Coordinated Disaster Relief

Strategic Minds Collective
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Scenario

- A country has been affected by a major natural disaster (National Pandemic)
- There are ***n*** provinces in the country
- Each province has their own relief organizations working to benefit their province the most
- The disaster affected each province **equally**
- Provinces have the choice to either focus all resources on themselves or coalesce with other provinces to benefit the country as a whole
- The way funds are added makes this game a ***superadditive coalition game***

Asymmetric

- Each province has a different budget to start with
- Thus groups of the same size may have different payoffs as well
- This reflects that fact that different provinces have different financial situations
  - Some provinces are poorer, some are richer
- The asymmetry of this scenario is what causes the collective action problem
  - Richer provinces have the potential to gain more resources should they act selfishly
  - The overall country would benefit if ***all*** provinces pooled their money into a grand coalition
  - However, because of the possible outcomes the result could be highly unfair towards poorer provinces

Fairness - Shapley Value

- Different team members contribute to a project's success
- The goal is to calculate a fair share of the reward to each team member based on their contribution to the team
- However, a player's contribution can vary based on when they join the team:
  - Example, if a player is the second to join a team, their contribution will likely be a lot
  - On the other hand, if they are the hundredth player, likely to not be that much
  - But this isn't the player's fault!
- The Shapley value accounts for this by calculating all possible ways a team could be formed
- Collaboration is accounted for so everyone is rewarded for their contribution and not just on their individual work

Fairness - Shapley Value

Mathematically defining the Shapley:

$\varphi_i(v)$ is Shapley value for i-th player given a payoff

n is the number of players

R is the collection of all possible orders of the players

P_i^R is the set of all players that occur before the i-th player in a given order

$$\varphi_i(v) = \frac{1}{n!} \sum_R \left[v(P_i^R \cup \{i\}) - v(P_i^R) \right]$$

Fairness - Shapley Value

- We want to ensure a fair outcome for all provinces.
- We choose the Shapley value to be the “fairest” outcome. Chosen because it depends on everyone’s marginal contributions.
- Four different properties that it (and only it fulfills all at once).
  - **Efficiency** , so the Shapley value for each player summed up is the total value of the grand coalition.
  - **Symmetry** , if two players contribute equal payoffs then their Shapley value is the same.
  - **Null player** , if a player doesn’t make any contribution to any coalition, their value is zero.
  - **Linearity** , the Shapley value given a payoff vector is linear for all players. (This also means we can represent the conversion of a payoff to a Shapley vector by a matrix)

Measure of unfairness

- We've discussed what a fair outcome would look like (Shapley value).
- But we've seen this isn't the only possible outcome in the core.

- Remember back to the example we presented last time. We had multiple outcomes like:
  - The Shapley value, (500, 200, 200)
  - But also the outcome, (900, 0, 0)
  - Also, (400, 250, 250)

- This last outcome seems unfair, but why?

Measure of unfairness

Something is unfair because an outcome deviates from what we'd expect

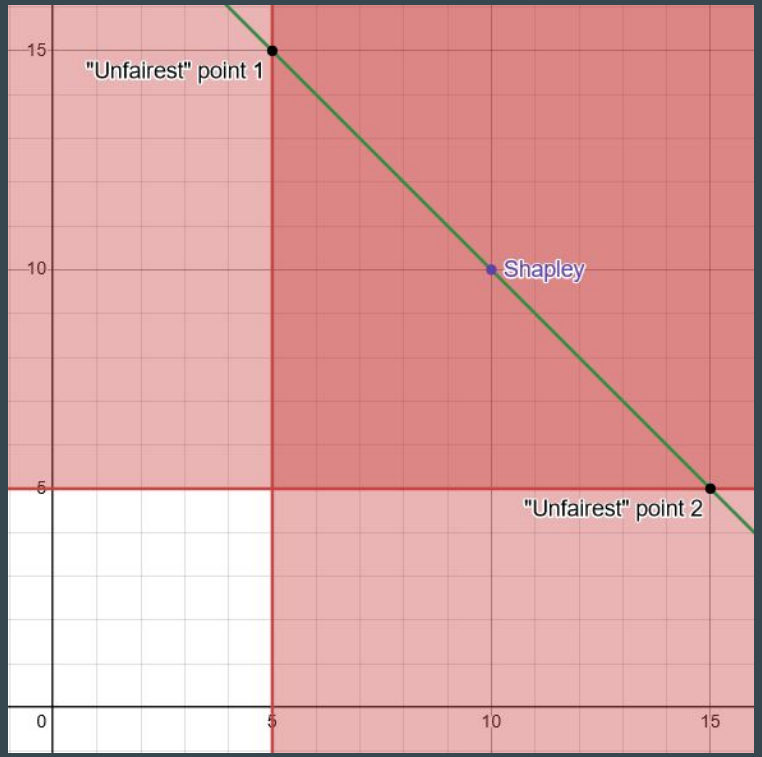
We expect the players to receive the Shapley value

However, they receive some other outcome

E.g. (10, 10) vs (15, 5)

We can measure the unfairness in the outcome by simply taking the distance between the Shapley and the given outcome!

- We use Euclidean distance to penalize larger deviations from the Shapley
- But could use other norms as well
- To measure the unfairness of the core (or a set of outcomes), we can simply find the point that is furthest away from the Shapley value and take the distance



Example

Some demos for you!

Mechanism

- An external government (like the state/federal government) could subsidize certain provinces (within a certain budget)
- Could collect a tax from each province and then redistribute it
  - This tax could be minimized to produce fair outcome with lowest cost
  - However this would shift the Shapley value
- Could enforce a contract such that it can penalize provinces that deviate too far from the Shapley
- Maybe there's a way the provinces can self-govern?