MATLAB CODE FOR LINEAR DYNAMIC RESPONSE HISTORY ANALYSIS OF MULTI-DEGREE OF FREEDOM SYSTEMS

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Description

Matlab code for the application of the linear dynamic response history analysis (linear DRHA) of multi-degree of freedom (MDOF) structures is presented. Two procedures to calculate the dynamic response of a MDOF system subject to dynamic loading are included:

(a) direct integration of equations of motion and

(b) the modal superposition of dynamic responses of SDOF systems equivalent to the eigenmodes to which a MDOF system is decomposed.

For the direct integration of equations of motion, the function LDRHA_DI_MDOF.m is used. See the following examples:

- example_Industrial_Building_DI_NPTEL.m
- example_Shear_Building_2_DI_NPTEL.m
- example_Shear_Frame_4A_DI_Chopra.m
- example_Shear_Frame_5_DI_Chopra.m

for more details.

The function LDRHA_DI_MDOF.m needs the damping matrix of the structure as user input. In the case that the damping of the structure is defined in terms of the critical damping ratios of its various eigenmodes and not by a damping matrix, the function CDM can be used to generate the classical damping matrix of the structure. Then this matrix can be input to the function LDRHA_DI_MDOF.m. See the following example:

- example_Damping_Chopra

for more details.

For the modal superposition procedure for the dynamic response history analysis the functions LDRHA_MS_MDOF.m and LDRHA_SDOF.m are used. The latter is called inside the former. See the following examples:

- example_Shear_Frame_5_MS_Chopra.m
- example_Industrial_Building_MS_NPTEL.m
- example_Shear_Building_2_MS_NPTEL.m
- example_Shear_Frame_4A_MS_Chopra.m

for more details.

All the above functions can be used for acceleration time histories of a constant time step size. If this is not the case, then the acceleration time history needs to be resampled by using the MATLAB program file function RESAMPLE.m. The user is encouraged to see the example

- example_Resampling_Nonuniform_Time_History.m

in this last case.

The dynamic response history analysis procedure with direct integration proceeds incrementally, by solving the MDOF system equations for each time step (LDRHA_DI_MDOF.m, LDRHA_SDOF.m).

The modal superposition dynamic response history analysis procedure (LDRHA_MS_MDOF.m) utilizes the following steps:

1. Define the structural properties.
    a. Determine the mass matrix m and stiffness matrix k.
    b. Estimate the modal damping ratios ζn.
2. Determine the natural frequencies ωn and modes φn.
3. Compute the response in each mode n by the following steps:
    a. Compute the dynamic response qn(t) of a SDOF system with natural frequency ωn and damping ratio ζn.
    b. Compute the nodal displacements from φn and qn(t)
    c. Compute the element forces associated with the nodal displacements from φn, k and qn(t)
4. Combine the contributions of all the n modes to determine the total response.

The present code is accompanied by 10 examples in which its application is presented. These examples are taken from various standard textbooks or other material. The results of the examples are verified by the results of the application of the present code.
The author is open to any suggestions or recommendations that the users may have.

Keywords: Dynamic loading, Response history, Structural design, Earthquake engineering, shock, modal superposition, direct integration, classical damping, resampling.

REFERENCES

[1] Chopra, A. K. 2019, Dynamics of Structures, Theory and Applications to Earthquake Engineering.

[2] MathWorks, Inc. MATLAB R2017b. Natick, MA: MathWorks, Inc.; 2017.

[3] National Programme on Technology Enhanced Learning (NPTEL).

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Dependency Report

The Dependency Report shows dependencies among MATLAB files in a folder (Learn More).

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Show child functions Show parent functions (current folder only)

Show subfunctions

Built-in functions and files in toolbox/matlab are not shown

Report for Folder C:\Users\pc\Desktop\LDRHA

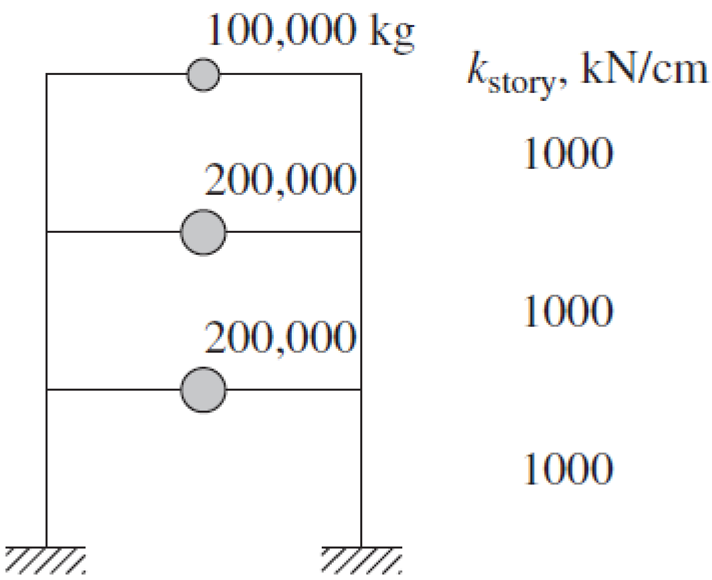
MATLAB File List	Children (called functions)	Parents (calling functions, current dir. only)
CDM		example_Damping_Chopra example_Industrial_Building_DI_NPTEL example_Shear_Building_2_DI_NPTEL example_Shear_Frame_4A_DI_Chopra example_Shear_Frame_5_DI_Chopra
LDRHA_DI_MDOF		example_Industrial_Building_DI_NPTEL example_Shear_Building_2_DI_NPTEL example_Shear_Frame_4A_DI_Chopra example_Shear_Frame_5_DI_Chopra
LDRHA_MS_MDOF	current dir : LDRHA_SDOF	example_Industrial_Building_MS_NPTEL example_Shear_Building_2_MS_NPTEL example_Shear_Frame_4A_MS_Chopra example_Shear_Frame_5_MS_Chopra
LDRHA_SDOF		LDRHA_MS_MDOF
Main		
example_Damping_Chopra	current dir : CDM	
example_Industrial_Building_DI_NPTEL	current dir : CDM current dir : LDRHA_DI_MDOF	
example_Industrial_Building_MS_NPTEL	current dir : LDRHA_MS_MDOF	
example_Resampling_Nonuniform_Time_History	toolbox :? Multiple class methods match resample.m	
example_Shear_Building_2_DI_NPTEL	current dir : CDM current dir : LDRHA_DI_MDOF	
example_Shear_Building_2_MS_NPTEL	current dir : LDRHA_MS_MDOF	
example_Shear_Frame_4A_DI_Chopra	current dir : CDM current dir : LDRHA_DI_MDOF	
example_Shear_Frame_4A_MS_Chopra	current dir : LDRHA_MS_MDOF	
example_Shear_Frame_5_DI_Chopra	current dir : CDM current dir : LDRHA_DI_MDOF	
example_Shear_Frame_5_MS_Chopra	current dir : LDRHA_MS_MDOF	
helpFun		help_CDM help_LDRHA_DI_MDOF help_LDRHA_MS_MDOF help_LDRHA_SDOF
help_CDM	current dir : helpFun	
help_LDRHA_DI_MDOF	current dir : helpFun	
help_LDRHA_MS_MDOF	current dir : helpFun	
help_LDRHA_SDOF	current dir : helpFun	

Classical damping matrix with superposition (Chopra, 2019)

Contents

Statement of the problem

- **Chopra (2019), Example 11.1:** The properties of a three-story shear building are given in Fig. E11.1.
- **Chopra (2019), Example 11.3:** Determine a damping matrix for the system of Fig. E11.1 by superposing the damping matrices for the first two modes, each with $\zeta_n = 0.05$.



Initialization of structural input data

Set the lateral stiffness of each storey in N/m.

```
k=[1e8;1e8;1e8];
```

Set the lumped mass at each floor in kg.

```
m=[2e5;2e5;1e5];
```

Calculation of structural properties

Calculate the stiffness matrix of the structure in N/m.

```
K=diag([k;0])+diag([0;k])-diag(k,1)-diag(k,-1);
K=K(2:end,2:end);
```

Calculate the mass matrix of the structure.

```
M=diag(m);
```

Set the damping ratios for the various eigenmodes

```
ksi=[0.05;0.05;0];
```

Construct the damping matrix

Classical damping matrix with modal superposition.

```
C = CDM(K,M,ksi);
```

Convert to kN-sec/cm.

```
C=1e-5*C
```

```
C =

    4.6022    0.6683   -1.7224
    0.6683    1.1575    0.6683
   -1.7224    0.6683    1.4399
```

Verify with Example 11.3 of Chopra (2019)

$$\begin{bmatrix} 4.60 & 0.668 & -1.72 \\ & 1.16 & 0.668 \\ (\text{sym}) & & 1.44 \end{bmatrix} \text{kN-sec/cm}$$

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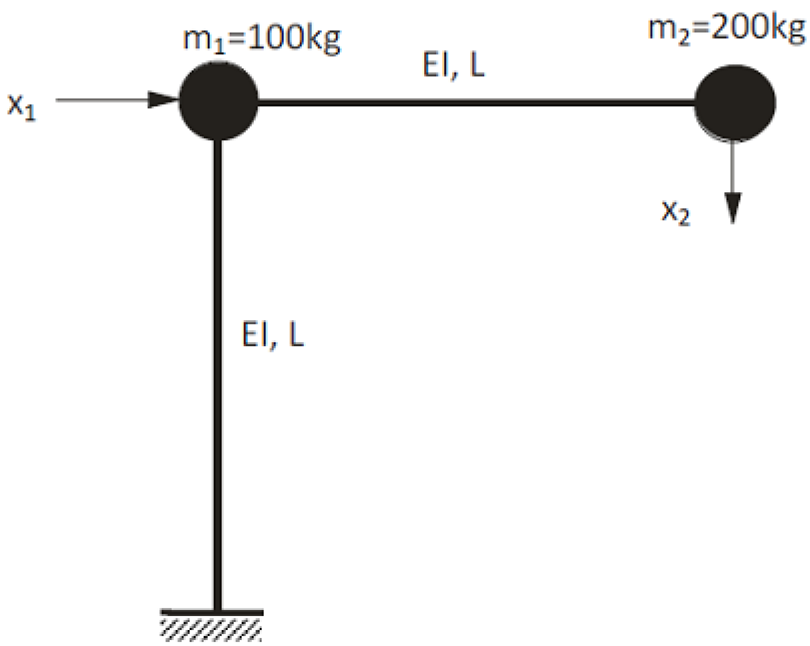
Industrial building, dynamic analysis with direct integration (NPTEL)

Contents

Statement of the problem

- This example comes from the Introduction to Earthquake Engineering - Web course of NPTEL (National Programme on Technology Enhanced Learning), Chapter 3, Dynamics of Earthquake Analysis, Example 3.7.
- An industrial structure is modeled as 2-DOF system as shown in the Figure below. Determine the horizontal and vertical displacement of the free end of the structure due to El-Centro, 1940 earthquake ground motion. Take EI =80 × 103 N.m2, L= 2m, m1= 100kg and m2= 200kg. The damping shall be considered as 2 percent.



Initialization of structural input data

Set the number of degrees of freedom of the structure, which is equal to the number of its storeys.

```
nDOFs=2;
```

Flexural rigidity

```
EI=80e3;
```

Length

```
L=2;
```

Calculation of structural properties

Calculate the stiffness matrix of the structure in N/m.

```
K=6*EI/(7*L^3)*[8,-3;-3,2];
```

Calculate the mass matrix of the structure.

```
M=[300,0;0,200];
```

Set the spatial distribution of the effective earthquake forces. Earthquake forces are applied at all dofs of the structure.

```
r=[1;0];
```

Load earthquake data

Earthquake acceleration time history of the El Centro earthquake (El Centro, 1940, El Centro Terminal Substation Building)

```
D=load('elcentro.dat');
dt=D(2,1)-D(1,1);
xgtt=9.81*D(:,2);
```

Set the critical damping ratio ($\xi = 0.05$)

```
ksi=0.02;
```

Time integration algorithm

```
AlgID='U0-V0-Opt';
```

Initial displacement

```
u0=zeros(nDOFs,1);
```

Initial velocity

```
ut0=zeros(nDOFs,1);
```

Minimum absolute value of the eigenvalues of the amplification matrix

```
rinf=1;
```

Dynamic Response History Analysis (DRHA) with direct integration

Define time

```
t=dt*(0:(numel(xgtt)-1));
```

Calculate the classical damping matrix of the structure

```
C = CDM(K,M,ksi*ones(nDOFs,1));
```

Perform DRHA analysis

```
[U,~,~,~] = LDRHA_DI_MDOF(K,C,M,r,dt,xgtt,AlgID,u0,ut0,rinf);
```

Horizontal displacement

```
UHeig=U(1,:);
```

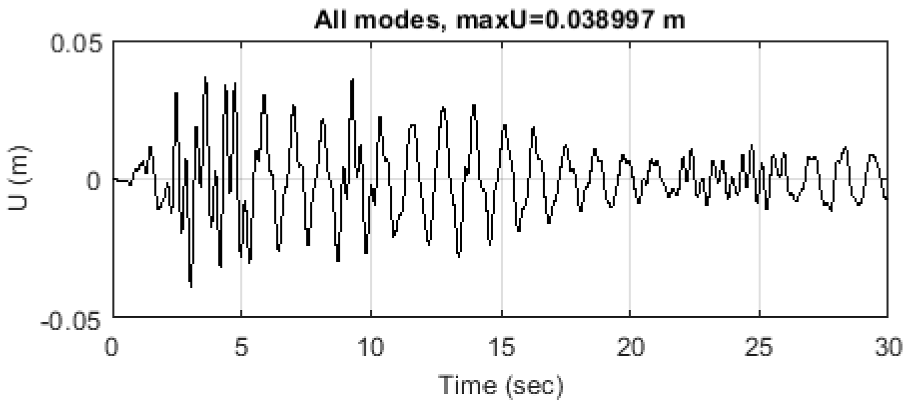
Vertical displacement

```
UVeig=U(2,:);
```

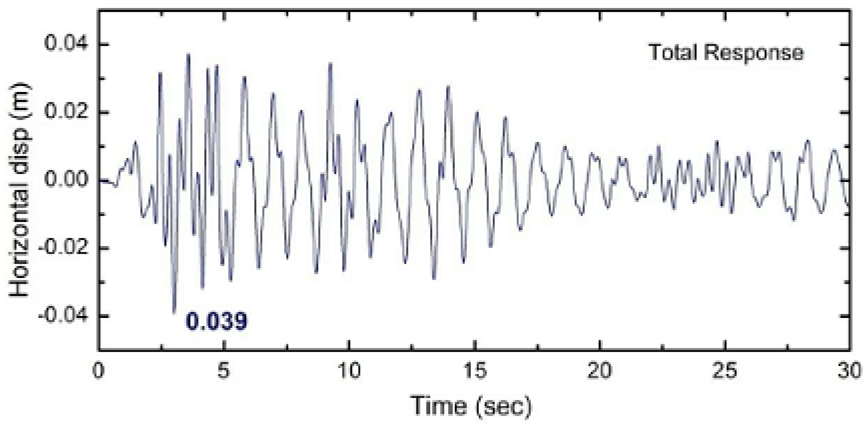
Horizontal displacement time history

Plot the contribution of all eigenmodes to the horizontal displacement time history.

```
FigHandle=figure('Name','Horizontal displacement','NumberTitle','off');
set(FigHandle,'Position',[50, 50, 500, 200]);
plot(t,UHeig,'LineWidth',1.,'Marker','.',...
    'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
grid on
xlim([0,30])
ylim([-0.05,0.05])
xlabel('Time (sec)','FontSize',10);
ylabel('U (m)','FontSize',10);
title(['All modes, maxU=',num2str(max(abs(UHeig))),' m'],...
    'FontSize',10)
```



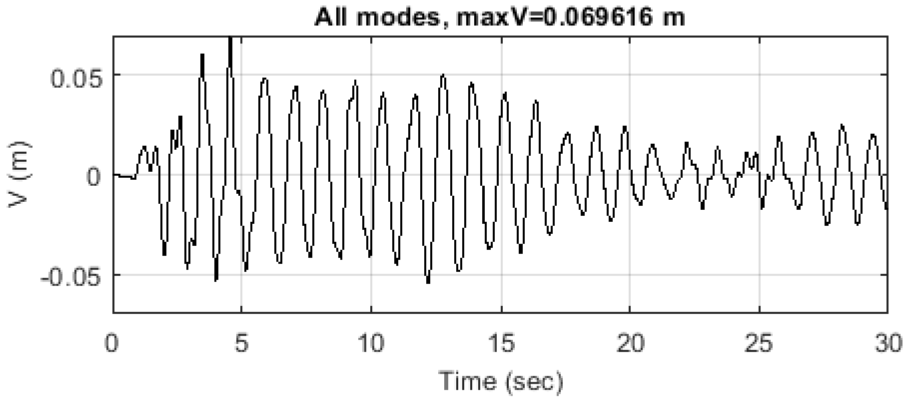
Verify with Figure 3.16 of Chapter 3, Dynamics of Earthquake Analysis, Example 3.7 of NPTEL



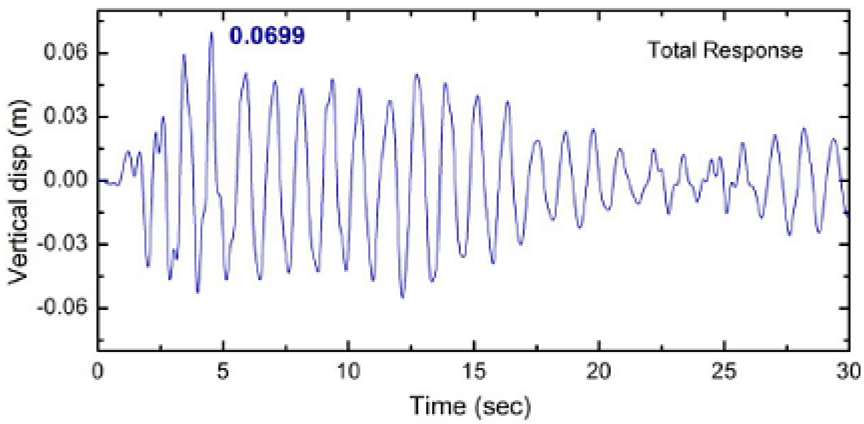
Vertical displacement time history

Plot the contribution of all eigenmodes to the vertical displacement time history.

```
FigHandle=figure('Name','Vertical displacement','NumberTitle','off');
set(FigHandle,'Position',[50, 50, 500, 200]);
plot(t,UVeig,'LineWidth',1.,'Marker','.',...
    'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
grid on
xlim([0,30])
ylim([-0.07,0.07])
xlabel('Time (sec)','FontSize',10);
ylabel('V (m)','FontSize',10);
title(['All modes, maxV=',num2str(max(abs(UVeig))),' m'],...
    'FontSize',10)
```



Verify with Figure 3.17 of Chapter 3, Dynamics of Earthquake Analysis, Example 3.7 of NPTEL



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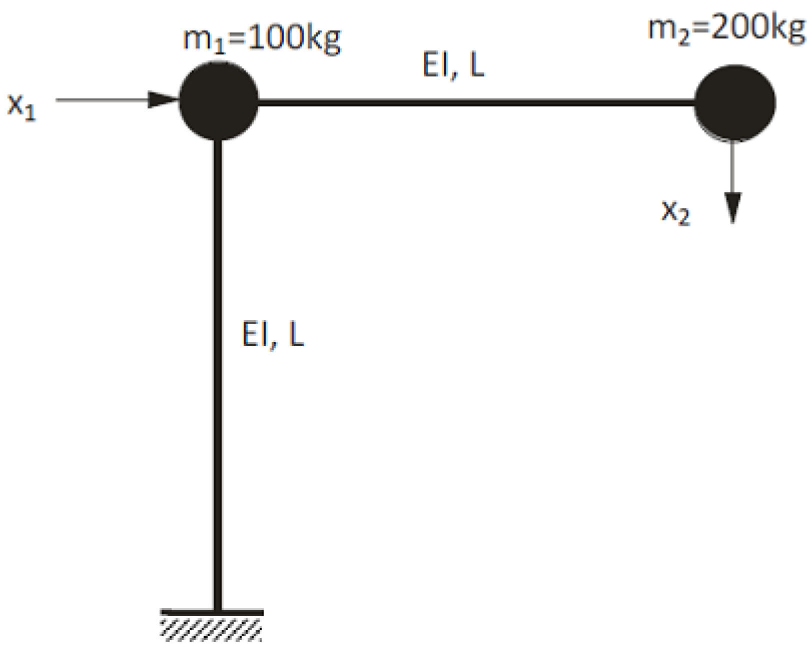
Industrial building dynamic analysis with modal superposition (NPTEL)

Contents

Statement of the problem

- This example comes from the Introduction to Earthquake Engineering - Web course of NPTEL (National Programme on Technology Enhanced Learning), Chapter 3, Dynamics of Earthquake Analysis, Example 3.7.
- An industrial structure is modeled as 2-DOF system as shown in the Figure below. Determine the horizontal and vertical displacement of the free end of the structure due to El-Centro, 1940 earthquake ground motion. Take EI =80 × 103 N.m2, L= 2m, m1= 100kg and m2= 200kg. The damping shall be considered as 2 percent.



Initialization of structural input data

Set the number of degrees of freedom of the structure, which is equal to the number of its storeys.

```
nDOFs=2;
```

Flexural rigidity

```
EI=80e3;
```

Length

```
L=2;
```

Calculation of structural properties

Calculate the stiffness matrix of the structure in N/m.

```
K=6*EI/(7*L^3)*[8,-3;-3,2];
```

Calculate the mass matrix of the structure.

```
M=[300,0;0,200];
```

Set the spatial distribution of the effective earthquake forces. Earthquake forces are applied at all dofs of the structure.

```
r=[1;0];
```

Load earthquake data

Earthquake acceleration time history of the El Centro earthquake (El Centro, 1940, El Centro Terminal Substation Building)

```
D=load('elcentro.dat');
dt=D(2,1)-D(1,1);
xgtt=9.81*D(:,2);
```

Set the critical damping ratio ($\xi = 0.05$)

```
ksi=0.02;
```

Time integration algorithm

```
AlgID='U0-V0-Opt';
```

Initial displacement

```
u0=zeros(nDOFs,1);
```

Initial velocity

```
ut0=zeros(nDOFs,1);
```

Minimum absolute value of the eigenvalues of the amplification matrix

```
rinf=1;
```

Dynamic Response History Analysis (DRHA) with modal superposition

Define time

```
t=dt*(0:(numel(xgtt)-1));
```

Perform DRHA analysis for each separate eigenmode to calculate the contribution of each eigenmode to the various frame responses

```
% Initialize
UVeig=cell(nDOFs+1,1);
UHeig=cell(nDOFs+1,1);
for i=1:nDOFs
    % DRHA analysis
    [U,~,~,~] = LDRHA_MS_MDOF(K,M,r,dt,xgtt,ksi,AlgID,u0,ut0,rinf,i);
    % Store the vertical displacement for each eigenmode
    UVeig{i}=U(2,:);
    % Store the horizontal displacement for each eigenmode
    UHeig{i}=U(1,:);
end
```

Perform DRHA analysis for all eigenmodes to calculate the contribution of all eigenmodes to the various frame responses

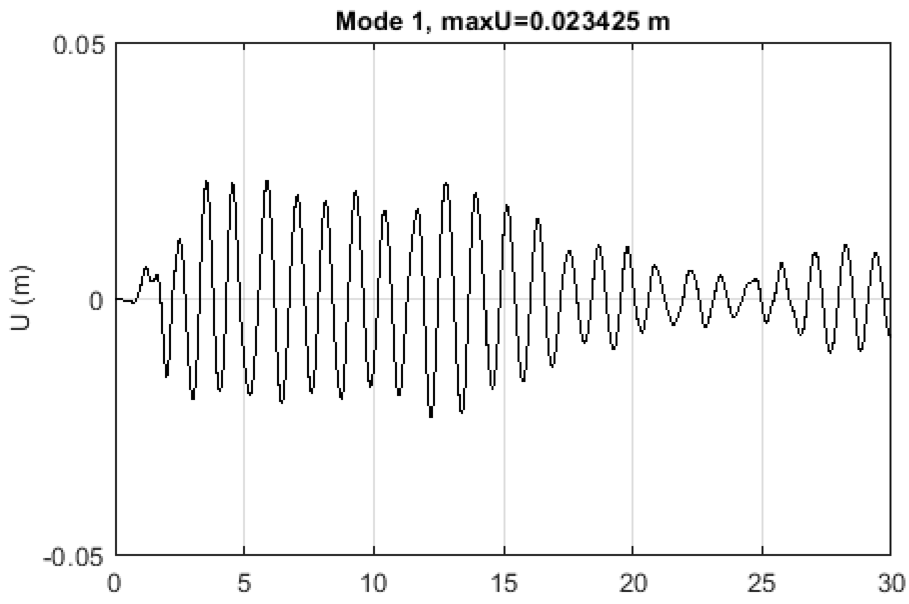
```
% Eigenmodes that are superposed
eigInd=(1:nDOFs)';
% DRHA analysis
[U,~,~,f] = LDRHA_MS_MDOF(K,M,r,dt,xgtt,ksi,AlgID,u0,ut0,rinf,eigInd);
% Vertical displacement for all eigenmodes
UVeig{nDOFs+1}=U(2,:);
% Horizontal displacement for all eigenmodes
UHeig{nDOFs+1}=U(1,:);
```

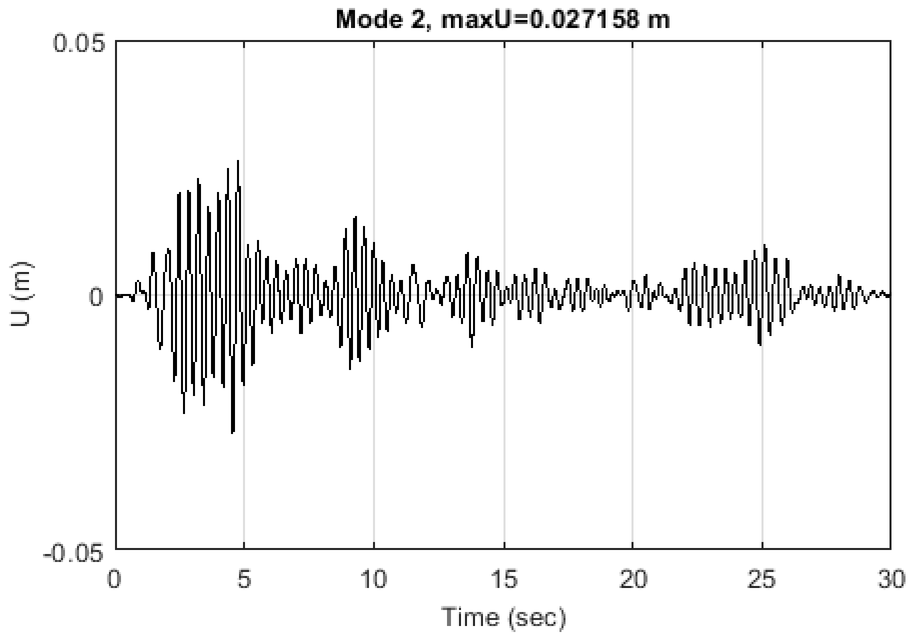
Horizontal displacement time history

Plot the contribution of each eigenmode to the horizontal displacement time history.

```
FigHandle=figure('Name','Horizontal displacement','NumberTitle','off');
set(FigHandle,'Position',[50, 50, 500, 750]);
for i=1:nDOFs
    subplot(nDOFs,1,i)
    plot(t,UHeig{i},'LineWidth',1.,'Marker','.',...
        'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
    grid on
    xlim([0,30])
    ylim([-0.05,0.05])
    ylabel('U (m)','FontSize',10);
    title(['Mode ',num2str(i),', maxU=',num2str(max(abs(UHeig{i}))),' m'],...
        'FontSize',10)
end
```

```
xlabel('Time (sec)','FontSize',10);
```

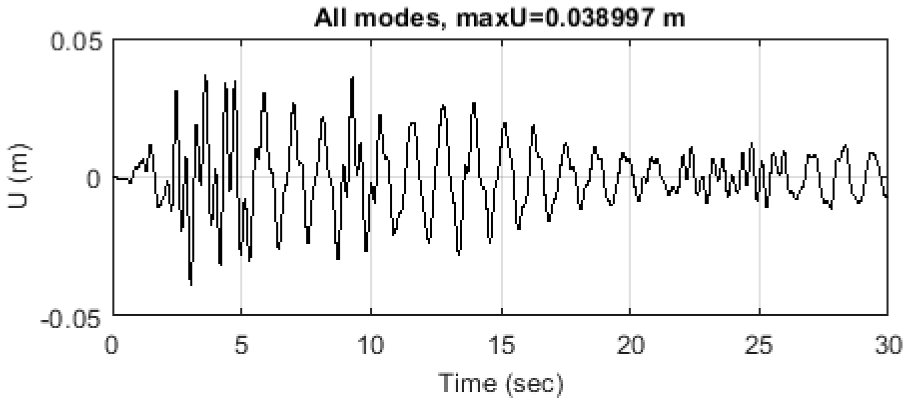




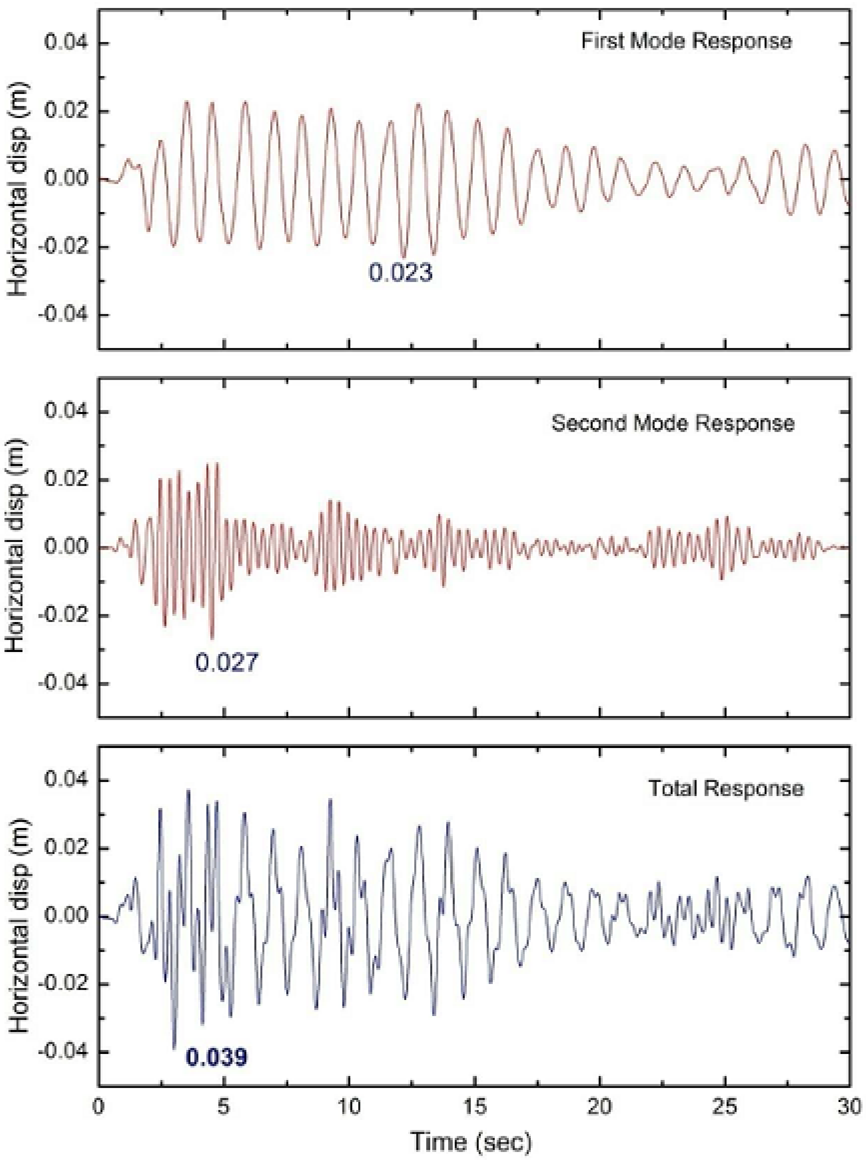
Plot the contribution of all eigenmodes to the horizontal displacement time history.

```
FigHandle=figure('Name','Horizontal displacement','NumberTitle','off');
set(FigHandle,'Position',[50, 50, 500, 200]);
plot(t,UHeig{nDOFs+1},'LineWidth',1.,'Marker','.',...
    'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
grid on
xlim([0,30])
ylim([-0.05,0.05])
```

```
xlabel('Time (sec)','FontSize',10);
ylabel('U (m)','FontSize',10);
title(['All modes, maxU=',num2str(max(abs(UHeig{nDOFs+1}))),' m'],...
    'FontSize',10)
```



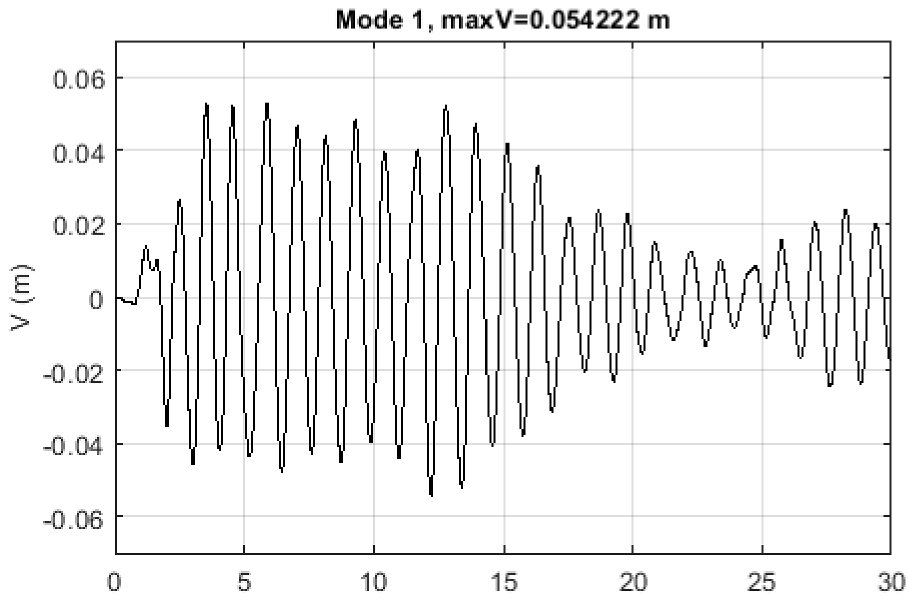
Verify with Figure 3.16 of Chapter 3, Dynamics of Earthquake Analysis, Example 3.7 of NPTEL

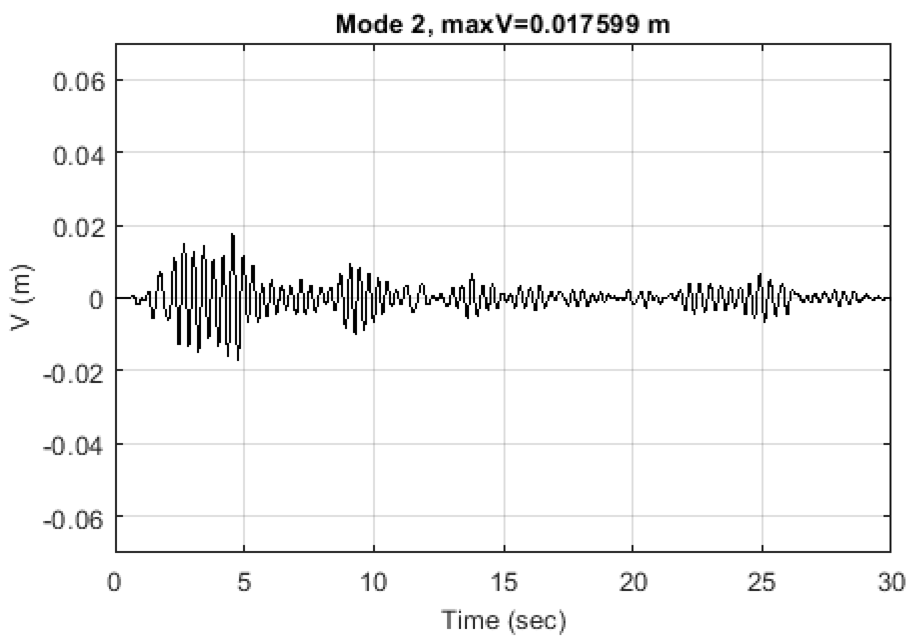


Vertical displacement time history

Plot the contribution of each eigenmode to the vertical displacement time history.

```
FigHandle=figure('Name','Vertical displacement','NumberTitle','off');
set(FigHandle,'Position',[50, 50, 500, 750]);
for i=1:nDOFs
    subplot(nDOFs,1,i)
    plot(t,UVeig{i},'LineWidth',1.,'Marker','.',...
        'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
    grid on
    xlim([0,30])
    ylim([-0.07,0.07])
    ylabel('V (m)','FontSize',10);
    title(['Mode ',num2str(i),', maxV=',num2str(max(abs(UVeig{i}))),' m'],...
        'FontSize',10)
end
xlabel('Time (sec)','FontSize',10);
```

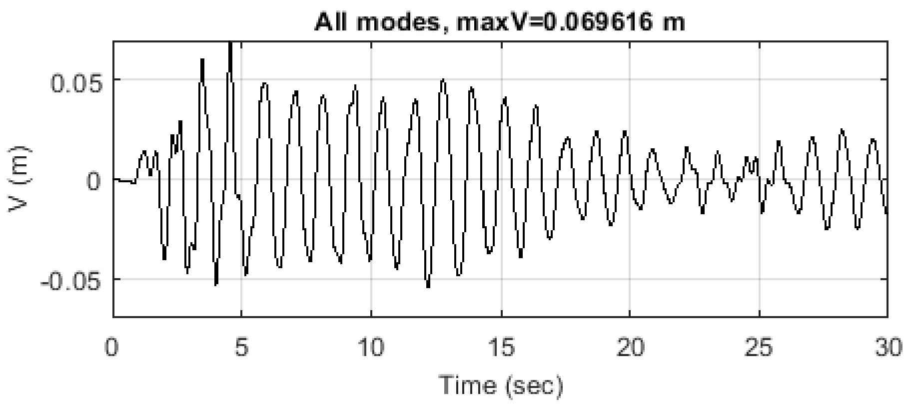




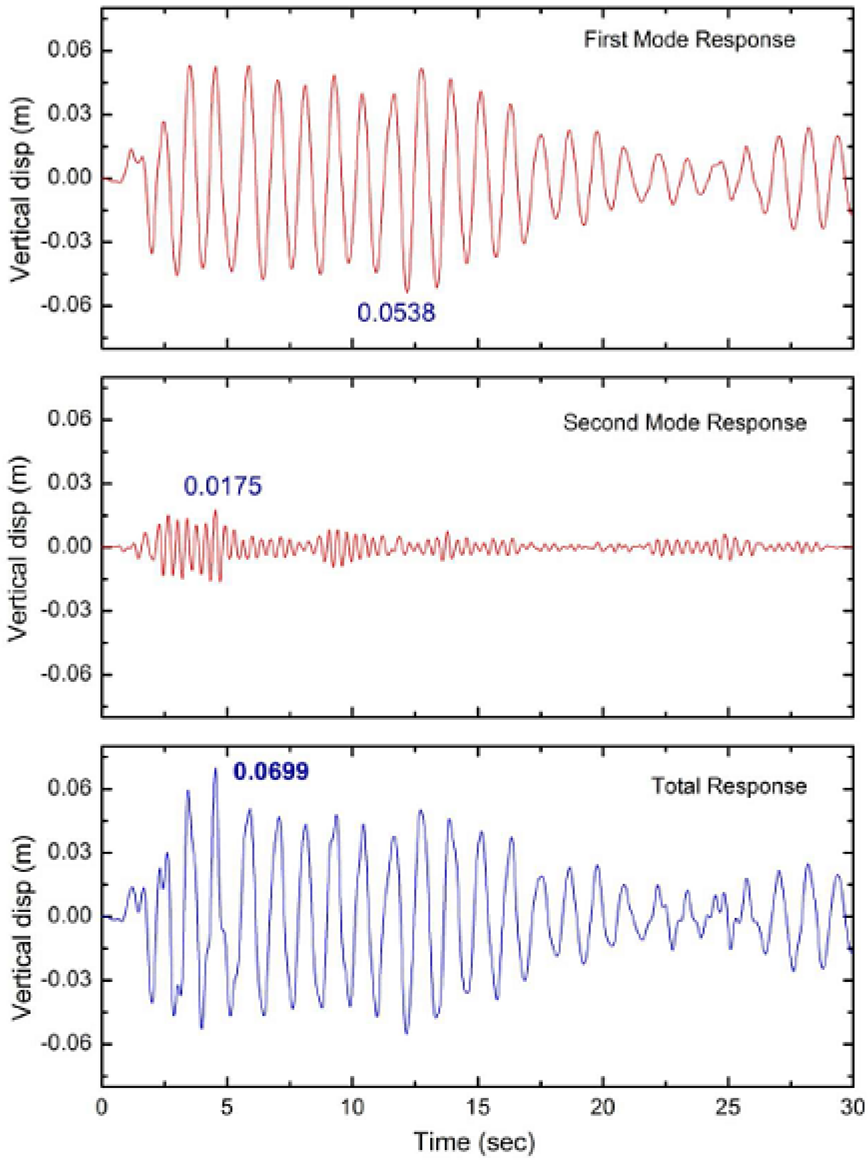
Plot the contribution of all eigenmodes to the vertical displacement time history.

```
FigHandle=figure('Name','Vertical displacement','NumberTitle','off');
set(FigHandle,'Position',[50, 50, 500, 200]);
plot(t,UVeig{nDOFs+1},'LineWidth',1.,'Marker','.',...
    'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
grid on
xlim([0,30])
ylim([-0.07,0.07])
xlabel('Time (sec)','FontSize',10);
ylabel('V (m)','FontSize',10);
```

```
title(['All modes, maxV=',num2str(max(abs(UVeig{nDOFs+1}))),' m'],...
    'FontSize',10)
```



Verify with Figure 3.17 of Chapter 3, Dynamics of Earthquake Analysis, Example 3.7 of NPTEL



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Resample acceleration time history with variable time step size

Contents

Statement of the problem

- The linear dynamic response history analysis (DRHA) algorithms that are included in this package work only for acceleration time histories defined in terms of time step with constant size. This may not be the case in acceleration time histories with nonuniform time step (i.e. variable time step size). In this case, the time history needs to be resampled, so that an equivalent time history is defined with constant time step size, suitable for use in the various functions of this package. Here an example is provided for converting an acceleration time history with nonuniform time step into an equivalent acceleration time history with uniform (constant size) time step.

Load earthquake data

Initial acceleration time history

```
D=load('elcentro_truncated.dat');
dt=D(2,1)-D(1,1);
t=D(:,1);
xgtt=D(:,2);
```

Define resampling parameters

Set the desired time step of the new acceleration time history that is produced after resampling.

```
dt_new=0.02;
```

Upsampling factor

```
p=1;
```

Downsampling factor

```
q=1;
```

Resampling procedure

Sample rate

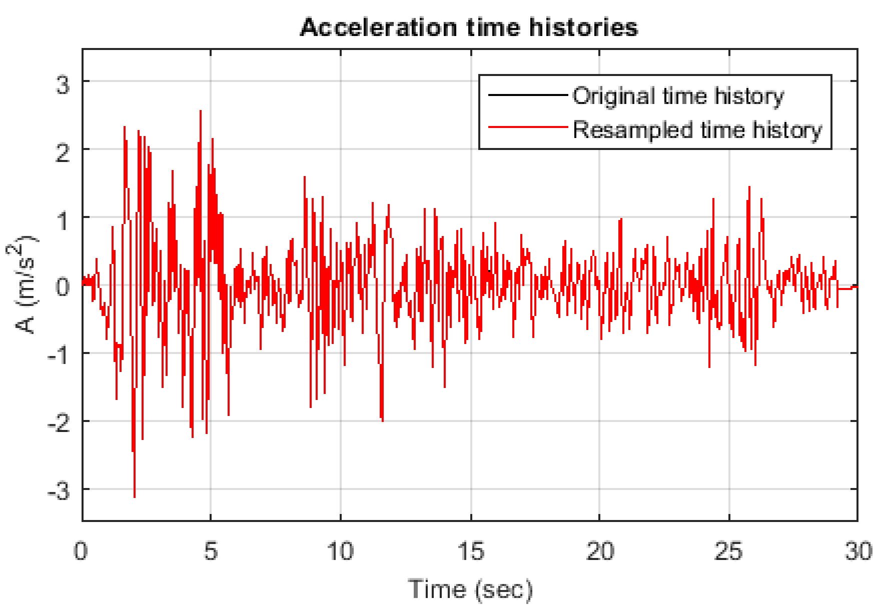
```
fs = 1/dt_new;
```

Resample the original acceleration time history into a new one with constant time step size equal to dt_new

```
[xgtt_new,t_new] = resample(xgtt,t,fs,p,q);
```

Plot original and new acceleration time history

```
FigHandle=figure('Name','Acceleration time histories','NumberTitle','off');
set(FigHandle,'Position',[50, 50, 500, 300]);
plot(t,xgtt,'LineWidth',1.,'Marker','.',...
    'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
hold on
plot(t_new,xgtt_new,'LineWidth',1.,'Marker','.',...
    'MarkerSize',1,'Color',[1 0 0],'markeredgecolor','r')
grid on
xlim([0,30])
ylim([-3.5,3.5])
xlabel('Time (sec)','FontSize',10);
ylabel('A (m/s^2)','FontSize',10);
title(['Acceleration time histories'],...
    'FontSize',10)
legend('Original time history','Resampled time history')
```



Check uniformity of the time step of the original acceleration time history

```
if any(diff(diff(t))>1e-14)
    disp('The original acceleration time history is nonuniform')
else
    disp('The original acceleration time history is uniform')
end
```

```
The original acceleration time history is nonuniform
```

Check uniformity of the time step of the resampled acceleration time history

```
if any(diff(diff(t_new))>1e-14)
    disp('The resampled acceleration time history is nonuniform')
else
    disp('The resampled acceleration time history is uniform')
end
```

```
The resampled acceleration time history is uniform
```

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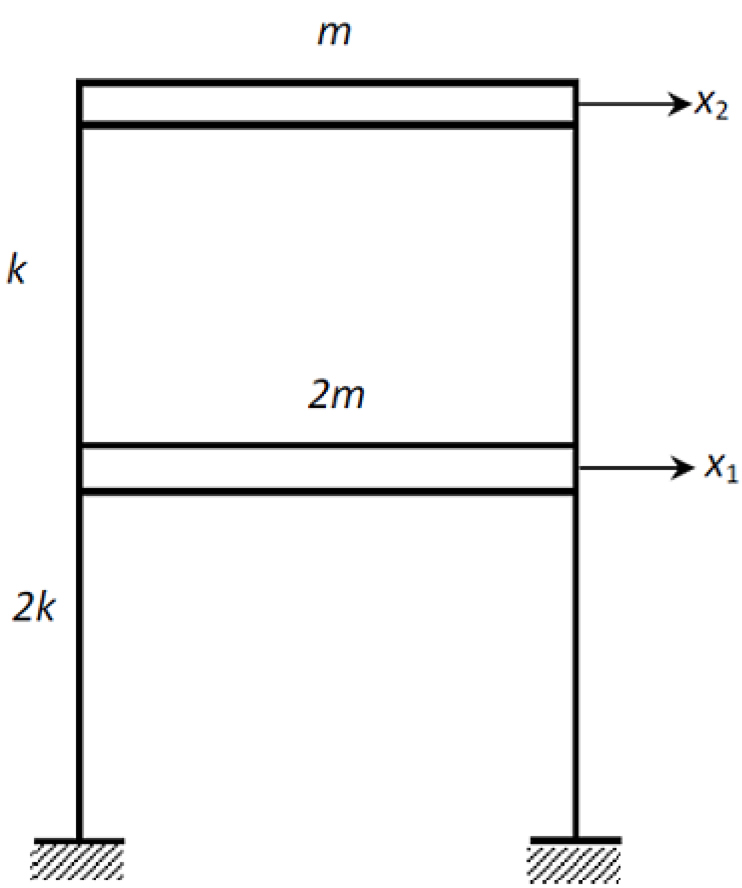
Two-storey shear frame, dynamic analysis with direct integration (NPTEL)

Contents

Statement of the problem

- This example comes from the Introduction to Earthquake Engineering - Web course of NPTEL (National Programme on Technology Enhanced Learning), Chapter 3, Dynamics of Earthquake Analysis, Example 3.6.
- A two-story building is modeled as 2-DOF system and rigid floors as shown in the following figure. Determine the top floor maximum displacement and base shear due to El-Centro, 1940 earthquake ground motion. Take the inter-story stiffness, k =197.392 × 103 N/m, the floor mass, m = 2500 kg and damping ratio as 2%.



Initialization of structural input data

Set the number of degrees of freedom of the structure, which is equal to the number of its storeys.

```
nDOFs=2;
```

Set the lateral stiffness of each storey in N/m.

```
k=197.392e3*[2;1];
```

Set the lumped mass at each floor in kg.

```
m=2500*[2;1];
```

Calculation of structural properties

Calculate the stiffness matrix of the structure in N/m.

```
K=diag([0;k])+diag([k;0])+diag(-k,1)+diag(-k,-1);
K=K(2:end,2:end);
```

Calculate the mass matrix of the structure.

```
M=diag(m);
```

Set the spatial distribution of the effective earthquake forces. Earthquake forces are applied at all dofs of the structure.

```
r=ones(nDOFs,1);
```

Load earthquake data

Earthquake acceleration time history of the El Centro earthquake (El Centro, 1940, El Centro Terminal Substation Building)

```
D=load('elcentro.dat');
dt=D(2,1)-D(1,1);
xgtt=9.81*D(:,2);
```

Set the critical damping ratio ($\xi = 0.05$)

```
ksi=0.02;
```

Time integration algorithm

```
AlgID='U0-V0-Opt';
```

Initial displacement

```
u0=zeros(nDOFs,1);
```

Initial velocity

```
ut0=zeros(nDOFs,1);
```

Minimum absolute value of the eigenvalues of the amplification matrix

```
rinf=1;
```

Dynamic Response History Analysis (DRHA) with modal superposition

Define time

```
t=dt*(0:(numel(xgtt)-1));
```

Calculate the classical damping matrix of the structure

```
C = CDM(K,M,ksi*ones(nDOFs,1));
```

Perform DRHA analysis

```
[U,~,~,f] = LDRHA_DI_MDOF(K,C,M,r,dt,xgtt,AlgID,u0,ut0,rinf);
```

Base shear time history for all eigenmodes

```
FBeig=sum(f,1);
```

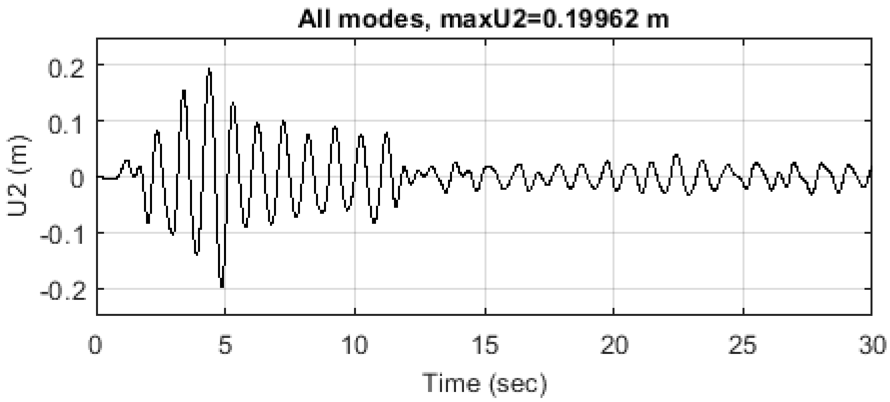
Roof displacement time history (2nd DOF) for all eigenmodes

```
Ueig=U(2,:);
```

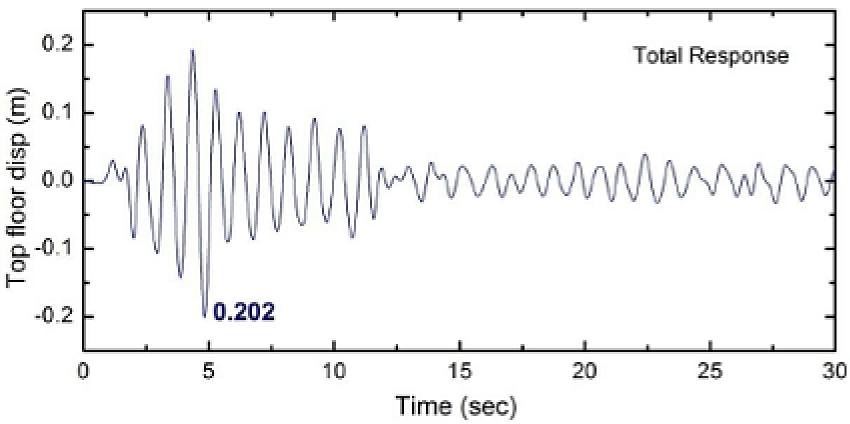
Roof displacement time history

Plot the contribution of all eigenmodes to the roof displacement time history. Convert displacements from m to cm.

```
FigHandle=figure('Name','Roof displacement','NumberTitle','off');
set(FigHandle,'Position',[50, 50, 500, 200]);
plot(t,Ueig,'LineWidth',1.,'Marker','.',...
    'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
grid on
xlim([0,30])
ylim([-0.25,0.25])
xlabel('Time (sec)','FontSize',10);
ylabel('U2 (m)','FontSize',10);
title(['All modes, maxU2=',num2str(max(abs(Ueig))),' m'],...
    'FontSize',10)
```



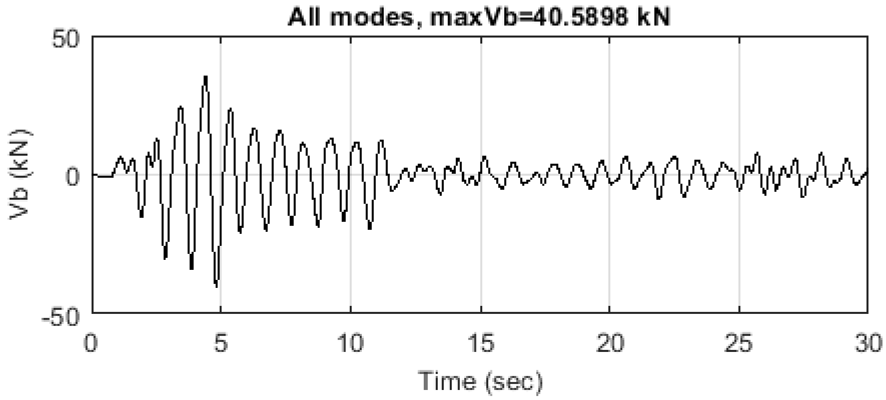
Verify with Figure 3.13 of Chapter 3, Dynamics of Earthquake Analysis, Example 3.6 of NPTEL



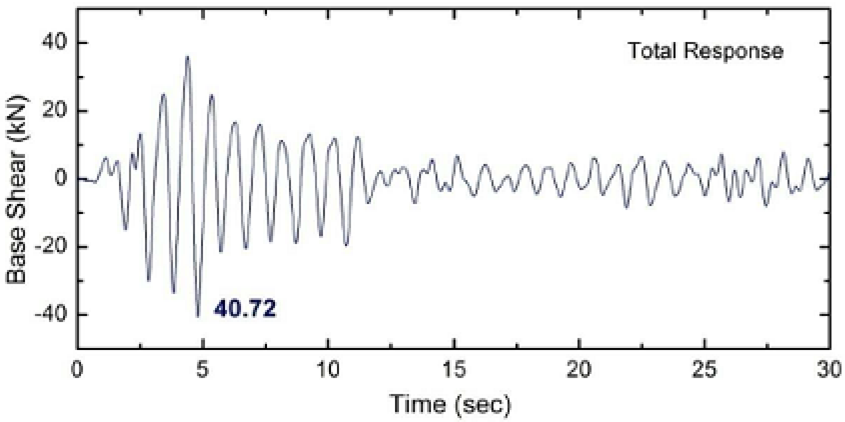
Base shear time history

Plot the contribution of all eigenmodes to the base shear time history. Convert forces from N to kN.

```
FigHandle=figure('Name','Base shear','NumberTitle','off');
set(FigHandle,'Position',[50, 50, 500, 200]);
plot(t,FBeig/1e3,'LineWidth',1.,'Marker','.',...
    'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
grid on
xlim([0,30])
ylim([-50,50])
xlabel('Time (sec)','FontSize',10);
ylabel('Vb (kN)','FontSize',10);
title(['All modes, maxVb=',num2str(max(abs(FBeig/1e3))),' kN'],...
    'FontSize',10)
```



Verify with Figure 3.14 of Chapter 3, Dynamics of Earthquake Analysis, Example 3.6 of NPTEL



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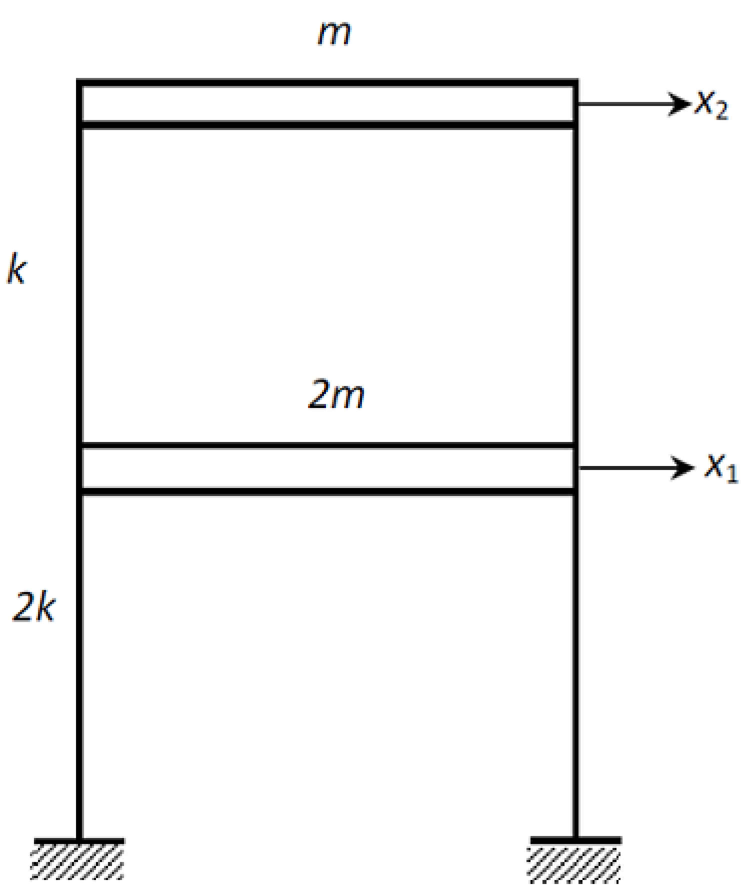
Two-storey shear frame dynamic analysis with modal superposition (NPTEL)

Contents

Statement of the problem

- This example comes from the Introduction to Earthquake Engineering - Web course of NPTEL (National Programme on Technology Enhanced Learning), Chapter 3, Dynamics of Earthquake Analysis, Example 3.6.
- A two-story building is modeled as 2-DOF system and rigid floors as shown in the following figure. Determine the top floor maximum displacement and base shear due to El-Centro, 1940 earthquake ground motion. Take the inter-story stiffness, k =197.392 × 103 N/m, the floor mass, m = 2500 kg and damping ratio as 2%.



Initialization of structural input data

Set the number of degrees of freedom of the structure, which is equal to the number of its storeys.

```
nDOFs=2;
```

Set the lateral stiffness of each storey in N/m.

```
k=197.392e3*[2;1];
```

Set the lumped mass at each floor in kg.

```
m=2500*[2;1];
```

Calculation of structural properties

Calculate the stiffness matrix of the structure in N/m.

```
K=diag([0;k])+diag([k;0])+diag(-k,1)+diag(-k,-1);
K=K(2:end,2:end);
```

Calculate the mass matrix of the structure.

```
M=diag(m);
```

Set the spatial distribution of the effective earthquake forces. Earthquake forces are applied at all dofs of the structure.

```
r=ones(nDOFs,1);
```

Load earthquake data

Earthquake acceleration time history of the El Centro earthquake (El Centro, 1940, El Centro Terminal Substation Building)

```
D=load('elcentro.dat');
dt=D(2,1)-D(1,1);
xgtt=9.81*D(:,2);
```

Set the critical damping ratio ($\xi = 0.05$)

```
ksi=0.02;
```

Time integration algorithm

```
AlgID='U0-V0-Opt';
```

Initial displacement

```
u0=zeros(nDOFs,1);
```

Initial velocity

```
ut0=zeros(nDOFs,1);
```

Minimum absolute value of the eigenvalues of the amplification matrix

```
rinf=1;
```

Dynamic Response History Analysis (DRHA) with modal superposition

Define time

```
t=dt*(0:(numel(xgtt)-1));
```

Perform DRHA analysis for each separate eigenmode to calculate the contribution of each eigenmode to the various frame responses

```
% Initialize
FBeig=cell(nDOFs+1,1);
Ueig=cell(nDOFs+1,1);
for i=1:nDOFs
    % DRHA analysis
    [U,~,~,f] = LDRHA_MS_MDOF(K,M,r,dt,xgtt,ksi,AlgID,u0,ut0,rinf,i);
    % Store the 2nd storey shear time history (2nd DOF) for each eigenmode
    FBeig{i}=sum(f,1);
    % Store the roof displacement time history (2nd DOF) for each eigenmode
    Ueig{i}=U(2,:);
end
```

Perform DRHA analysis for all eigenmodes to calculate the contribution of all eigenmodes to the various frame responses

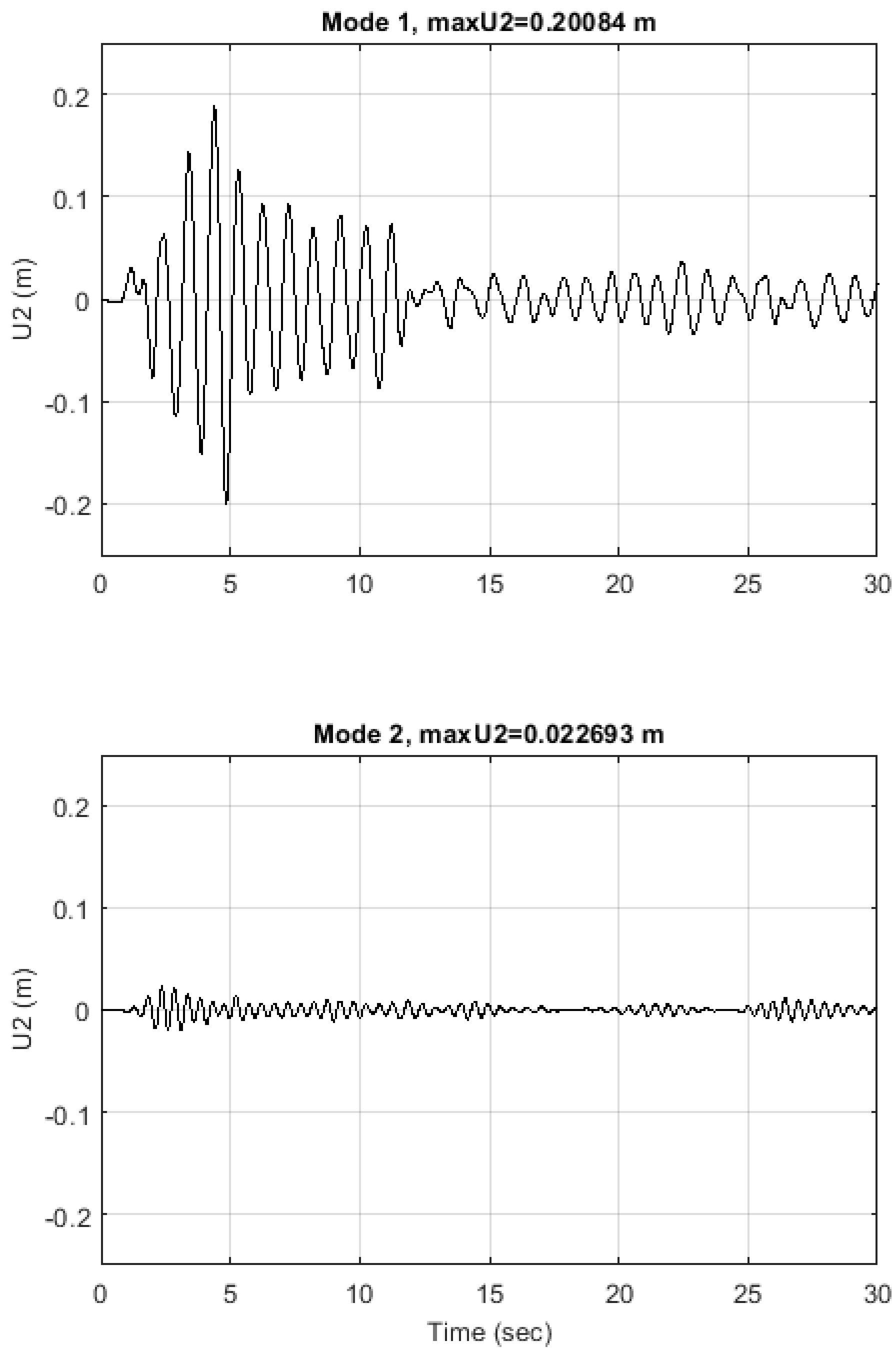
```
% Eigenmodes that are superposed
eigInd=(1:nDOFs)';
% DRHA analysis
[U,~,~,f] = LDRHA_MS_MDOF(K,M,r,dt,xgtt,ksi,AlgID,u0,ut0,rinf,eigInd);
% Base shear time history for all eigenmodes
FBeig{nDOFs+1}=sum(f,1);
% Roof displacement time history (2nd DOF) for all eigenmodes
Ueig{nDOFs+1}=U(2,:);
```

Roof displacement time history

Plot the contribution of each eigenmode to the roof displacement time history.

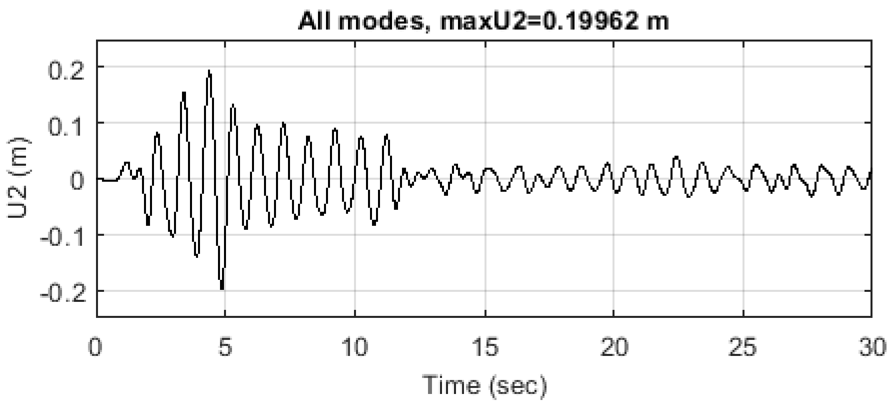
```
FigHandle=figure('Name','Roof displacement','NumberTitle','off');
set(FigHandle,'Position',[50, 50, 500, 750]);
for i=1:nDOFs
    subplot(nDOFs,1,i)
    plot(t,Ueig{i},'LineWidth',1.,'Marker','.',...
        'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
```

```
    grid on
    xlim([0,30])
    ylim([-0.25,0.25])
    ylabel('U2 (m)','FontSize',10);
    title(['Mode ',num2str(i),', maxU2=',num2str(max(abs(Ueig{i}))),' m'],...
        'FontSize',10)
end
xlabel('Time (sec)','FontSize',10);
```

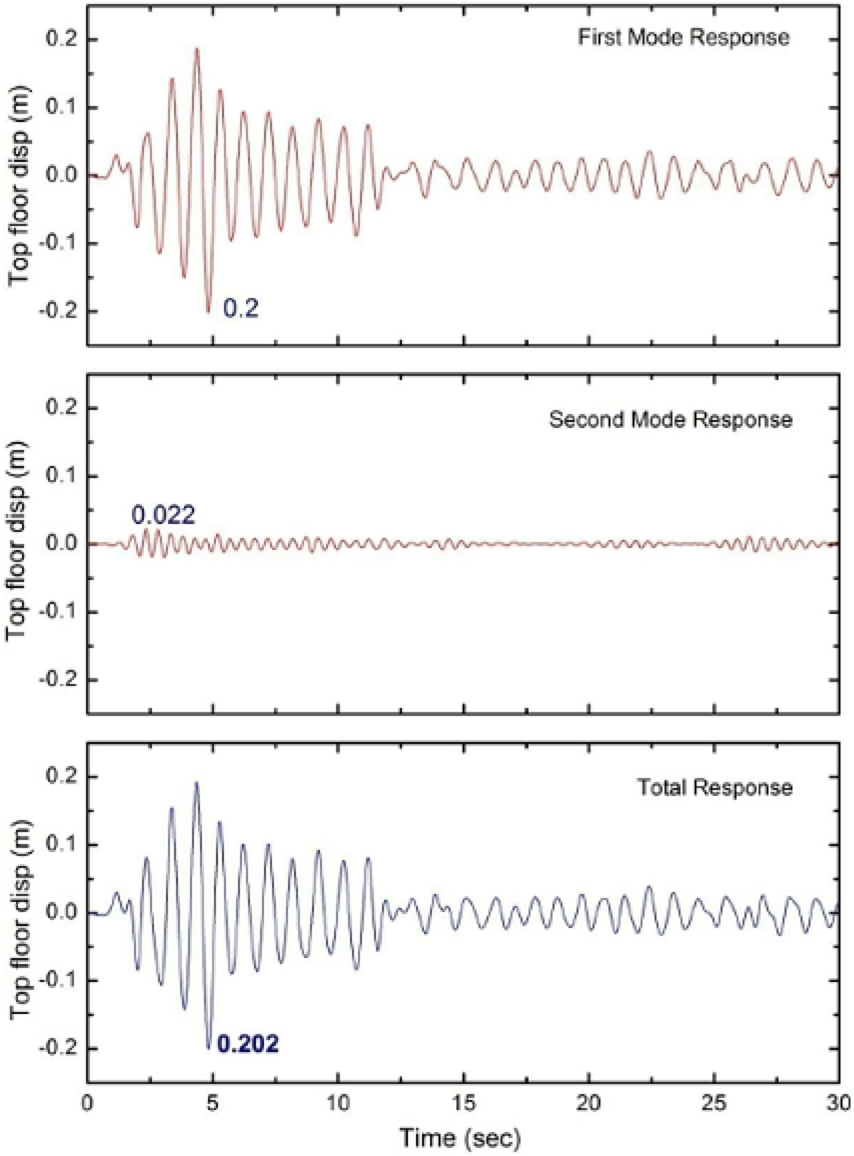


Plot the contribution of all eigenmodes to the roof displacement time history. Convert displacements from m to cm.

```
FigHandle=figure('Name','Roof displacement','NumberTitle','off');
set(FigHandle,'Position',[50, 50, 500, 200]);
plot(t,Ueig{nDOFs+1},'LineWidth',1.,'Marker','.',...
    'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
grid on
xlim([0,30])
ylim([-0.25,0.25])
xlabel('Time (sec)','FontSize',10);
ylabel('U2 (m)','FontSize',10);
title(['All modes, maxU2=',num2str(max(abs(Ueig{nDOFs+1}))),' m'],...
    'FontSize',10)
```



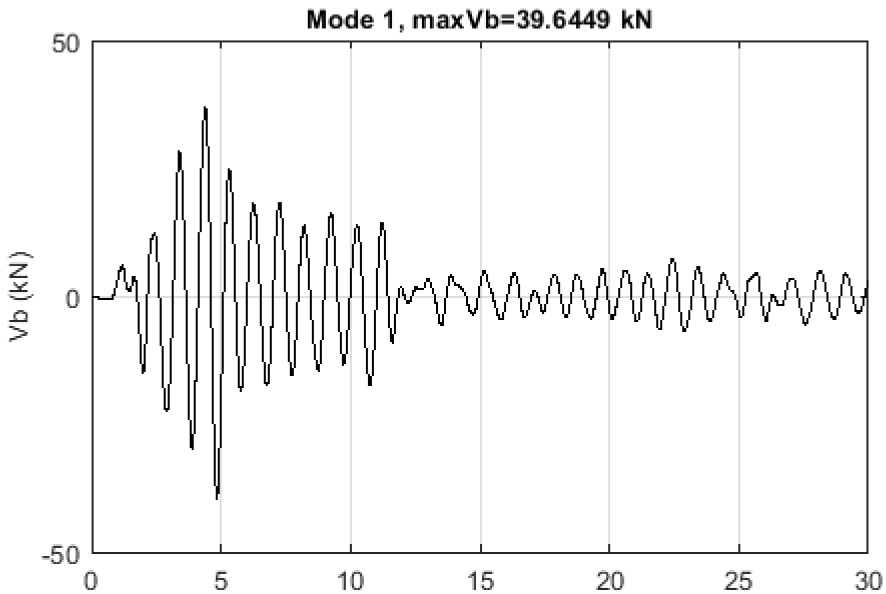
Verify with Figure 3.13 of Chapter 3, Dynamics of Earthquake Analysis, Example 3.6 of NPTEL

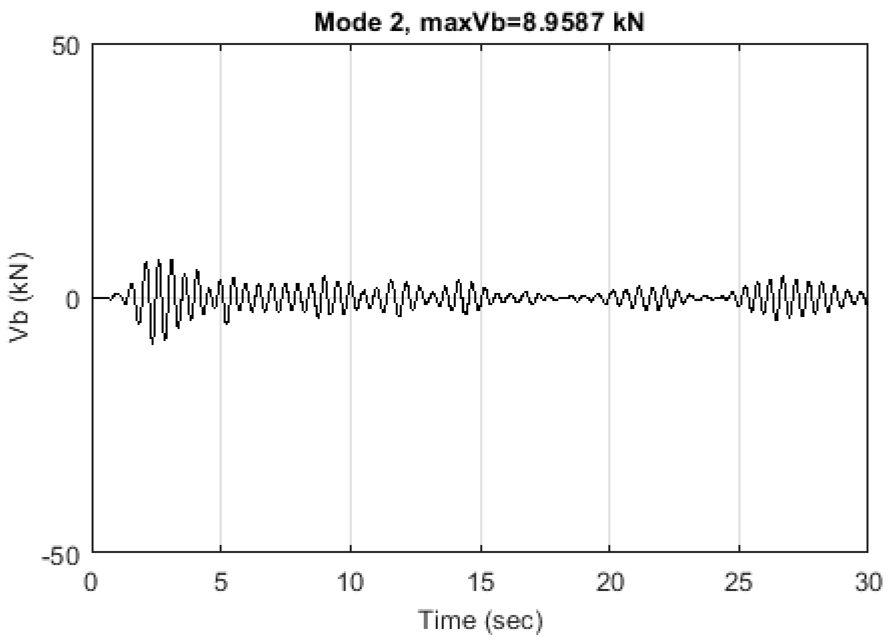


Base shear time history

Plot the contribution of each eigenmode to the base shear time history. Convert forces from N to kN.

```
FigHandle=figure('Name','Base shear','NumberTitle','off');
set(FigHandle,'Position',[50, 50, 500, 750]);
for i=1:nDOFs
    subplot(nDOFs,1,i)
    plot(t,FBeig{i}/1e3,'LineWidth',1.,'Marker','.',...
        'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
    grid on
    xlim([0,30])
    ylim([-50,50])
    ylabel('Vb (kN)','FontSize',10);
    title(['Mode ',num2str(i),', maxVb=',num2str(max(abs(FBeig{i}/1e3))),' kN'],...
        'FontSize',10)
end
xlabel('Time (sec)','FontSize',10);
```

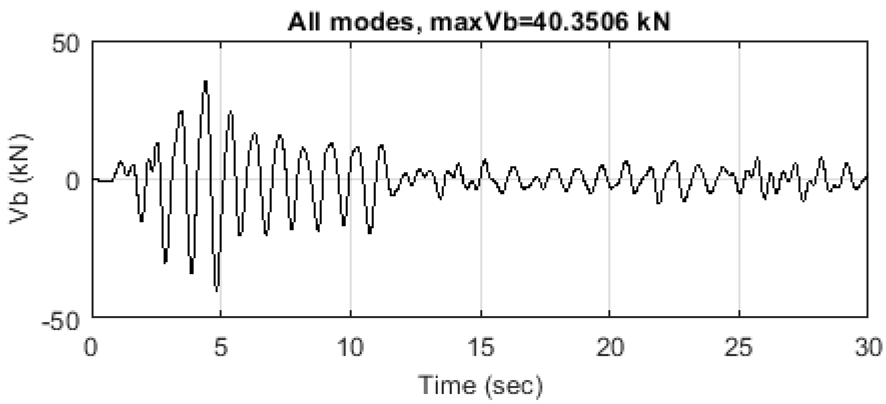




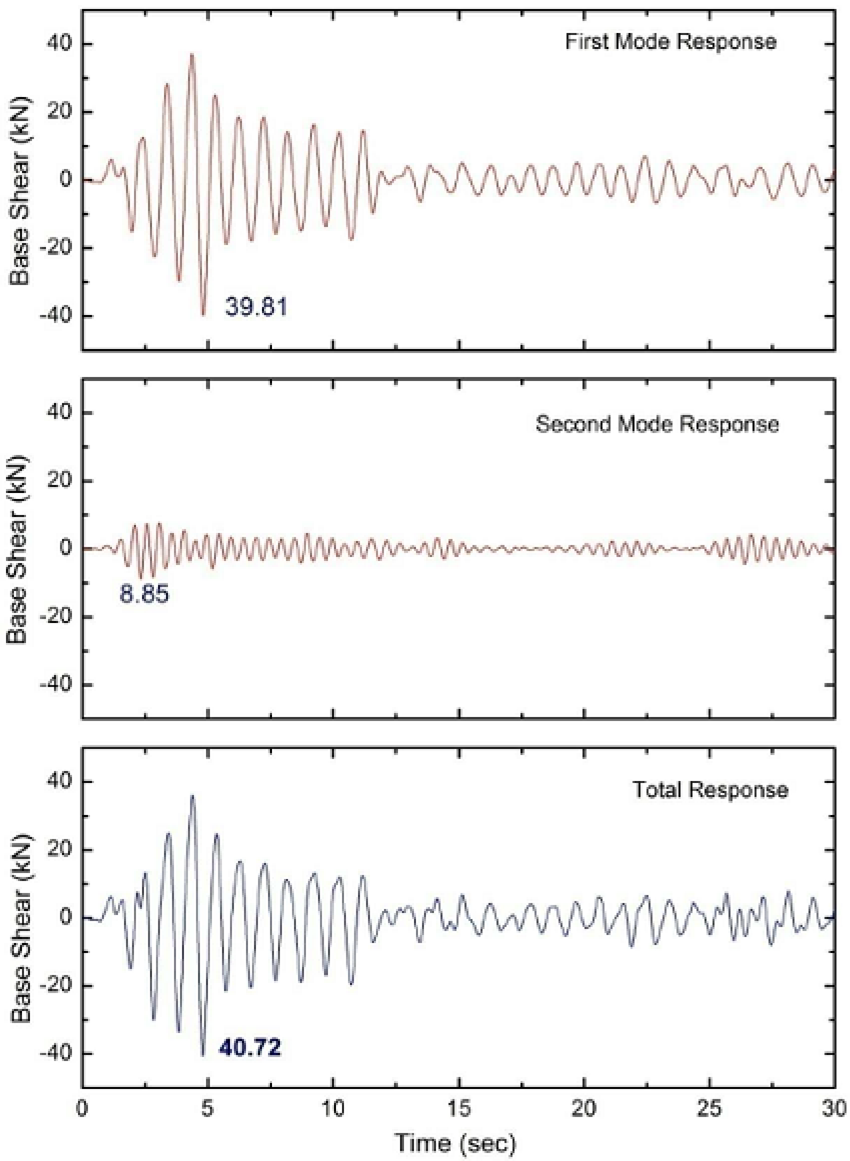
Plot the contribution of all eigenmodes to the base shear time history. Convert forces from N to kN.

```
FigHandle=figure('Name','Base shear','NumberTitle','off');
set(FigHandle,'Position',[50, 50, 500, 200]);
plot(t,FBeig{nDOFs+1}/1e3,'LineWidth',1.,'Marker','.',...
    'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
grid on
xlim([0,30])
ylim([-50,50])
xlabel('Time (sec)','FontSize',10);
ylabel('Vb (kN)','FontSize',10);
```

```
title(['All modes, maxVb=',num2str(max(abs(FBeig{nDOFs+1}/1e3))),' kN'],...
    'FontSize',10)
```



Verify with Figure 3.14 of Chapter 3, Dynamics of Earthquake Analysis, Example 3.6 of NPTEL



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Four-story frame with an appendage, dynamic analysis with direct integration (Chopra, 2019)

Contents

Statement of the problem

- **Chopra (2019), Section 13.2.7:** Consider a four-story building with a light appendage-a penthouse, a small housing for mechanical equipment, an advertising billboard, or the like. This example is presented because it brings out certain special response features representative of a system with two natural frequencies that are close.
- **Chopra (2019), Section 13.2.7:** The lumped masses at the first four floors are $m_j = m = 45Mg$ (=0.45kN-sec^2/cm) and the appendage mass is $m_5 = 0.01m$. The lateral stiffness of each of the first four stories is $k_j = k = 39.431kN/cm.$, the appendage stiffness $k_5 = 0.0012k$. The height of each story and the appendage is 4 m. The damping ratio for all natural modes is $\zeta_n = 0.05$. The response of this system to the El Centro ground motion is determined

Initialization of structural input data

Set the storey height of the structure in m.

```
h=4;
```

Set the number of degrees of freedom of the structure, which is equal to the number of its storeys.

```
nDOFs=5;
```

Set the lateral stiffnesses of all storeys in N/m.

```
k=[3.9431e6;3.9431e6;3.9431e6;3.9431e6;0.0012*3.9431e6];
```

Set the lumped mass at each floor in kg.

```
m=[45e3;45e3;45e3;45e3;0.01*45e3];
```

Calculation of structural properties

Calculate the stiffness matrix of the structure in N/m.

```
K=diag([0;k])+diag([k;0])-diag(k,1)-diag(k,-1);
K=K(2:end,2:end);
```

Calculate the mass matrix of the structure.

```
M=diag(m);
```

Set the spatial distribution of the effective earthquake forces. Earthquake forces are applied at all dofs of the structure.

```
r=ones(5,1);
```

Load earthquake data

Earthquake acceleration time history of the El Centro earthquake (El Centro, 1940, El Centro Terminal Substation Building)

```
D=load('elcentro.dat');
dt=D(2,1)-D(1,1);
xgtt=9.81*D(:,2);
```

Set the critical damping ratio ($\xi = 0.05$)

```
ksi=0.05;
```

Time integration algorithm

```
AlgID='U0-V0-Opt';
```

Initial displacement

```
u0=zeros(nDOFs,1);
```

Initial velocity

```
ut0=zeros(nDOFs,1);
```

Minimum absolute value of the eigenvalues of the amplification matrix

```
rinf=1;
```

Dynamic Response History Analysis (DRHA) with direct integration

Define time

```
t=dt*(0:(numel(xgtt)-1));
```

Calculate the classical damping matrix of the structure

```
C = CDM(K,M,ksi*ones(nDOFs,1));
```

Perform DRHA analysis

```
[~,~,~,f] = LDRHA_DI_MDOF(K,C,M,r,dt,xgtt,AlgID,u0,ut0,rinf);
```

Base shear time history

```
FBeig=sum(f,1);
```

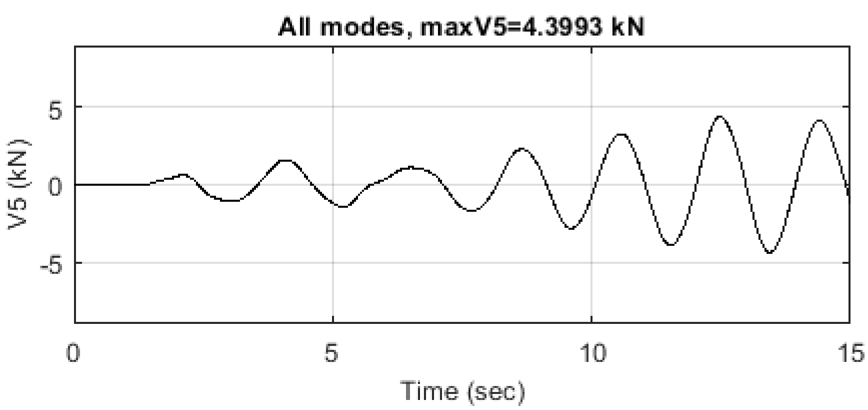
Appendage shear time history

```
Feig=f(5,:);
```

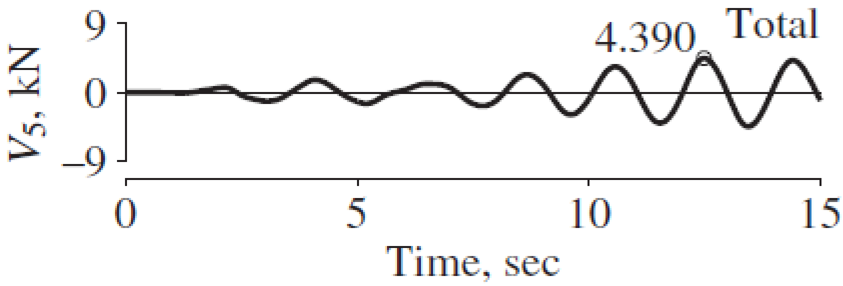
Appendage shear time history

Plot the appendage shear time history.

```
FigHandle=figure('Name','Appendage shear','NumberTitle','off');
set(FigHandle,'Position',[50, 50, 500, 200]);
plot(t,Feig/1e3,'LineWidth',1.,'Marker','.',...
    'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
grid on
xlim([0,15])
ylim([-9,9])
xlabel('Time (sec)','FontSize',10);
ylabel('V5 (kN)','FontSize',10);
title(['All modes, maxV5=',num2str(max(abs(Feig/1e3))),' kN'],...
    'FontSize',10)
```



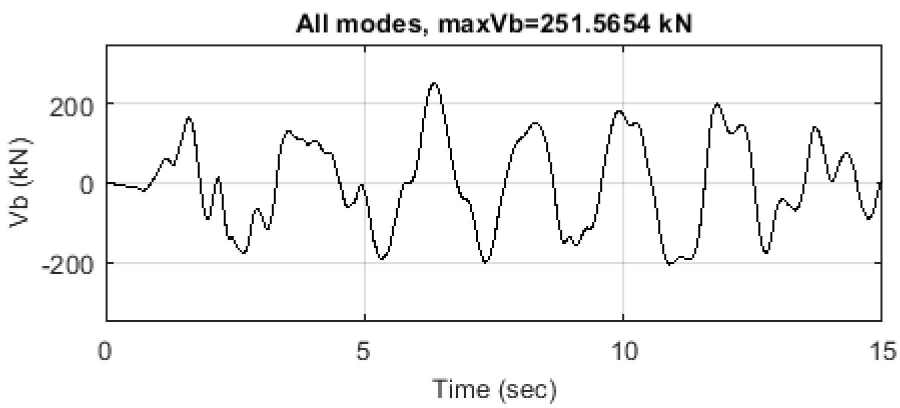
Verify with Figure 13.2.11 (right) of Chopra (2019)



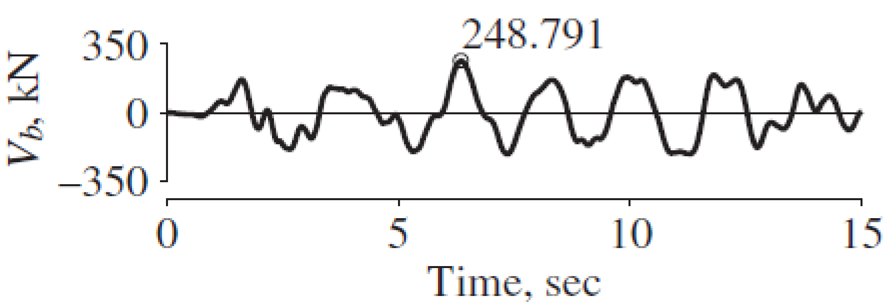
Base shear time history

Plot the base shear time history.

```
FigHandle=figure('Name','Base shear','NumberTitle','off');
set(FigHandle,'Position',[50, 50, 500, 200]);
plot(t,FBeig/1e3,'LineWidth',1.,'Marker','.',...
    'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
grid on
xlim([0,15])
ylim([-350,350])
xlabel('Time (sec)','FontSize',10);
ylabel('Vb (kN)','FontSize',10);
title(['All modes, maxVb=',num2str(max(abs(FBeig/1e3))),' kN'],...
    'FontSize',10)
```



Verify with Figure 13.2.11 (left) of Chopra (2019)



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Four-story frame with an appendage dynamic analysis with modal superposition (Chopra, 2019)

Contents

Statement of the problem

- **Chopra (2019), Section 13.2.7:** Consider a four-story building with a light appendage-a penthouse, a small housing for mechanical equipment, an advertising billboard, or the like. This example is presented because it brings out certain special response features representative of a system with two natural frequencies that are close.
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Initialization of structural input data

Set the storey height of the structure in m.

```
h=4;
```

Set the number of degrees of freedom of the structure, which is equal to the number of its storeys.

```
nDOFs=5;
```

Set the lateral stiffnesses of all storeys in N/m.

```
k=[3.9431e6;3.9431e6;3.9431e6;3.9431e6;0.0012*3.9431e6];
```

Set the lumped mass at each floor in kg.

```
m=[45e3;45e3;45e3;45e3;0.01*45e3];
```

Calculation of structural properties

Calculate the stiffness matrix of the structure in N/m.

```
K=diag([0;k])+diag([k;0])-diag(k,1)-diag(k,-1);
K=K(2:end,2:end);
```

Calculate the mass matrix of the structure.

```
M=diag(m);
```

Set the spatial distribution of the effective earthquake forces. Earthquake forces are applied at all dofs of the structure.

```
r=ones(5,1);
```

Load earthquake data

Earthquake acceleration time history of the El Centro earthquake (El Centro, 1940, El Centro Terminal Substation Building)

```
D=load('elcentro.dat');
dt=D(2,1)-D(1,1);
xgtt=9.81*D(:,2);
```

Set the critical damping ratio ($\xi = 0.05$)

```
ksi=0.05;
```

Time integration algorithm

```
AlgID='U0-V0-Opt';
```

Initial displacement

```
u0=zeros(nDOFs,1);
```

Initial velocity

```
ut0=zeros(nDOFs,1);
```

Minimum absolute value of the eigenvalues of the amplification matrix

```
rinf=1;
```

Dynamic Response History Analysis (DRHA) with modal superposition

Define time

```
t=dt*(0:(numel(xgtt)-1));
```

Perform DRHA analysis for each separate eigenmode to calculate the contribution of each eigenmode to the various frame responses

```
% Initialize
FBeig=cell(5,1);
Feig=cell(5,1);
for i=1:5
    % DRHA analysis
    [U,~,~,f] = LDRHA_MS_MDOF(K,M,r,dt,xgtt,ksi,AlgID,u0,ut0,rinf,i);
    % Store the base shear time history for each eigenmode
    FBeig{i}=sum(f,1);
    % Store the appendage shear time history for each eigenmode
    Feig{i}=f(5,:);
end
```

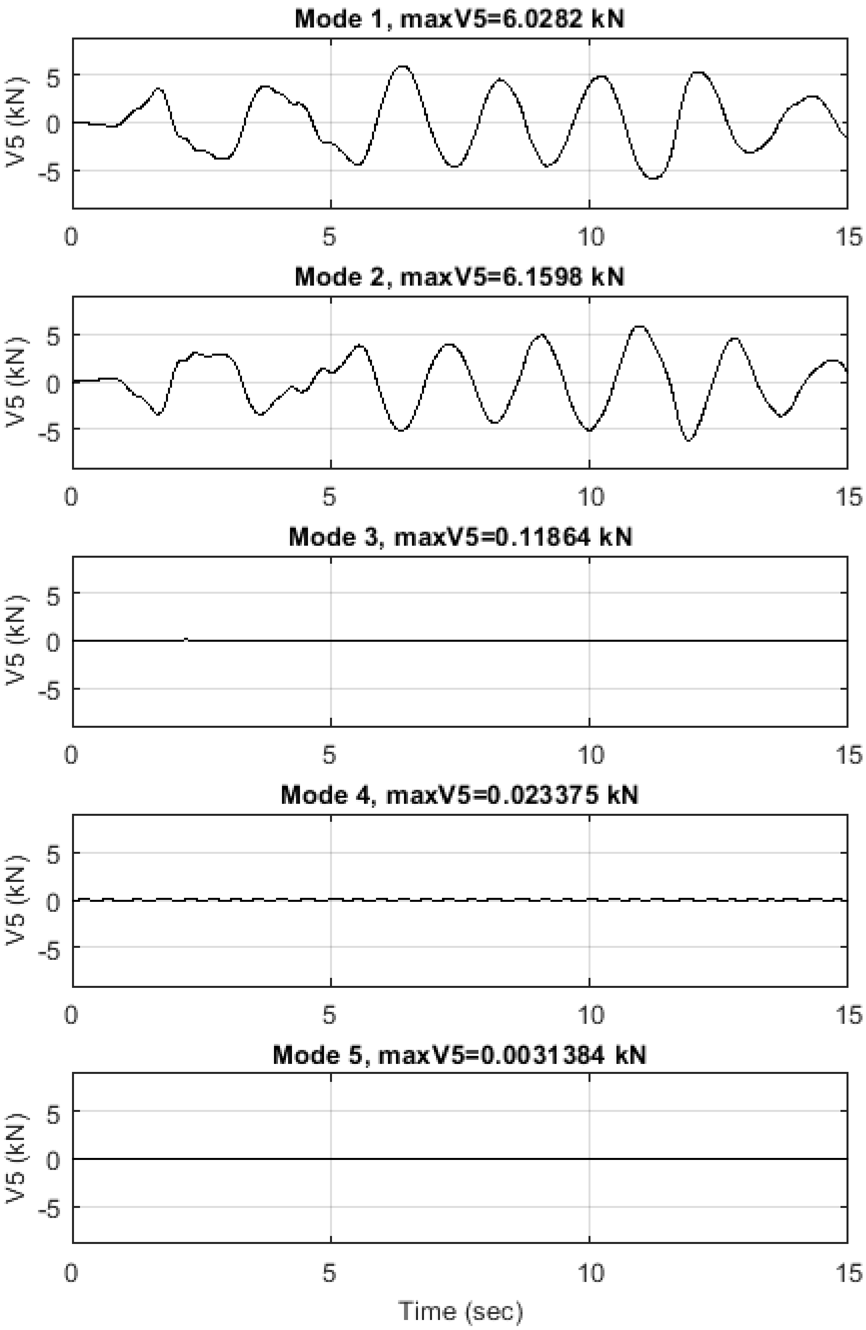
Perform DRHA analysis for all eigenmodes to calculate the contribution of all eigenmodes to the various frame responses

```
% Eigenmodes that are superposed
eigInd=[1;2;3;4;5];
% DRHA analysis
[U,~,~,f] = LDRHA_MS_MDOF(K,M,r,dt,xgtt,ksi,AlgID,u0,ut0,rinf,eigInd);
% Base shear time history for all eigenmodes
FBeig{6}=sum(f,1);
% Appendage shear time history for all eigenmodes
Feig{6}=f(5,:);
```

Appendage shear time history

Plot the contribution of each eigenmode to the appendage shear time history.

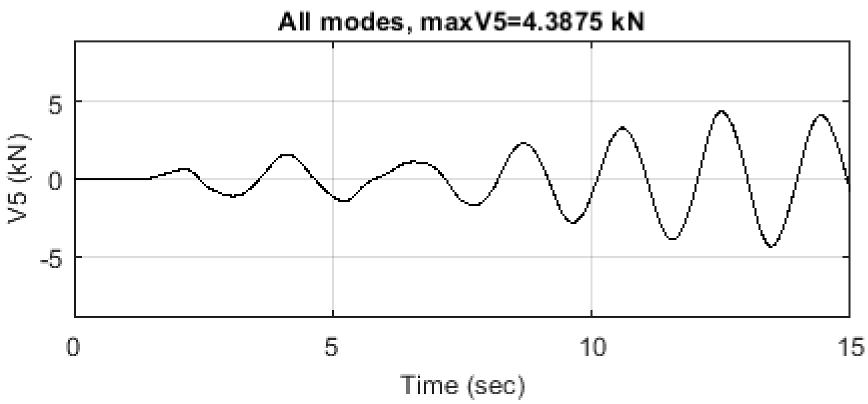
```
FigHandle=figure('Name','Appendage shear','NumberTitle','off');
set(FigHandle,'Position',[50, 50, 500, 750]);
for i=1:5
    subplot(nDOFs,1,i)
    plot(t,Feig{i}/1e3,'LineWidth',1.,'Marker','.',...
        'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
    grid on
    xlim([0,15])
    ylim([-9,9])
    ylabel('V5 (kN)','FontSize',10);
    title(['Mode ',num2str(i),', maxV5=',num2str(max(abs(Feig{i}/1e3))),' kN'],...
        'FontSize',10)
end
xlabel('Time (sec)','FontSize',10);
```



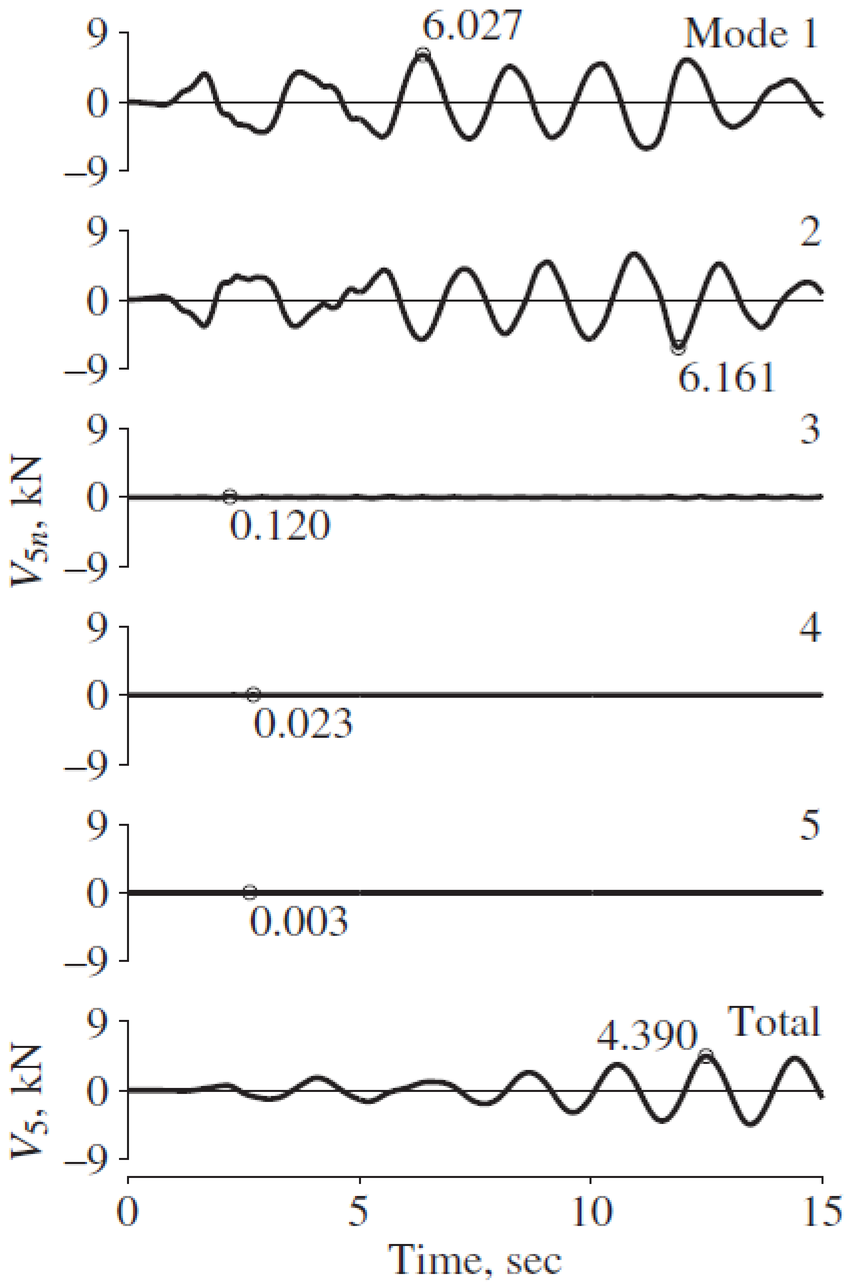
Plot the contribution of all eigenmodes to the appendage shear time history.

```
FigHandle=figure('Name','Appendage shear','NumberTitle','off');
set(FigHandle,'Position',[50, 50, 500, 200]);
plot(t,Feig{6}/1e3,'LineWidth',1.,'Marker','.',...
    'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
grid on
xlim([0,15])
ylim([-9,9])
xlabel('Time (sec)','FontSize',10);
ylabel('V5 (kN)','FontSize',10);
```

```
title(['All modes, maxV5=',num2str(max(abs(Feig{6}/1e3))),' kN'],...
    'FontSize',10)
```



Verify with Figure 13.2.11 (right) of Chopra (2019)

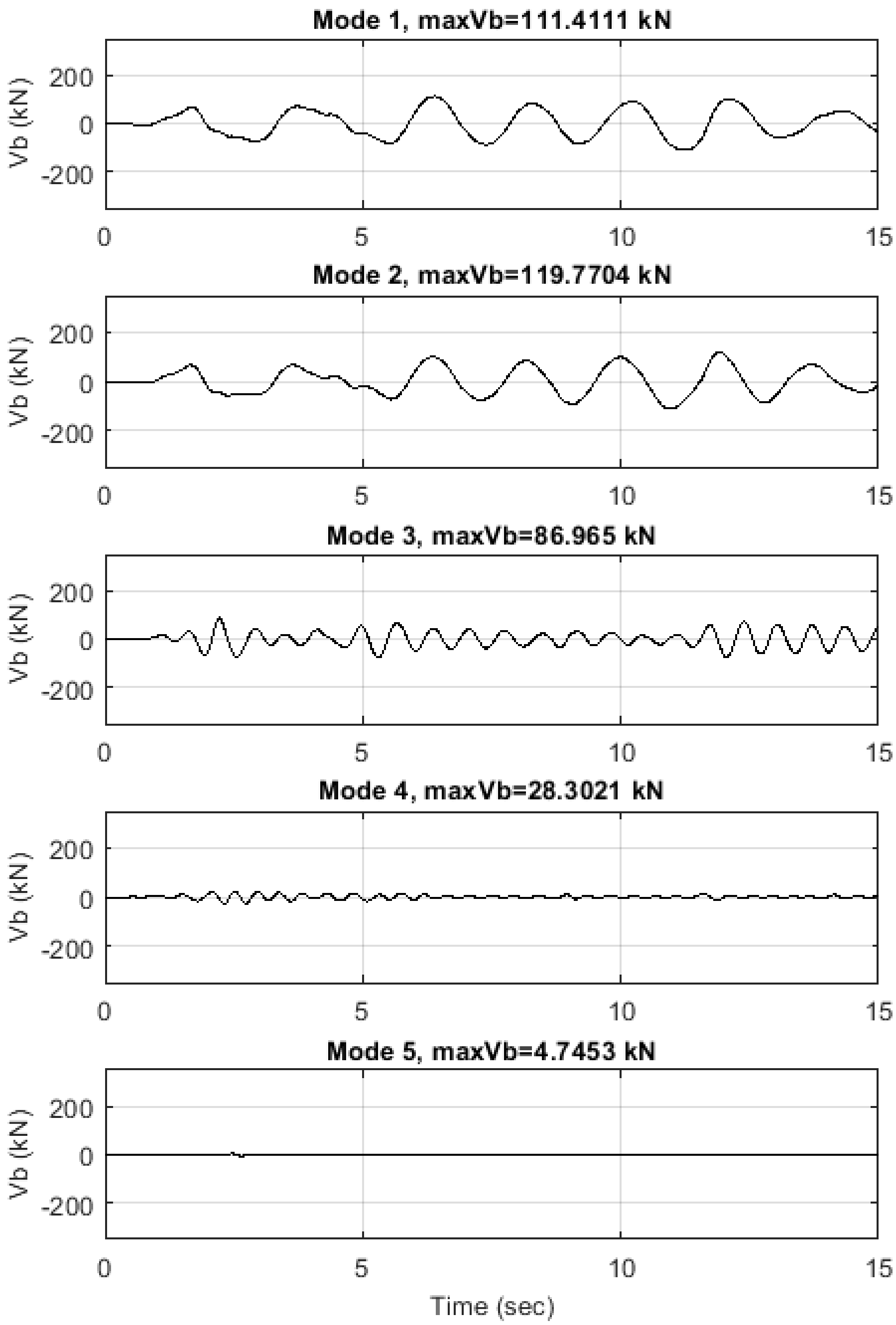


Base shear time history

Plot the contribution of each eigenmode to the base shear time history.

```
FigHandle=figure('Name','Base shear','NumberTitle','off');
set(FigHandle,'Position',[50, 50, 500, 750]);
for i=1:5
    subplot(nDOFs,1,i)
    plot(t,FBeig{i}/1e3,'LineWidth',1.,'Marker','.',...
        'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
    grid on
    xlim([0,15])
    ylim([-350,350])
    ylabel('Vb (kN)','FontSize',10);
    title(['Mode ',num2str(i),', maxVb=',num2str(max(abs(FBeig{i}/1e3))),' kN'],...
        'FontSize',10)
end
```

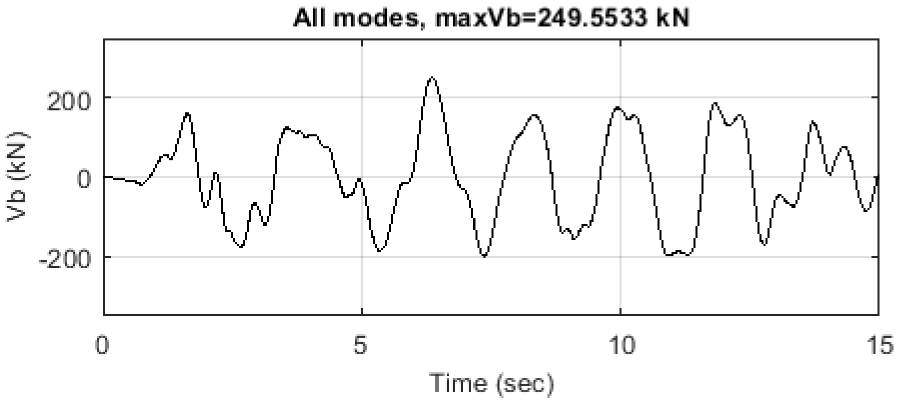
```
xlabel('Time (sec)','FontSize',10);
```



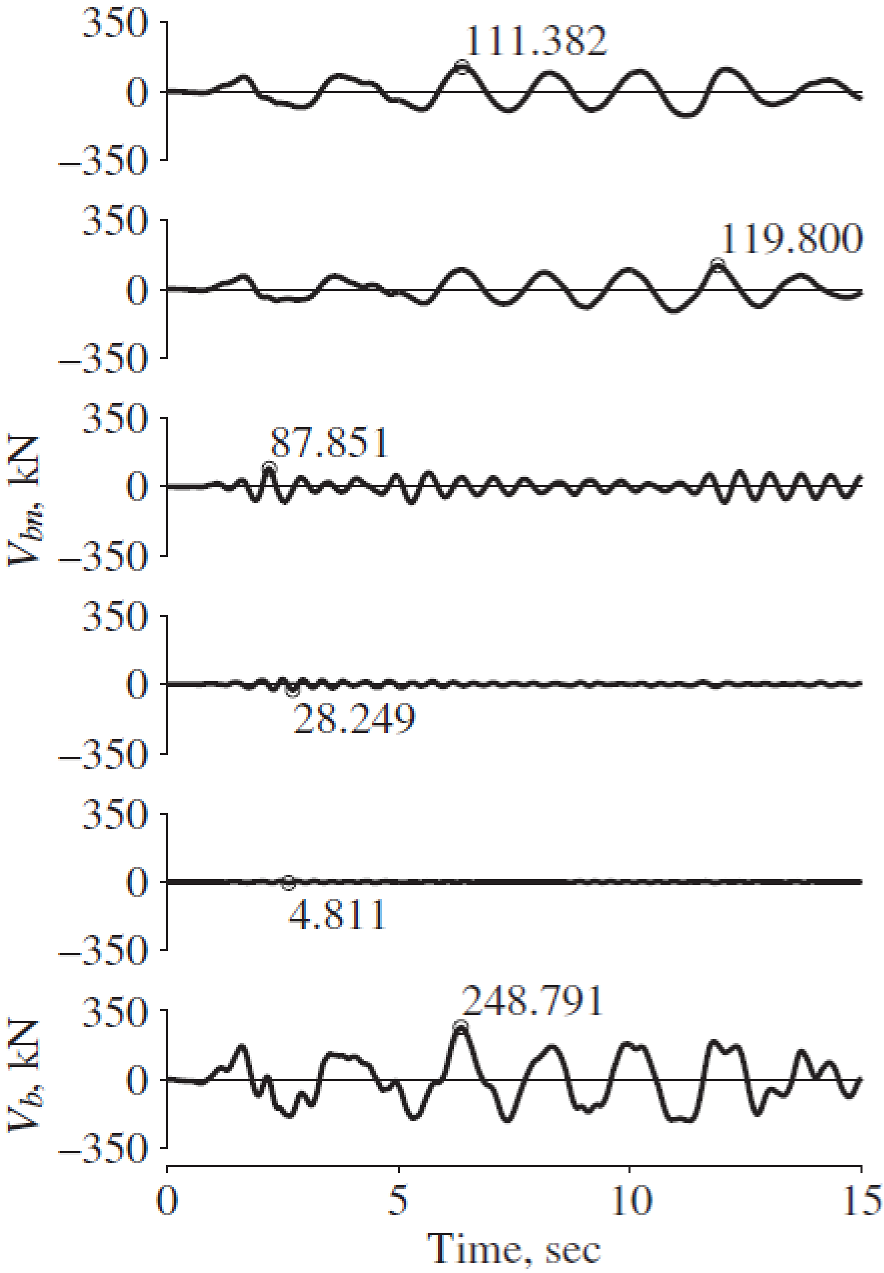
Plot the contribution of all eigenmodes to the base shear time history.

```
FigHandle=figure('Name','Base shear','NumberTitle','off');
set(FigHandle,'Position',[50, 50, 500, 200]);
plot(t,FBeig{6}/1e3,'LineWidth',1.,'Marker','.',...
    'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
grid on
xlim([0,15])
ylim([-350,350])
```

```
xlabel('Time (sec)','FontSize',10);
ylabel('Vb (kN)','FontSize',10);
title(['All modes, maxVb=',num2str(max(abs(FBeig{6}/1e3))),' kN'],...
    'FontSize',10)
```



Verify with Figure 13.2.11 (left) of Chopra (2019)



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Five-storey shear frame, dynamic analysis with direct integration (Chopra, 2019)

Contents

Statement of the problem

- **Chopra (2019), Section 13.2.6:** Consider the five-story shear frame of Fig. 12.8.1, subjected to the El Centro ground motion.
- **Chopra (2019), Section 13.2.6:** The structure is subjected to the El Centro ground motion (Chopra (2012), Fig. 6.1.4). The lumped mass $m_j = m = 45Mg$ (=0.45kN-sec^2/cm) at each floor, the lateral stiffness of each story is $k_j = k = 54.82kN/cm.$, and the height of each story is 4 m. The damping ratio for all natural modes is $\zeta_n = 0.05$.

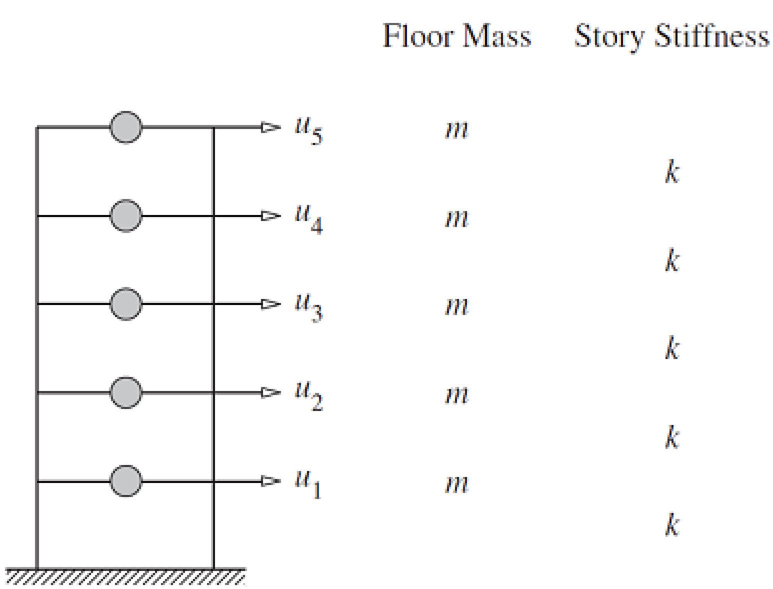


Figure 12.8.1 Uniform five-story shear building.

Initialization of structural input data

Set the storey height of the structure in m.

```
h=4;
```

Set the number of degrees of freedom of the structure, which is equal to the number of its storeys.

```
nDOFs=5;
```

Set the lateral stiffness of each storey in N/m.

```
k=5.482e6;
```

Set the lumped mass at each floor in kg.

```
m=45e3;
```

Calculation of structural properties

Calculate the stiffness matrix of the structure in N/m.

```
K=k*(diag([2*ones(nDOFs-1,1);1])+diag(-ones(nDOFs-1,1),1)+diag(-ones(nDOFs-1,1),-1));
```

Calculate the mass matrix of the structure.

```
M=m*eye(nDOFs);
```

Set the spatial distribution of the effective earthquake forces. Earthquake forces are applied at all dofs of the structure.

```
r=ones(5,1);
```

Load earthquake data

Earthquake acceleration time history of the El Centro earthquake (El Centro, 1940, El Centro Terminal Substation Building)

```
D=load('elcentro.dat');
dt=D(2,1)-D(1,1);
xgtt=9.81*D(:,2);
```

Set the critical damping ratio ($\xi = 0.05$)

```
ksi=0.05;
```

Time integration algorithm

```
AlgID='U0-V0-Opt';
```

Initial displacement

```
u0=zeros(nDOFs,1);
```

Initial velocity

```
ut0=zeros(nDOFs,1);
```

Minimum absolute value of the eigenvalues of the amplification matrix

```
rinf=1;
```

Dynamic Response History Analysis (DRHA) with direct integration

Define time

```
t=dt*(0:(numel(xgtt)-1));
```

Calculate the classical damping matrix of the structure

```
C = CDM(K,M,ksi*ones(nDOFs,1));
```

Perform DRHA analysis

```
[U,~,~,f] = LDRHA_DI_MDOF(K,C,M,r,dt,xgtt,AlgID,u0,ut0,rinf);
```

Base shear time history

```
FBeig=sum(f,1);
```

5th storey shear time history (5th DOF)

```
Feig=f(5,:);
```

Roof displacement time history (5th DOF)

```
Ueig=U(5,:);
```

Base moment time history

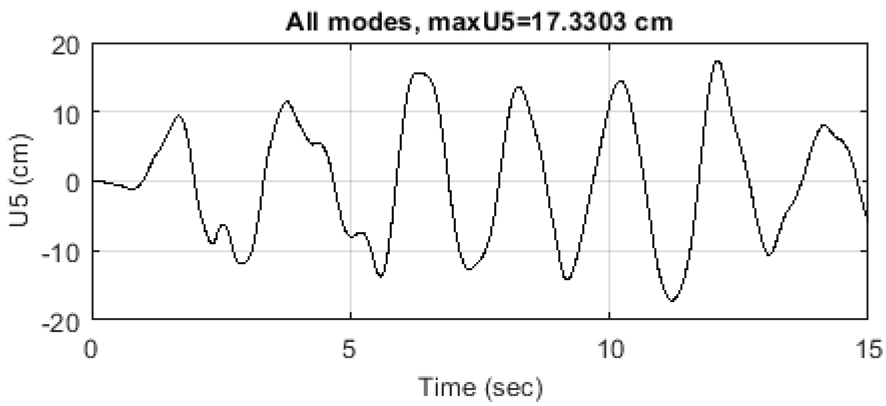
```
MBeig=sum(f.*repmat((h:h:5*h)',1,size(f,2)),1);
```

Roof displacement time history

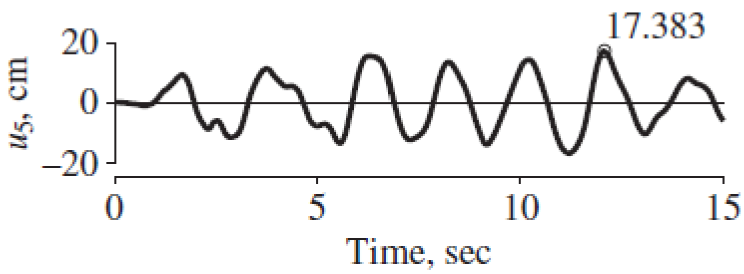
Plot the roof displacement time history. Convert displacements from m to cm.

```
FigHandle=figure('Name','Roof displacement','NumberTitle','off');
```

```
set(FigHandle,'Position',[50, 50, 500, 200]);
plot(t,100*Ueig,'LineWidth',1.,'Marker','.',...
    'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
grid on
xlim([0,15])
ylim([-20,20])
xlabel('Time (sec)','FontSize',10);
ylabel('U5 (cm)','FontSize',10);
title(['All modes, maxU5=',num2str(max(abs(100*Ueig))),' cm'],...
    'FontSize',10)
```



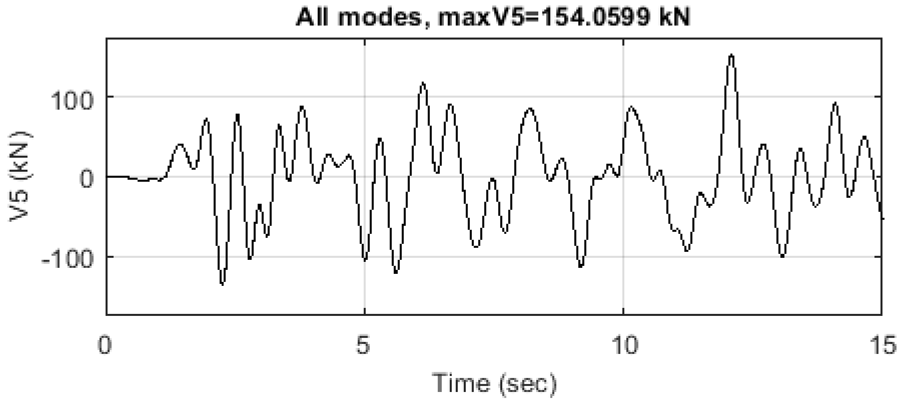
Verify with Figure 13.2.8 (left) of Chopra (2019)



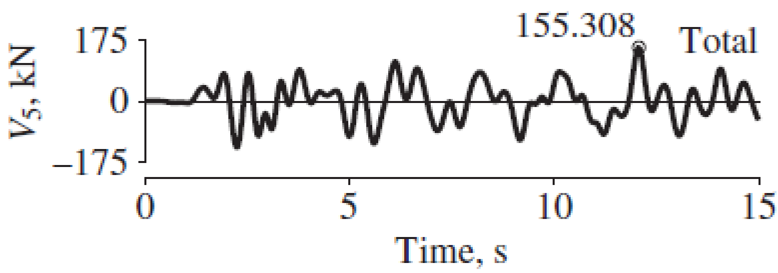
Fifth-story shear time history

Plot the fifth-story shear time history. Convert forces from N to kN.

```
FigHandle=figure('Name','Fifth-story shear','NumberTitle','off');
set(FigHandle,'Position',[50, 50, 500, 200]);
plot(t,Feig/1e3,'LineWidth',1.,'Marker','.',...
    'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
grid on
xlim([0,15])
ylim([-175,175])
xlabel('Time (sec)','FontSize',10);
ylabel('V5 (kN)','FontSize',10);
title(['All modes, maxV5=',num2str(max(abs(Feig/1e3))),' kN'],...
    'FontSize',10)
```



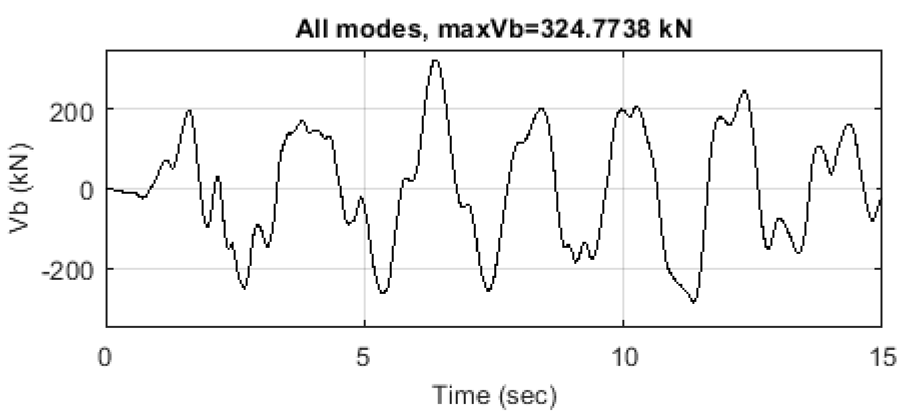
Verify with Figure 13.2.7 (right) of Chopra (2019)



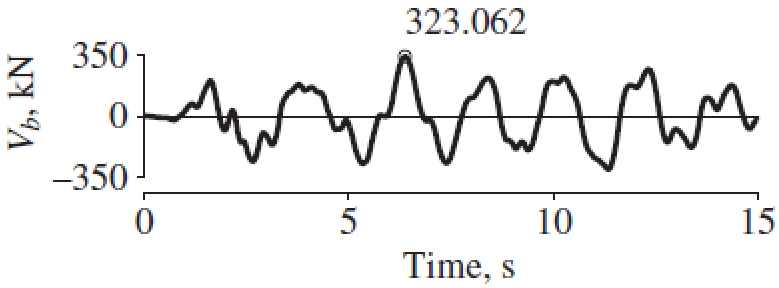
Base shear time history

Plot the base shear time history. Convert forces from N to kN.

```
FigHandle=figure('Name','Base shear','NumberTitle','off');
set(FigHandle,'Position',[50, 50, 500, 200]);
plot(t,FBeig/1e3,'LineWidth',1.,'Marker','.',...
    'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
grid on
xlim([0,15])
ylim([-350,350])
xlabel('Time (sec)','FontSize',10);
ylabel('Vb (kN)','FontSize',10);
title(['All modes, maxVb=',num2str(max(abs(FBeig/1e3))),' kN'],...
    'FontSize',10)
```



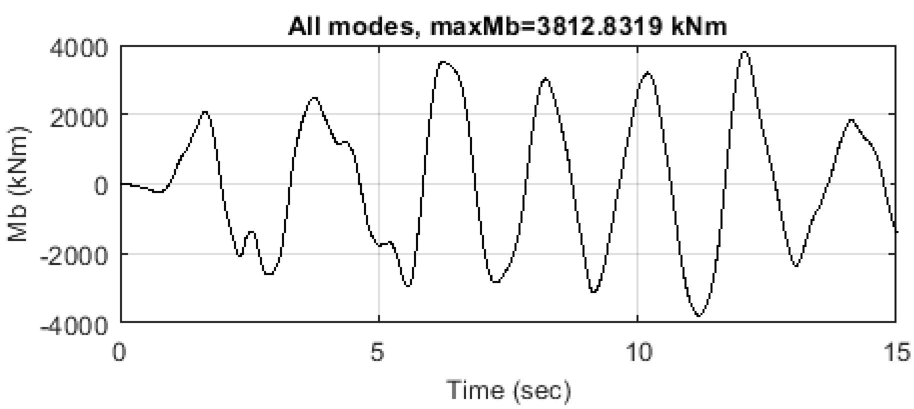
Verify with Figure 13.2.7 (left) of Chopra (2019)



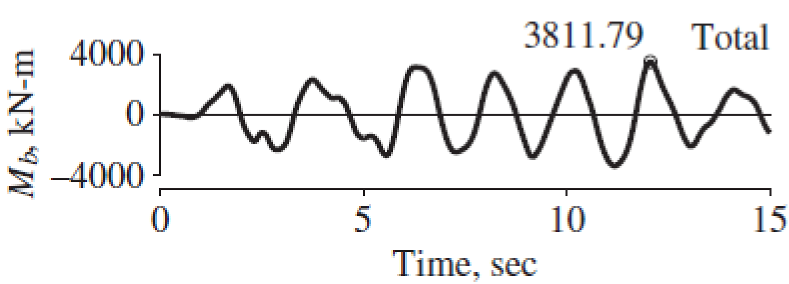
Base moment time history

Plot the base moment time history. Convert moments from Nm to kNm.

```
FigHandle=figure('Name','Base moment','NumberTitle','off');
set(FigHandle,'Position',[50, 50, 500, 200]);
plot(t,MBeig/1e3,'LineWidth',1.,'Marker','.',...
    'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
grid on
xlim([0,15])
ylim([-4000,4000])
xlabel('Time (sec)','FontSize',10);
ylabel('Mb (kNm)','FontSize',10);
title(['All modes, maxMb=',num2str(max(abs(MBeig/1e3))),' kNm'],...
    'FontSize',10)
```



Verify with Figure 13.2.8 (right) of Chopra (2019)



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Five-storey shear frame dynamic analysis with modal superposition (Chopra, 2019)

Contents

Statement of the problem

- **Chopra (2019), Section 13.2.6:** Consider the five-story shear frame of Fig. 12.8.1, subjected to the El Centro ground motion.
- **Chopra (2019), Section 13.2.6:** The structure is subjected to the El Centro ground motion (Chopra (2012), Fig. 6.1.4). The lumped mass $m_j = m = 45Mg$ (=0.45kN-sec^2/cm) at each floor, the lateral stiffness of each story is $k_j = k = 54.82kN/cm$., and the height of each story is 4 m. The damping ratio for all natural modes is $\zeta_n = 0.05$.

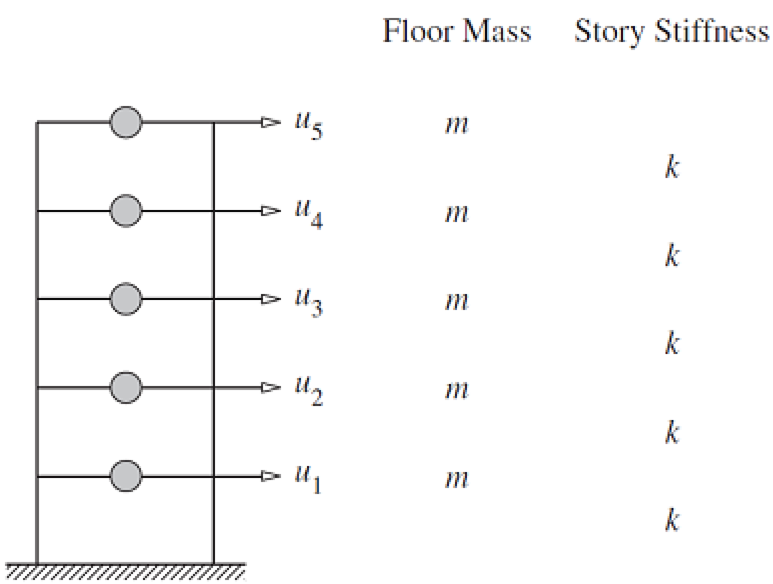


Figure 12.8.1 Uniform five-story shear building.

Initialization of structural input data

Set the storey height of the structure in m.

```
h=4;
```

Set the number of degrees of freedom of the structure, which is equal to the number of its storeys.

```
nDOFs=5;
```

Set the lateral stiffness of each storey in N/m.

```
k=5.482e6;
```

Set the lumped mass at each floor in kg.

```
m=45e3;
```

Calculation of structural properties

Calculate the stiffness matrix of the structure in N/m.

```
K=k*(diag([2*ones(nDOFs-1,1);1])+diag(-ones(nDOFs-1,1),1)+diag(-ones(nDOFs-1,1),-1));
```

Calculate the mass matrix of the structure.

```
M=m*eye(nDOFs);
```

Set the spatial distribution of the effective earthquake forces. Earthquake forces are applied at all dofs of the structure.

```
r=ones(5,1);
```

Load earthquake data

Earthquake acceleration time history of the El Centro earthquake (El Centro, 1940, El Centro Terminal Substation Building)

```
D=load('elcentro.dat');
dt=D(2,1)-D(1,1);
xgtt=9.81*D(:,2);
```

Set the critical damping ratio ($\xi = 0.05$)

```
ksi=0.05;
```

Time integration algorithm

```
AlgID='U0-V0-Opt';
```

Initial displacement

```
u0=zeros(nDOFs,1);
```

Initial velocity

```
ut0=zeros(nDOFs,1);
```

Minimum absolute value of the eigenvalues of the amplification matrix

```
rinf=1;
```

Dynamic Response History Analysis (DRHA) with modal superposition

Define time

```
t=dt*(0:(numel(xgtt)-1));
```

Perform DRHA analysis for each separate eigenmode to calculate the contribution of each eigenmode to the various frame responses

```
% Initialize
FBeig=cell(5,1);
Feig=cell(5,1);
Ueig=cell(5,1);
MBeig=cell(5,1);
for i=1:5
    % DRHA analysis
    [U,~,~,f] = LDRHA_MS_MDOF(K,M,r,dt,xgtt,ksi,AlgID,u0,ut0,rinf,i);
    % Store the 5th storey shear time history (5th DOF) for each eigenmode
    FBeig{i}=sum(f,1);
    % Store the 5th storey shear time history (5th DOF) for each eigenmode
    Feig{i}=f(5,:);
    % Store the roof displacement time history (5th DOF) for each eigenmode
    Ueig{i}=U(5,:);
    % Base moment time history for each eigenmode
    MBeig{i}=sum(f.*repmat((h:h:5*h)',1,size(f,2)),1);
end
```

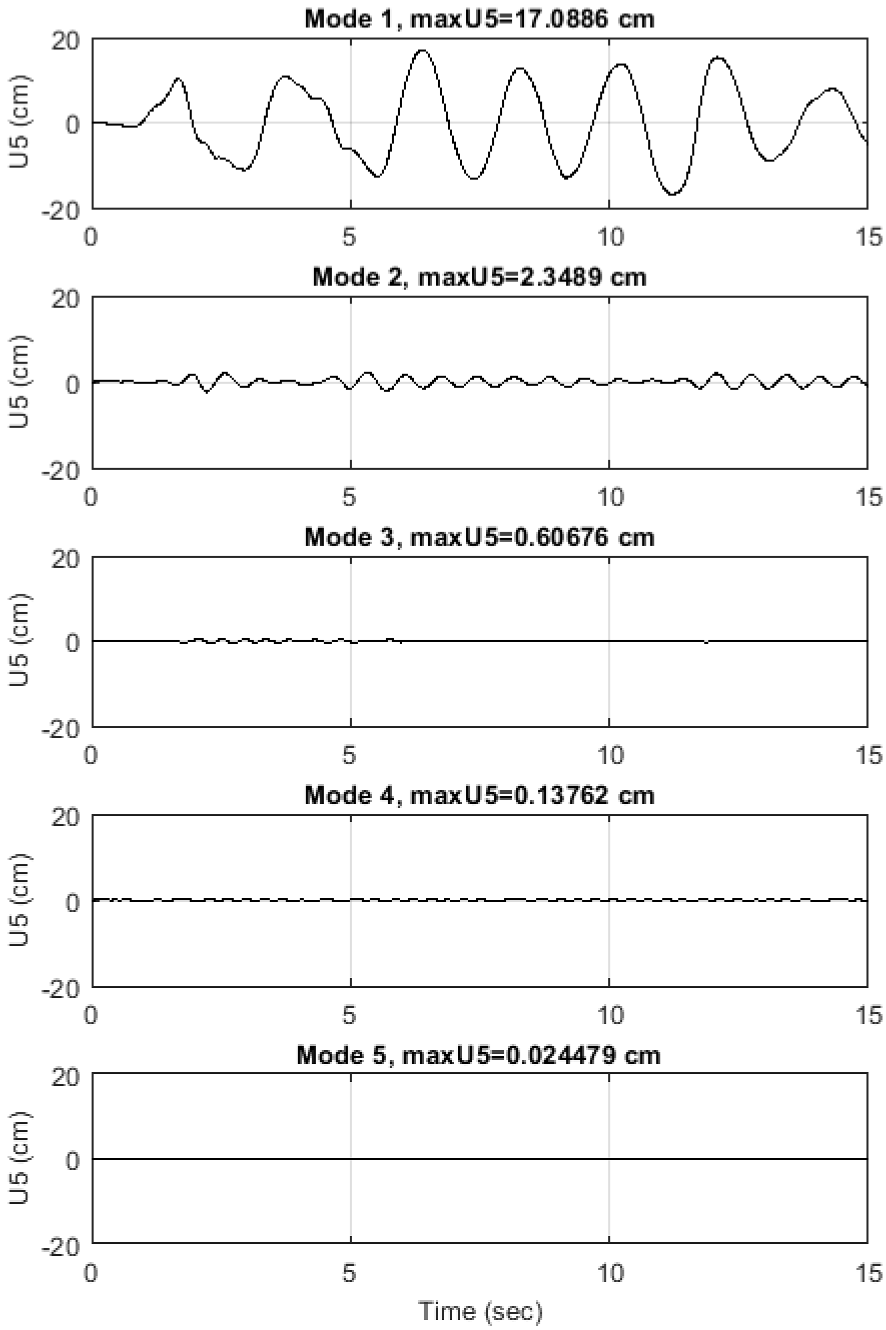
Perform DRHA analysis for all eigenmodes to calculate the contribution of all eigenmodes to the various frame responses

```
% Eigenmodes that are superposed
eigInd=[1;2;3;4;5];
% DRHA analysis
[U,~,~,f] = LDRHA_MS_MDOF(K,M,r,dt,xgtt,ksi,AlgID,u0,ut0,rinf,eigInd);
% Base shear time history for all eigenmodes
FBeig{6}=sum(f,1);
% 5th storey shear time history (5th DOF) for all eigenmodes
Feig{6}=f(5,:);
% Roof displacement time history (5th DOF) for all eigenmodes
Ueig{6}=U(5,:);
% Base moment time history for all eigenmodes
MBeig{6}=sum(f.*repmat((h:h:5*h)',1,size(f,2)),1);
```

Roof displacement time history

Plot the contribution of each eigenmode to the roof displacement time history. Convert displacements from m to cm.

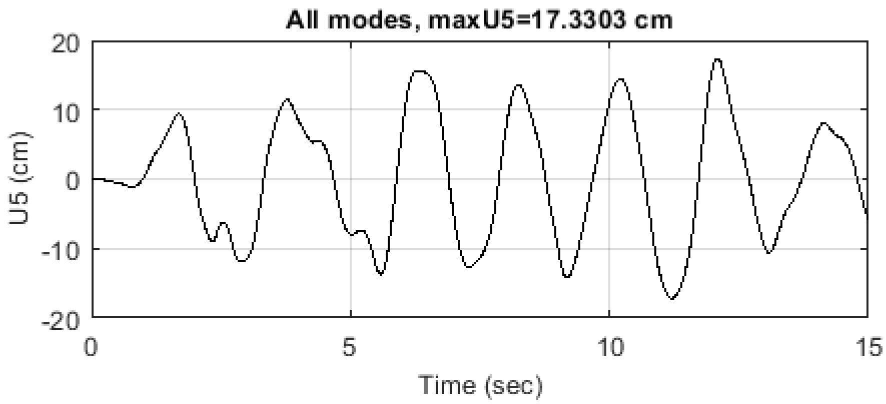
```matlab
FigHandle=figure('Name','Roof displacement','NumberTitle','off');
set(FigHandle,'Position',[50, 50, 500, 750]);
for i=1:5
    subplot(nDOFs,1,i)
    plot(t,100*Ueig{i},'LineWidth',1.,'Marker','.',...
        'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
    grid on
    xlim([0,15])
    ylim([-20,20])
    ylabel('U5 (cm)','FontSize',10);
    title(['Mode ',num2str(i),', maxU5=',num2str(max(abs(100*Ueig{i}))),' cm'],...
        'FontSize',10)
end
xlabel('Time (sec)','FontSize',10);
```



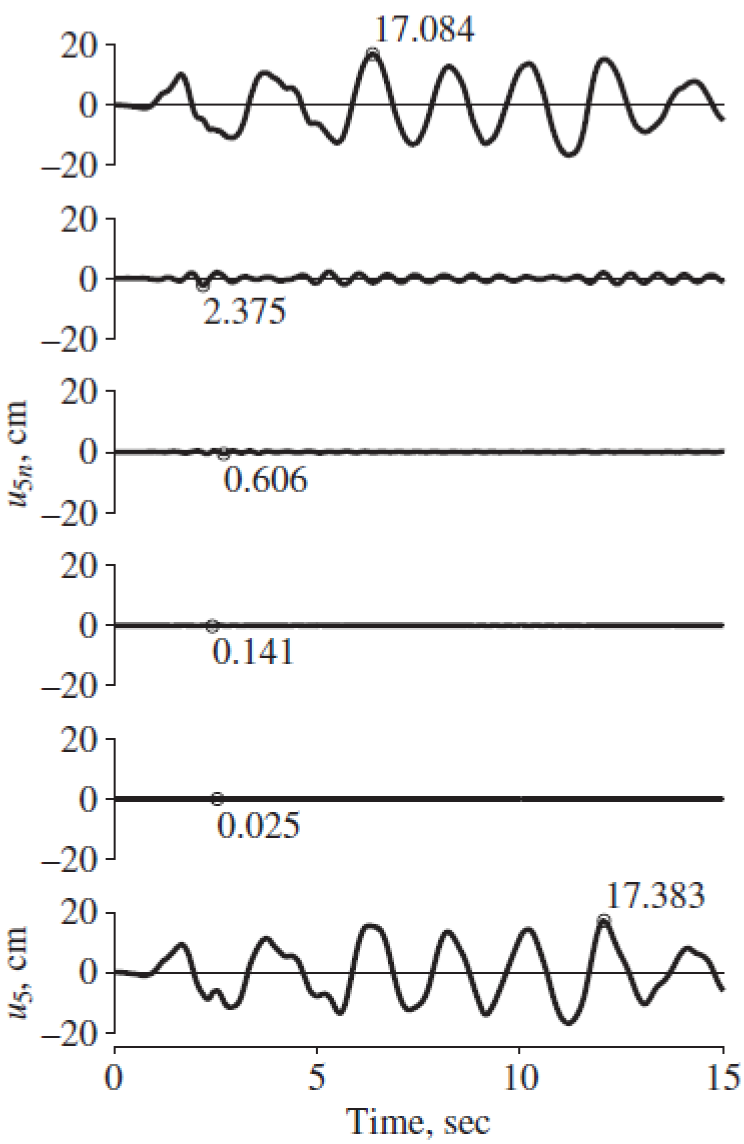
Plot the contribution of all eigenmodes to the roof displacement time history. Convert displacements from m to cm.

```
FigHandle=figure('Name','Roof displacement','NumberTitle','off');
set(FigHandle,'Position',[50, 50, 500, 200]);
plot(t,100*Ueig{6},'LineWidth',1.,'Marker','.',...
    'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
grid on
xlim([0,15])
ylim([-20,20])
xlabel('Time (sec)','FontSize',10);
ylabel('U5 (cm)','FontSize',10);
```

```
title(['All modes, maxU5=',num2str(max(abs(100*Ueig{6}))),' cm'],...
    'FontSize',10)
```



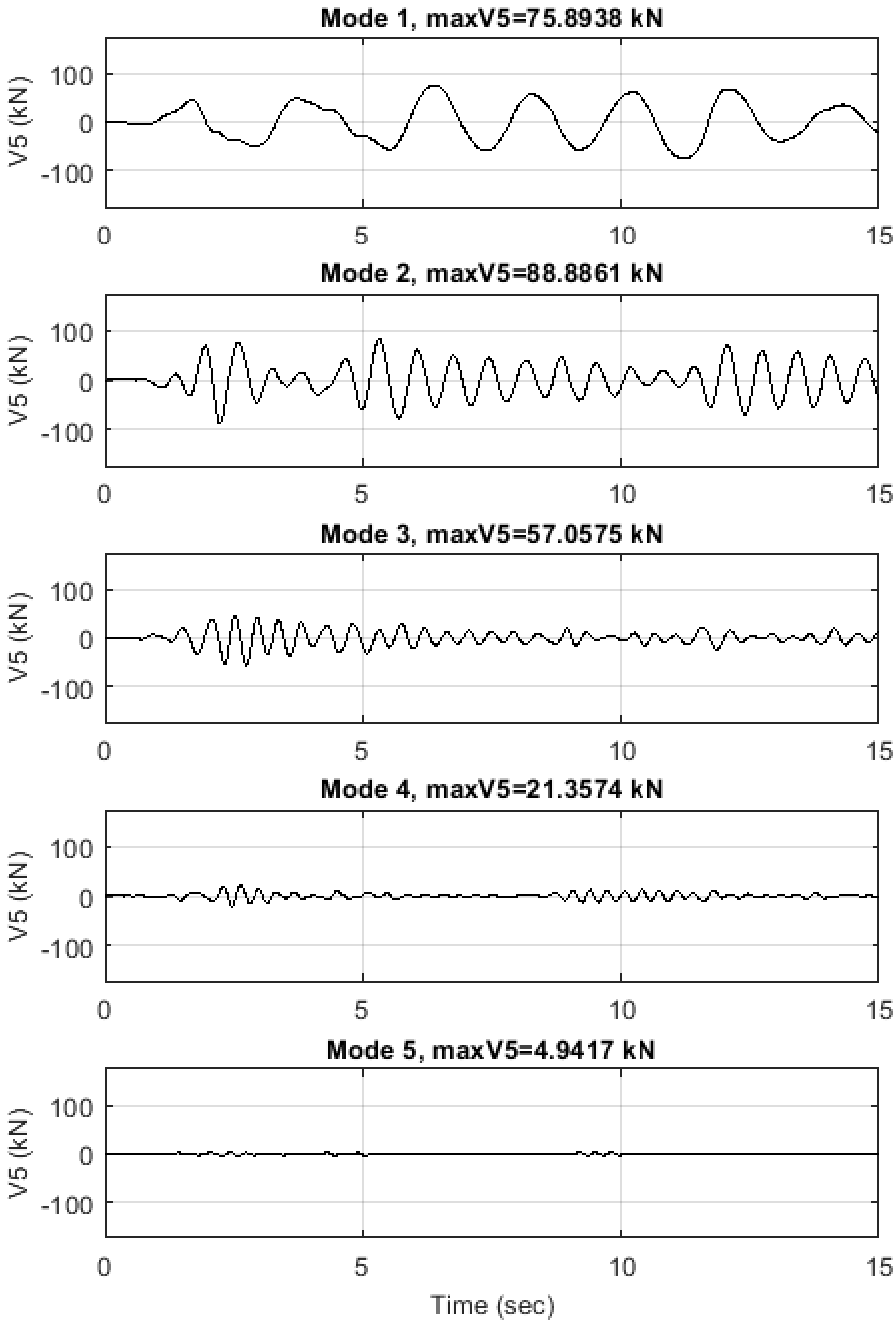
Verify with Figure 13.2.8 (left) of Chopra (2019)



Fifth-story shear time history

Plot the contribution of each eigenmode to the fifth-story shear time history. Convert forces from N to kN.

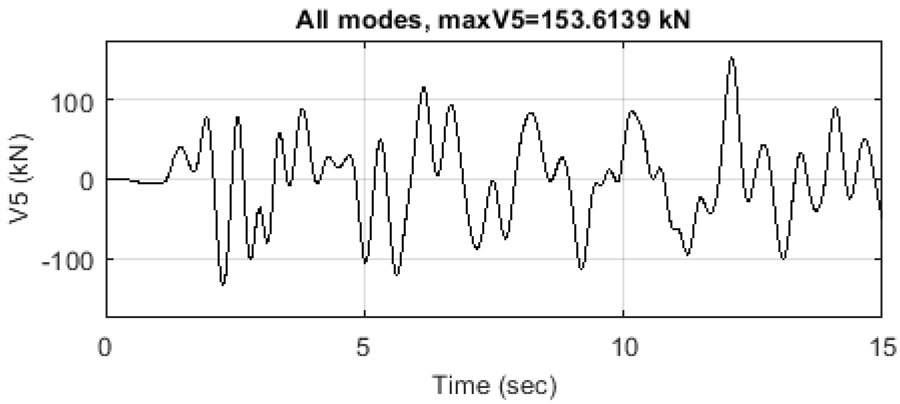
```
FigHandle=figure('Name','Fifth-story shear','NumberTitle','off');
set(FigHandle,'Position',[50, 50, 500, 750]);
for i=1:5
    subplot(nDOFs,1,i)
    plot(t,Feig{i}/1e3,'LineWidth',1.,'Marker','.',...
        'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
    grid on
    xlim([0,15])
    ylim([-175,175])
    ylabel('V5 (kN)','FontSize',10);
    title(['Mode ',num2str(i),', maxV5=',num2str(max(abs(Feig{i}/1e3))),' kN'],...
        'FontSize',10)
end
xlabel('Time (sec)','FontSize',10);
```



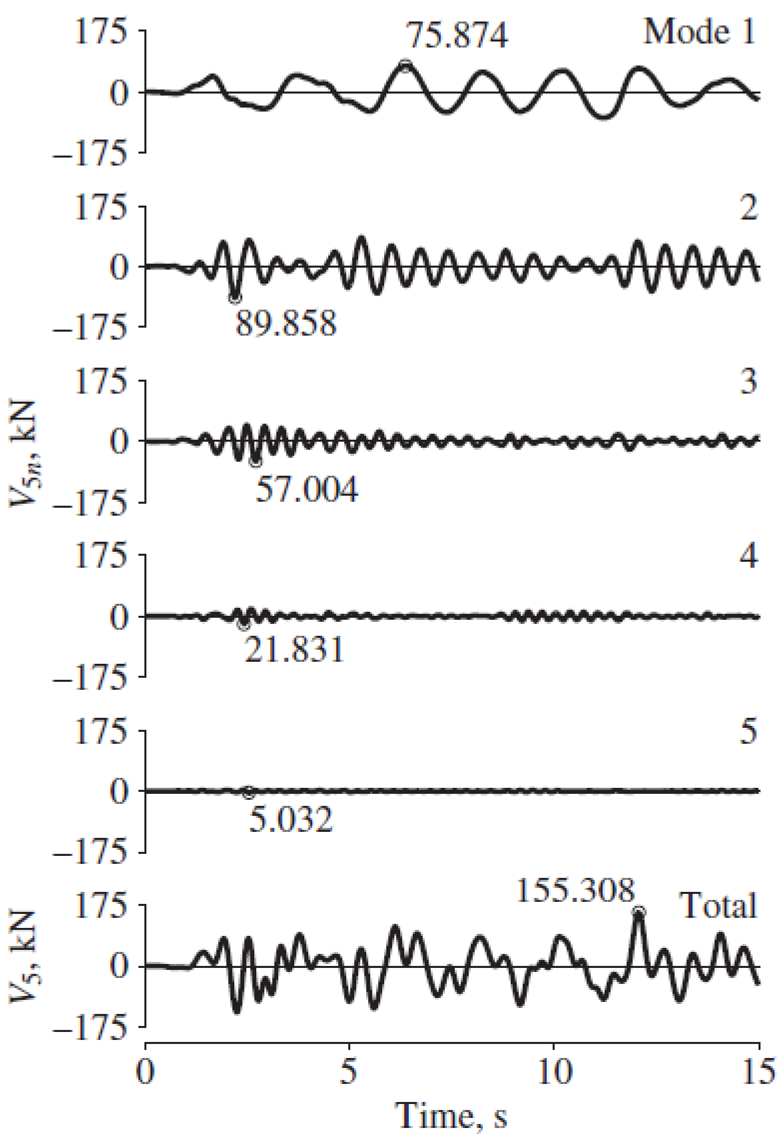
Plot the contribution of all eigenmodes to the fifth-story shear time history. Convert forces from N to kN.

```
FigHandle=figure('Name','Fifth-story shear','NumberTitle','off');
set(FigHandle,'Position',[50, 50, 500, 200]);
plot(t,Feig{6}/1e3,'LineWidth',1.,'Marker','.',...
    'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
grid on
xlim([0,15])
ylim([-175,175])
xlabel('Time (sec)','FontSize',10);
ylabel('V5 (kN)','FontSize',10);
```

```
title(['All modes, maxV5=',num2str(max(abs(Feig{6}/1e3))),' kN'],...
    'FontSize',10)
```



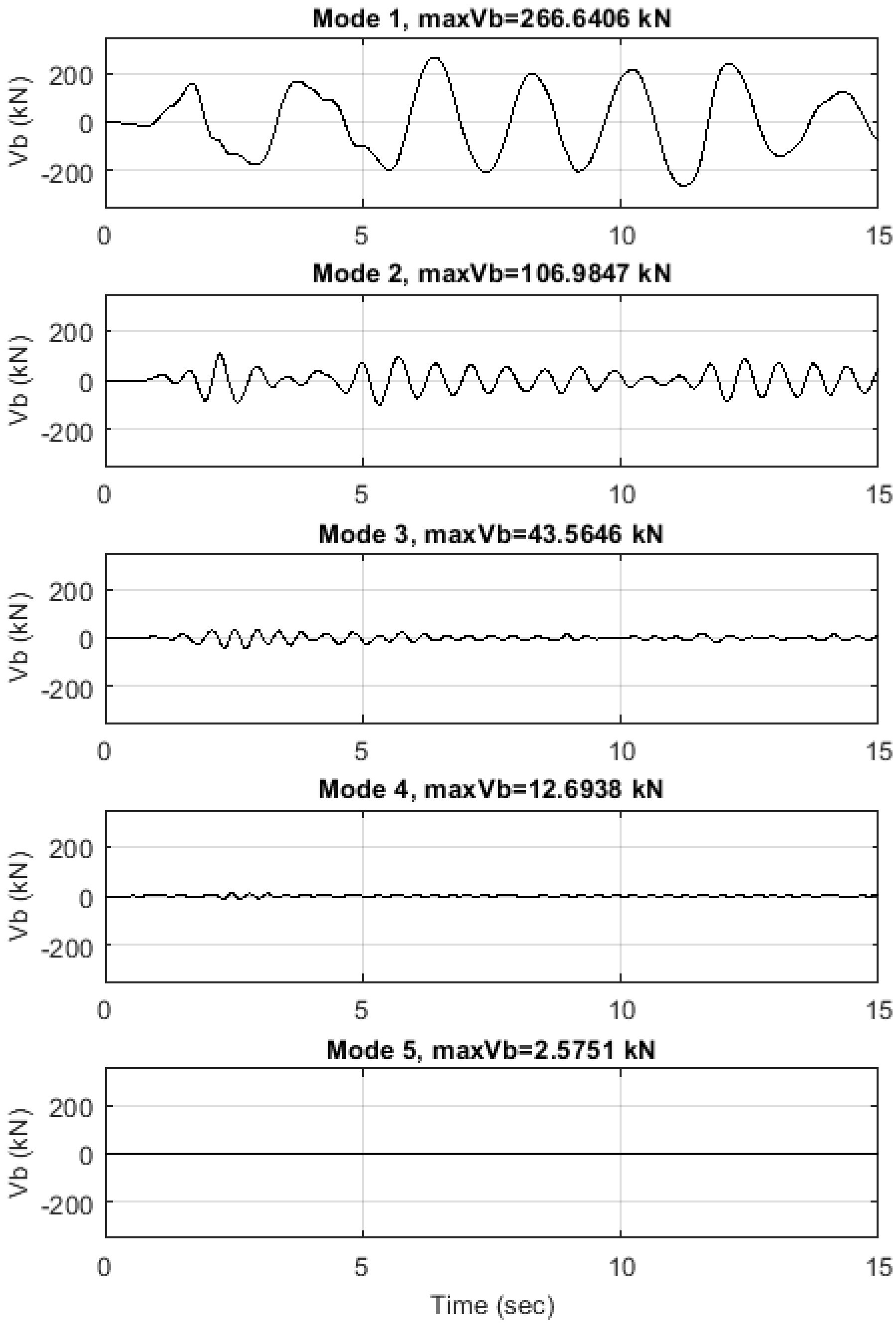
Verify with Figure 13.2.7 (right) of Chopra (2019)



Base shear time history

Plot the contribution of each eigenmode to the base shear time history. Convert forces from N to kN.

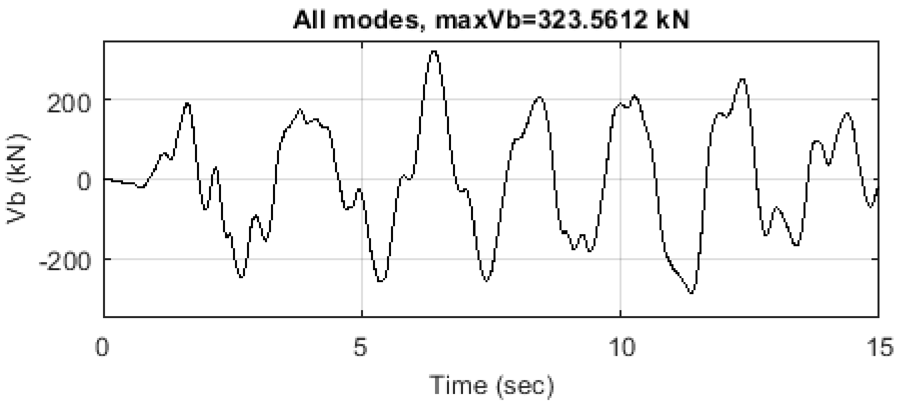
```
FigHandle=figure('Name','Base shear','NumberTitle','off');
set(FigHandle,'Position',[50, 50, 500, 750]);
for i=1:5
    subplot(nDOFs,1,i)
    plot(t,FBeig{i}/1e3,'LineWidth',1.,'Marker','.',...
        'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
    grid on
    xlim([0,15])
    ylim([-350,350])
    ylabel('Vb (kN)','FontSize',10);
    title(['Mode ',num2str(i),', maxVb=',num2str(max(abs(FBeig{i}/1e3))),' kN'],...
        'FontSize',10)
end
xlabel('Time (sec)','FontSize',10);
```



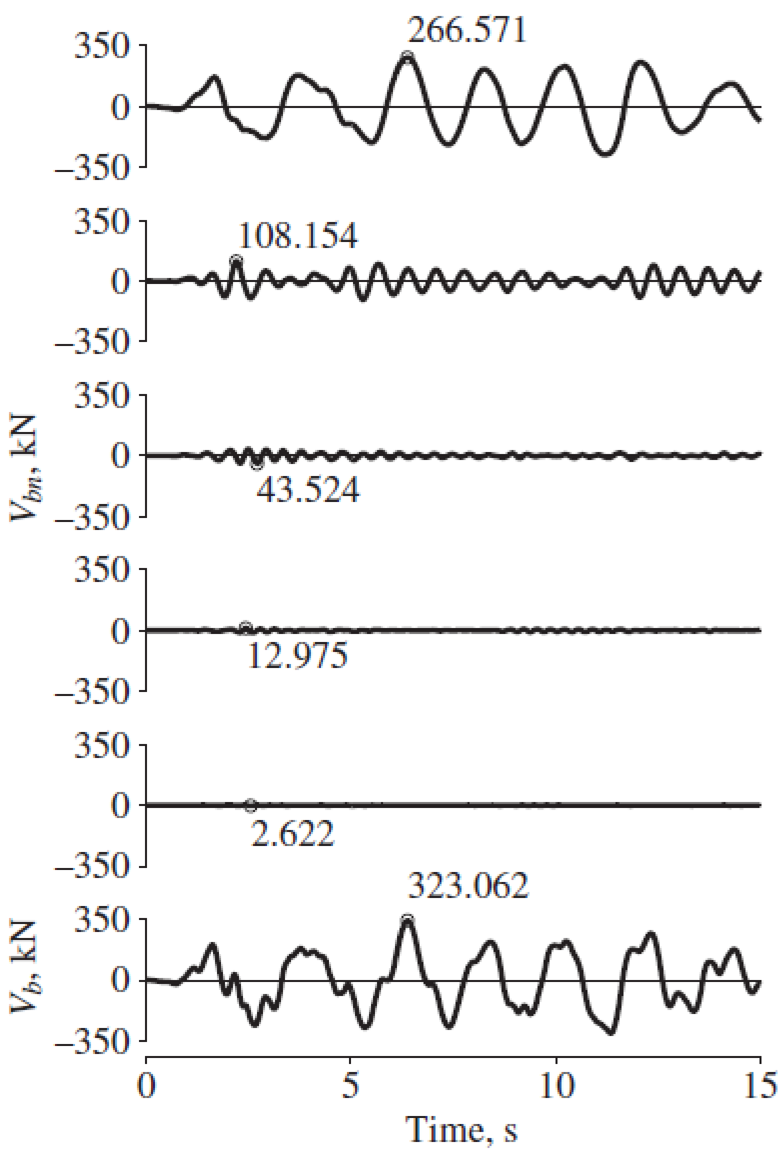
Plot the contribution of all eigenmodes to the base shear time history. Convert forces from N to kN.

```
FigHandle=figure('Name','Base shear','NumberTitle','off');
set(FigHandle,'Position',[50, 50, 500, 200]);
plot(t,FBeig{6}/1e3,'LineWidth',1.,'Marker','.',...
    'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
grid on
xlim([0,15])
ylim([-350,350])
xlabel('Time (sec)','FontSize',10);
ylabel('Vb (kN)','FontSize',10);
```

```
title(['All modes, maxVb=',num2str(max(abs(FBeig{6}/1e3))),' kN'],...
    'FontSize',10)
```



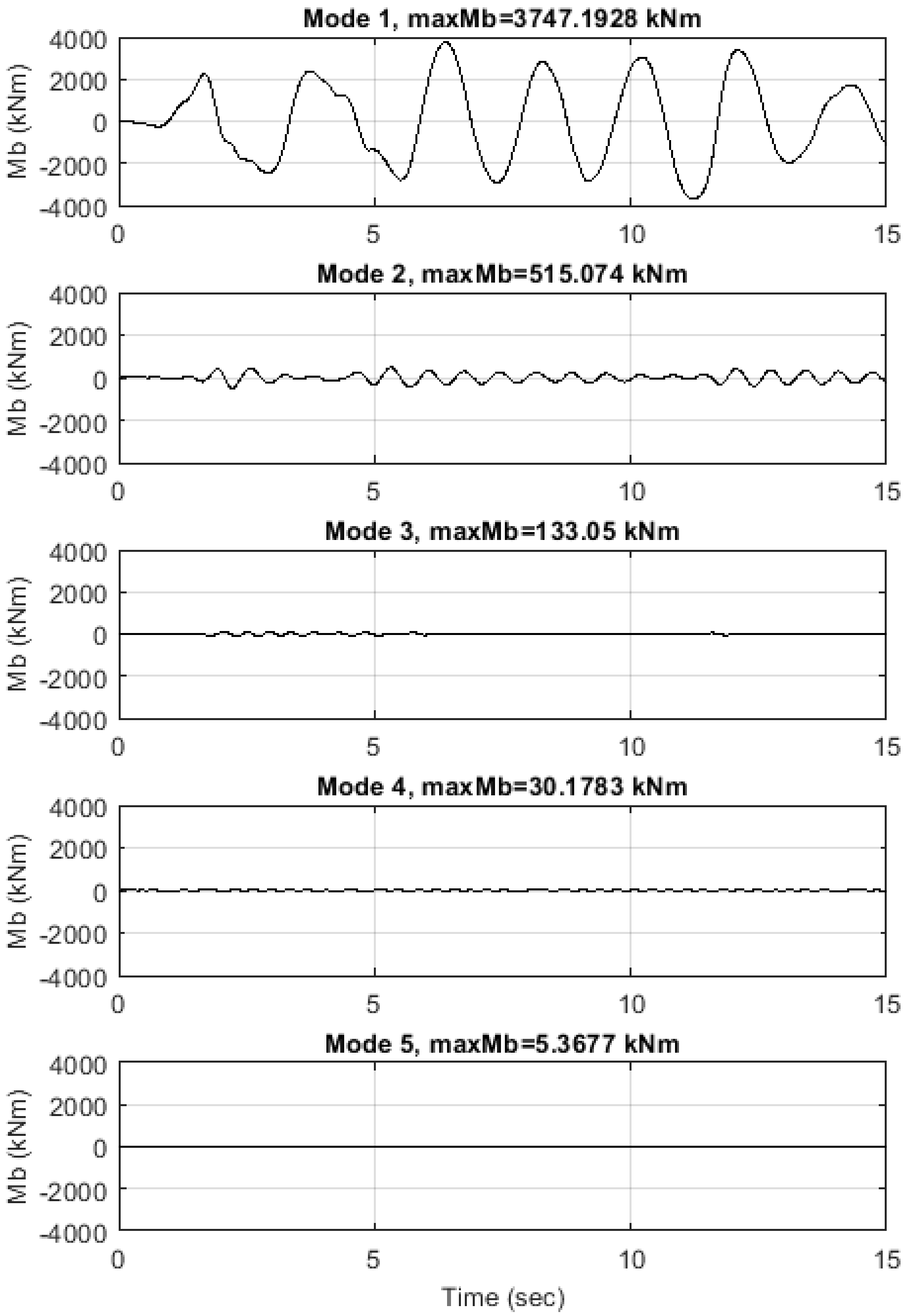
Verify with Figure 13.2.7 (left) of Chopra (2019)



Base moment time history

Plot the contribution of each eigenmode to the base moment time history. Convert moments from Nm to kNm.

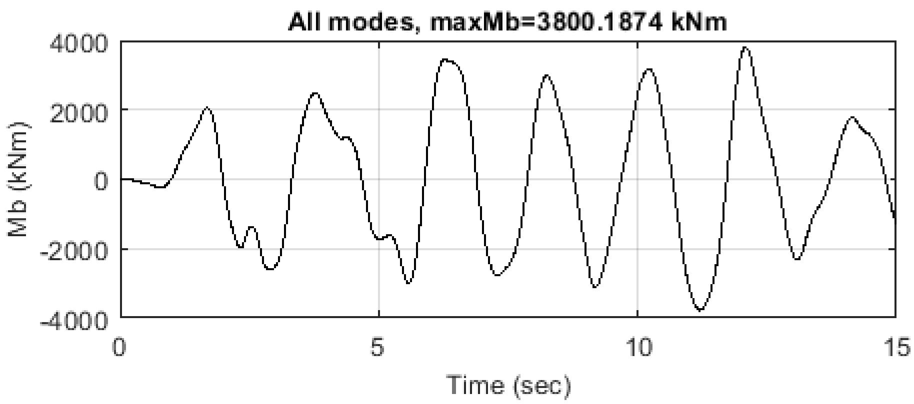
```
FigHandle=figure('Name','Base moment','NumberTitle','off');
set(FigHandle,'Position',[50, 50, 500, 750]);
for i=1:5
    subplot(nDOFs,1,i)
    plot(t,MBeig{i}/1e3,'LineWidth',1.,'Marker','.',...
        'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
    grid on
    xlim([0,15])
    ylim([-4000,4000])
    ylabel('Mb (kNm)','FontSize',10);
    title(['Mode ',num2str(i),', maxMb=',num2str(max(abs(MBeig{i}/1e3))),' kNm'],...
        'FontSize',10)
end
xlabel('Time (sec)','FontSize',10);
```



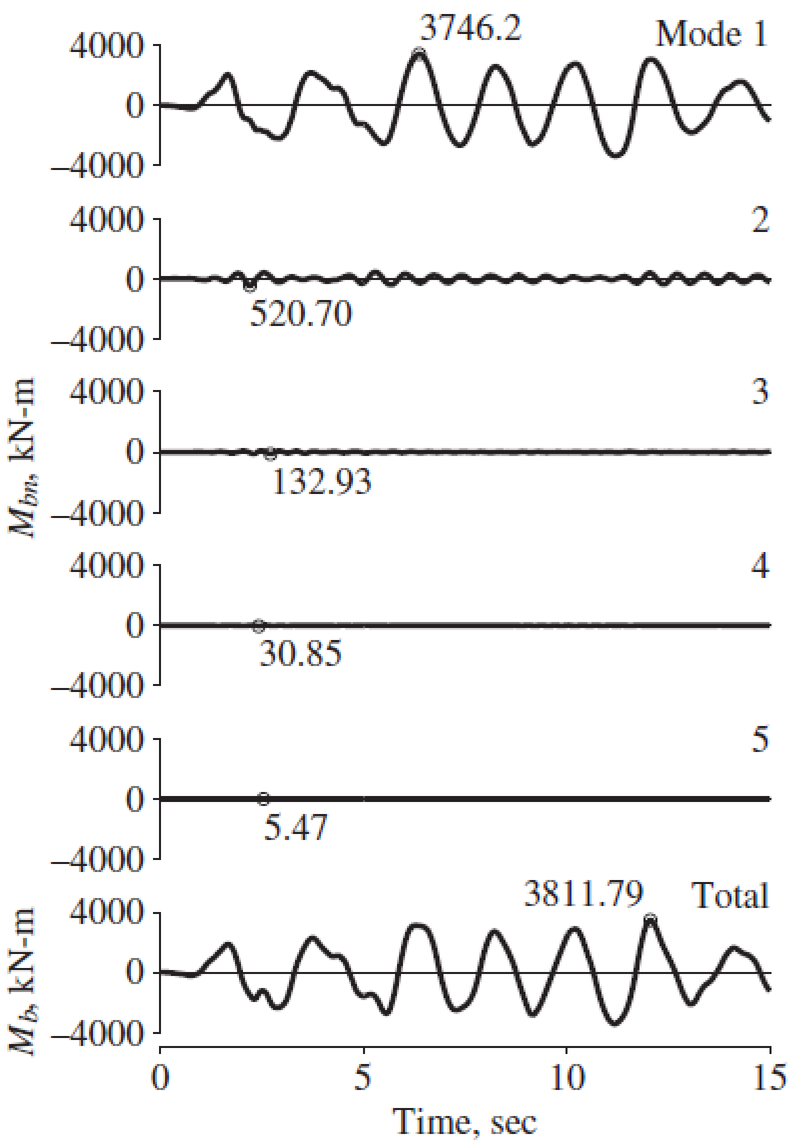
Plot the contribution of all eigenmodes to the base moment time history. Convert moments from Nm to kNm.

```
FigHandle=figure('Name','Base moment','NumberTitle','off');
set(FigHandle,'Position',[50, 50, 500, 200]);
plot(t,MBeig{6}/1e3,'LineWidth',1.,'Marker','.',...
    'MarkerSize',1,'Color',[0 0 0],'markeredgecolor','k')
grid on
xlim([0,15])
ylim([-4000,4000])
xlabel('Time (sec)','FontSize',10);
ylabel('Mb (kNm)','FontSize',10);
```

```
title(['All modes, maxMb=',num2str(max(abs(MBeig{6}/1e3))),' kNm'],...
    'FontSize',10)
```



Verify with Figure 13.2.8 (right) of Chopra (2019)



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CDM

Documentation of the CDM function.

```
helpFun('CDM')
```

```
 Classical Damping Matrix of a MDOF system

 C = CDM(K,M,ksi)

 Description
     Construct the classical damping matrix of a MDOF system by
     superposition of modal damping matrices, based on the ratio of
     critical damping, KSI, given its stiffness and mass matrices, K and M
     respectively. If the damping matrix of a linear system satisfies the
     identity c*m^(?1)*k=k*m^(?1)*c the system is said to possess
     classical damping. This function contains the preferred model for
     nonlinear RHA of buildings because it eliminates the spurious damping
     forces that may be encountered in the Rayleigh or Caughey damping
     models.

 Input parameters
     K [double(:ndofs x :ndofs)] is the stiffness matrix of the system
         containing only the free degrees of freedom.
     M [double(:ndofs x :ndofs)] is the mass matrix of the structure
         containing only the free degrees of freedom.
     KSI [double(:ndofs x 1)]: ratio of critical damping of the MDOF
         system for each eigenmode

 Output parameters
     C [double(:ndofs x :ndofs)] is the damping matrix of the structure
         containing only the free degrees of freedom.

________________________________________________________________________
```



```
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 _______________________________________________________________________
```


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LDRHA_DI_MDOF

Documentation of the LDRHA_DI_MDOF function.

```
helpFun('LDRHA_DI_MDOF')
```

```
Linear Dynamic Response History Analysis of a MDOF system with Direct
Integration

[U,UT,UTT,FS] = LDRHA_DI_MDOF(K,C,M,R,DT,XGTT,ALGID,U0,UT0,RINF)

Description
    Determine the time history of the linear structural response of a
    Multi-DOF (MDOF) system by direct time integration.

Input parameters
    K [double(:ndofs x :ndofs)] is the stiffness of the system containing
        only the free degrees of freedom.
    C [double(:ndofs x :ndofs)] is the damping matrix of the structure
        containing only the free degrees of freedom.
    M [double(:ndofs x :ndofs)] is the mass matrix of the structure
        containing only the free degrees of freedom.
    R [double(:ndofs x 1)]: influence vector. It determines the spatial
        distribution of the effective earthquake forces
    DT [double(1 x 1)]: uniform time step of acceleration time history
        XGTT
    XGTT [double(:nstep x 2)] 2-column vector of the acceleration history
        of the excitation imposed at the base. The first column contains
        time and the second column contains acceleration. nstep is the
        number of time steps of the dynamic response.
    ALGID [char(1 x :inf)]: algorithm to be used for the time
        integration. It can be one of the following strings for superior
        optimally designed algorithms:
            'generalized a-method': The generalized a-method (Chung &
            Hulbert, 1993)
            'HHT a-method': The Hilber-Hughes-Taylor method (Hilber,
            Hughes & Taylor, 1977)
            'WBZ': The Wood–Bossak–Zienkiewicz method (Wood, Bossak &
            Zienkiewicz, 1980)
            'U0-V0-Opt': Optimal numerical dissipation and dispersion
            zero order displacement zero order velocity algorithm
            'U0-V0-CA': Continuous acceleration (zero spurious root at
            the low frequency limit) zero order displacement zero order
            velocity algorithm
            'U0-V0-DA': Discontinuous acceleration (zero spurious root at
            the high frequency limit) zero order displacement zero order
            velocity algorithm
            'U0-V1-Opt': Optimal numerical dissipation and dispersion
            zero order displacement first order velocity algorithm
            'U0-V1-CA': Continuous acceleration (zero spurious root at
            the low frequency limit) zero order displacement first order
            velocity algorithm
            'U0-V1-DA': Discontinuous acceleration (zero spurious root at
            the high frequency limit) zero order displacement first order
            velocity algorithm
            'U1-V0-Opt': Optimal numerical dissipation and dispersion
```

```
            first order displacement zero order velocity algorithm
            'U1-V0-CA': Continuous acceleration (zero spurious root at
            the low frequency limit) first order displacement zero order
            velocity algorithm
            'U1-V0-DA': Discontinuous acceleration (zero spurious root at
            the high frequency limit) first order displacement zero order
            velocity algorithm
            'Newmark ACA': Newmark Average Constant Acceleration method
            'Newmark LA': Newmark Linear Acceleration method
            'Newmark BA': Newmark Backward Acceleration method
            'Fox-Goodwin': Fox-Goodwin formula
    U0 [double(:ndofs x 1)]: initial displacement of the MDOF system
    UT0 [double(:ndofs x 1)]: initial velocity of the MDOF system
    RINF [double(1 x 1)]: minimum absolute value of the eigenvalues of
        the amplification matrix. For the amplification matrix see
        eq.(61) in Zhou & Tamma (2004).

Output parameters
    U ([ndofs x nstep]): displacement time history.
    UT ([ndofs x nstep]): velocity time history.
    UTT ([ndofs x nstep]): acceleration time history.
    FS ([ndofs x nstep]): equivalent static force time history.

Notation in the code
    u=displacement
    un=displacement after increment n
    ut=velocity
    utn=velocity after increment n
    utt=acceleration
    uttn=acceleration after increment n
```


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LDRHA_MS_MDOF

Documentation of the LDRHA_MS_MDOF function.

```
helpFun('LDRHA_MS_MDOF')
```

```
Linear Dynamic Response History Analysis (DRHA) of a MDOF system with
eigenmode superposition

[U,V,A,F] = LDRHA_MS_MDOF(K,M,R,DT,XGTT,KSI,ALGID,U0,UT0,RINF,EIGIND)

Description
    Determine the time history of the linear structural response of a
    Multi-DOF (MDOF) system by linear superposition of its modal
    responses.

Input parameters
    K [double(:ndofs x :ndofs)] is the stiffness matrix of the system
        containing only the free degrees of freedom.
    M [double(:ndofs x :ndofs)] is the mass matrix of the structure
        containing only the free degrees of freedom.
    R [double(:ndofs x 1)]: influence vector. It determines the spatial
        distribution of the effective earthquake forces
    DT [double(1 x 1)]: uniform time step of acceleration time history
        XGTT
    XGTT [double(:nstep x 2)] 2-column vector of the acceleration history
        of the excitation imposed at the base. The first column contains
        time and the second column contains acceleration. nstep is the
        number of time steps of the dynamic response.
    KSI [double(1 x 1)]: ratio of critical damping of the MDOF system
    ALGID [char(1 x :inf)]: algorithm to be used for the time
        integration. It can be one of the following strings for superior
        optimally designed algorithms:
            'generalized a-method': The generalized a-method (Chung &
            Hulbert, 1993)
            'HHT a-method': The Hilber-Hughes-Taylor method (Hilber,
            Hughes & Taylor, 1977)
            'WBZ': The Wood–Bossak–Zienkiewicz method (Wood, Bossak &
            Zienkiewicz, 1980)
            'U0-V0-Opt': Optimal numerical dissipation and dispersion
            zero order displacement zero order velocity algorithm
            'U0-V0-CA': Continuous acceleration (zero spurious root at
            the low frequency limit) zero order displacement zero order
            velocity algorithm
            'U0-V0-DA': Discontinuous acceleration (zero spurious root at
            the high frequency limit) zero order displacement zero order
            velocity algorithm
            'U0-V1-Opt': Optimal numerical dissipation and dispersion
            zero order displacement first order velocity algorithm
            'U0-V1-CA': Continuous acceleration (zero spurious root at
            the low frequency limit) zero order displacement first order
            velocity algorithm
            'U0-V1-DA': Discontinuous acceleration (zero spurious root at
            the high frequency limit) zero order displacement first order
            velocity algorithm
            'U1-V0-Opt': Optimal numerical dissipation and dispersion
```

```
            first order displacement zero order velocity algorithm
            'U1-V0-CA': Continuous acceleration (zero spurious root at
            the low frequency limit) first order displacement zero order
            velocity algorithm
            'U1-V0-DA': Discontinuous acceleration (zero spurious root at
            the high frequency limit) first order displacement zero order
            velocity algorithm
            'Newmark ACA': Newmark Average Constant Acceleration method
            'Newmark LA': Newmark Linear Acceleration method
            'Newmark BA': Newmark Backward Acceleration method
            'Fox-Goodwin': Fox-Goodwin formula
    U0 [double(:ndofs x 1)]: initial displacement of the MDOF system
    UT0 [double(:ndofs x 1)]: initial velocity of the MDOF system
    RINF [double(1 x 1)]: minimum absolute value of the eigenvalues of
        the amplification matrix. For the amplification matrix see
        eq.(61) in Zhou & Tamma (2004).
    EIGIND [double(:inf x 1)] is the eigenmode indicator. Only the
        eigenmode numbers that are contained in EIGIND are taken into
        account for the modal superposition.

Output parameters
    U ([ndofs x nstep]): displacement time history.
    V ([ndofs x nstep]): velocity time history.
    A ([ndofs x nstep]): acceleration time history.
    F ([ndofs x nstep]): equivalent static force time history.
```

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LDRHA_SDOF

Documentation of the LDRHA_SDOF function.

```
helpFun('LDRHA_SDOF')
```

```
 Linear Dynamic Response History Analysis (FLDRHA) of a SDOF system

 [U,UT,UTT,F] = LDRHA_SDOF(K,M,DT,XGTT,KSI,ALGID,U0,UT0,RINF)

 Description
     Linear direct time integration of second order differential
     equation of motion of dynamic response of linear elastic Single DOF
     systems. The General Single Step Single Solve (GSSSS) family of
     algorithms published by X.Zhou & K.K.Tamma (2004) is employed for
     direct time integration of the general linear dynamic response
     structural Single Degree of Freedom (SDOF) dynamic problem. The
     optimal numerical dissipation and dispersion zero order displacement
     zero order velocity algorithm designed according to the above journal
     article, is used in this routine. This algorithm encompasses the
     scope of Linear Multi-Step (LMS) methods and is limited by the
     Dahlquist barrier theorem (Dahlquist,1963).
     The force - displacement - velocity relation of the SDOF structure is
     linear. For the time integration of the equations of motion the
     signal processing function FILTER is used which is much faster than a
     loop over the time increments. However, this function is applicable
     only for time steps of equal size DT. If this is not the case,
     resampling can be used to reduce the excitation time history into a
     time history with equal time steps, using the function RESAMPLE, as
     follows:
         fs = 100; % Sample rate
         p=1; % upsampling factor
         q=1; % downsampling factor
         [xgtt_new,time_new] = resample(xgtt_old,time_old,fs,p,q);

 Input parameters
     K [double(1 x 1)]: stiffness of the SDOF system.
     M [double(1 x 1)]: mass of the SDOF system.
     DT [double(1 x 1)]: uniform time step of acceleration time history
         XGTT
     XGTT [double(:nstep x 2)]: column vector of the acceleration history
         of the excitation imposed at the base. nstep is the number of
         time steps of the dynamic response.
     KSI [double(1 x 1)]: ratio of critical damping of the SDOF system
     ALGID [char(1 x :inf)]: algorithm to be used for the time
         integration. It can be one of the following strings for superior
         optimally designed algorithms:
             'generalized a-method': The generalized a-method (Chung &
             Hulbert, 1993)
             'HHT a-method': The Hilber-Hughes-Taylor method (Hilber,
             Hughes & Taylor, 1977)
             'WBZ': The Wood–Bossak–Zienkiewicz method (Wood, Bossak &
             Zienkiewicz, 1980)
             'U0-V0-Opt': Optimal numerical dissipation and dispersion
             zero order displacement zero order velocity algorithm
             'U0-V0-CA': Continuous acceleration (zero spurious root at
```

```
            the low frequency limit) zero order displacement zero order
            velocity algorithm
            'U0-V0-DA': Discontinuous acceleration (zero spurious root at
            the high frequency limit) zero order displacement zero order
            velocity algorithm
            'U0-V1-Opt': Optimal numerical dissipation and dispersion
            zero order displacement first order velocity algorithm
            'U0-V1-CA': Continuous acceleration (zero spurious root at
            the low frequency limit) zero order displacement first order
            velocity algorithm
            'U0-V1-DA': Discontinuous acceleration (zero spurious root at
            the high frequency limit) zero order displacement first order
            velocity algorithm
            'U1-V0-Opt': Optimal numerical dissipation and dispersion
            first order displacement zero order velocity algorithm
            'U1-V0-CA': Continuous acceleration (zero spurious root at
            the low frequency limit) first order displacement zero order
            velocity algorithm
            'U1-V0-DA': Discontinuous acceleration (zero spurious root at
            the high frequency limit) first order displacement zero order
            velocity algorithm
            'Newmark ACA': Newmark Average Constant Acceleration method
            'Newmark LA': Newmark Linear Acceleration method
            'Newmark BA': Newmark Backward Acceleration method
            'Fox-Goodwin': Fox-Goodwin formula
    U0 [double(1 x 1)]: initial displacement of the SDOF system
    UT0 [double(1 x 1)]: initial velocity of the SDOF system
    RINF [double(1 x 1)]: minimum absolute value of the eigenvalues of
        the amplification matrix. For the amplification matrix see
        eq.(61) in Zhou & Tamma (2004).

Output parameters
    U [double(:nstep x 1)]: time-history of displacement
    UT [double(:nstep x 1)]: time-history of velocity
    UTT [double(:nstep x 1)]: time-history of acceleration
```

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