

Due: Monday, February 4, 2019, in class

1. Solve the Towers of Hanoi game for the following graph $G=(V,E)$ with $V=\{\text{Start, Aux1, Aux2, Aux3, Dest}\}$ and $E = \{(\text{Start,Aux1}), (\text{Aux1,Aux2}), (\text{Aux2,Aux3}), (\text{Aux3,Aux1}), (\text{Aux3,Dest})\}$.

(a) Design an algorithm and determine the time and space complexities of moving n disks from Start to Dest.

(b) Implement this algorithm whereby your program prints out each of the moves of every disk. Show the output for $n=1, 2, 3, 4, 5, 6, 7, 8, 9$, and 10. (If the output is too long, print out only the first 100 and the last 100 moves.)

2. Determine for the following code how many pages are transferred between disk and main memory, assuming each page has 2048 words, the active memory set size is 4096 (i. e., at any time no more than 4096 pages may be in main memory), and the replacement strategy is LRU (the Least Recently Used page is always replaced); also assume that all two-dimensional arrays are of size (1:4096, 1:4096), with each array element occupying one word, $N=4096$

```
for I := 1 to 4096 do
```

```
  for J := 1 to 4096 do
```

```
    { A[I,J] := A[I,J] * B[I,J] ; B[I,J] := C[N-I+1,J] * B[I,J] }
```

provided the arrays are mapped into the main memory space

(a) in row-major order,

(b) in column-major order.

3. Consider QuickSort on the array $A[1:n]$ and assume that the pivot element x (used to split the array $A[lo:hi]$ into two portions such that all elements in the left portion $A[lo:m]$ are $\leq x$ and all elements in the right portion $A[m:hi]$ are $\geq x$) is the penultimate element of the array to be split (i. e., $A[hi-1]$).

Construct an infinite sequence of numbers for n and construct an assignment of the numbers $1 \dots n$ to the n array elements that causes QuickSort, with the stated choice of pivot, to

(a) execute optimally (that is $A[lo:m]$ and $A[m:hi]$ are always of equal size)

(b) execute in the slowest possible way.

4. Memory fragmentation in C: Design, implement, and execute a C-program that does the following: It allocates memory for a sequence of $3m$ arrays of size 800,000 elements each; then it explicitly deallocates all even-numbered arrays and allocates a sequence of m arrays of size 900,000 elements each. Measure the amounts of time your program requires for the allocation of the first sequence and for the second sequence. Choose m so that you exhaust almost all of the main memory available to your program. Explain your timings!!

5. Implement a binary search function in **three substantially different** programming languages. In each program (identical, except for the programming language), carry out the same 10,000,000 unsuccessful searches for eight different-sized arrays, namely arrays of sizes 128, 512, 2048, 8192, 32768, 131072, 524288, and 2,097,152. Measure in each of the three programs the time it takes to do the 10,000,000 searches for each of the eight arrays. Compare these timings to the theoretical timings the algorithm binary search provides. Are there differences between the three programs? Explain your timings and observations!!

Percentage points:

1: 20%

2: 15%

3: 20%

4: 20%

5: 25%

$O(n^2)$
INFINITE
SEQUENCE
 2^k 3^k