UFABC

Cálculo Numérico
GRUPO 2
Lista 1

Professor André Pierro de Camargo	**Data** 13/07/2021

Grupo		
Nome	**RA**	**Turma**
Débora Vilela Bueno	11201812044	A2
Gabriela Rocha	11201812290	B2
George Harrison Rocha	11201810309	A2
Larissa Rodrigues de Almeida	11201812076	A2

Exercício 1

1.

Exercício 1.1

$$\phi(x) = 2 + h^2 q - \frac{1 - \frac{h^2}{4}p^2}{x} \qquad sendo\ p = 10, q = 0, h = \frac{1}{10}$$

$$\phi'(x) = \frac{1 - \frac{h^2}{4}p^2}{x^2}$$

O menor valor que uma função pode assumir é ou o menor valor entre seus extremos ou o menor valor de seus pontos críticos
Podemos então, calcular os Pontos Críticos dessa função igualando sua derivada a zero

$$\frac{1 - \frac{h^2}{4}p^2}{x^2} = 0$$

$$\frac{1 - \frac{\left(\frac{1}{10}\right)^2}{4}10^2}{x^2} = 0$$

$$\frac{\frac{3}{4}}{x^2} = 0, \text{não possui solução para os } \mathbb{R}$$

Como nossa derivada igualada a zero não apresenta solução, nosso único candidato para ponto mínimo é 1, que é extremo do intervalo [1,∞[; entretanto 1 pode ser tanto ponto mínimo quanto máximo da nossa função, por isso, se faz necessário verificarmos estudando um ponto ao redor de 1.

Para $x = 1$,

$$\phi(1) = 2 + \left(\frac{1}{10}\right)^2 .0 - \frac{1 - \frac{\left(\frac{1}{10}\right)^2}{4}10^2}{1}$$

$$\phi(1) = 2 - \frac{3}{4}$$

$$\phi(1) = \frac{8}{4} - \frac{3}{4}$$

$$\phi(1) = \frac{5}{4} = 1{,}25$$

Para $x = 2$,

$$\phi(2) = 2 + \left(\frac{1}{10}\right)^2 0 - \frac{1 - \frac{\left(\frac{1}{10}\right)^2}{4}10^2}{2}$$

$$\phi(2) = 2 - \left(\frac{3}{4} . \frac{1}{2}\right)$$

$$\phi(2) = \frac{16}{8} - \frac{3}{8}$$

$$\phi(2) = \frac{13}{8} = 1{,}625$$

Como $\phi(1) < \phi(2)$, *podemos concluir que 1 é o ponto mínimo da nossa função e, consequentemente,* $\phi(x) \geq 1$ *para todo* $x \geq 1$

2.

Exercício 1.2

$$\phi(x) = 2 + h^2 q - \left(1 - \frac{h^2}{4}p^2\right)x^{-1}$$

$$\phi'(x) = \left(1 - \frac{h^2}{4}p^2\right)x^{-2} = \frac{1 - \frac{h^2}{4}p^2}{x^2}$$

$$max\,|\phi'(x)| \le \frac{3}{4} \qquad x \in [1, \infty[$$

Perceba que, no intervalo $[1, \infty[$, $\phi'(x)$ *atinge valor máximo quando* $x = 1$

$$\phi'(1) = \frac{1 - \frac{h^2}{4}p^2}{1^2} \qquad sendo\ p = 10, q = 0, h = \frac{1}{10}$$

$$\phi'(1) = 1 - \frac{\left(\frac{1}{10}\right)^2}{4}10^2$$

$$\phi'(1) = 1 - \frac{1}{4} \Leftrightarrow \phi'(1) = \frac{3}{4}$$

Portanto, a inequação $max\,|\phi'(x)| \le \frac{3}{4}$ *é válida*

3.

a.

f(1): 0.25

f(2): -0.375

Como f(1)f(2) < 0 então conclui-se, pelo teorema visto em aula, que existe ao menos um número α ∈ [1, 2] tal que f(α) = 0

b. k = 65

c.

α0: 2.0000000000000000
α1: 1.6250000000000000
α2: 1.5384615384615383
α3: 1.5125000000000000
α4: 1.5041322314049586
α5: 1.5013736263736264
α6: 1.5004574565416284
α7: 1.5001524390243901
α8: 1.5000508078447312
α9: 1.5000169353746104
α10: 1.5000056450611359
α11: 1.5000018816799638
α12: 1.5000006272258677
α13: 1.5000002090752018
α14: 1.5000000696917242
α15: 1.5000000232305737
α16: 1.5000000077435245
α17: 1.5000000025811748
α18: 1.5000000008603915
α19: 1.5000000002867973
α20: 1.5000000000955991
α21: 1.5000000000318663
α22: 1.5000000000106222
α23: 1.5000000000035407
α24: 1.5000000000011802
α25: 1.5000000000003935
α26: 1.5000000000001312
α27: 1.5000000000000437
α28: 1.5000000000000147
α29: 1.5000000000000049
α30: 1.5000000000000016
α31: 1.5000000000000004
α32: 1.5000000000000002
α33: 1.5000000000000000
α34: 1.5000000000000000
α35: 1.5000000000000000
α36: 1.5000000000000000
α37: 1.5000000000000000
α38: 1.5000000000000000
α39: 1.5000000000000000
α40: 1.5000000000000000
α41: 1.5000000000000000
α42: 1.5000000000000000
α43: 1.5000000000000000
α44: 1.5000000000000000
α45: 1.5000000000000000
α46: 1.5000000000000000
α47: 1.5000000000000000
α48: 1.5000000000000000
α49: 1.5000000000000000
α50: 1.5000000000000000
α51: 1.5000000000000000
α52: 1.5000000000000000
α53: 1.5000000000000000
α54: 1.5000000000000000
α55: 1.5000000000000000
α56: 1.5000000000000000
α57: 1.5000000000000000
α58: 1.5000000000000000
α59: 1.5000000000000000
α60: 1.5000000000000000
α61: 1.5000000000000000
α62: 1.5000000000000000
α63: 1.5000000000000000
α64: 1.5000000000000000
α65: 1.5000000000000000

d.

$$\phi(x) = x \qquad sendo\ p = 10, q = 0, h = \frac{1}{10}$$

$$\phi(x) = 2 + h^2q - \frac{1 - \frac{h^2}{4}p^2}{x}$$

$$\phi(x) = 2 + \frac{1}{10}^{2} .0 - \frac{1 - \frac{\left(\frac{1}{10}\right)^2}{4} 10^2}{x}$$

$$\phi(x) = 2 - \frac{1 - \frac{1}{4}}{x}$$

$$\phi(x) = 2 - \frac{0{,}75}{x}$$

$$\phi(x) = \frac{2x\ - 0{,}75}{x}$$

$Como\ \phi(x) = x\ \longrightarrow x = \frac{2x - 0{,}75}{x}$

$x^2 = 2x - 0{,}75$

$x^2 - 2x + 0{,}75 = 0\ \longrightarrow\ \Delta = 1$

$x = \frac{2 \pm 1}{2}$

$x' = 1{,}5;\ x = 0{,}5 \quad \therefore \quad \alpha' = 1{,}5;\ \ \alpha'' = 0{,}5$

$|\alpha_k - \alpha| < \varepsilon;\ \alpha_k = 1{,}5;\ \varepsilon = 10^{-8};\ \alpha' = 1{,}5$

$|1{,}5 - 1{,}5| < 10^{-8}$

$0\ < 10^{-8}$

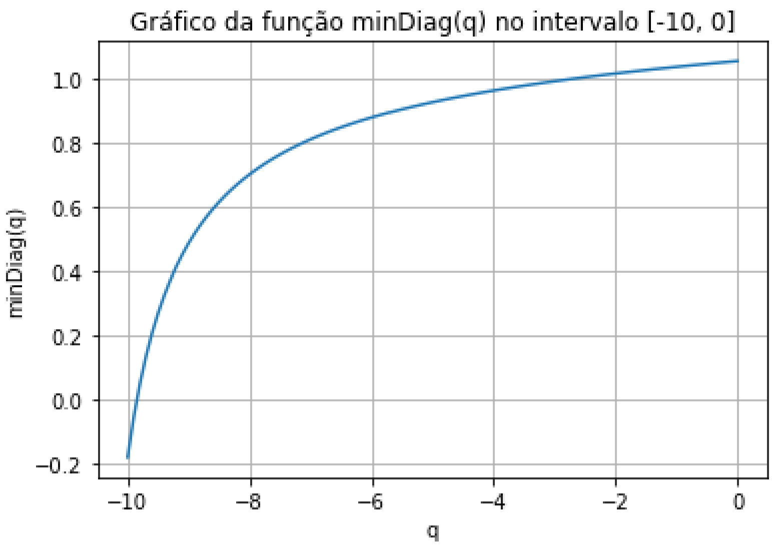
$Quando\ \alpha$ *é* $igual\ a$ 1,5; $a\ aproxima$*ção* $\alpha_k\ satisfaz\ \ |\alpha_k - \alpha| < \varepsilon$

Exercício 2

1.

```
q       minDiag(q)
-10: -0.179150823019887
 -9:  0.485512432446793
 -8:  0.699706671288196
 -7:  0.808794313514776
 -6:  0.876852010300157
 -5:  0.924605103546851
 -4:  0.960790776148577
 -3:  0.989734213026507
 -2:  1.013823615943179
 -1:  1.034486941101393
  0:  1.052631578947369
```

2.



3.

```
TODAS as iterações geradas pelo método das secantes
NumIter             iter                   new iter               f(new iter)              controle
      1 -9.9000000000000004e+00 -9.8564652550845651e+00 -7.3407154511546668e-03  5.3886103334728339e-05
      2 -9.8564652550845651e+00 -9.8496652627026506e+00 -3.4513819216175357e-04  1.1912037158755988e-07
      3 -9.8496652627026506e+00 -9.8493297740087922e+00 -2.2986959129500661e-06  5.2838985118376776e-12
      4 -9.8493297740087922e+00 -9.8493275245990901e+00 -7.2456973754242426e-10 -1.0383901446714872e-16
```

Exercício 3

1.

	Iteracao 1	Iteracao 2	Iteracao 3	Iteracao 4	Iteracao 5	Iteracao 6	Iteracao 7
[1,]	0.0290644802652380	0.0620052935694767	0.0942388309120125	0.1241726062941370	0.1512207802452441	0.1752420973695945	0.1963141055302150
[2,]	0.0691327545681070	0.1367811518733645	0.1996030478501398	0.2563689435190467	0.3067823867398521	0.3510061269059115	0.3894334810145116
[3,]	0.1090326105039824	0.2086431377251687	0.2978437230181141	0.3765980429194793	0.4453889143330034	0.5049642422219076	0.5561892325860118
[4,]	0.1414038221156857	0.2657868416166712	0.3743023255735278	0.4682600120523476	0.5490667768473830	0.6181451073704249	0.6763672791002174
[5,]	0.1614316237476618	0.2999720553488989	0.4184063603650486	0.5192043265561841	0.6046034176628649	0.6755690687817991	0.7341187488507362
[6,]	0.1663712362360925	0.3064133596918088	0.4239999661026689	0.5224199359753503	0.6022769367162729	0.6669327469252623	0.7193984291259470
[7,]	0.1553654188901599	0.2837096595771482	0.3894652948094806	0.4716617752443491	0.5363890172333400	0.5879488568453124	0.6293657144889896
[8,]	0.1293134267456481	0.2336757536287431	0.3077612120763389	0.3637470180374474	0.4072996392519406	0.4417553523434036	0.4693127180155668
[9,]	0.0906805618863348	0.1404077852167054	0.1757084942731767	0.2023849661333476	0.2231371952800029	0.2395548719558561	0.2526855803072653

2.

```
  n            En
 100: 4.1123689875410818e-05
 200: 1.0280859061628789e-05
 300: 4.5692645669337395e-06
 400: 2.5702107859792989e-06
 500: 1.6449350030978138e-06
 600: 1.1423147612266149e-06
 700: 8.3924883620412061e-07
 800: 6.4254891829484961e-07
 900: 5.0769272430528645e-07
1000: 4.1122264926585217e-07
```