Machine Learning Lab 1

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In [28]:

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
from matplotlib import cm
```

1. Load data

In [32]:

```
data_row1 = np.genfromtxt('../Data/lab1/ex1data1.txt', delimiter=',
data1 = pd.DataFrame(data_row1, columns=list(['Population', 'Profit
data1.head()
```

Out[32]:

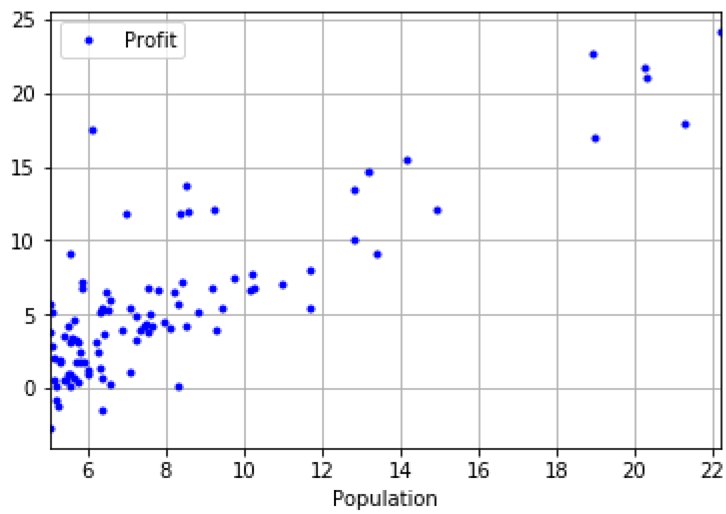
	Population	Profit
0	6.1101	17.5920
1	5.5277	9.1302
2	8.5186	13.6620
3	7.0032	11.8540
4	5.8598	6.8233

2. Build a graph of the dependence of the profit of the restaurant on the population of the city in which it is located.

In [5]:

```python
plt.figure()
data1.plot(x='Population', y='Profit',  style=['b.'])
plt.grid(True)
plt.show()
```

```
<Figure size 432x288 with 0 Axes>
```



3. Cost function

In [12]:

```python
def compute_cost(X, y, theta):
    h = [np.matmul(x, theta.T).sum() for x in X]
    return np.power(h - y, 2).sum() / (2 * m)
```

In [10]:

```python
m = data_row1.shape[0] # Size of training set
n = data_row1.shape[1] # Size of feature vector
X1 = data1[['Population']]
X1.insert(0, 'theta_0', 1)
y1 = data1['Profit']
# theta the coefficients of the linear equation
theta = np.zeros((1, n))
X1.head()
```

Out[10]:

	theta_0	Population
0	1	6.1101
1	1	5.5277
2	1	8.5186
3	1	7.0032
4	1	5.8598

In [13]:

```python
cost = compute_cost(X1.to_numpy(), y1.to_numpy(), theta)
print('Cost =', cost)
```

```
Cost = 32.072733877455676
```

4. Implement the gradient descent function to select model parameters. Build the resulting model (function) together with the graph

In [83]:
```
'''
function [theta, J_history] = gradient_dscent(X, y, theta, alpha, nu
taking num_iters gradient steps with learning rate alpha
'''
def gradient_descent(X, y, theta, alpha, number_iterations):
    m = y.shape[0]
    n = X.shape[1]
    j_history = []
    for i in range(0, number_iterations):
        deltas = np.zeros(n)
        for j in range(0, n):
            xj = X[:, j]
            h = [np.matmul(x, theta.T)[0] for x in X]
            deltas[j] = ((h - y) * xj).sum() * alpha / m
        theta[0] -= deltas
        j_history.append(compute_cost(X, y, theta))

    return theta, j_history
```

In [17]:
```
iterations = 1500
alpha = 0.01
(theta, j_history) = gradient_descent(X1.to_numpy(), y1.to_numpy(),
```

In [18]:
```
print('gradient descent thera: ', theta)
```

```
gradient descent thera:  [[-3.63029144  1.16636235]]
```

In [53]:
```
print('for population = 35,000, profit prediction: ', (np.matmul([1,
print('for population = 70,000, profit prediction', (np.matmul([1, 7
```

```
for population = 35,000, profit prediction:  4519.767867701772
for population = 70,000, profit prediction 45342.45012944714
```

In [54]:
```
h = [np.matmul(x, theta.T).sum() for x in X1.to_numpy()]
data1_plot = data1.join(pd.DataFrame({'Linear regression': h}))
```

In [30]:
```
plt.figure()
ax = data1_plot.plot(x='Population', y='Profit',  style=['b.'])
data1_plot.plot(x='Population', y='Linear regression', ax=ax)
plt.grid(True)
plt.show()
```

```
<Figure size 432x288 with 0 Axes>
```

5. Build a three-dimensional graph of the dependence of the loss function on the parameters of the model (θ0 and θ1) as a surface and as contours (contour plot).

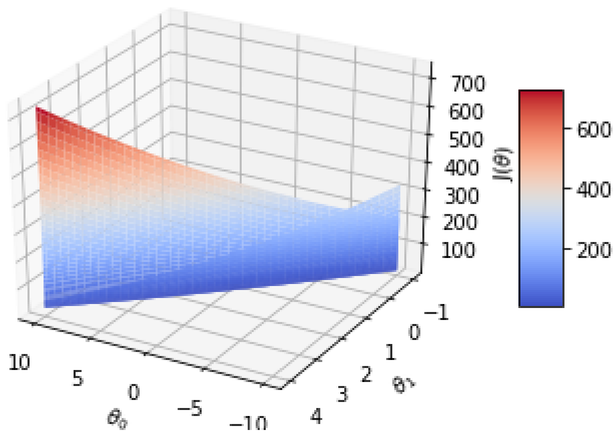
In [36]:
```
theta0_vals = np.linspace(-10, 10, num=100)
theta1_vals = np.linspace(-1, 4, num=100)
# initialize J values to a matrix of 0
J_vals = np.zeros((theta0_vals.size, theta1_vals.size))
```

In [37]:
```
# Fill J values
for i in range(0, theta0_vals.size):
    for j in range(0, theta1_vals.size):
        t = np.array([[theta0_vals[i], theta1_vals[j]]])
        J_vals[i, j] = computeCost(X1.to_numpy(), y1.to_numpy(), t)

J_vals = J_vals.T
```

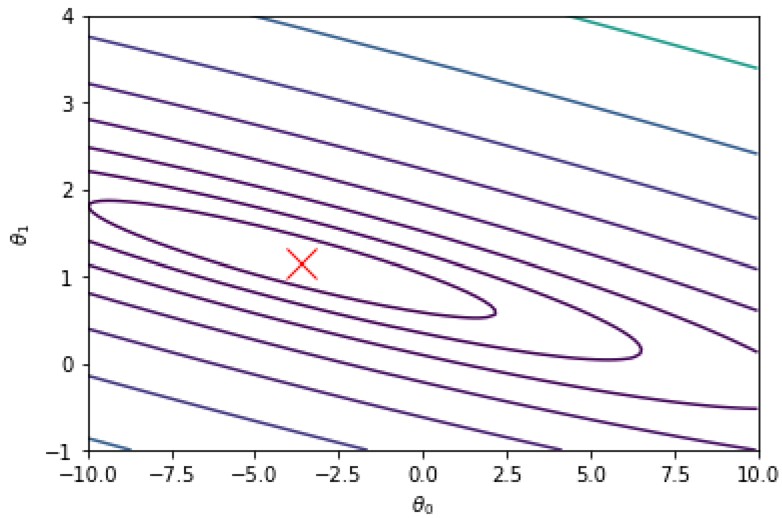
In [40]:

```python
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
surf=ax.plot_surface(theta0_vals,theta1_vals,J_vals,cmap="coolwarm")
fig.colorbar(surf, shrink=0.5, aspect=5)
ax.set_xlabel(r'$\theta_0$')
ax.set_ylabel(r'$\theta_1$')
ax.set_zlabel(r'J($\theta$)')
ax.view_init(30,120)#rotate for better angle
plt.show()
```



In [55]:

```
# Contour plot
plt.figure()
# Plot J_vals as 15 contours spaced logarithmically between 0.01 and
ax = plt.contour(theta0_vals, theta1_vals, J_vals, np.logspace(-2, 3
#plt.clabel(ax, inline=0, fontsize=2)
plt.xlabel(r'$\theta_0$')
plt.ylabel(r'$\theta_1$')
plt.plot(theta[0, 0], theta[0, 1], 'rx', linewidth=1, markersize=15)
plt.show()
```



6. Load Data

In [76]:

```
data_row2 = np.genfromtxt('../Data/lab1/ex1data2.txt', delimiter=',
data2 = pd.DataFrame(data_row2, columns=list(['size', 'number', 'pri
data2.head()
```

Out[76]:

	size	number	price
0	2104.0	3.0	399900.0
1	1600.0	3.0	329900.0
2	2400.0	3.0	369000.0
3	1416.0	2.0	232000.0
4	3000.0	4.0	539900.0

7.Normalize the signs. Did this affect the rate of convergence of the gradient descent? Give the answer in the form of a graph.

In [84]:

```
def normalization(X):
    norm = (X - X.mean(axis=0)) / X.std(axis=0)
    mu = X.mean(axis=0)
    sigma = X.std(axis=0)
    return norm, mu, sigma

```

In [79]:
```python
X = data2[['size', 'number']]
X_norm, mu, sigma = normalization(X)
X_norm.describe()
```

Out[79]:

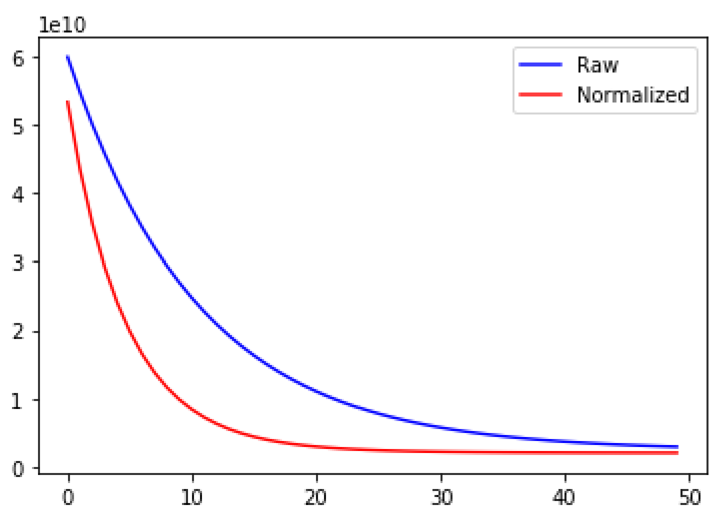
	size	number
count	4.700000e+01	4.700000e+01
mean	3.779483e-17	2.185013e-16
std	1.000000e+00	1.000000e+00
min	-1.445423e+00	-2.851859e+00
25%	-7.155897e-01	-2.236752e-01
50%	-1.417900e-01	-2.236752e-01
75%	3.376348e-01	1.090417e+00
max	3.117292e+00	2.404508e+00

In [80]:
```python
y = data2['price']
m = y.size
n = data_row2.shape[1]
X.insert(0, 'theta_0', 1)
X_norm.insert(0, 'theta_0', 1)

theta1 = np.zeros((1, n))
theta2 = np.zeros((1, n))
```

In [81]:
```python
(theta1, j_history) = gradient_descent(X.to_numpy(), y.to_numpy(), t
(theta2, j_norm_history) = gradient_descent(X_norm.to_numpy(), y.to_
```

In [82]:
```python
p1 = plt.plot(range(0, len(j_history)), j_history, color='blue')
plt.legend('Raw')
p2 = plt.plot(range(0, len(j_norm_history)), j_norm_history, color=
plt.legend((p1[0], p2[0]), ('Raw', 'Normalized'))
plt.show()
```



8. Implement the loss functions J(θ) and gradient descent for the case of multivariate

8. Implement the loss functions J(θ) and gradient descent for the case of multivariate linear regression using vectorization.

In [86]:

```
def gradient_descent2(X, y, theta, alpha, number_iterations):
    m = y.shape[0]
    j_history = []
    XT = X.T
    for i in range(0, number_iterations):
        h = [np.matmul(x, theta.T)[0] for x in X]
        loss = h - y
        cost = np.sum(loss ** 2) / (2 * m)
        gradient = np.matmul(XT, loss) / m
        theta[0] -= alpha * gradient
        j_history.append(cost)

    return theta, j_history
```

In [87]:

```
iterations = 400
alpha = 0.01
theta_GD = np.zeros((1, n))

(theta_GD, j_history) = gradient_descent2(X_norm.to_numpy(), y.to_nu
print('Theta :', theta_GD)
```

```
Theta : [[334302.06399328 100087.11600585   3673.54845093]]
```

In [92]:

```
price = np.array([1, (1650 - mu[0]) / sigma[0], (4 - mu[1]) / sigma
print('price prediction for 4 rooms house: ', price)
```

```
price prediction for 4 rooms house:  [294141.99995712]
```

9.Show that vectorization gives a performance boost

In [96]:

```
from timeit import default_timer as timer

iterations = 1000
alpha = 0.02
theta = np.zeros((1, n))

start = timer()
(theta, j_history) = gradient_descent(X_norm.to_numpy(), y.to_numpy(
end = timer()
exec_time1 = end-start
print('theta', theta)
print('time excuted:', exec_time1)
```

```
theta [[340412.65900156 110620.78816241  -6639.21215439]]
time excuted: 0.7502348749985686
```

In [95]:

```
theta = np.zeros((1, n))

start = timer()
(theta, j_history) = gradient_descent2(X_norm.to_numpy(), y.to_numpy
end = timer()
exec_time2 = end-start

print('theta', theta)
print('time excuted:', exec_time2)
```

```
theta [[340412.65900156 110620.78816241  -6639.21215439]]
time excuted: 0.17354223399888724
```

In [105]:

```
diff = (exec_time1 / exec_time2)
print(diff,'faster')
```

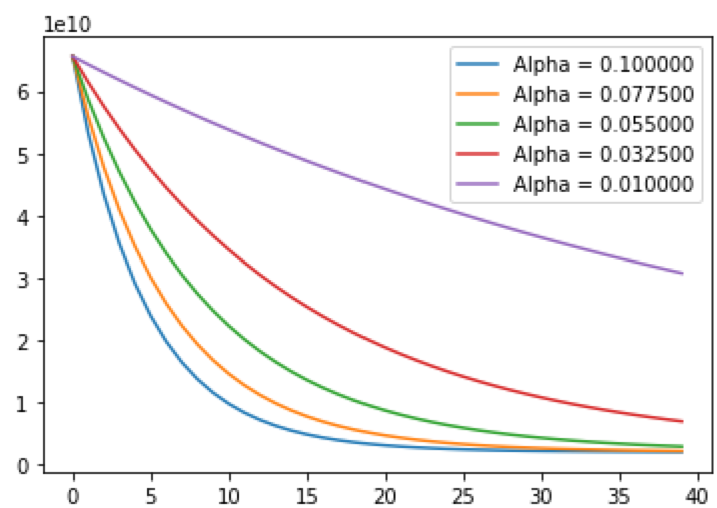
```
4.323067980117043 faster
```

10. Try changing the parameter α (learning factor). How does the graph of the loss function change depending on the number of iterations of the gradient descent? Draw the result as a graph.

In [101]:

```
alphas = np.linspace(0.1, 0.01, num=5)
plots = []
for alpha in alphas:
    theta = np.zeros((1, n))
    (theta, j_history) = gradient_descent2(X_norm.to_numpy(), y.to_r
    p = plt.plot(range(0, len(j_history)), j_history)
    plots.append(p[0])

plt.legend(plots, ["Alpha = %f" % (x) for x in alphas])
plt.show()
```



11.Build a model using an analytical solution that can be obtained by the least squares method. Compare the results of this model with the model obtained by gradient descent.

In [109]:

```
# computes the closed-form solution to linear regression using the 
def normal_equations(X, y):
    XX = np.asmatrix(X)
    XT = XX.T
    return ((XT @ XX).I @ XT) @ y
```

In [115]:

```
theta_A = normal_equations(X.to_numpy(), y.to_numpy())
print('theta from normal equations', theta_A)
print('theta from the normal normalized gradient descent: %s' % (the

price = np.array([1, 1200, 4]) @ theta_A.T
print('price prediction of 1200 sq. ft, 4 rooms house (using normal 

price = np.array([1, (1200 - mu[0]) / sigma[0], (4 - mu[1]) / sigma
print('price prediction of 1200 sq. ft, 4 rooms house (using gradien
```

```
theta from normal equations [[89597.9095428    139.21067402 -8738.0191
1233]]
theta from the normal normalized gradient descent: [[334302.06399328 1
00087.11600585   3673.54845093]]
price prediction of 1200 sq. ft, 4 rooms house (using normal equation
s): [[221698.64191464]]
price prediction of 1200 sq. ft, 4 rooms house (using gradient descen
t): [237467.6966757]
```