

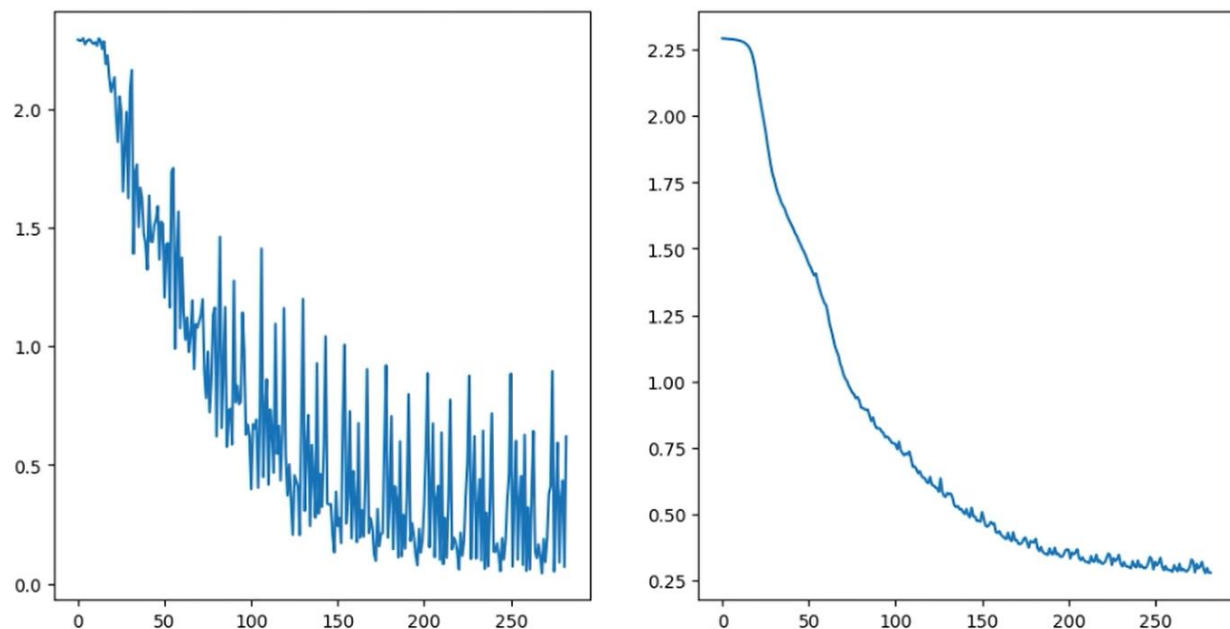
Note: we didn't implement the softmax_backward function, because we only need to use it once, so we did it in the L_layer_model.

Note: in L_layer_model we return 1 extra item which is a list containing the training/validation costs/ accuracy, Epoch, training step. Which we used to compare between different variations of our model.

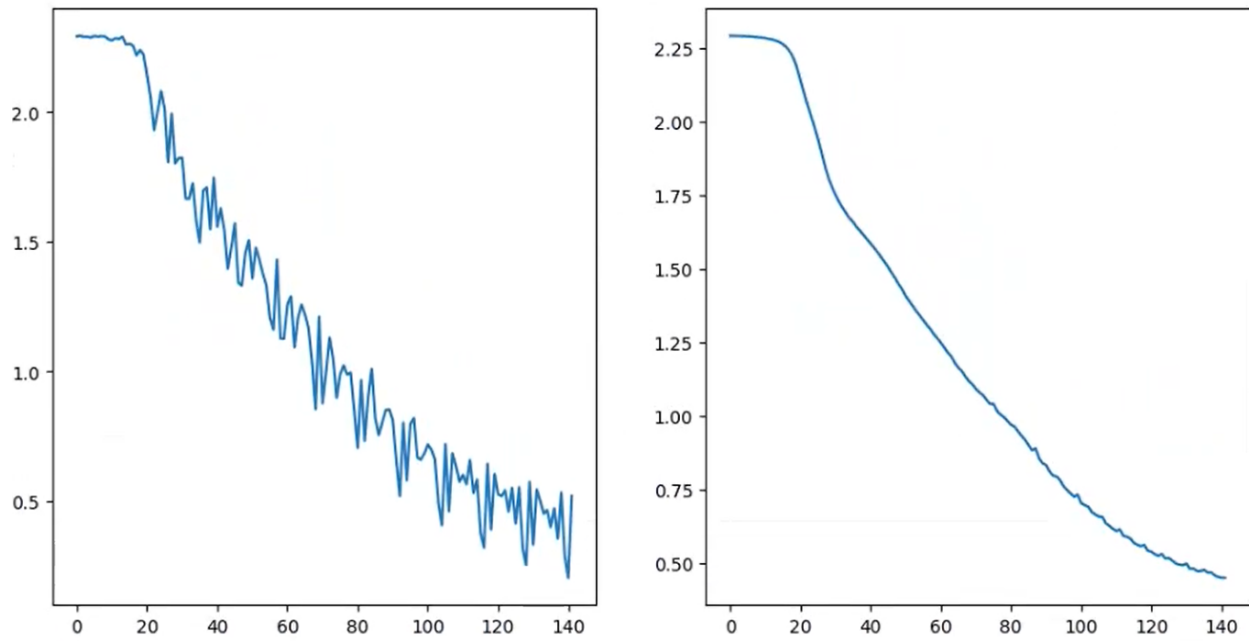
Question 4)

We ran our network on 6 different batch sizes ([16,32,64,128,256,512]) with the same parameters which were (layers = [784, 20, 7, 5, 10], learning rate = 0.009, num epochs = 100, use batchnorm = False) and got the following results:

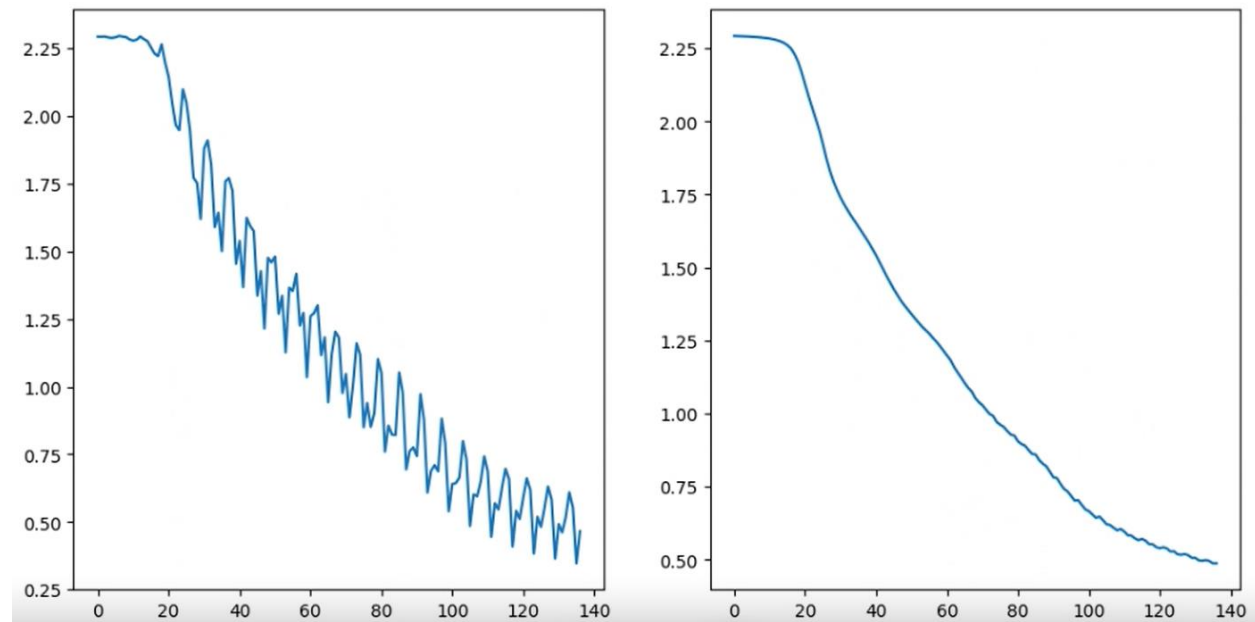
Iteration By Train Cost Vs. Iteration By Validation Cost on batch size of 16



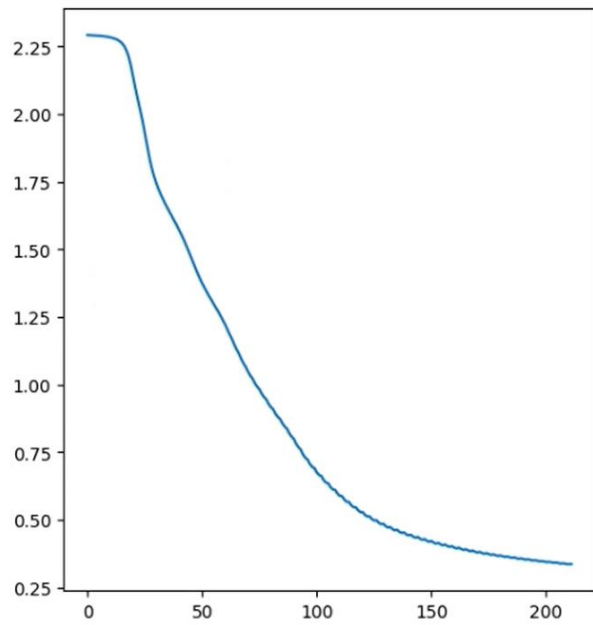
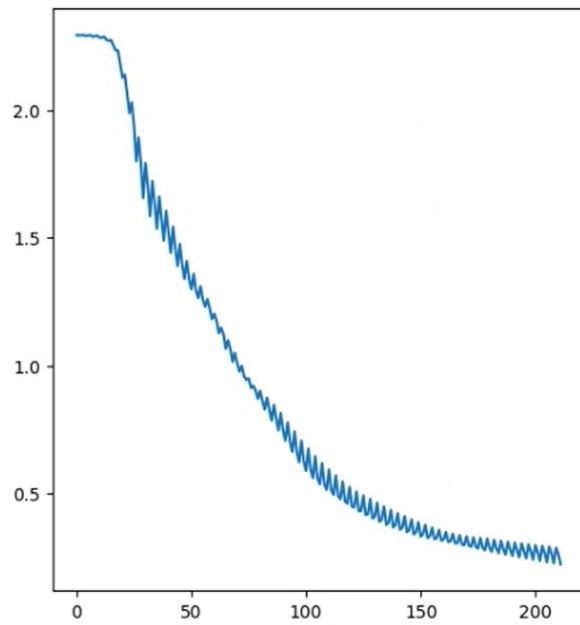
Iteration By Train Cost Vs. Iteration By Validation Cost on batch size of 32



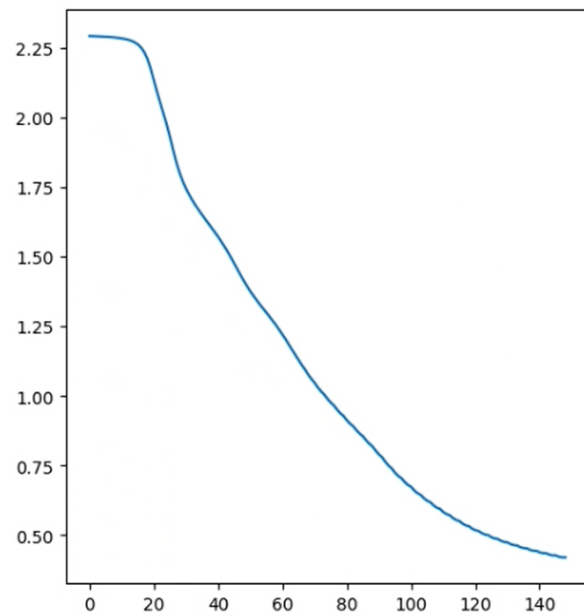
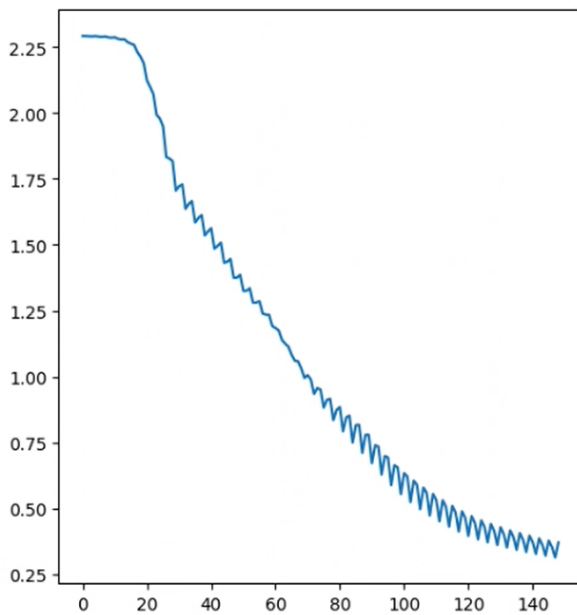
Iteration By Train Cost Vs. Iteration By Validation Cost on batch size of 64



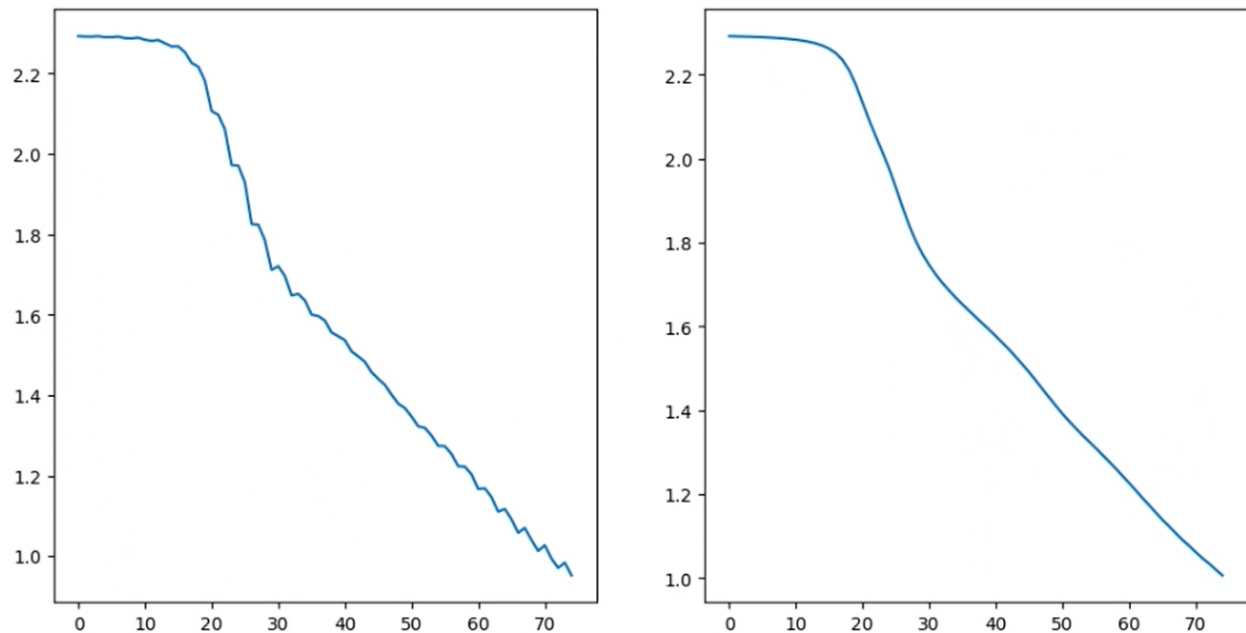
Iteration By Train Cost Vs. Iteration By Validation Cost on batch size of 128



Iteration By Train Cost Vs. Iteration By Validation Cost on batch size of 256



Iteration By Train Cost Vs. Iteration By Validation Cost on batch size of 512



Summary:

	Batch Size	Epoch Number	Iteration	Train Time in sec	Train Cost \
0	16.0	11.0	28300.0	37.532196	0.619773
1	32.0	11.0	14200.0	21.220486	0.520210
2	64.0	22.0	13700.0	24.760588	0.465770
3	128.0	70.0	21200.0	44.589424	0.223600
4	256.0	99.0	14900.0	46.293001	0.369444
5	512.0	99.0	7501.0	38.954504	0.951898

	Train Accuracy	Validation Cost	Validation Accuracy	Test Accuracy
0	0.933932	0.279818	0.919063	0.9212
1	0.889245	0.451611	0.872292	0.8796
2	0.872708	0.487337	0.859479	0.8638
3	0.919714	0.336514	0.905417	0.9063
4	0.891901	0.420726	0.881250	0.8831
5	0.655547	1.006298	0.652604	0.6595

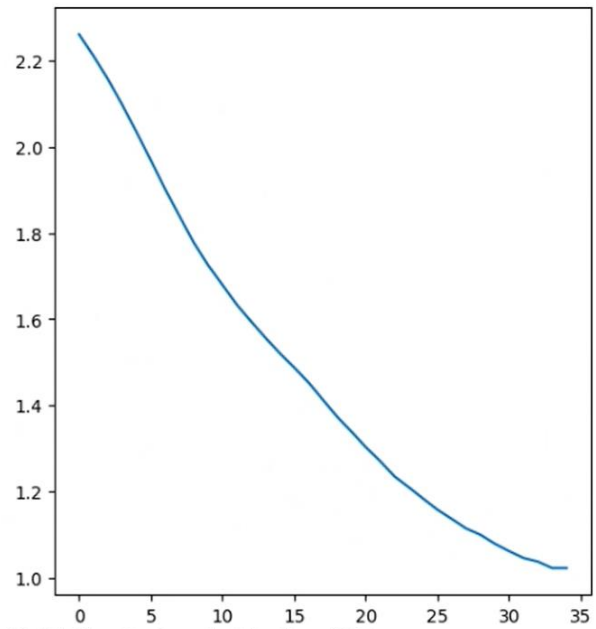
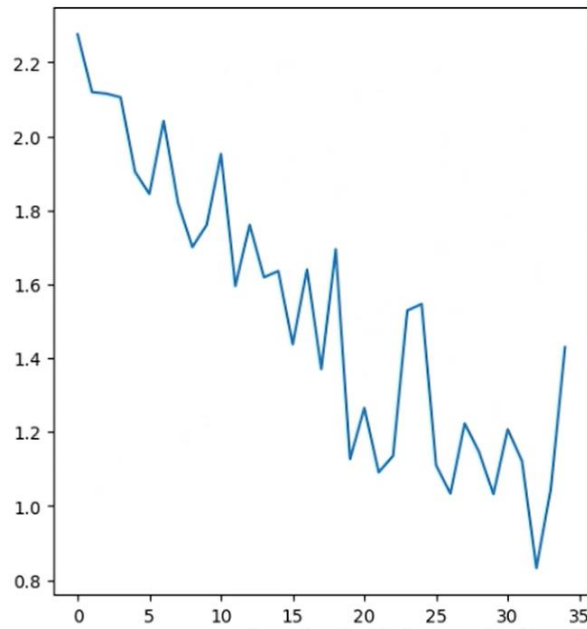
From the following table we found that the best batch size is 16, it received the highest test accuracy of 0.9212 and a runtime of 37.5 sec (note: if we wanted value to runtime more highly, we would choose batch size 32, since it got a similar test accuracy of ~ 0.88 with almost half the running time of 16).

Across all batch sizes, we can see from the graphs that our network isn't bias to overfitting but is learning the images in an appropriate way considering the overall accuracies displayed in the table above, we may attribute these accuracy numbers to the good initialization of the weights.

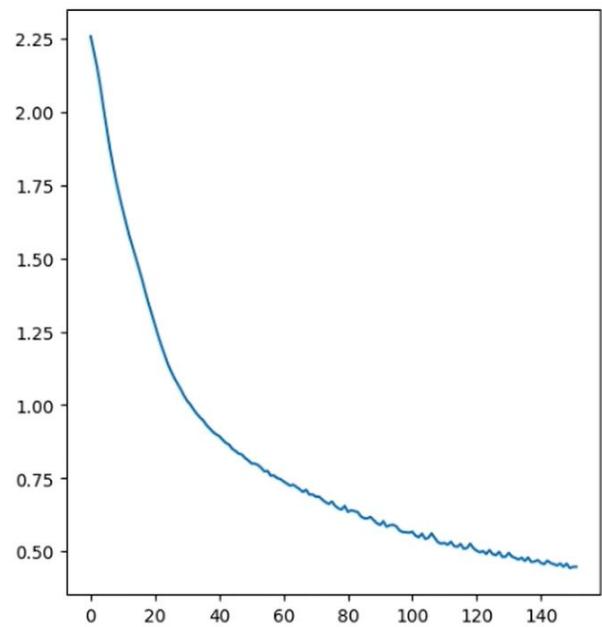
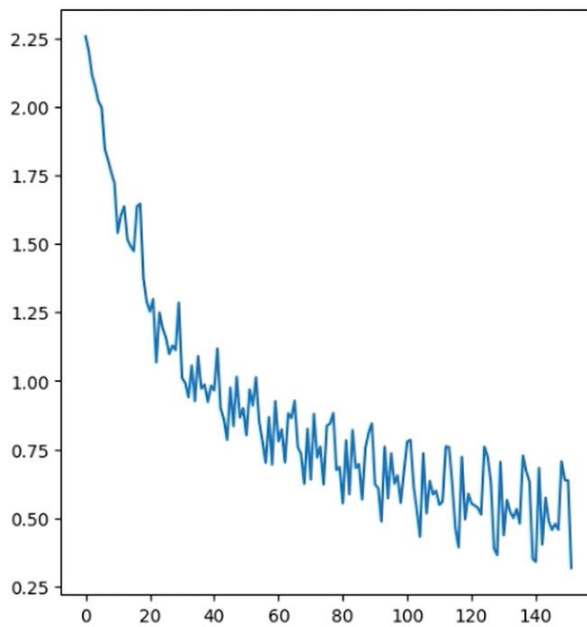
Question 5)

We ran our network on 6 different batch sizes ([16,32,64,128,256,512]) with the same parameters which were (layers = [784, 20, 7, 5, 10], learning rate = 0.009, num epochs = 100, use batchnorm = True) and got the following results:

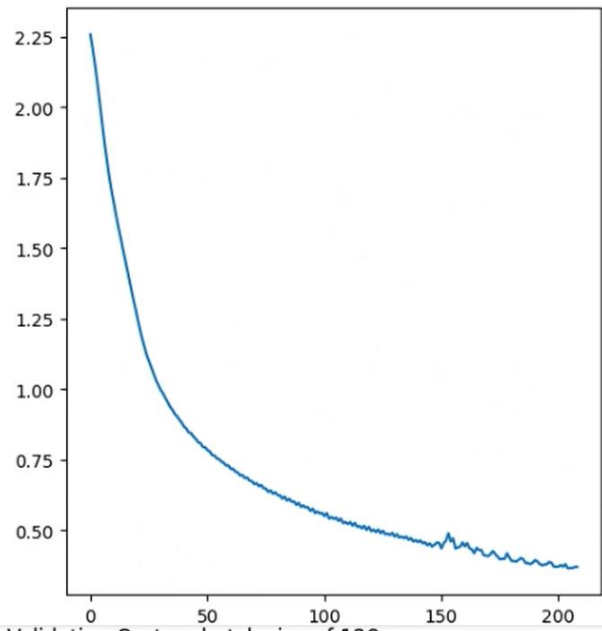
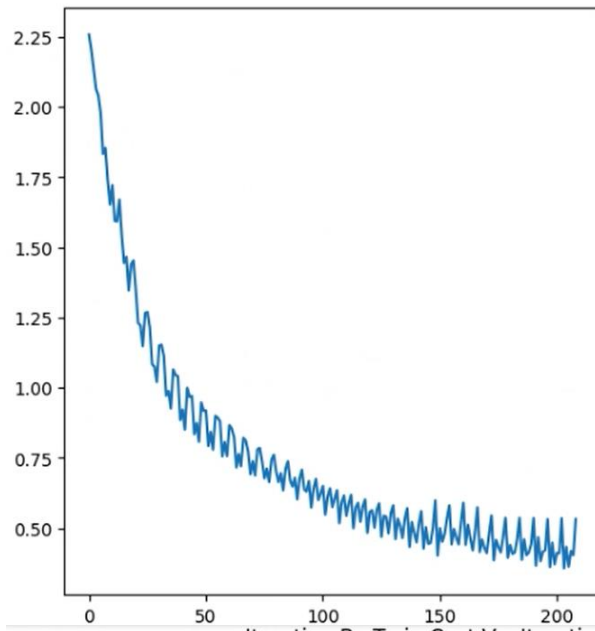
Iteration By Train Cost Vs. Iteration By Validation Cost on batch size of 16



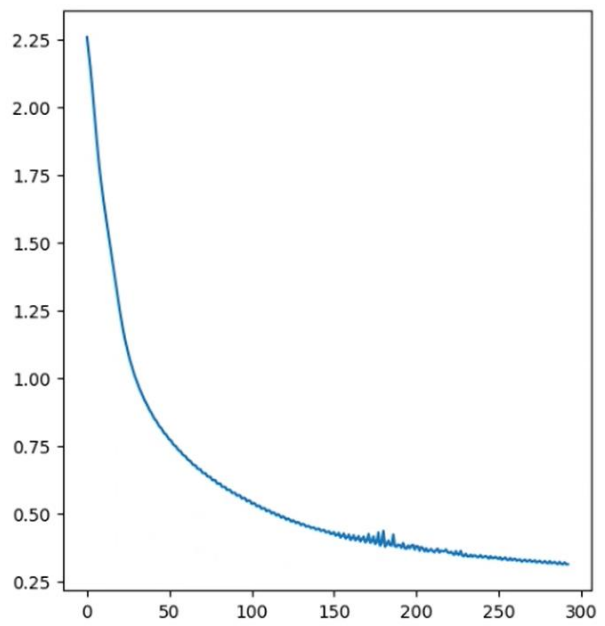
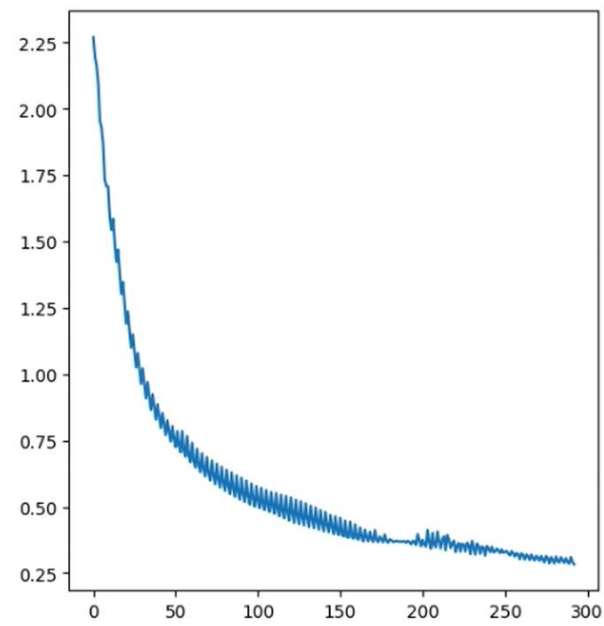
Iteration By Train Cost Vs. Iteration By Validation Cost on batch size of 32



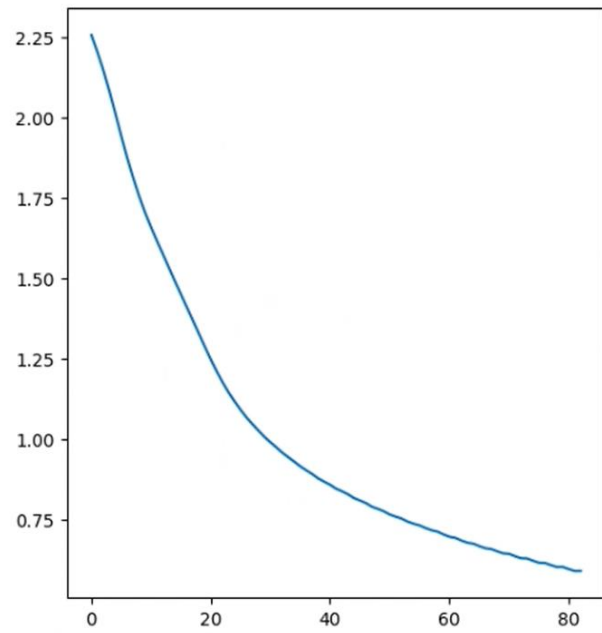
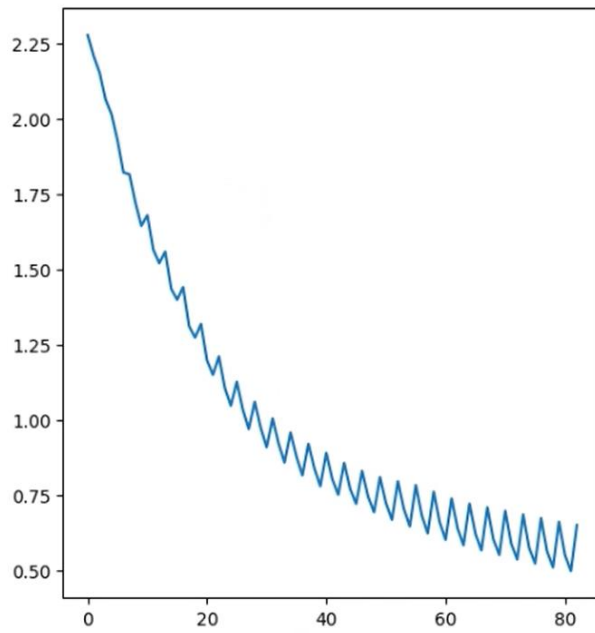
Iteration By Train Cost Vs. Iteration By Validation Cost on batch size of 64



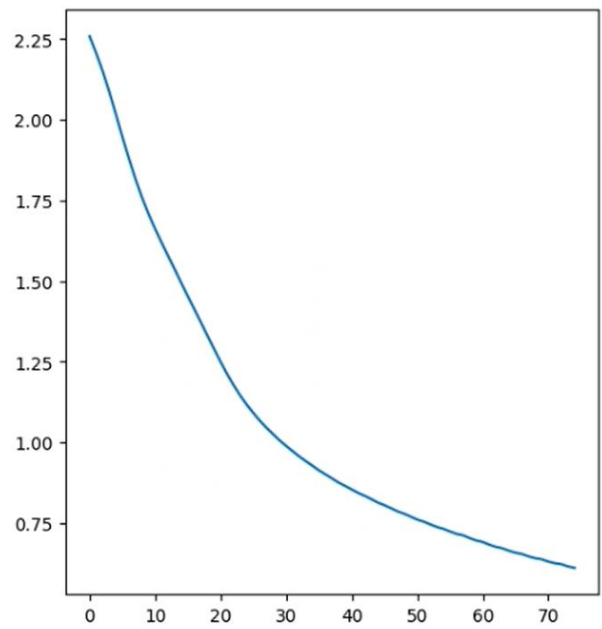
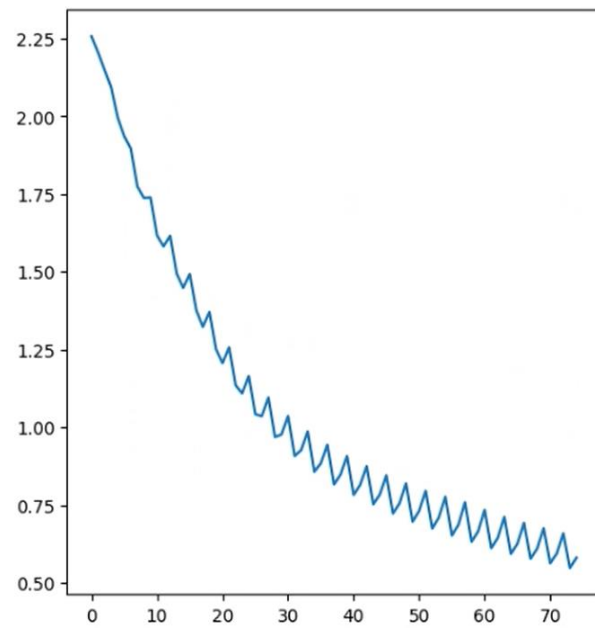
Iteration By Train Cost Vs. Iteration By Validation Cost on batch size of 128



Iteration By Train Cost Vs. Iteration By Validation Cost on batch size of 256



Iteration By Train Cost Vs. Iteration By Validation Cost on batch size of 512



Summary:

	Batch Size	Epoch Number	Iteration	Train Time in sec	Train Cost \
0	16.0	1.0	3500.0	8.445564	1.429712
1	32.0	12.0	15200.0	24.535761	0.317321
2	64.0	34.0	20900.0	39.912460	0.531921
3	128.0	97.0	29300.0	69.197941	0.283816
4	256.0	55.0	8300.0	28.682476	0.651894
5	512.0	99.0	7501.0	41.360658	0.580793

	Train Accuracy	Validation Cost	Validation Accuracy	Test Accuracy
0	0.631823	1.022430	0.637083	0.6316
1	0.884349	0.447353	0.876354	0.8827
2	0.905573	0.370209	0.894896	0.8987
3	0.925625	0.313215	0.906875	0.9130
4	0.858307	0.589825	0.854271	0.8569
5	0.853958	0.611586	0.850625	0.8545

From the summary table above, we concluded that the optimal batch size to use with `batch_norm = True` is 128 since it has the highest accuracy of 0.913 with a runtime of 69.2 seconds.

If we are deciding by the runtime, then we would recommend using the batch size 32, it has similar accuracy of 0.8827 (difference of 0.03 in accuracy) with a much faster runtime of 24.5 which is 2.8 times faster than batch size 128.

The difference in runtime is attributed to the fact that we do batch normalization, the bigger the batch, the longer our model needs to normalize it.

Note: The main contributor to the runtime values is the epoch number.

Question 6)

The changes we made on the functions so that we can do L2 norm:

added a new parameter for

`L_layer_model/L_model_backward/Linear_activation_backward/Linear_backward / Compute_cost` called `lamda` with a default value of 0.

When the user sets the value of `lamda` to be other than 0 in `L_layer_model`, we send its value through the functions until we reach `Linear_backward`, in which we only use it to calculate the derivative of `w` (`dw`) by multiplying `lamda` with the old value of `W` and adding it to `dw`.

```
def Linear_backward(dZ, cache, lamda=0):
    derative_dict = {}
    A = cache["A"].T
    W = cache["W"].T

    derative_dict["dx"] = torch.matmul(W, dZ)
    derative_dict["dw"] = torch.matmul(dZ, A) + lamda * cache["W"]
    derative_dict["db"] = torch.sum(dZ, dim=1, keepdim=True)

    return derative_dict
```


We also send lamda to the compute_cost function in which we include the regularization term into the cost function.

```
def compute_cost(AL, Y, parameters, lamda=0):
    num_samples = AL.shape[1]
    L2_regularization = 0
    # Calculate the negative log likelihood loss
    sum_res = torch.sum(Y * torch.log(AL + 0.001), dim=0)
    cost = -1 * torch.mean(sum_res).item()
    # convert params dict to params vec
    if lamda != 0:
        parameters = param_dict_2vec(parameters)
        L2_regularization = (lamda / (2*num_samples)) * (torch.sum(torch.square(parameters))).item()
    loss = cost + L2_regularization
    return loss
```

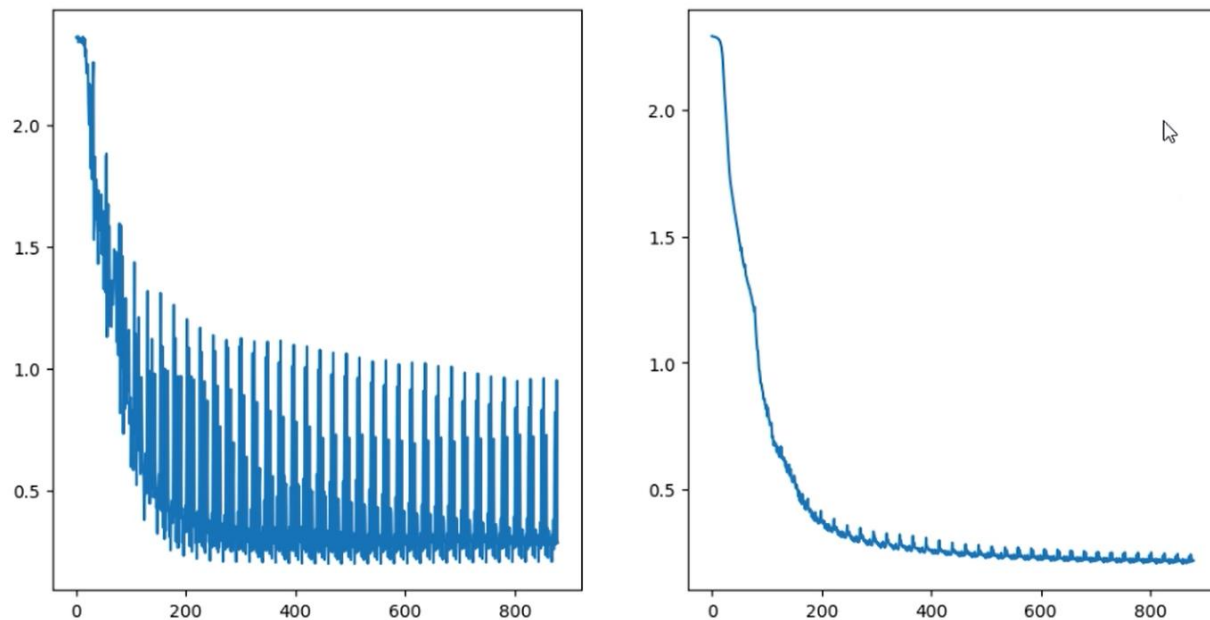
Note: we tried multiple lambda values ([0.2, 0.15, 0.1 0.05]) and decided to go with 0.05 since it gave us the best results.

(The reason why 0.05 is our optimal value is because our network before L2 wasn't bias to overfitting, so using L2 optimization didn't improve our performance that much. That's why using higher lambda values backfired, and we received lower performances)

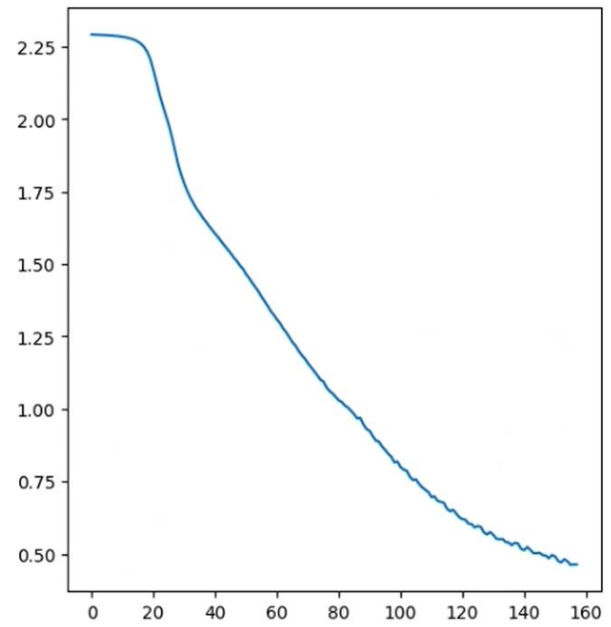
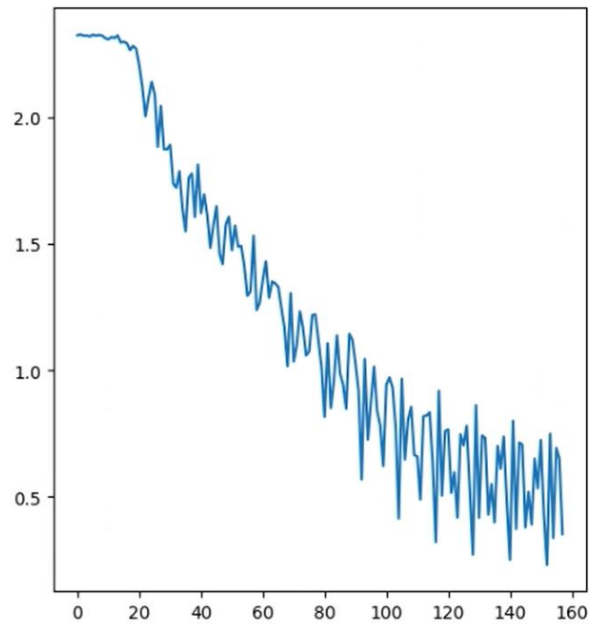
Part 1 (without batch norm):

We ran our network with the same parameters as before, this time with lambda = 0.05 and batch norm = False

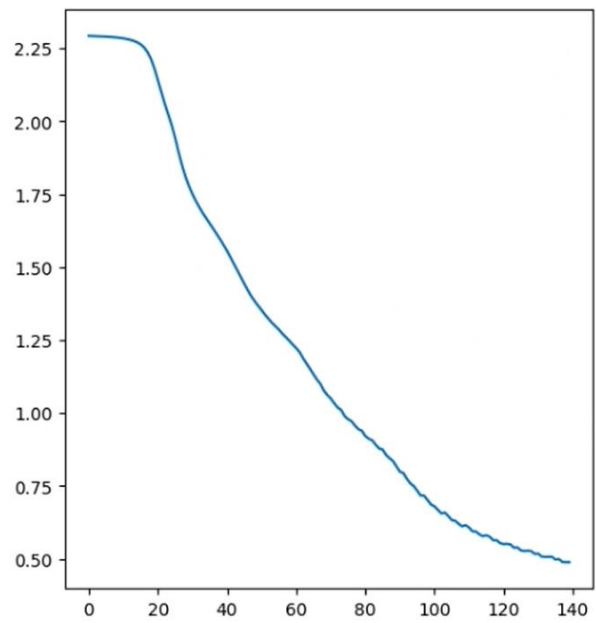
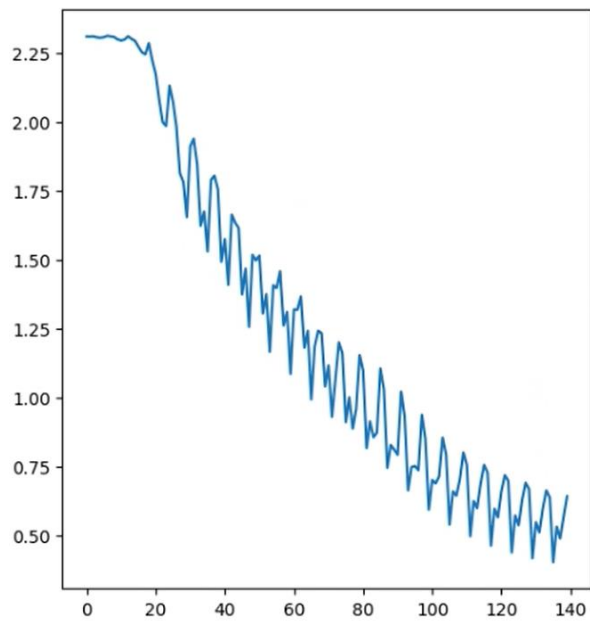
Iteration By Train Cost Vs. Iteration By Validation Cost on batch size of 16 and using lamda val of 0.05



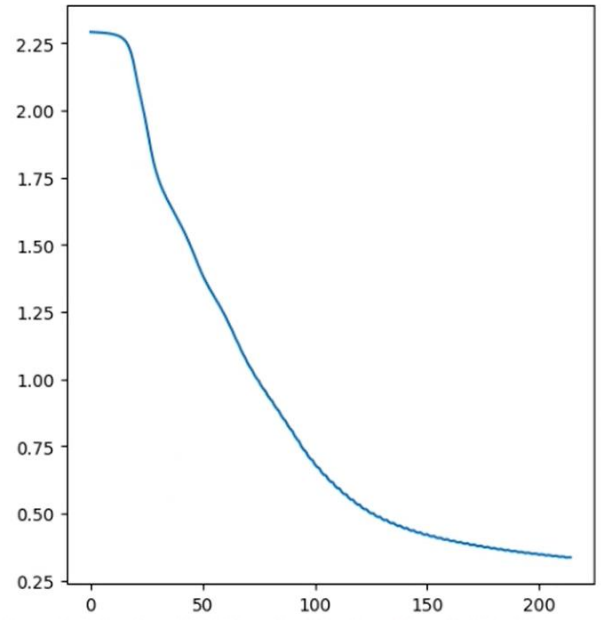
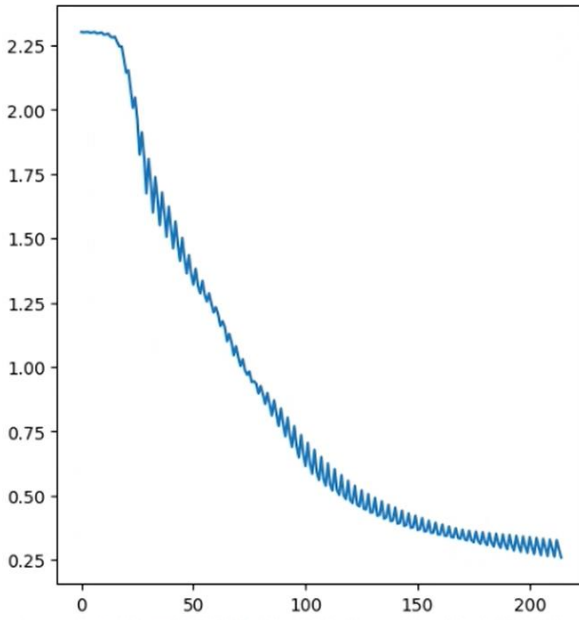
Iteration By Train Cost Vs. Iteration By Validation Cost on batch size of 32 and using lamda val of 0.05



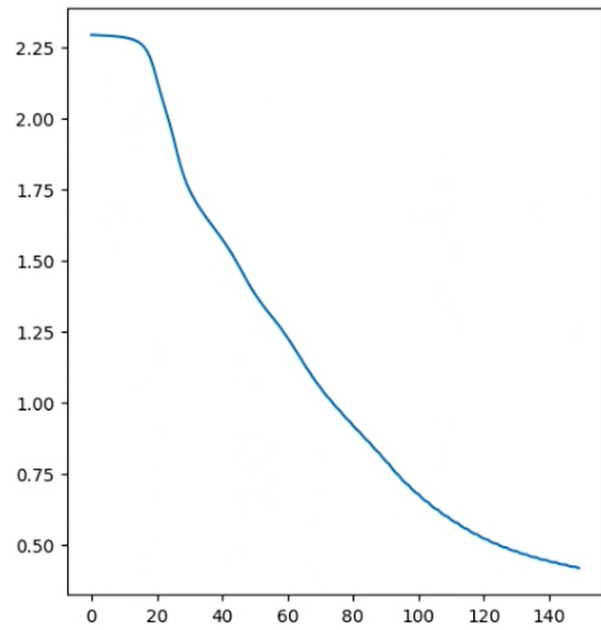
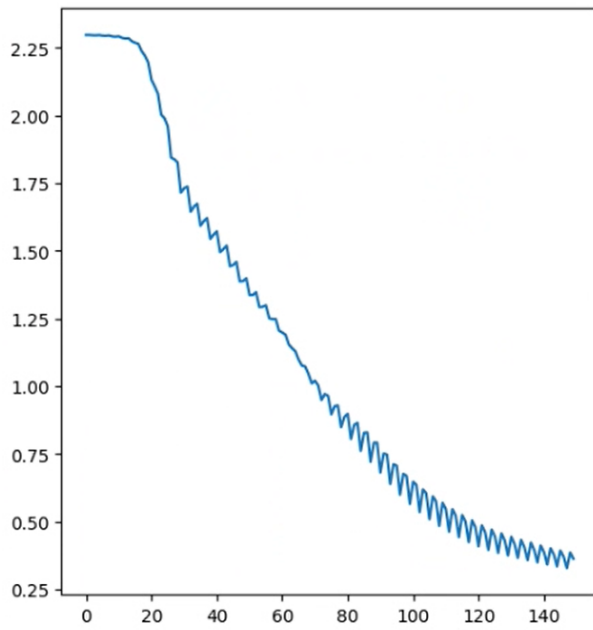
Iteration By Train Cost Vs. Iteration By Validation Cost on batch size of 64 and using lamda val of 0.05



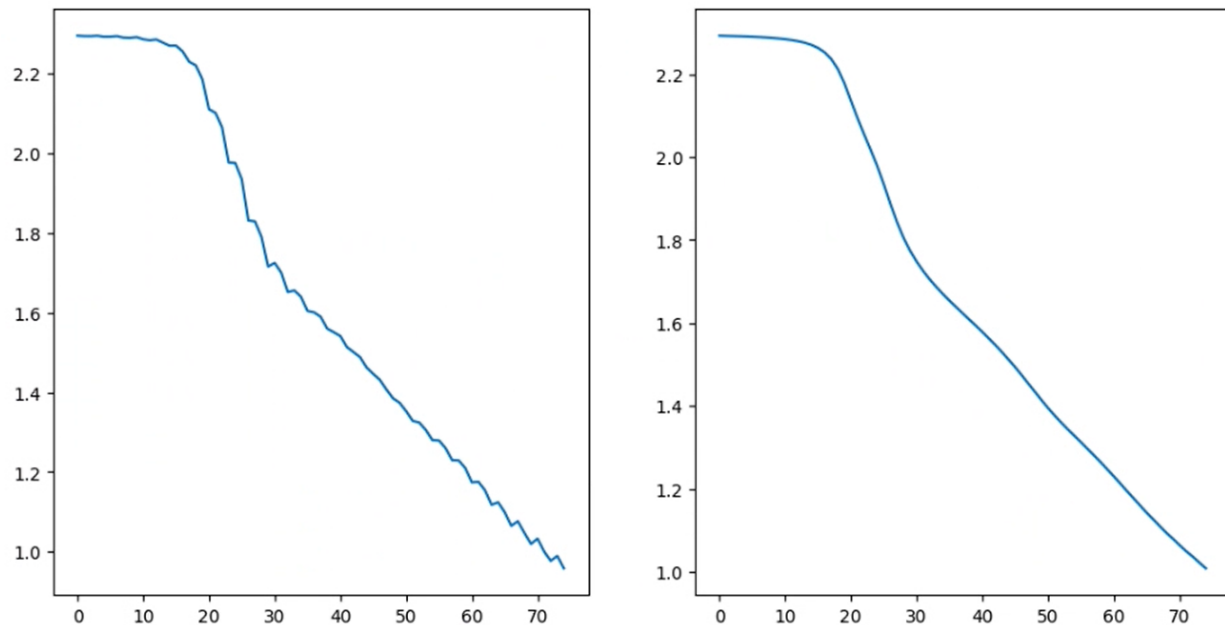
Iteration By Train Cost Vs. Iteration By Validation Cost on batch size of 128 and using lamda val of 0.05



Iteration By Train Cost Vs. Iteration By Validation Cost on batch size of 256 and using lamda val of 0.05



Iteration By Train Cost Vs. Iteration By Validation Cost on batch size of 512 and using lamda val of 0.05



Summary:

	Batch Size	Epoch Number	Iteration	Train Time in sec	Lamda	Train Cost \
0	16.0	36.0	88000.0	113.101533	0.05	0.285722
1	32.0	13.0	15800.0	22.902976	0.05	0.353336
2	64.0	23.0	14000.0	25.200291	0.05	0.641019
3	128.0	71.0	21500.0	45.270340	0.05	0.259270
4	256.0	99.0	15001.0	45.935024	0.05	0.362785
5	512.0	99.0	7501.0	38.593479	0.05	0.958572

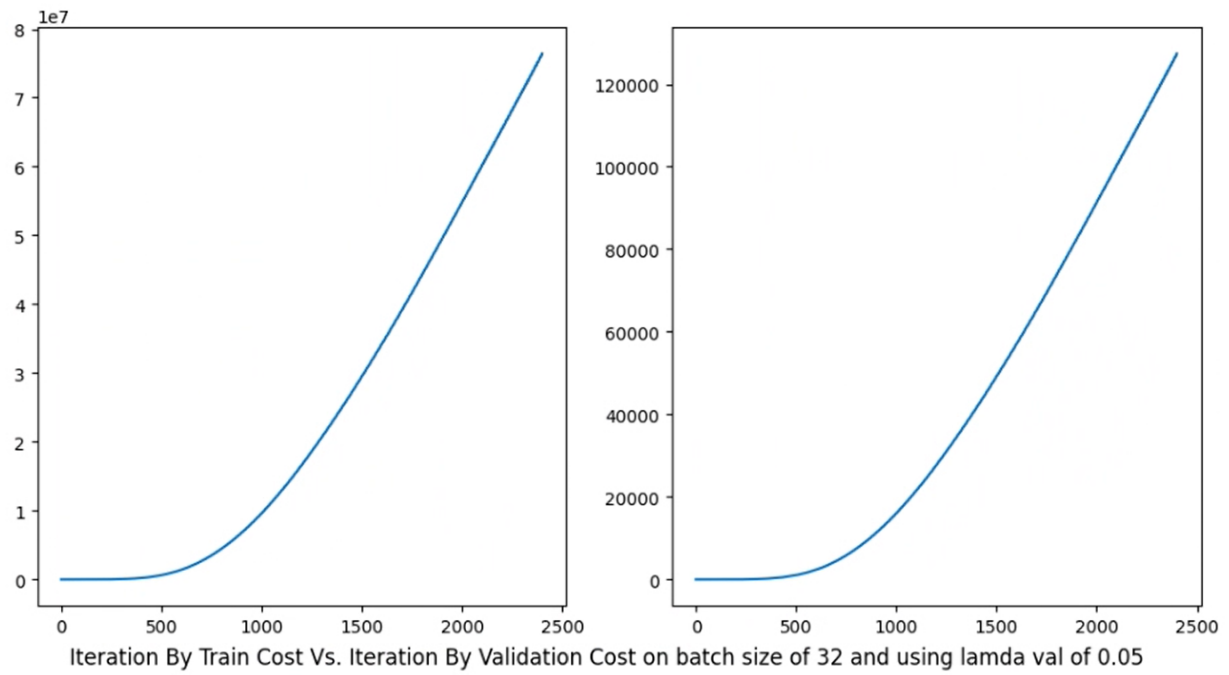
	Train Accuracy	Validation Cost	Validation Accuracy	Test Accuracy
0	0.954844	0.218033	0.934792	0.9388
1	0.885964	0.463098	0.872500	0.8793
2	0.870078	0.488456	0.858125	0.8624
3	0.919479	0.337129	0.907292	0.9069
4	0.894714	0.417856	0.883021	0.8862
5	0.655729	1.008669	0.652396	0.6588

The optimal value for batch size for the test accuracy is easily 16, we got an accuracy of almost 0.94 but it has a very big runtime of 113 seconds, compared to batch size 128 which has an accuracy of 0.907 with a much shorter runtime of 45 seconds (more than twice as fast).

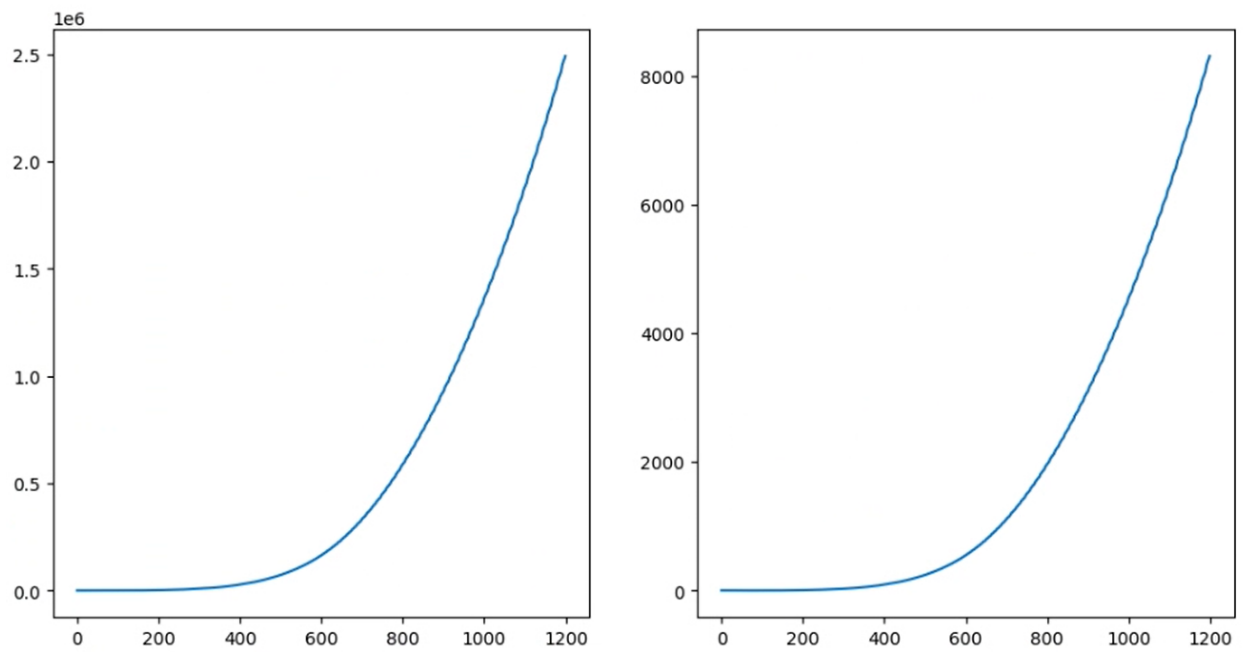
Part 2 (with batch norm):

We ran our network with the same parameters, this time with batch norm = True and with lambda = 0.05 (same as before)

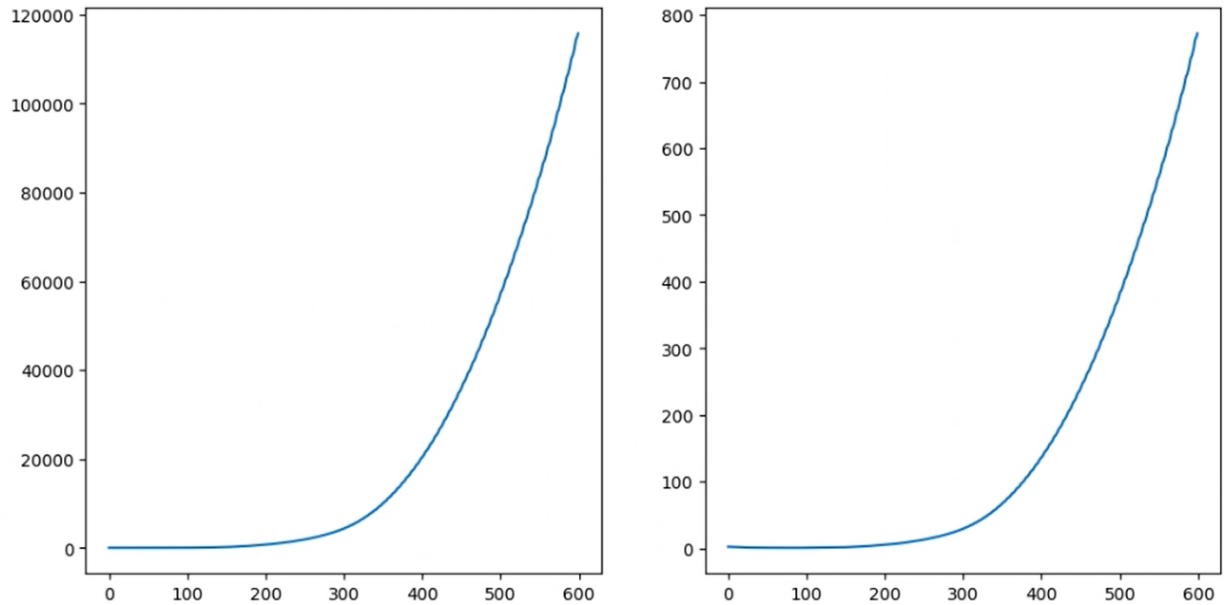
Iteration By Train Cost Vs. Iteration By Validation Cost on batch size of 16 and using lamda val of 0.05



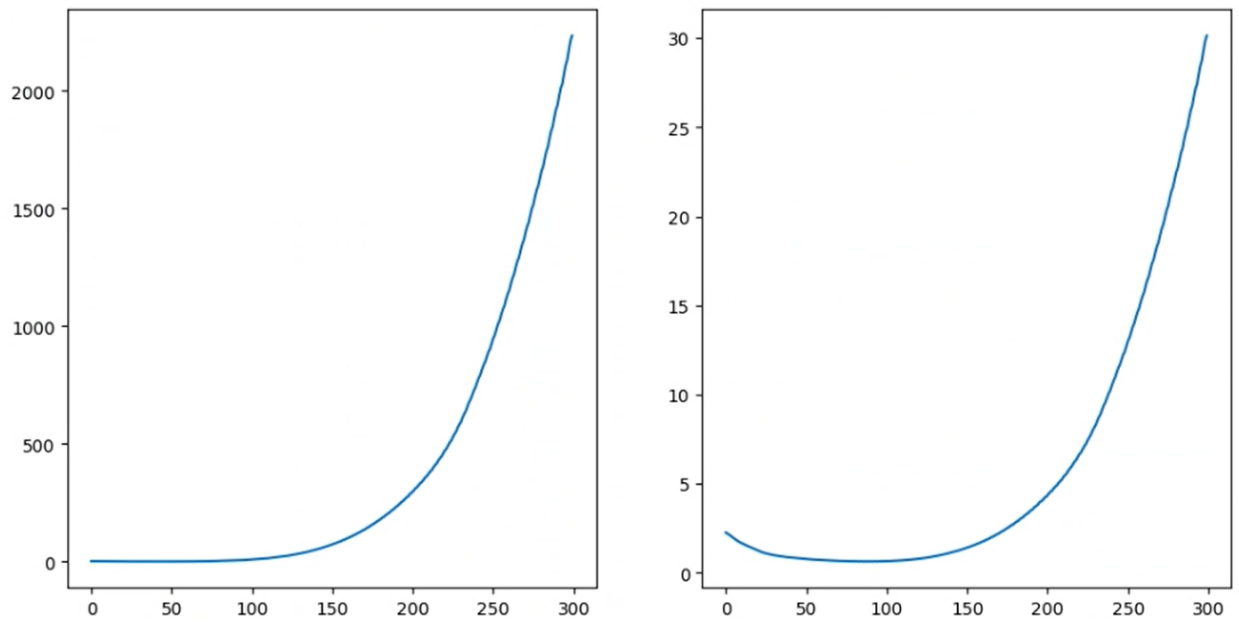
Iteration By Train Cost Vs. Iteration By Validation Cost on batch size of 32 and using lamda val of 0.05



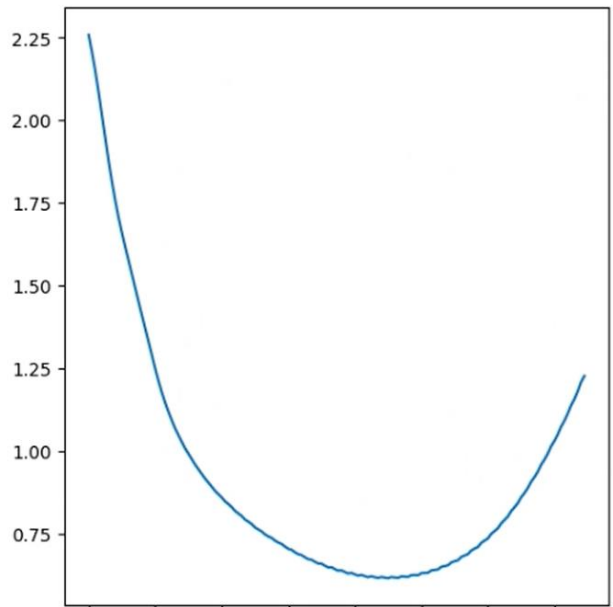
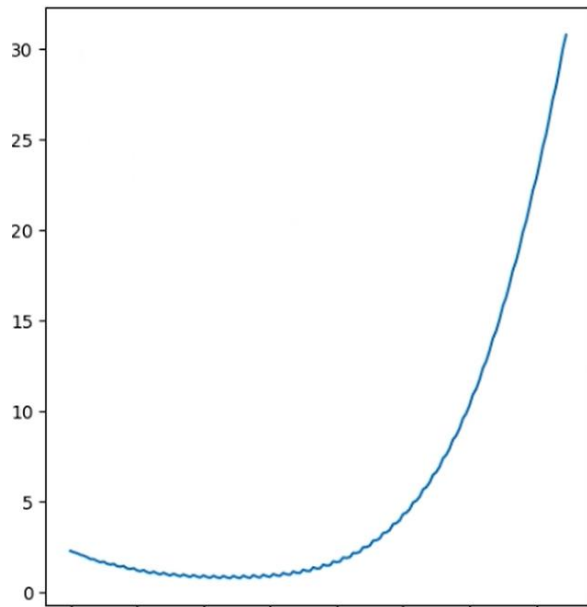
Iteration By Train Cost Vs. Iteration By Validation Cost on batch size of 64 and using lamda val of 0.05



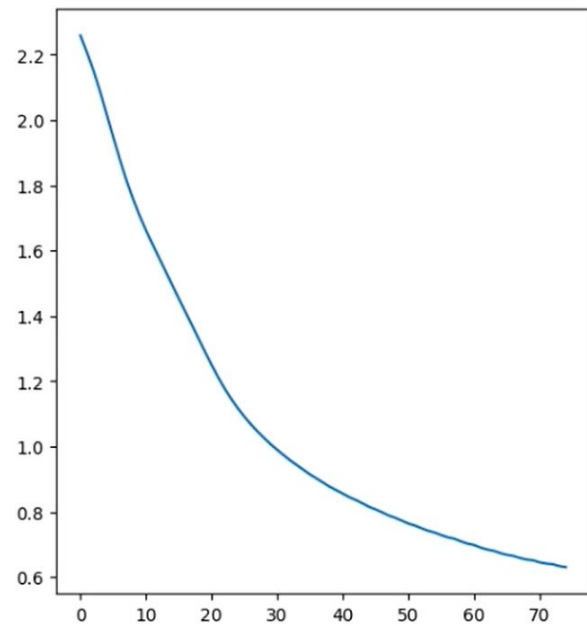
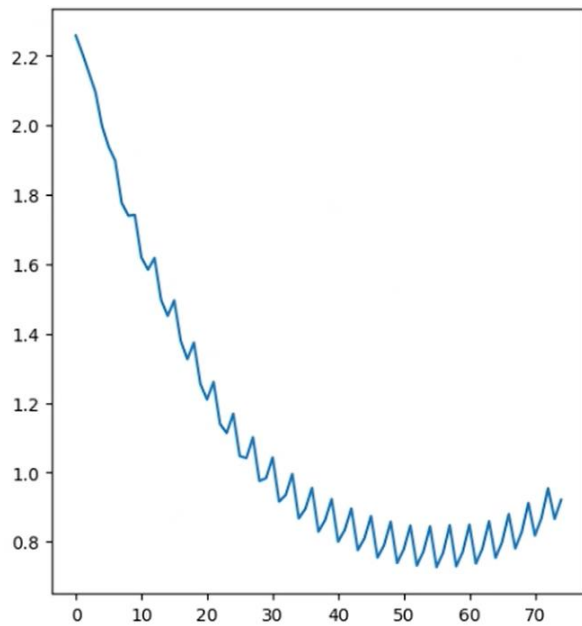
Iteration By Train Cost Vs. Iteration By Validation Cost on batch size of 128 and using lamda val of 0.05



Iteration By Train Cost Vs. Iteration By Validation Cost on batch size of 256 and using lamda val of 0.05



Iteration By Train Cost Vs. Iteration By Validation Cost on batch size of 512 and using lamda val of 0.05



Note: a possible reason why some of the graphs are increasing in cost instead of decreasing is because by normalizing the activations, it can sometimes lead to changes in the scale of the loss. The regularization effects of L2 regularization and the normalization effects of batch normalization might be interacting in a way that causes the raw loss values to increase while still improving generalization.

Summary:

	Batch Size	Epoch Number	Iteration	Train Time in sec	Lamda	Train Cost	Train Accuracy	Validation Cost	Validation Accuracy	Test Accuracy
0	16.0	99.0	240001.0	351.298554	0.05	7.640679e+07	0.859010	127345.154841	0.857187	0.8587
1	32.0	99.0	120001.0	193.917766	0.05	2.492274e+06	0.913385	8307.908510	0.900625	0.9090
2	64.0	99.0	60001.0	111.943309	0.05	1.157698e+05	0.925391	772.108136	0.906875	0.9148
3	128.0	99.0	30001.0	72.241061	0.05	2.234877e+03	0.915937	30.136424	0.898229	0.9027
4	256.0	99.0	15001.0	50.719430	0.05	3.076232e+01	0.903802	1.227017	0.891458	0.9006
5	512.0	99.0	7501.0	41.783917	0.05	9.208192e-01	0.853750	0.630801	0.850417	0.8539

Looking at the test accuracy we can say that the optimal batch size is 64 since it got the highest accuracy of 0.915 with a runtime of 112 seconds.

Weight comparison with/without L2:

```

for k in wb.keys():
    # Element-wise difference between two weight matrices of the same layer
    difference_matrix = wb1[k] - wb[k]
    # Compute the sum to obtain a scalar value
    total_difference = torch.sum(difference_matrix).item()
    print(f"difference between {k} with and without using l2 function: {total_difference}")

```

```

difference between W1 with and without using l2 function: -3.431547229449886
difference between B1 with and without using l2 function: 0.23569400110644045
difference between W2 with and without using l2 function: 1.9620542896365956
difference between B2 with and without using l2 function: -0.7003459205611358
difference between W3 with and without using l2 function: -0.7935287373372641
difference between B3 with and without using l2 function: -0.06697204202139297
difference between W4 with and without using l2 function: -0.32699345761892407
difference between B4 with and without using l2 function: -7.327471962526033e-14

```

Looking at the difference between the weights, we can see that there is a minimal difference between them, which can be attributed to the fact that our network wasn't bias to overfitting and thus didn't improve that much upon regularization.

Note:

Here we put a sample of the progress of our model (via printing the training cost every 100 step), we couldn't fit it all into this document, but they are printed in the .ipynb code file

```

The cost on trianing step number: 100, the training loss: 2.36082
The cost on trianing step number: 200, the training loss: 2.35626
The cost on trianing step number: 300, the training loss: 2.35804
The cost on trianing step number: 400, the training loss: 2.36526
The cost on trianing step number: 500, the training loss: 2.3404
The cost on trianing step number: 600, the training loss: 2.35181
The cost on trianing step number: 700, the training loss: 2.35841
The cost on trianing step number: 800, the training loss: 2.35786
The cost on trianing step number: 900, the training loss: 2.34628
The cost on trianing step number: 1000, the training loss: 2.34124
The cost on trianing step number: 1100, the training loss: 2.34834
The cost on trianing step number: 1200, the training loss: 2.33656
The cost on trianing step number: 1300, the training loss: 2.36247
The cost on trianing step number: 1400, the training loss: 2.3523
The cost on trianing step number: 1500, the training loss: 2.32803
The cost on trianing step number: 1600, the training loss: 2.35461
The cost on trianing step number: 1700, the training loss: 2.28364
The cost on trianing step number: 1800, the training loss: 2.31228
The cost on trianing step number: 1900, the training loss: 2.25225
The cost on trianing step number: 2000, the training loss: 2.21288
The cost on trianing step number: 2100, the training loss: 2.24249
The cost on trianing step number: 2200, the training loss: 2.25012
The cost on trianing step number: 2300, the training loss: 2.09842
The cost on trianing step number: 2400, the training loss: 2.09353
The cost on trianing step number: 85500, the training loss: 0.2948
The cost on trianing step number: 85600, the training loss: 0.28067
The cost on trianing step number: 85700, the training loss: 0.36283
The cost on trianing step number: 85800, the training loss: 0.23388
The cost on trianing step number: 85900, the training loss: 0.7289
The cost on trianing step number: 86000, the training loss: 0.23629
The cost on trianing step number: 86100, the training loss: 0.34537
The cost on trianing step number: 86200, the training loss: 0.22418
The cost on trianing step number: 86300, the training loss: 0.32592
The cost on trianing step number: 86400, the training loss: 0.33476
The cost on trianing step number: 86500, the training loss: 0.29598
The cost on trianing step number: 86600, the training loss: 0.30383
The cost on trianing step number: 86700, the training loss: 0.27811
The cost on trianing step number: 86800, the training loss: 0.3018
The cost on trianing step number: 86900, the training loss: 0.20795
The cost on trianing step number: 87000, the training loss: 0.32107
The cost on trianing step number: 87100, the training loss: 0.29947
The cost on trianing step number: 87200, the training loss: 0.38101
The cost on trianing step number: 87300, the training loss: 0.25921
The cost on trianing step number: 87400, the training loss: 0.64516
The cost on trianing step number: 87500, the training loss: 0.82241
The cost on trianing step number: 87600, the training loss: 0.27884
The cost on trianing step number: 87700, the training loss: 0.43086
The cost on trianing step number: 87800, the training loss: 0.95347
The cost on trianing step number: 87900, the training loss: 0.29797
The cost on trianing step number: 88000, the training loss: 0.28572

```