Problem solving session – linear and nonlinear models for classification

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Exercise 1
Use the script `Classification.m` to generate three classes. The script generates samples $\mathbf{x} = [x_1, x_2]$ by sampling from a two dimensional Gaussian distribution specified by a mean vector $\boldsymbol{\mu} = [\mu_1, \mu_2]$ and a co-variance matrix:

$$\sum = \begin{bmatrix} \sigma_{11}^2 & \sigma_{12}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 \end{bmatrix} \tag{1}$$

Each generated sample is then associated with its corresponding class, $\mathcal{C}_k$, where $k = 1, 2$ and 3. For instance if $\mathbf{x}$ is sampled from Gaussian distribution 1, it belongs to class 1, i.e. $\mathcal{C}_1$. The objective of this exercise is to determine a discriminant function $\mathbf{y}(\mathbf{x}) = \widetilde{\mathbf{W}}^T\widetilde{\mathbf{x}}$ to classify an unknown sample $\mathbf{x}$ to its corresponding class.

We start by only considering two classes, i.e. class 1 and 2.

1. Allocate 80% of the data–set for training and the rest for the testing
2. On the training data–set, employ the least squares method to determine the weight matrix $\widetilde{\mathbf{W}}$
3. Perform classification on the test data–set by computing $\mathbf{y}(\mathbf{x}) = \widetilde{\mathbf{W}}^T\widetilde{\mathbf{x}}$ and assigning the corresponding class
4. Count the number of classification errors

Next, we consider all three classes and the objective is to determine a discriminant function $\mathbf{y}(\mathbf{x}) = \widetilde{\mathbf{W}}^T\widetilde{\mathbf{x}}$ to classify an unknown sample $\mathbf{x}$ to its corresponding class.

1. Modify the data set for two classes to form a data set for three classes. Make sure that you employ the correct class labeling scheme as well as random shuffling of the data within the data set.
2. Allocate 80% of the data–set for training and the rest for the testing
3. On the training data, employ the least squares method to determine the weight matrix $\widetilde{\mathbf{W}}$
4. Perform classification on the test data–set by computing $\mathbf{y}(\mathbf{x}) = \widetilde{\mathbf{W}}^T\widetilde{\mathbf{x}}$ and assigning a class
5. Count the number of classification errors

Exercise 2
The objective of this exercise is to demonstrate that the perceptron can be used to learn class labeling and perform accurate classification. It should be kept in mind that the perceptron can only be used for two class probelems.

1. Choose the proper class labeling (assignment) and form the data–set. The classes are given by sampling from the two dimensional Gaussian distribution (see Exercise 1)
2. Implement the iterative algorithm for learning the perceptron's weights. The implementation should be performed on the training data–set
3. Plot the evolution of the weights as a function of iterations. Comment on the results!
4. Evaluate the accuracy of the implementation on the test data–set
5. Count the number of classification errors

Exercise 3
Consider classification problem involving only two classes and demonstrate that the logistic regression can be used to perform accurate classification.