COSC 3320 **Exam 1**

Open Books and Notes

Wednesday, February 13, 2019, 4:00 – 5:25 pm

1. Design a function (algorithm) that solves the Towers of Hanoi game for the following graph G=(V,E) with V={Start, Aux1, Aux2, Aux3, Dest } and E = {(Start, Aux1), (Aux1, Aux2), (Aux2, Aux3), (Aux3, Dest), (Dest, Start) }. Estimate the time complexity of your function, in terms of the number n of disks to be moved.

2. Determine for the following code how many pages are transferred between disk and main memory. Assume each page has 1024 words, the active memory set size is 512 (i. e., at any time no more than 512 pages may be in main memory), and the replacement strategy is LRU (the Least Recently Used page is always replaced); also assume that all 2D arrays are of size (1:2048, 1:2048), with each array element occupying one word,

```
for I := 1 to 2048 do
  for J :=1 to 2048 do
    { A[J,I]:=A[J,I]*B[I,J] }
```

provided the arrays are mapped into the main memory space in **row**-major order.

3. Consider QuickSort on the array A[1:n] and assume that the pivot element x (used to split the array A[lo:hi] into two portions such that all elements in the left portion A[lo:m] are ≤x and all elements in the right portion A[m:hi] are ≥x) is the **last** element of the array to be split (i. e., A[lo]).

For a specific value of n at least 13 (**your** choice), construct an assignment of the numbers 1...n to the n array elements that causes QuickSort, with the stated choice of pivot, to

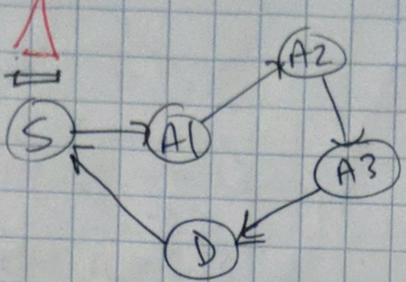
(a) execute optimally (that is A[lo:m] and A[m:hi] are always of equal size)

(b) execute in the slowest possible way.

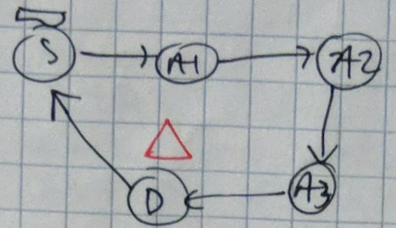
4. Consider the problem of multiplying n rectangular matrices discussed in class. Assume, in contrast to what we did in class, that we want to determine the **maximum** number of scalar multiplications that one might need (that is, compute the maximum of all possible parenthesizations).

Formulate precisely an algorithm that determines this value. Then carry out your method on the following product to show what is the worst-possible parenthesization and how many scalar multiplications are required to carry it out: $M_{9,2}*M_{2,9}*M_{9,1}*M_{1,8}*M_{8,5}$.

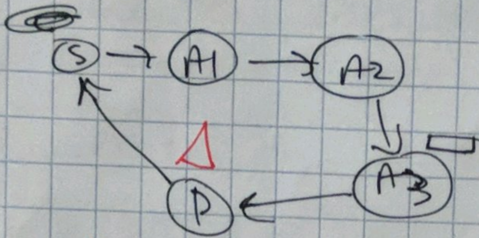
Percentage points:	**1: 25%**	**2: 25%**	**3: 25%**	**4: 25%**



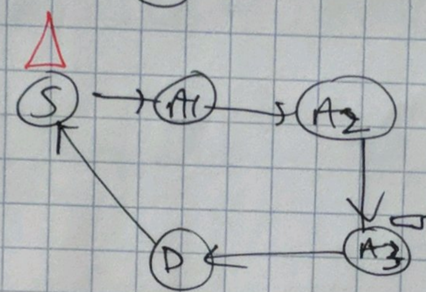
[redacted]



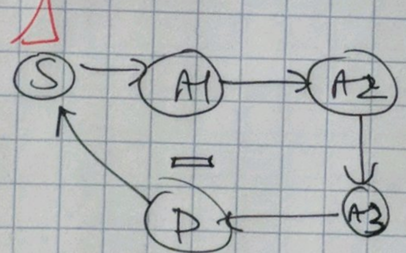
4 HOPS F(n-1)



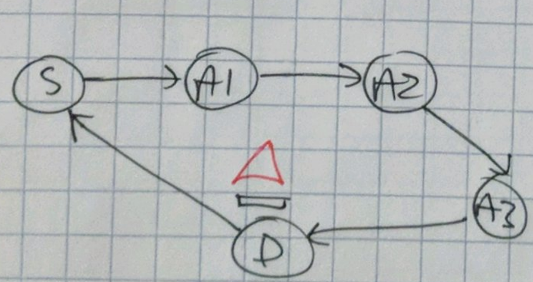
S → A3 O(1)



1 HOP F(n-1)



A3 → D O(1)



4 HOPS F(n-1)

q_1 23 q_2 25. Q_3 25 G_4 25.

F(n, S, D)

if (n == 1)

S → D

else if ~~else~~ $n \geq 2$ {

move $n-1$ disks from S to D // 1^{st} RECUR. CALL

S → A3 // ~~O(1)~~ nth disk to A3

move $n-1$ disks from D to S // 2nd RECUR. CALL

A3 → D // nth disk to D

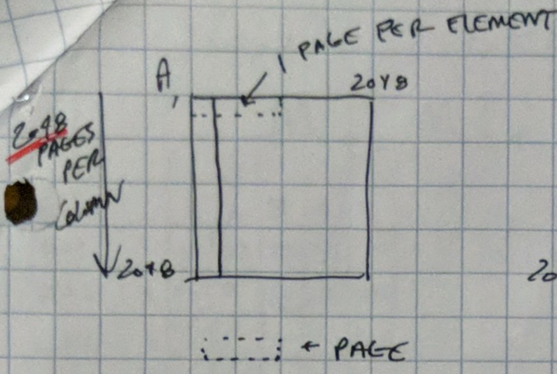
move $n-1$ disks from S to D // 3rd RECUR. CALL

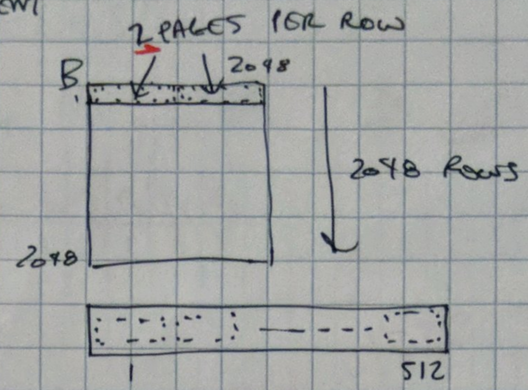
}

$r = 3 \quad d = 1$

$$T(n) = 3T(n-1) + O(1) \Rightarrow O(r^{n/d}) = O(3^n)$$

$O(6^{\frac{n}{2}})$ -2





$\underline{2048}\ W * 2048$

$\underline{2048}\ R * 2048$

$\cancel{2048}\ \underline{2}\ R * 2048$

$(\underline{2048} + \underline{2048} + \underline{2})\,2048 = 8,392,704$ TRANSFERS

$n\log(n)$

BEST CASE: MEDIAN OF THE ~~B~~ SUB ARRAY IS THE LAST ELEMENT AND CHOSEN AS THE PIVOT

n=3 [1, 3, 2]

n=7 [1, 3, 2, 5, 7, 6, 4]

n=15 [1, 3, 2, 5, 7, 6, 4, 9, 11, 10, 13, 15, 14, 12, 8]

EXECUTE: [1, 3, 2, 5, 7, 6, 4] 8 [9, 11, 10, 13, 15, 14, 12]

[1, 3, 2] 4 [5, 7, 6] 8 [9, 11, 10] 12 [13, 15, 14]

[1] 2 [3] 4 [5] 6 [7] 8 [9] 10 [11] 12 [13] 14 [15]

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

EACH SUB ARRAY IS OF EQUAL SIZE AND MEDIAN IS ALWAYS THE PIVOT

THUS, THE CHOSEN ARRAY IS OPTIMAL

WORST CASE: LARGEST ELEMENT IS THE LAST ELEMENT OF THE SUB ARRAY AND CHOSEN AS PIVOT

n=15 [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]

EXECUTE: [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14] 15

[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13] 14 15

.

.

.

[1] 2 3 4 5 6 7 8 9 10 11 12 13 14 15

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

~~SINCE EACH SUB ARRAY IS OF EQUAL SIZE AND LARGEST~~

LARGEST ELEMENT IS ALWAYS CHOSEN AS PIVOT AS SHOWN

THUS, THE CHOSEN ARRAY HAS THE WORST TIME COMPLEXITY

$$S[i,j] = S[i,k] + S[k+1,j] + P_i \cdot P_{k+1} \cdot P_{j+1}$$

$$M_{9,2} \; M_{2,9} \; M_{9,1} \; M_{1,8} \; M_{8,5}$$

		1	2	3	4	5
k=1	1	0	162	243	882	1332
k=2	2		0	18	216	1242
k=3	3			0	72	360
k=4	4				0	40
k=5	5					0

P_1	P_2	P_3	P_4	P_5	P_6
9	2	9	1	8	5

MAX 1332 SCALAR MULTIPLICATIONS

$k=1 \quad S[1,2] = P_1 \cdot P_2 \cdot P_3 = 162$

$k=2 \quad S[2,3] = P_2 \cdot P_3 \cdot P_4 = 18$

$k=3 \quad S[3,4] = P_3 \cdot P_4 \cdot P_5 = 72$

$k=4 \quad S[4,5] = P_4 \cdot P_5 \cdot P_6 = 40$

$S[1,3] = \max(36, 243) = 243$

$k=1, \quad S[1,3] = S[\overset{0}{1,1}] + S[\overset{18}{2,3}] + \overset{18}{P_1 P_2 P_4} = 36$

$k=2 \quad S[1,3] = S[\overset{162}{1,2}] + S[\overset{0}{3,3}] + \overset{81}{P_1 P_3 P_4} = \underline{243}$

$S[2,4] = \max(216, 34) = 216$

$k=2 \quad S[2,4] = S[\overset{0}{2,2}] + S[\overset{72}{3,4}] + \overset{144}{P_2 P_3 P_5} = \underline{216}$

$k=3 \quad S[2,4] = S[\overset{18}{2,3}] + S[\overset{0}{4,4}] + \overset{16}{P_2 P_4 P_5} = 34$

$S[3,5] = \max(112, 360) = 360$

$k=3 \quad S[3,5] = S[\overset{0}{3,3}] + S[\overset{40}{4,5}] + \overset{72}{P_3 P_4 P_6} = 112$

$k=4 \quad S[3,5] = S[\overset{72}{3,4}] + S[\overset{0}{5,5}] + P_3 P_5 P_6 = \underline{360}$

$S[1,4] = \max(360, 882, 315) = 882$

$k=1 \quad S[1,4] = S[\overset{0}{1,1}] + S[\overset{216}{2,4}] + \overset{144}{P_1 P_2 P_5} = 360$

$k=2 \quad S[1,4] = S[\overset{162}{1,2}] + S[\overset{72}{3,4}] + \overset{648}{P_1 P_3 P_5} = \underline{882}$

$k=3 \quad S[1,4] = S[\overset{243}{1,3}] + S[\overset{0}{4,4}] + \overset{72}{P_1 P_4 P_5} = 315$

$S[2,5] = \max(927, 328, 1242) = 1242$

$k=2 \quad S[2,5] = S[\overset{162}{1,2}] + S[\overset{360}{3,5}] + \overset{405}{P_1 P_3 P_6} = 927$

$k=3 \quad S[2,5] = S[\overset{243}{1,3}] + S[\overset{40}{4,5}] + \overset{45}{P_1 P_4 P_6} = 328$

$k=4 \quad S[2,5] = S[\overset{882}{1,4}] + S[\overset{0}{5,5}] + \overset{360}{P_1 P_5 P_6} = \underline{1242}$

$S[1,5] = \max(1332, 927, 328, 1242) = 1332$

$k=1 \quad S[1,5] = S[\overset{0}{1,1}] + S[\overset{1242}{2,5}] + p_1 \overset{90}{p_2} p_6 = \underline{1332}$

$k=2 \quad S[1,5] = S[\overset{162}{1,2}] + S[\overset{360}{3,5}] + p_1 \overset{405}{p_3} p_6 = 927$

$k=3 \quad S[1,5] = S[\overset{243}{1,3}] + S[\overset{40}{4,5}] + p_1 \overset{45}{p_4} p_6 = 328$

$k=4 \quad S[1,5] = S[\overset{882}{1,4}] + S[\overset{0}{5,5}] + p_1 \overset{360}{p_5 p_6} = 1242$

WORST POSSIBLE PARENTHISAZATION

$\cancel{M_{9,2} \; M_{9,1}}$

$M_{9,2}((M_{2,9}(M_{9,1} \; M_{1,8}))M_{8,5})$

9,8

2,8 2,5

144

72 98 80 90 =

$) \times M_{8,5}$