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THE NORM IN TAXICAB GEOMETRY

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Abstract

In this paper, we will define the inner-product and the norm in taxicab geometry and then we will discuss this inner-product geometrically.

1. Introduction

Defining metric, inner-product and norm is a fundamental concept for a new space. For Euclidean space these are all known, very well. In 1975, by using a different metric in R^2

$$d_T(A,B) = |x_1 - x_2| + |y_1 - y_2|$$

for $A = (x_1, y_1), B = (x_2, y_2)$, E. F. Krause has defined a new geometry, named by taxicab geometry. He mentioned in his book, *Taxicab Geometry* , that the taxicab geometry is a non-Euclidean geometry. It differs from Euclidean geometry in just one axiom (side-angle-side axiom), it has a wide range of applications in the urban world, and it is easy to understand [4, 5].

It is known that, geometrically, the inner product of two vectors in Euclidean geometry is the multiplication of length of one vector and the length of the projection vector of the other vector onto this vector [3]. Namely, let α, β be two vectors and θ be the angle between them. Then, the inner-product of these two vectors, geometrically, can be written by the equation

$$< \alpha, \beta >_E = \| \alpha \|_E \quad \| \beta \|_E \quad \cos_E \theta \ .$$

In this paper, we will define an inner-product and the norm in taxicab geometry. Then we will give the geometrical approach.

2. The Inner-Product

First of all, we note that all vectors, we are dealing with, passing through the origin, and the orientation will be counterclockwise direction. Thus, the vectors on coordinate axes will be taken as in the next quadrant.

Definition 2.1 *Let $\alpha = (a_1, a_2)$, $\beta = (b_1, b_2)$ be two vectors in R^2. Then*

$$< \alpha, \beta >_T = \begin{cases} (i) & |a_1b_1| + |a_2b_2|, \quad \alpha, \beta \text{ are in the same quadrant} \\ (ii) & -|a_1b_1| + |a_2b_2|, \quad \alpha, \beta \text{ are in the neighbor quadrants, and} \\ & \qquad a_1b_1 < 0, a_2b_2 > 0 \\ (iii) & |a_1b_1| - |a_2b_2|, \quad \alpha, \beta \text{ are in the neighbor quadrants, and} \\ & \qquad a_1b_1 > 0, a_2b_2 < 0 \\ (iv) & -|a_1b_1| - |a_2b_2|, \quad \alpha, \beta \text{ are in the opposite quadrants} \end{cases}$$

defines the inner-product of α and β in the taxicab geometry.

Theorem 2.2 *The inner-product of two vectors in taxicab geometry is positive definite, symmetric, and two-linear.*

Proof: 1- $\forall \alpha = (a_1, a_2) \in R^2$, since it will be always in the same quadrant, the equation (i) holds. Thus,

$$\begin{aligned} \langle \alpha, \alpha \rangle &= |a_1a_1| + |a_2a_2| \\ &= \left|a_1^2\right| + \left|a_2^2\right| \geq 0 \end{aligned}$$

and obviously,

$$\langle \alpha, \alpha \rangle = 0 \Leftrightarrow a_1 = 0 \text{ and } a_2 = 0. \text{ That is, } \alpha = 0.$$

2- $\forall\ \alpha = (a_1, a_2)$, $\beta = (b_1, b_2) \in R^2$

$$\begin{aligned} \langle \alpha, \beta \rangle &= \pm |a_1b_1| \pm |a_2b_2| \\ &= \pm |b_1a_1| \pm |b_2a_2| \\ &= \langle \beta, \alpha \rangle \end{aligned}$$

To prove the two-linearity, we first give the following diagram that will give us which equation we need to use.

$\alpha \setminus \beta$	I	II	III	IV
I	(i)	(iii)	(iv)	(ii)
II	(iii)	(i)	(ii)	(iv)
III	(iv)	(ii)	(i)	(iii)
IV	(ii)	(iv)	(iii)	(i)

where I, II, III and IV are the first, second, third, and fourth quadrants, respectively.

Notation : From now on, $\alpha \in I,\ \beta \in II$ will be read "α is in the first quadrant", "β is in the second quadrant", etc., respectively.

Before giving the two-linearity, we first note that, in general, there are three cases.

A. Vectors are to be in the same quadrant.

B. Vectors are to be in the neighbor quadrants.

C. Vectors are to be in the opposite quadrants.

Each case also has many stages. We will give at least one stage for each case. We also note that the rest of the proof is similar and the authors have checked all cases.

3. Let $\alpha = (a_1, a_2),\ \beta = (b_1, b_2) \in R^2,\ r \in R$. We need to show that

$$\langle r\,\vec{\alpha}, \vec{\beta}\rangle = \langle \vec{\alpha}, r\,\vec{\beta}\rangle = r\langle \vec{\alpha}, \vec{\beta}\rangle$$

A. Let α and β be in the same quadrant. There are two cases depending on the sign of r.

(a). $\forall\ r \in R^+$, since $r\,\vec{\alpha}$ and $\vec{\beta}$ will still be in the same quadrant, the equation (i) holds. So,

$$\begin{aligned} \langle r\,\vec{\alpha}, \vec{\beta}\rangle &= |ra_1b_1| + |ra_2b_2| \\ &= |r|\,(|a_1b_1| + |a_2b_2|) \\ &= r\langle \alpha, \beta\rangle \end{aligned}$$

(b). $\forall\ r \in R^-$, while α is in the first quadrant, $r\alpha$ is in the third quadrant. So, $r\,\vec{\alpha}$ and $\vec{\beta}$ are in opposite quadrants. Thus, the equation (iv) holds. So,

$$
\begin{aligned}
\langle r\,\vec{\alpha},\vec{\beta}\rangle &= -\left|ra_1b_1\right| - \left|ra_2b_2\right| \\
&= -\left|r\right|\left(\left|a_1b_1\right| + \left|a_2b_2\right|\right) \\
&= -\left(-r\right)\left(\left|a_1b_1\right| + \left|a_2b_2\right|\right) \\
&= r\langle\vec{\alpha},\vec{\beta}\rangle
\end{aligned}
$$

B. Let $\vec{\alpha}$ and $\vec{\beta}$ be in the neighbor quadrants. Since $r\,\vec{\alpha}$ and $\vec{\beta}$ will still be in the neighbor quadrants (ii) or (iii) holds.

For instance, let $\vec{\alpha} \in I$, $\vec{\beta} \in II$.

(a). $\forall\ r \in R^+$, since $r\,\vec{\alpha} \in I$, (ii) holds. Thus,

$$
\begin{aligned}
\langle r\,\vec{\alpha},\vec{\beta}\rangle &= -\left|ra_1b_1\right| + \left|ra_2b_2\right| \\
&= \left|r\right|\left(-\left|a_1b_1\right| + \left|a_2b_2\right|\right) \\
&= r\langle\vec{\alpha},\vec{\beta}\rangle
\end{aligned}
$$

(b). $\forall\ r \in R^-$, $r\,\vec{\alpha} \in III$, (ii) holds. Thus

$$
\begin{aligned}
\langle r\,\vec{\alpha},\vec{\beta}\rangle &= -\left(-\left|ra_1b_1\right| + \left|ra_2b_2\right|\right) \\
&= -\left|r\right|\left(-\left|a_1b_1\right| + \left|a_2b_2\right|\right) \\
&= -(-r)\left(-\left|a_1b_1\right| + \left|a_2b_2\right|\right) \\
&= r\langle\vec{\alpha},\vec{\beta}\rangle
\end{aligned}
$$

C. Let α and β be in the opposite quadrants. Again, there are two cases depending on the sign of r.

(a). $\forall\ r \in R^+$, since $r\,\vec{\alpha}$ and β will still be in the opposite quadrants, the equation (iv) holds. Thus,

$$
\begin{aligned}
\langle r\,\vec{\alpha},\vec{\beta}\rangle &= -\left|ra_1b_1\right| - \left|ra_2b_2\right| \\
&= \left|r\right|\left(-\left|a_1b_1\right| - \left|a_2b_2\right|\right) \\
&= r\langle\vec{\alpha},\vec{\beta}\rangle
\end{aligned}
$$

(b). $\forall\ r \in R^-$, since $r\,\vec{\alpha}$ and β will be in the same quadrant, the equation (i) holds. Thus,

$$
\begin{aligned}
\langle r\,\vec{\alpha},\vec{\beta}\rangle &= (|ra_1b_1| + |ra_2b_2|) \\
&= |r|\,(|a_1b_1| + |a_2b_2|) \\
&= r\,(-\,|a_1b_1| - |a_2b_2|) \\
&= r\langle\vec{\alpha},\vec{\beta}\rangle
\end{aligned}
$$

The proof of $\langle \alpha, r\beta\rangle$ is exactly the same as above.

4. Let $\alpha = (a_1, a_2)$, $\beta = (b_1, b_2)$, $\gamma = (c_1, c_2)$ be three vectors in R^2. Now, to finish the proof we need to show

$$
\begin{aligned}
\langle \alpha+\beta,\gamma\rangle = \langle \alpha,\gamma\rangle + \langle \beta,\gamma\rangle \\
\langle \alpha,\beta+\gamma\rangle = \langle \alpha,\beta\rangle + \langle \alpha,\gamma\rangle
\end{aligned}
$$

Again, in general, there are three cases:

A. All three vectors are in the same quadrant.

B. Any two vectors are in the same quadrant.

C. All three vectors are in the different quadrants.

As in the proof of "3", each case has many stages. We will prove at least one stage for each case.

A. Let α, β, γ be in the same quadrant. Since $\alpha+\beta$ and γ will be in the same quadrant, (i) holds. Then,

$$
\begin{aligned}
\langle \alpha+\beta,\gamma\rangle &= |(a_1+b_1)c_1| + |(a_2+b_2)c_2| \\
&= |a_1+b_1|\,|c_1| + |a_2+b_2|\,|c_2| \\
&= (|a_1| + |b_1|)\,|c_1| + (|a_2| + |b_2|)\,|c_2| \\
&= |a_1c_1| + |b_1c_1| + |a_2c_2| + |b_2c_2| \\
&= |a_1c_1| + |a_2c_2| + |b_1c_1| + |b_2c_2| \\
&= \langle \alpha,\gamma\rangle + \langle \beta,\gamma\rangle
\end{aligned}
$$

B. Let any two vectors be in the same quadrant. In this case, there are many subcases. Let us prove the following subcase.

Subcase 1: Let $\alpha, \gamma \in I$, $\beta \in II$. Depending on the length and position of α and β, $\alpha+\beta$ will be either in I or II.

(a). Let $\alpha+\beta \in I$ (Figure 1.). Since $\gamma \in I$, the equation (i) holds. So,

$$\begin{aligned}
\langle \alpha+\beta, \gamma \rangle &= |(a_1+b_1)c_1| + |(a_2+b_2)c_2| \\
&= |a_1+b_1|\,|c_1| + |a_2+b_2|\,|c_2| \\
&= (|a_1| - |b_1|)\,|c_1| + (|a_2| + |b_2|)\,|c_2| \\
&= |a_1c_1| - |b_1c_1| + |a_2c_2| + |b_2c_2| \\
&= |a_1c_1| + |a_2c_2| + |b_2c_2| - |b_1c_1| \\
&= \langle \alpha, \gamma \rangle + \langle \beta, \gamma \rangle
\end{aligned}$$

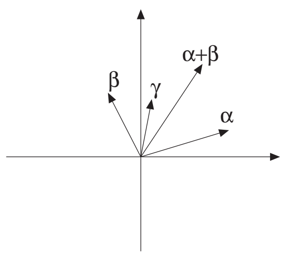

Figure 1.

(b). Let $\alpha+\beta \in II$ (Figure 2.). Since $\gamma \in I$, the equation (ii) holds. So,

$$\begin{aligned}
\langle \alpha+\beta, \gamma \rangle &= -|(a_1+b_1)c_1| + |(a_2+b_2)c_2| \\
&= -|a_1+b_1|\,|c_1| + |a_2+b_2|\,|c_2| \\
&= -(|b_1| - |a_1|)\,|c_1| + (|a_2| + |b_2|)\,|c_2| \\
&= -|b_1c_1| + |a_1c_1| + |a_2c_2| + |b_2c_2| \\
&= |a_1c_1| + |a_2c_2| + |b_2c_2| - |b_1c_1| \\
&= \langle \alpha, \gamma \rangle + \langle \beta, \gamma \rangle
\end{aligned}$$

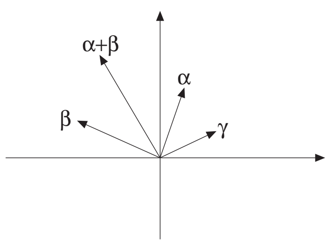

Figure 2.

C. Let all three vectors be in the different quadrants. Depending on the length and position of α and β, $\alpha+\beta$ can be in the same quadrant, neighbor quadrants, or opposite quadrants with γ. Let us prove these subcases.

Subcase 2 : (a). Let $\alpha \in I,\ \beta \in III,\ \gamma \in II$. From Figure 3., $\alpha+\beta$ and γ are in the same quadrant. So, the equation (i) holds.

$$\begin{aligned}
\langle \alpha+\beta, \gamma \rangle &= |(a_1+b_1)c_1| + |(a_2+b_2)c_2| \\
&= |a_1+b_1|\,|c_1| + |a_2+b_2|\,|c_2| \\
&= (|b_1| - |a_1|)\,|c_1| + (|a_2| - |b_2|)\,|c_2| \\
&= -|a_1c_1| + |b_1c_1| + |a_2c_2| - |b_2c_2| \\
&= \langle \alpha, \gamma \rangle + \langle \beta, \gamma \rangle
\end{aligned}$$

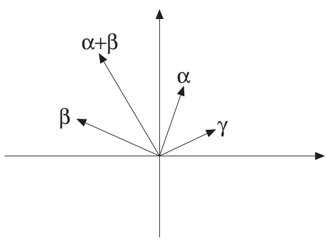

Figure 3.

(b). Let $\alpha \in I,\ \beta \in II,\ \gamma \in III$. From Figure 4., $\alpha+\beta$ and γ are in the neighbor quadrants. So, the equation (iii) holds.

$$\begin{aligned}
\langle \alpha+\beta, \gamma \rangle &= |(a_1+b_1)c_1| - |(a_2+b_2)c_2| \\
&= |a_1+b_1|\,|c_1| - |a_2+b_2|\,|c_2| \\
&= (|a_1| - |b_1|)\,|c_1| - (|a_2| + |b_2|)\,|c_2| \\
&= |b_1c_1| - |a_1c_1| - |a_2c_2| - |b_2c_2| \\
&= \langle \alpha, \gamma \rangle + \langle \beta, \gamma \rangle
\end{aligned}$$

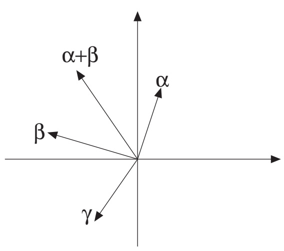

Figure 4.

(c). Let $\alpha \in I,\ \beta \in II,\ \gamma \in III$. From Figure 5., $\alpha+\beta$ and γ are in the opposite quadrants. So, the equation (iv) holds.

$$\begin{aligned}
\langle \alpha+\beta, \gamma \rangle &= -\left|(a_1+b_1)c_1\right| - \left|(a_2+b_2)c_2\right| \\
&= -\left|a_1+b_1\right|\left|c_1\right| - \left|a_2+b_2\right|\left|c_2\right| \\
&= -(\left|a_1\right| - \left|b_1\right|)\left|c_1\right| - (\left|a_2\right| + \left|b_2\right|)\left|c_2\right| \\
&= -\left|a_1c_1\right| + \left|b_1c_1\right| - \left|a_2c_2\right| - \left|b_2c_2\right| \\
&= -\left|a_1c_1\right| - \left|a_2c_2\right| + \left|b_1c_1\right| - \left|b_2c_2\right| \\
&= \langle \alpha, \gamma \rangle + \langle \beta, \gamma \rangle
\end{aligned}$$

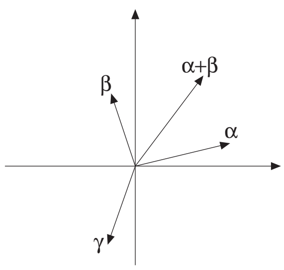

Figure 5.

3. The Norm

As it is known, in Euclidean geometry, the norm of a vector α, is defined by

$$\|\alpha\|_E = \sqrt{\langle \alpha, \alpha \rangle_E}\ .$$

In taxicab geometry, we define the norm of a vector as follows:

Definition 3.1 *Let $\alpha = (a_1, a_2) \in R^2$ be any vector. Then,*

$$\|\alpha\|_T = \sqrt{\langle \alpha, \alpha \rangle_T + 2\left|a_1a_2\right|}$$

defines the norm of α in taxicab geometry. Obviously,

$$\|\alpha\|_T = \sqrt{a_1^2 + a_2^2 + 2\left|a_1a_2\right|} = \left|a_1\right| + \left|a_2\right| = d_T(\alpha, 0).$$

As in the Euclidean geometry, the norm in taxicab geometry satisfies the following properties.

Theorem 3.2 *Let* $\alpha = (a_1, a_2), \beta = (b_1, b_2), \gamma = (c_1, c_2) \in R^2$ *and* $r \in R$. *Then,*

$$\begin{array}{lll} (i) & \|\alpha\|_T & \geq 0 \\ (ii) & \|r\alpha\|_T & = |r|\,\|\alpha\|_T\,,\ r \in R \\ (iii) & \|\alpha+\beta\|_T & \leq \|\alpha\|_T + \|\beta\|_T \\ (iv) & \|\alpha-\beta\|_T & \geq \|\alpha\|_T - \|\beta\|_T \\ (v) & \|\alpha-\beta\|_T & \leq \|\alpha\|_T + \|\beta\|_T \\ (vi) & \|\alpha-\beta\|_T & \leq \|\alpha-\gamma\|_T + \|\gamma-\beta\|_T \end{array}$$

Proof: (i) and (ii) are obvious.

(iii)

$$\begin{aligned} \|\alpha+\beta\|_T &= \sqrt{\langle \alpha+\beta, \alpha+\beta\rangle + 2\,|(a_1+b_1)(a_2+b_2)|} \\ &= \sqrt{(|a_1+b_1| + |a_2+b_2|)^2} \\ &= |a_1+b_1| + |a_2+b_2| \\ &\leq (|a_1|+|a_2|) + (|b_1|+|b_2|) = \|\alpha\|_T + \|\beta\|_T \end{aligned}$$

(iv)

$$\begin{aligned} \|\alpha-\beta\|_T &= \sqrt{\langle \alpha-\beta, \alpha-\beta\rangle + 2\,|(a_1-b_1)(a_2-b_2)|} \\ &= \sqrt{(|a_1-b_1| + |a_2-b_2|)^2} \\ &= |a_1-b_1| + |a_2-b_2| \\ &\geq (|a_1|+|a_2|) - (|b_1|+|b_2|) = \|\alpha\|_T - \|\beta\|_T \end{aligned}$$

(v)

$$\begin{aligned} \|\alpha-\beta\|_T &= \sqrt{\langle \alpha-\beta, \alpha-\beta\rangle + 2\,|(a_1-b_1)(a_2-b_2)|} \\ &= \sqrt{(|a_1-b_1| + |a_2-b_2|)^2} \\ &= |a_1+(-b_1)| + |a_2+(-b_2)| \\ &\leq (|a_1|+|a_2|) + (|b_1|+|b_2|) = \|\alpha\|_T + \|\beta\|_T \end{aligned}$$

(vi)

$$\begin{aligned} \|\alpha-\beta\|_T &= \|\alpha-\beta+\gamma-\gamma\|_T \\ &= \|(\alpha-\gamma)+(\gamma-\beta)\|_T \\ &\leq \|\alpha-\gamma\|_T + \|\gamma-\beta\|_T \end{aligned}$$

4. Geometrical Approach

Before giving the geometrical meaning of this inner-product, let us give some trigonometric equalities.

As it is mentioned in [1], the reduction formulas in Euclidean geometry hold, but the addition and subtraction formulas do not hold in taxicab geometry. It is proved, for instance, that,

$$\begin{aligned}
\cos_T(\pi_T/2-\theta) &= \sin_T\theta\\
\sin_T(\pi_T/2-\theta) &= \cos_T\theta\\
\cos_T(\pi_T/2+\theta) &= -\sin_T\theta\\
\sin_T(\pi_T/2+\theta) &= \cos_T\theta\\
\cos_T(\pi_T-\theta) &= -\cos_T\theta\\
\sin_T(\pi_T-\theta) &= \sin_T\theta\\
\cos_T(\pi_T+\theta) &= -\cos_T\theta\\
\sin_T(\pi_T+\theta) &= -\sin_T\theta
\end{aligned}$$

where $\pi_T = 4$ [5], and

$$cos_T(\theta_2-\theta_1) = \begin{cases}
1+\cos_T\theta_2-\cos_T\theta_1 & \alpha\in I,\beta\in I,\gamma\in I;\\
 & \alpha\in II,\beta\in II,\gamma\in I;\\
 & \alpha\in I,\beta\in II,\gamma\in I;\\
 & \alpha\in I,\beta\in II,\gamma\in II;\\
1-\cos_T\theta_2+\cos_T\theta_1 & \alpha\in III,\beta\in III,\gamma\in I;\\
 & \alpha\in IV,\beta\in IV,\gamma\in I;\\
 & \alpha\in III,\beta\in IV,\gamma\in II;\\
 & \alpha\in III,\beta\in IV,\gamma\in I;\\
-1+\cos_T\theta_2+\cos_T\theta_1 & \alpha\in I,\beta\in III,\gamma\in III;\\
 & \alpha\in I,\beta\in IV,\gamma\in III;\\
 & \alpha\in I,\beta\in IV,\gamma\in IV;\\
 & \alpha\in II,\beta\in IV,\gamma\in III;\\
-1-\cos_T\theta_2-\cos_T\theta_1 & \alpha\in I,\beta\in III,\gamma\in II;\\
 & \alpha\in II,\beta\in III,\gamma\in I;\\
 & \alpha\in II,\beta\in III,\gamma\in II;\\
 & \alpha\in II,\beta\in IV,\gamma\in II;
\end{cases}$$

where θ_1, θ_2, and $\theta_2-\theta_1$ represents the angles of α, β, and γ, respectively, with respect to positive $x-$axis.

As it is known, the geometrical approach of inner-product in Euclidean geometry is

$$\langle \alpha, \beta \rangle_E = \|\alpha\|_E \ \ \|\beta\|_E \ \ \cos_E \theta$$

where $\alpha, \beta \in R^2$ and θ is the angle between them [2].

The geometrical approach of inner-product in taxicab geometry is as follows:

Definition 4.1 *Let* $\alpha = (a_1, a_2), \beta = (b_1, b_2) \in R^2$ *and* θ *be angle between* α *and* β*.*

Define *the taxicab constant,* R_T;

$$R_T := \begin{cases} 2\,|a_1 b_2| & , \quad \alpha \in II, \beta \in II, \gamma \in I; \\ & \quad\ \ \alpha \in IV, \beta \in IV, \gamma \in I; \\ \\ -2\,|a_1 b_2| & , \quad \alpha \in I, \beta \in III, \gamma \in II; \\ & \quad\ \ \alpha \in II, \beta \in IV, \gamma \in III; \\ \\ 2\,|a_2 b_1| & , \quad \alpha \in I, \beta \in I, \gamma \in I; \\ & \quad\ \ \alpha \in III, \beta \in III, \gamma \in I; \\ \\ -2\,|a_2 b_1| & , \quad \alpha \in I, \beta \in III, \gamma \in III; \\ & \quad\ \ \alpha \in II, \beta \in IV, \gamma \in II; \\ \\ 0 & , \quad \alpha \in I, \beta \in II, \gamma \in I; \\ & \quad\ \ \alpha \in I, \beta \in II, \gamma \in II; \\ & \quad\ \ \alpha \in I, \beta \in IV, \gamma \in III; \\ & \quad\ \ \alpha \in I, \beta \in IV, \gamma \in IV; \\ & \quad\ \ \alpha \in II, \beta \in III, \gamma \in I; \\ & \quad\ \ \alpha \in II, \beta \in III, \gamma \in II; \\ & \quad\ \ \alpha \in III, \beta \in IV, \gamma \in I; \\ & \quad\ \ \alpha \in III, \beta \in IV, \gamma \in II; \end{cases}$$

Then, we have

$$\langle \alpha, \beta \rangle_T = \|\alpha\|_T \ \ \|\beta\|_T \ \ \cos_T \theta - R_T \ .$$

Since the trigonometric equalities are different in taxicab geometry, we need R_T to make the geometrical approach the same as in Euclidean geometry.

Let us prove this for one subcase.

Subcase 1 : Let $\alpha = (a_1, a_2),\ \beta = (b_1, b_2) \in I,$ as in Figure 6. Obviously,

$$< \alpha, \beta >_T = |a_1 a_2| + |b_1 b_2| \ .$$

On the other hand,

$$\begin{aligned}
\|\alpha\|_T &= |a_1| + |a_2| \\
\|\beta\|_T &= |b_1| + |b_2| \\
\cos_T \theta &= \cos_T(\theta_2 - \theta_1) \\
&= 1 + \cos_T \theta_2 - \cos_T \theta_1 \\
&= 1 + \tfrac{|b_1|}{\|\beta\|_T} - \tfrac{|a_1|}{\|\alpha\|_T} \ .
\end{aligned}$$

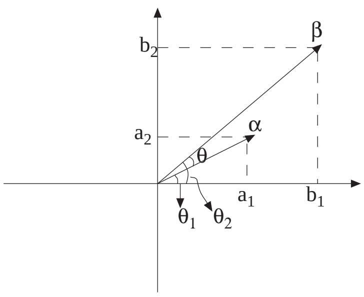

Figure 6.

Thus,

$$\|\alpha\|_T \ \ \|\beta\|_T \cos_T \theta \quad = \quad |a_1 b_1| + |a_2 b_2| + 2|a_2 b_1| \ .$$

From definition of R_T, since $\theta_1, \theta_2, \theta_2 - \theta_1 \in [0, 2],\ R_T = 2|a_2 b_1|$. So,

$$\|\alpha\|_T \ \ \|\beta\|_T \cos_T \theta - R_T = |a_1 b_1| + |a_2 b_2| = < \alpha, \beta >_T$$

as desired.

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