```
> restart;with(LinearAlgebra):
```

A

```
> ea1:=<0,1,2,2,0>:ea2:=<1,1,4,0,0>:ea3:=<1,2,6,2,1>:ea4:=<-1,2,2,
  6,-1>:ea5:=<2,-2,0,1,0>:ea6:=<-2,-2,1,0,0>:
```

```
> eMe[1,2,3,0]:=<ea1|ea2|ea3|<0,0,0,0,0>>
```

$$eMe_{1,\,2,\,3,\,0} := \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 2 & 0 \\ 2 & 4 & 6 & 0 \\ 2 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{1.1}$$

```
> ReducedRowEchelonForm(eMe[1,2,3,0])
```

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{1.2}$$

```
> eMe[1,2,3,4]:=<ea1|ea2|ea3|ea4>
```

$$eMe_{1,\,2,\,3,\,4} := \begin{bmatrix} 0 & 1 & 1 & -1 \\ 1 & 1 & 2 & 2 \\ 2 & 4 & 6 & 2 \\ 2 & 0 & 2 & 6 \\ 0 & 0 & 1 & -1 \end{bmatrix} \tag{1.3}$$

```
> ReducedRowEchelonForm(eMe[1,2,3,4])
```

$$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{1.4}$$

```
> (4*ea1+0*ea2+(-1)*ea3)=ea4
```

$$\begin{bmatrix} -1 \\ 2 \\ 2 \\ 6 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 2 \\ 6 \\ -1 \end{bmatrix} \tag{1.5}$$

B

```
> eq1:=ea1/sqrt(ea1.ea1)
```

$$eq1 := \begin{bmatrix} 0 \\ \frac{1}{3} \\ \frac{2}{3} \\ \frac{2}{3} \\ 0 \end{bmatrix} \tag{2.1}$$

```
> w2:=ea2-(ea2.eq1)*eq1
```

$$w2 := \begin{bmatrix} 1 \\ 0 \\ 2 \\ -2 \\ 0 \end{bmatrix} \tag{2.2}$$

```
> eq2:=w2/sqrt(w2.w2)
```

$$eq2 := \begin{bmatrix} \frac{1}{3} \\ 0 \\ \frac{2}{3} \\ -\frac{2}{3} \\ 0 \end{bmatrix} \tag{2.3}$$

```
> w3:=ea3-(ea3.eq2)*eq2-(ea3.eq1)*eq1
```

$$w3 := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{2.4}$$

```
> eq3:=w3/sqrt(w3.w3)
```

$$eq3 := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{2.5}$$

```
> eMe[1,2,3,q1]:=<ea1|ea2|ea3|eq1>
```

$$eMe_{1,\,2,\,3,\,q1} := \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 2 & \frac{1}{3} \\ 2 & 4 & 6 & \frac{2}{3} \\ 2 & 0 & 2 & \frac{2}{3} \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{2.6}$$

```
> ReducedRowEchelonForm(eMe[1,2,3,q1])
```

$$\begin{bmatrix} 1 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{2.7}$$

```
> eMe[1,2,3,q2]:=<ea1|ea2|ea3|eq2>
```

$$eMe_{1,\,2,\,3,\,q2} := \begin{bmatrix} 0 & 1 & 1 & \frac{1}{3} \\ 1 & 1 & 2 & 0 \\ 2 & 4 & 6 & \frac{2}{3} \\ 2 & 0 & 2 & -\frac{2}{3} \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{2.8}$$

```
> ReducedRowEchelonForm(eMe[1,2,3,q2])
```

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{1}{3} \\ 0 & 1 & 0 & \frac{1}{3} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{2.9}$$

```
> eMe[1,2,3,q3]:=<ea1|ea2|ea3|eq3>
```

$$eMe_{1,\,2,\,3,\,q3} := \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 2 & 0 \\ 2 & 4 & 6 & 0 \\ 2 & 0 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \tag{2.10}$$

```
> ReducedRowEchelonForm(eMe[1,2,3,q3])
```

$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{2.11}$$

C

```
> eMe[1,2,3]:=<ea1|ea2|ea3>
```

$$eMe_{1,\,2,\,3} := \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 2 \\ 2 & 4 & 6 \\ 2 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.1}$$

```
> Transpose(eMe[1,2,3]).ea5
```

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{3.2}$$

```
> Transpose(eMe[1,2,3]).ea6
```

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{3.3}$$

```
> ea5.ea6
```

$$0 \tag{3.4}$$

```
> Q:=GramSchmidt([ea1,ea2,ea3,ea5,ea6],normalized)
```

$$Q := \left[\begin{bmatrix} 0 \\ \frac{1}{3} \\ \frac{2}{3} \\ \frac{2}{3} \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{1}{3} \\ 0 \\ \frac{2}{3} \\ -\frac{2}{3} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} \frac{2}{3} \\ -\frac{2}{3} \\ 0 \\ \frac{1}{3} \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{2}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \\ 0 \\ 0 \end{bmatrix} \right] \tag{3.5}$$

```
> eFq:=<Q[1]|Q[2]|Q[3]|Q[4]|Q[5]>
```

$$eFq := \begin{bmatrix} 0 & \frac{1}{3} & 0 & \frac{2}{3} & -\frac{2}{3} \\ \frac{1}{3} & 0 & 0 & -\frac{2}{3} & -\frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & 0 & 0 & \frac{1}{3} \\ \frac{2}{3} & -\frac{2}{3} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (3.6)$$

D

```
> F1:=<1,0,0,0,0>:
  F0:=<0,0,0,0,0>:
  F2:=<0,-1,0,0,0>:
  F3:=<0,0,-1,0,0>:
```

```
> qMq:=<F1|F2|F3|F0|F0>
```

$$qMq := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (4.1)$$

```
> qa5:=eFq^(-1).ea5
```

$$qa5 := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 3 \\ 0 \end{bmatrix} \quad (4.2)$$

```
> qa6:=eFq^(-1).ea6
```

$$qa6 := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 3 \end{bmatrix} \quad (4.3)$$

```
> qa1:=eFq^(-1).ea1
```

$$qa1 := \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{4.4}$$

```
> qa2:=eFq^(-1).ea2
```

$$qa2 := \begin{bmatrix} 3 \\ 3 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{4.5}$$

```
> (qa2-qa1)*(-1)
```

$$\begin{bmatrix} 0 \\ -3 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{4.6}$$

```
> qa3:=eFq^(-1).ea3
```

$$qa3 := \begin{bmatrix} 6 \\ 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} \tag{4.7}$$

```
> qa4:=eFq^(-1).ea4
```

$$qa4 := \begin{bmatrix} 6 \\ -3 \\ -1 \\ 0 \\ 0 \end{bmatrix} \tag{4.8}$$

```
> V1:=<a,b,c,0,0>:
  V2:=<f,g,h,0,0>:
> V1.V2=(qMq.V1).(qMq.V2)
```

$$\bar{a}\,f + \bar{b}\,g + \bar{c}\,h = \bar{a}\,f + \bar{b}\,g + \bar{c}\,h \tag{4.9}$$

```
> V3:=<a,b,c,d,e>:
  V4:=<f,g,h,i,j>:
> V3.V4;
  (qMq.V3).(qMq.V4)
```

$$\bar{a}\,f + \bar{b}\,g + \bar{c}\,h + \bar{d}\,i + \bar{e}\,j$$
$$\bar{a}\,f + \bar{b}\,g + \bar{c}\,h \tag{4.10}$$

E

```
> eFe:=eFq.qMq.eFq^(-1)
```

$$eFe := \begin{bmatrix} -\frac{1}{9} & 0 & -\frac{2}{9} & \frac{2}{9} & 0 \\ 0 & \frac{1}{9} & \frac{2}{9} & \frac{2}{9} & 0 \\ -\frac{2}{9} & \frac{2}{9} & 0 & \frac{8}{9} & 0 \\ \frac{2}{9} & \frac{2}{9} & \frac{8}{9} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix} \tag{5.1}$$

```
> Transpose(eFe)
```

$$\begin{bmatrix} -\frac{1}{9} & 0 & -\frac{2}{9} & \frac{2}{9} & 0 \\ 0 & \frac{1}{9} & \frac{2}{9} & \frac{2}{9} & 0 \\ -\frac{2}{9} & \frac{2}{9} & 0 & \frac{8}{9} & 0 \\ \frac{2}{9} & \frac{2}{9} & \frac{8}{9} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix} \tag{5.2}$$