```
> restart:
  with(plots):
```

A

```
> f:=(x,y)->5/4*x^2*y-1/4*x^4-y^2+1
```

$$f := (x, y) \mapsto \frac{5}{4} \cdot x^2 \cdot y - \frac{1}{4} \cdot x^4 - y^2 + 1 \tag{1.1}$$

```
> r:=(u)->(u,1/2*u^2)
```

$$r := u \mapsto \left(u, \frac{u^2}{2} \right) \tag{1.2}$$

```
> h:=u->f(r(u)):
  'h(u)'=h(u)
```

$$h(u) = \frac{u^4}{8} + 1 \tag{1.3}$$

```
> vector_r:=u-><u,1/2*u^2,h(u)>
```

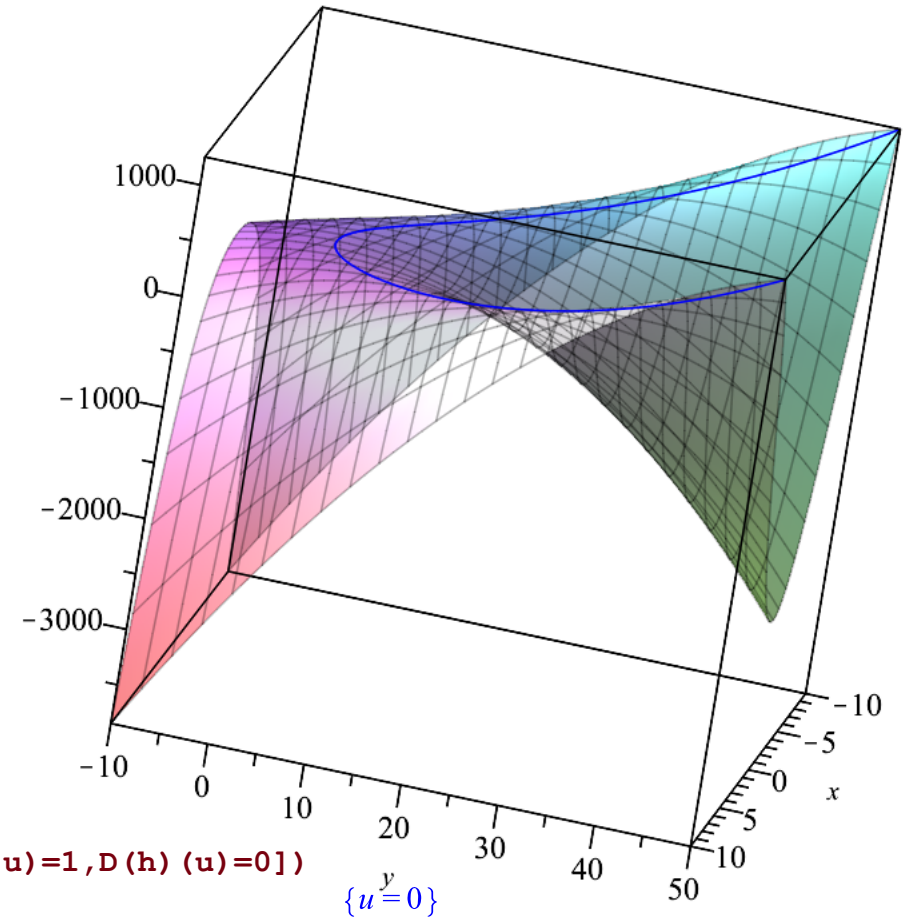
$$vector\_r := u \mapsto \left\langle u, \frac{u^2}{2}, h(u) \right\rangle \tag{1.4}$$

```
> sp1:=spacecurve(vector_r(u),u=-10..10,color=blue):

> p1:=plot3d(f(x,y),x=-10...10,y=-10..50,transparency=0.25,title=
  "h(u) løftet på f(x,y)"):
```

```
> display(p1,sp1,orientation=[39, 71, 29] )
```

h(u) løftet på f(x,y)



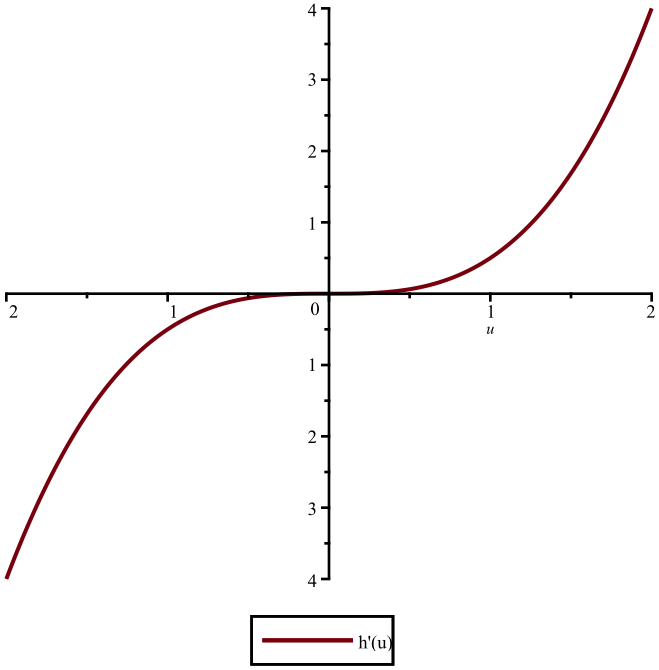

```
> solve([h(u)=1,D(h)(u)=0])
```

$$\{u = 0\}$$

```
> 'h´(u)'=D(h)(u)
```

$$h'(u) = \frac{u^3}{2} \qquad \textbf{(1.6)}$$

```
> plot(D(h)(u),u=-2..2,legend="h'(u)")
```



B

```
> A:=(0,-1):
  B:=(0,0):
```

```
> fxx:=unapply(D[1,1](f)(A),x)
```

$$fxx := x \mapsto \frac{5}{2} \tag{2.1}$$

```
> fxy:=unapply(D[1,2](f)(A),x)
```

$$fxy := x \mapsto 0 \tag{2.2}$$

```
> fyy:=unapply(D[2,2](f)(A),x)
```

$$fyy := x \mapsto -2 \tag{2.3}$$

```
> fyx:=unapply(D[2,1](f)(A),x)
```

$$fyx := x \mapsto 0 \tag{2.4}$$

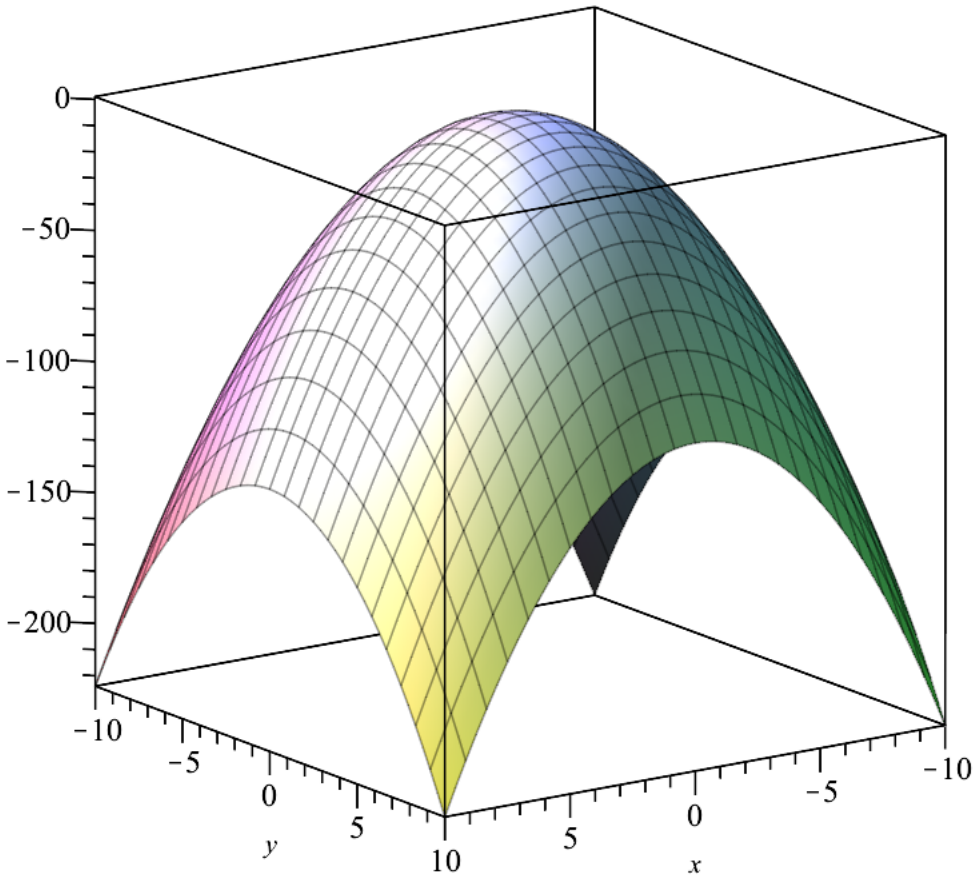
```
> H:=<fxx(x,y),fyx(x,y)|fxy(x,y),fyy(x,y)>
```

$$H := \begin{bmatrix} \frac{5}{2} & 0 \\ 0 & -2 \end{bmatrix} \tag{2.5}$$

```
> P2:=unapply(mtaylor(f(x,y),[x=0,y=-1],3),[x,y]):
  'P[2]'=expand(P2(x,y))
```

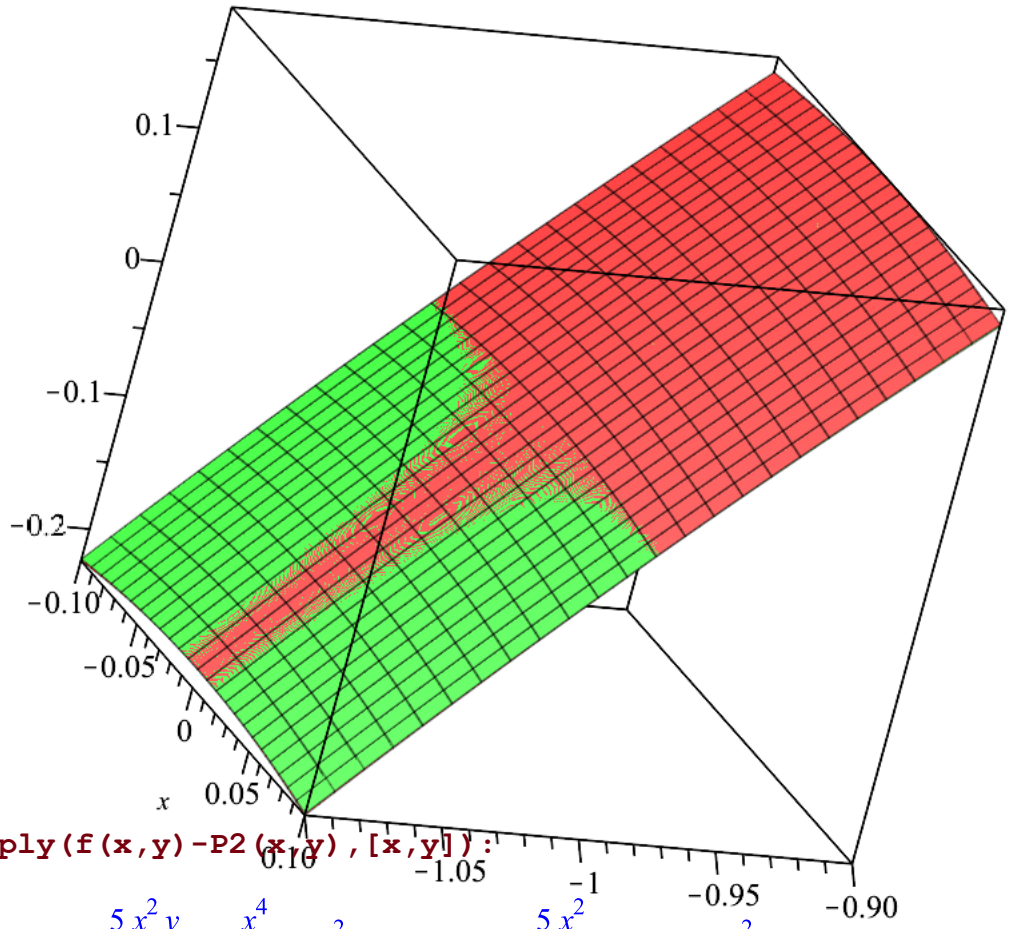
$$P_2 = 1 - \frac{5\,x^2}{4} - y^2 \tag{2.6}$$

```
> plot3d(P2(x,y))
```




```
> plotf:=plot3d(f(x,y),x=-1/10...1/10,y=-11/10..-9/10,color=red):
> plotP2:=plot3d(P2(x,y),x=-1/10...1/10,y=-11/10..-9/10,color=
  green):
```

```
> display(plotf,plotP2,orientation=[11,65,36])
```




```
> R:=unapply(f(x,y)-P2(x,y),[x,y]):
  R(x,y)
```

$$\frac{5\,x^2\,y}{4} - \frac{x^4}{4} - y^2 - 1 - 2\,y + \frac{5\,x^2}{4}_{y} + (y+1)^2$$

```
> expand(R(x,y))
```

$$\frac{5}{4}\,x^2\,y - \frac{1}{4}\,x^4 + \frac{5}{4}\,x^2 \tag{2.8}$$

```
> lign1:=D[1](R)(x,y)=0;
  lign2:=D[2](R)(x,y)=0;
```

$$lign1 := \frac{5}{2}\,y\,x - x^3 + \frac{5}{2}\,x = 0$$

$$lign2 := \frac{5\,x^2}{4} = 0 \tag{2.9}$$

```
> solve([lign1,lign2])
```

$$\{x = 0, y = y\} \tag{2.10}$$

```
> diff(R(-1/10,y),y);
  R(-1/10,%)
```

$$\frac{1}{80}$$

$$\frac{2021}{160000} \tag{2.11}$$

```
> diff(R(1/10,y),y):
  R(1/10,%)
```

$$\frac{2021}{160000} \tag{2.12}$$

```
> diff(R(x,-11/10),x):
  solve(%,x);
```

$$0,\ \frac{I}{2},\ -\frac{I}{2} \tag{2.13}$$

```
> diff(R(x,-9/10),x):
  solve(%,x);
```

$$0,\ \frac{1}{2},\ -\frac{1}{2} \tag{2.14}$$

```
> max(abs(R(-1/10,-11/10)),abs(R(1/10,-11/10)),abs(R(-1/10,-9/10)
  ),abs(R(1/10,-9/10)))
```

$$\frac{51}{40000} \tag{2.15}$$

```
> abs(R(1/10,-11/10))
```

$$\frac{51}{40000} \tag{2.16}$$

C

```
> fxxb:=unapply(D[1,1](f)(B),x)
```

$$fxxb := x \mapsto 0 \tag{3.1}$$

```
> fxyb:=unapply(D[1,2](f)(B),x)
```

$$fxyb := x \mapsto 0 \tag{3.2}$$

```
> fyyb:=unapply(D[2,2](f)(B),x)
```

$$fyyb := x \mapsto -2 \tag{3.3}$$

```
> fyxb:=unapply(D[2,1](f)(B),x)
```

$$fyxb := x \mapsto 0 \tag{3.4}$$

```
> HB:=<fxxb(x,y),fyxb(x,y)|fxyb(x,y),fyyb(x,y)>
```

$$HB := \begin{bmatrix} 0 & 0 \\ 0 & -2 \end{bmatrix} \tag{3.5}$$

D

```
> restart:
  with(LinearAlgebra):
```

```
> f:=(x,y)->5/4*x^2*y-1/4*x^4-y^2+1
```

$$f := (x, y) \mapsto \frac{5}{4} \cdot x^2 \cdot y - \frac{1}{4} \cdot x^4 - y^2 + 1 \tag{4.1}$$

```
> f(x,a*x)
```

$$\frac{5}{4} x^3 a - \frac{1}{4} x^4 - a^2 x^2 + 1 \tag{4.2}$$

```
> diff(f(x,a*x),x);
  solve(%=0)
```

$$\frac{15}{4} x^2 a - x^3 - 2 a^2 x$$

$$\{a = a, x = 0\}, \left\{a = \left(\frac{15}{16} + \frac{\sqrt{97}}{16}\right) x, x = x\right\}, \left\{a = \left(\frac{15}{16} - \frac{\sqrt{97}}{16}\right) x, x = x\right\} \tag{4.3}$$

```
> diff(diff(f(x,a*x),x),x);
  subs(x=0,%)
```

$$\frac{15}{2} a x - 3 x^2 - 2 a^2$$

$$-2 a^2 \tag{4.4}$$

```
> diff(f(0,y),y);
```

$$-2 y \tag{4.5}$$

```
> diff(diff(f(0,y),y),y);
```

$$-2 \tag{4.6}$$