



Cálculo Numérico  
**GRUPO 2**  
**Lista 2**

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<b>Grupo</b>		
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## Exercício 1

### Tarefa 1

1

1.0000000000000000  
0.7688000000000000  
0.3865801600000001  
-0.0171952883199999  
-0.3130696609057276  
-0.4194820888917828  
-0.3303546374685648  
-0.1121148364807320  
0.1274467709980197  
0.2847330011201696

### Tarefa 2

1

3.68238772807672e-16  
-9.42477765070660e+00  
1.50000004934803e+01  
1.14313901976492e+01  
-5.74851558112778e+00  
-2.16429994342159e+00  
6.52532906614688e-01  
1.64254141130614e-01  
-3.54941476485151e-02

2

Calculando  $F_j$ :

$$\begin{aligned}F_j''(x) &= f_j(x) = \sum_{i=0}^j \binom{j}{i} \binom{j+i}{i} \left(\frac{x-1}{2}\right)^i \\F_j'(x) &= \int F_j''(x) = \sum_{i=0}^j \binom{j}{i} \binom{j+i}{i} \int \left(\frac{x-1}{2}\right)^i \\F_j'(x) &= \int F_j''(x) = \sum_{i=0}^j \binom{j}{i} \binom{j+i}{i} \frac{1}{2^i} \int (x-1)^i \\F_j'(x) &= \int F_j''(x) = \sum_{i=0}^j \binom{j}{i} \binom{j+i}{i} \frac{(x-1)^{i+1}}{(i+1)2^i} \\F_j(x) &= \int F_j'(x) = \sum_{i=0}^j \binom{j}{i} \binom{j+i}{i} \int \frac{(x-1)^{i+1}}{(i+1)2^i} \\F_j(x) &= \int F_j'(x) = \sum_{i=0}^j \binom{j}{i} \binom{j+i}{i} \frac{1}{(i+1)2^i} \int (x-1)^{i+1} \\F_j(x) &= \int F_j'(x) = \sum_{i=0}^j \binom{j}{i} \binom{j+i}{i} \frac{(x-1)^{i+2}}{(i+1)(i+2)2^i}\end{aligned}$$

-1.0000000000000000  
-1.39723253269819  
0.99985871629146  
1.39623936211975  
-1.0000000000000000

### 3

k	$E_{m,k}$	
2:	2.11503493182779e+00	16: 8.54097117475305e-08
3:	4.65926518784316e-01	17: 9.16407574269584e-08
4:	1.29124376281809e-01	18: 1.16811843975384e-07
5:	2.69084772919949e-02	19: 1.49951993488884e-07
6:	5.33968432466758e-03	20: 4.27778902079012e-07
7:	9.09028062504869e-04	21: 3.45682398533720e-06
8:	1.43031107841418e-04	22: 8.87621154244123e-06
9:	2.02639258984338e-05	23: 1.13171540656598e-03
10:	2.65731734505614e-06	24: 9.82341935441555e-03
11:	3.53055053414764e-07	25: 1.61649104520109e+01
12:	6.40173075661110e-08	26: 8.06008356702353e+02
13:	8.18064185281742e-08	27: 2.75318390686568e+04
14:	8.17986203216492e-08	28: 6.31996211648756e+05
15:	8.54131190219931e-08	29: 2.83238351837420e+07
		30: 5.53545981854637e+08

### Tarefa 3

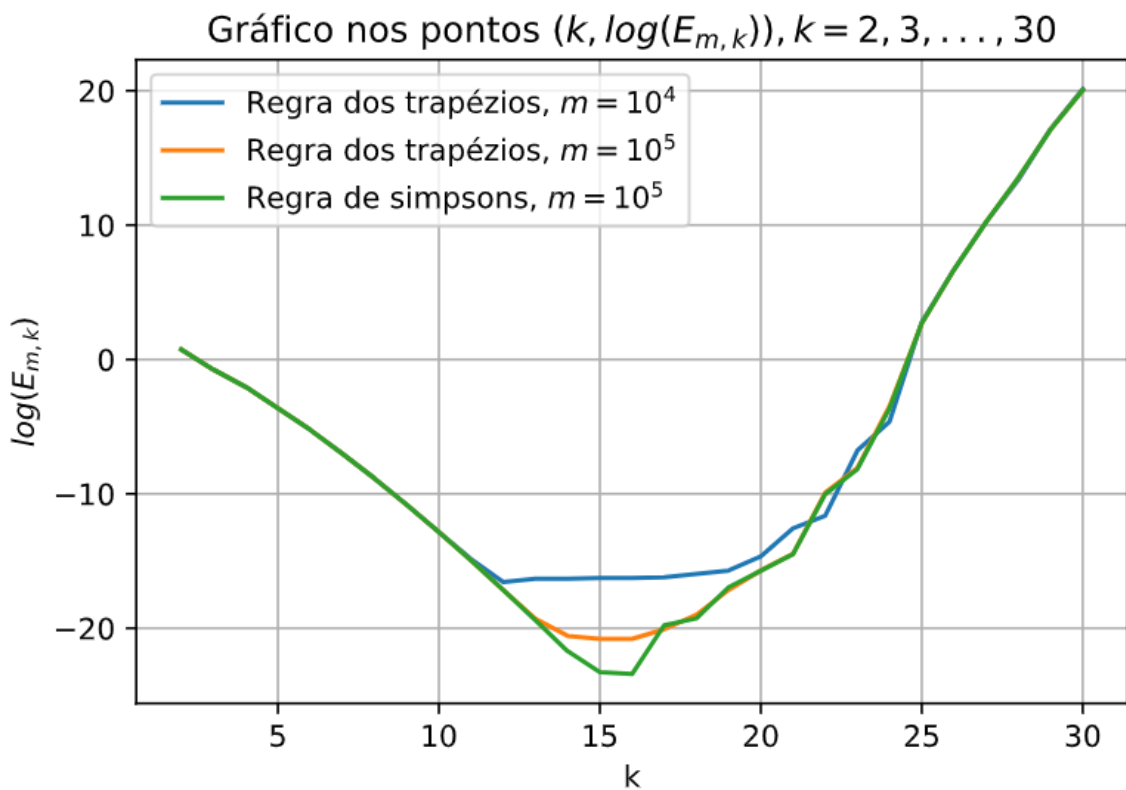
#### 1

k	$E_{m,k}$	
2:	2.11503492469818e+00	16: 9.26677068413539e-10
3:	4.65926464544369e-01	17: 1.90554283463484e-09
4:	1.29124439127210e-01	18: 5.54045820333471e-09
5:	2.69084511087114e-02	19: 3.52349232013438e-08
6:	5.33968375352623e-03	20: 1.51336258191748e-07
7:	9.08993954670978e-04	21: 5.19428687839607e-07
8:	1.43100759776971e-04	22: 4.91237719886239e-05
9:	2.02307850438732e-05	23: 3.11854119275257e-04
10:	2.64888284640108e-06	24: 3.13908388204767e-02
11:	3.18983395197758e-07	25: 1.46038575116924e+01
12:	3.49643232144814e-08	26: 7.92886577361134e+02
13:	4.11725054050294e-09	27: 2.79339942297951e+04
14:	1.17126508492049e-09	28: 7.36587510863195e+05
15:	9.29157750739762e-10	29: 2.70155667257183e+07
		30: 5.04067924555524e+08

#### 2

k	$E_{m,k}$	
2:	2.11503492462615e+00	16: 6.76469991134354e-11
3:	4.65926463996488e-01	17: 2.57319254792776e-09
4:	1.29124439761996e-01	18: 4.27741986275265e-09
5:	2.69084508442367e-02	19: 4.35734730519499e-08
6:	5.33968374774285e-03	20: 1.46938716083511e-07
7:	9.08993610154563e-04	21: 5.04027302783427e-07
8:	1.43101463317530e-04	22: 4.44097319431958e-05
9:	2.02304504324236e-05	23: 2.83294860920602e-04
10:	2.64879763656189e-06	24: 2.61204323414179e-02
11:	3.18669965637675e-07	25: 1.48496450634439e+01
12:	3.57451284127563e-08	26: 7.87244507788824e+02
13:	3.73634190253824e-09	27: 2.79479769930506e+04
14:	3.71193520365409e-10	28: 7.48088065720730e+05
15:	7.75705055744424e-11	29: 2.68074731799837e+07
		30: 4.99992309400118e+08

3



A regra de integração mais eficiente é a regra de simpsons, pois atinge os menores valores de erros.

Quando  $k > 20$  temos uma instabilidade que faz o erro calculado crescer, isso acontece devido a limitação da máquina, quando atinge valores extremamente pequenos.

#### Tarefa 4

1

Matriz:

```
[15, 135, 1495]
[135, 1495, 18495]
[1495, 18495, 243847]
```

Vetor de termos independentes:

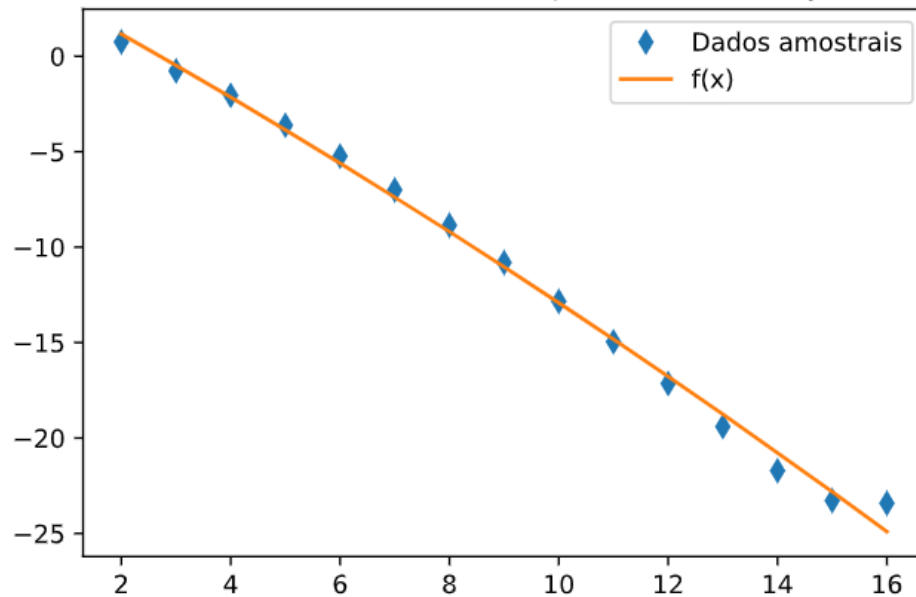
```
[-170.33636413008873 -2054.4245235020608 -26431.99577106991]
```

2

[4.345932955328876, -1.5567567737710966, -0.016965347157808588]

3

Gráfico com os dados amostrais e a função de mínimos quadrados obtida



## Exercício 2

1

t_j	x_j	y_j	z_j
0.00	0.7416000000000000	0.7416000000000000	-0.4832000000000000
0.02	0.731170970533011	0.745950320550440	-0.477121291083451
0.04	0.658402867431580	0.799590053264362	-0.457992920695942
0.06	0.557003355652040	0.869250617463324	-0.426253973115364
0.08	0.451011369794600	0.931476398956622	-0.382487768751223
0.10	0.356477564467200	0.970952668723200	-0.327430233190400
0.12	0.282974254183295	0.978997057978758	-0.261971312162053
0.14	0.234942884003160	0.952207445980743	-0.187150329983903
0.16	0.212887051254730	0.891259096050379	-0.104146147305108
0.18	0.214419077796467	0.799843854417347	-0.014262932213814
0.20	0.235168111411200	0.683744205619200	0.081087682969600
0.22	0.269557714046352	0.550034957314017	0.180407328639631
0.24	0.311460873742406	0.406405306491320	0.282133819766274
0.26	0.354740356217940	0.260594018192671	0.384665625589388
0.28	0.393682291206025	0.119930426979885	0.486387281814090
0.30	0.423330867763200	-0.009026049484800	0.585695181721600
0.32	0.439731991898758	-0.120755216255752	0.681023224356994
0.34	0.440093738998478	-0.210961578700892	0.770867839702414
0.36	0.422871412643429	-0.276683365134823	0.853811952491394
0.38	0.387785000550931	-0.316332488617931	0.928547488067000
0.40	0.335776797491200	-0.329672862924800	0.993896065433600
0.42	0.268916944159673	-0.317744508558813	1.048827564399140
0.44	0.190264610111475	-0.282740905563377	1.092476295451903

0.46	0.103692527990930	-0.227847070753713	1.124154542762782
0.48	0.013682565415507	-0.157045857866726	1.143363292451219
0.50	-0.074900000000000	-0.074900000000000	1.149800000000000
0.52	-0.157045857866726	0.013682565415508	1.143363292451219
0.54	-0.227847070753713	0.103692527990931	1.124154542762782
0.56	-0.282740905563377	0.190264610111475	1.092476295451903
0.58	-0.317744508558813	0.268916944159673	1.048827564399140
0.60	-0.329672862924800	0.335776797491200	0.993896065433600
0.62	-0.316332488617931	0.387785000550931	0.928547488067001
0.64	-0.276683365134823	0.422871412643430	0.853811952491393
0.66	-0.210961578700890	0.440093738998478	0.770867839702414
0.68	-0.120755216255751	0.439731991898757	0.681023224356994
0.70	-0.009026049484800	0.423330867763200	0.585695181721600
0.72	0.119930426979885	0.393682291206027	0.486387281814089
0.74	0.260594018192671	0.354740356217941	0.384665625589388
0.76	0.406405306491319	0.311460873742402	0.282133819766274
0.78	0.550034957314018	0.269557714046355	0.180407328639630
0.80	0.683744205619202	0.235168111411202	0.081087682969599
0.82	0.799843854417350	0.214419077796466	-0.014262932213815
0.84	0.891259096050379	0.212887051254730	-0.104146147305109
0.86	0.952207445980745	0.234942884003161	-0.187150329983902
0.88	0.978997057978760	0.282974254183292	-0.261971312162053
0.90	0.970952668723199	0.356477564467198	-0.327430233190403
0.92	0.931476398956624	0.451011369794601	-0.382487768751223
0.94	0.869250617463319	0.557003355652036	-0.426253973115366
0.96	0.799590053264361	0.658402867431575	-0.457992920695944
0.98	0.745950320550441	0.731170970533021	-0.477121291083450
1.00	0.741600000000009	0.741600000000002	-0.483200000000000

2

j	$t_j^*$	$px(t_j^*)+py(t_j^*)+pz(t_j^*)$			
0	0.00	1.000000000000000	22	0.44	1.000000000000007
1	0.02	1.000000000000002	23	0.46	0.999999999999998
2	0.04	1.000000000000002	24	0.48	0.999999999999996
3	0.06	1.000000000000002	25	0.50	1.000000000000000
4	0.08	1.000000000000000	26	0.52	1.000000000000009
5	0.10	1.000000000000000	27	0.54	1.000000000000002
6	0.12	1.000000000000000	28	0.56	1.000000000000009
7	0.14	0.999999999999999	29	0.58	0.999999999999999
8	0.16	1.000000000000002	30	0.60	1.000000000000000
9	0.18	1.000000000000002	31	0.62	1.000000000000000
10	0.20	1.000000000000000	32	0.64	0.999999999999998
11	0.22	1.000000000000000	33	0.66	1.000000000000007
12	0.24	0.999999999999999	34	0.68	0.999999999999998
13	0.26	1.000000000000002	35	0.70	0.999999999999989
14	0.28	0.999999999999998	36	0.72	1.000000000000007
15	0.30	0.999999999999998	37	0.74	0.999999999999996
16	0.32	1.000000000000004	38	0.76	0.999999999999957
17	0.34	1.000000000000000	39	0.78	1.000000000000022
18	0.36	0.999999999999998	40	0.80	1.000000000000027
19	0.38	1.000000000000002	41	0.82	1.000000000000011
20	0.40	1.000000000000000	42	0.84	1.000000000000000
21	0.42	1.000000000000000	43	0.86	1.000000000000047
			44	0.88	0.999999999999989

45	0.90	0.99999999999999947	48	0.96	0.99999999999999916
46	0.92	1.00000000000000016	49	0.98	1.00000000000000111
47	0.94	0.99999999999999895	50	1.00	1.00000000000000107

3

Gráfico no pontos  $(p_x(t_j^*), p_y(t_j^*), p_z(t_j^*))$ ,  $t_j^* = j/50, j = 0, 1, \dots, 50$

