Question 3

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(a) Best case: pivot is always the median of the subarray causing the complexity to be $O(n\log_2(n))$

Pattern for each sub array: $\left[a_1, a_2, a_3, \dots, a_{\left\lfloor\frac{k}{2}\right\rfloor}, a_{\left\lceil\frac{k}{2}\right\rceil} + 1, a_{\left\lceil\frac{k}{2}\right\rceil} + 2, \dots, a_{k-1}, a_{\left\lceil\frac{k}{2}\right\rceil}, a_k\right]$

$a_1 \le a_2 \le \cdots \le a_k$ where $k = 1,3,7$

This pattern holds for any array of size…

- $S(n) = 2 \cdot S(n-1) + 1$ where $S(0) = 0$ and an integer $n \ge 1$
  Equivalent alternative: $2^k - 1$ where an integer $k \ge 1$
- $\{1,3,5,7,15,31,\dots\}$

Examples

i. $[k]$ where k can be any integer and is to be treated as a_k
  $n\log_2(n) = 1\log_2(1) = 0$

ii. [1,2,3]
  $n\log_2(n) = 3\log_2(3) = 4.75$

iii. [1,2,3,5,6,4,7]
  $n\log_2(n) = 7\log_2(7) = 19.65$

iv. [1,2,3,5,6,4,7,9,10,11,13,14,12,8,15]
  1,2,3,5,6,4,7 (pattern from iii.)
  9,10,11,13,14,12,15 (pattern from iii.)
  8 splits each of these subarrays

  $n\log_2(n) = 15\log_2(15) = 58.6$

v. [1,2,3,5,6,4,7,9,10,11,13,14,12,8,15,17,18,19,21,22,20,23,25,26,27,29,30,28,24,16,31]
  1,2,3,5,6,4,7,9,10,11,13,14,12,8,15 (from iv.)
    8 splits this subarray into equal subarrays
  17,18,19,21,22,20,23,25,26,27,29,30,28,31 (from iv.)
    24 splits this subarray into equal subarrays
  16 splits each of these subarrays

  $n\log_2(n) = 31\log_2(31) = 153.58$

(b) Worst case: pivot is always the largest element in each subarray causing the complexity to be $O(n^2)$

Pattern for each array: $[2,3,4,\dots,n-1,n,1]$ where the size of the array is at least 3

Examples

- [2,3,1]
  $n^2 = 3^2 = 9$
- [2,3,4,5,1]
  $n^2 = 5^2 = 25$

…

- $[2,3,\dots,17,1]$
  $n^2 = 17^2 = 289$

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