are subtracted, the uncertainty is also the sum of the *absolute* uncertainties.
- When quantities are multiplied, the *total percentage* uncertainty is the sum of the *percentage* uncertainties.
- When quantities are divided, the *total percentage* uncertainty is also the sum of the *percentage* uncertainties.
- When a quantity is raised to the power *n*, the *total percentage* uncertainty is *n* multiplied by the *percentage* uncertainty – for example, for a quantity x^2, total percentage uncertainty = 2 × percentage uncertainty in x.

Example surface area:
What is the surface area including the uncertainty?



(6) M - identify systematic and random errors in measurements.
(7) S - Describe the relationship between pressure and temperature
(8) C - Explain uncertainty and calculate absolute and percentage uncertainty.

Plenary

Back to the starter.... How think is a piece of paper.

$$\text{percentage uncertainty} = \frac{\text{uncertainty}}{\text{measured value}} \times 100\%$$

1. Measure with the micrometer in **mm**
2. Record the value with the absolute uncertainty 0.107 ± 0.0005
3. Calculate the percentage uncertainty. 0.107 ± 0.46% mm
4. Repeat the process but this time measure 64 thicknesses of paper. What do you notice about the percentage uncertainty?

R = 0.01 mm

0.06 mm ± 0.005 mm absolute uncertainty.

% uncertainty = $\frac{AU}{Value} \times 100 = 8.3\%$