

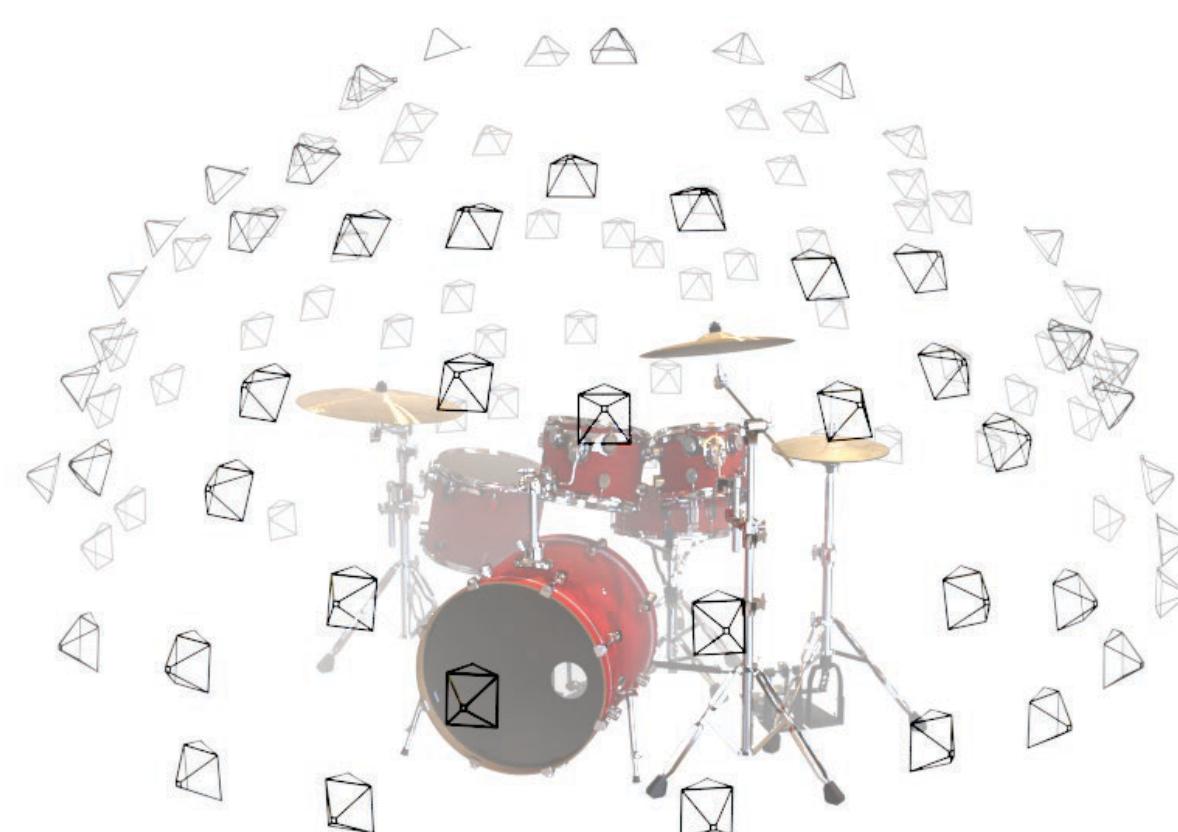
RENDERING

Inverse Graphics & Differentiable Rendering

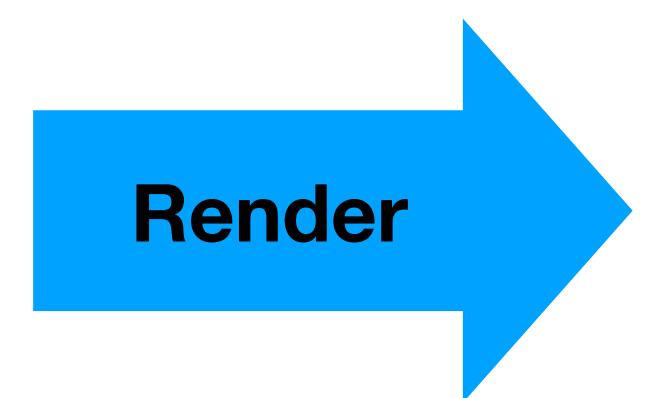
Rendering (Graphics): Given 3D Scene + Camera parameters, yield images



3D Scene



Camera Poses



Images

Inverse Graphics: Given Images, Infer Camera Poses & 3D Scene!



Reconstruct

Images

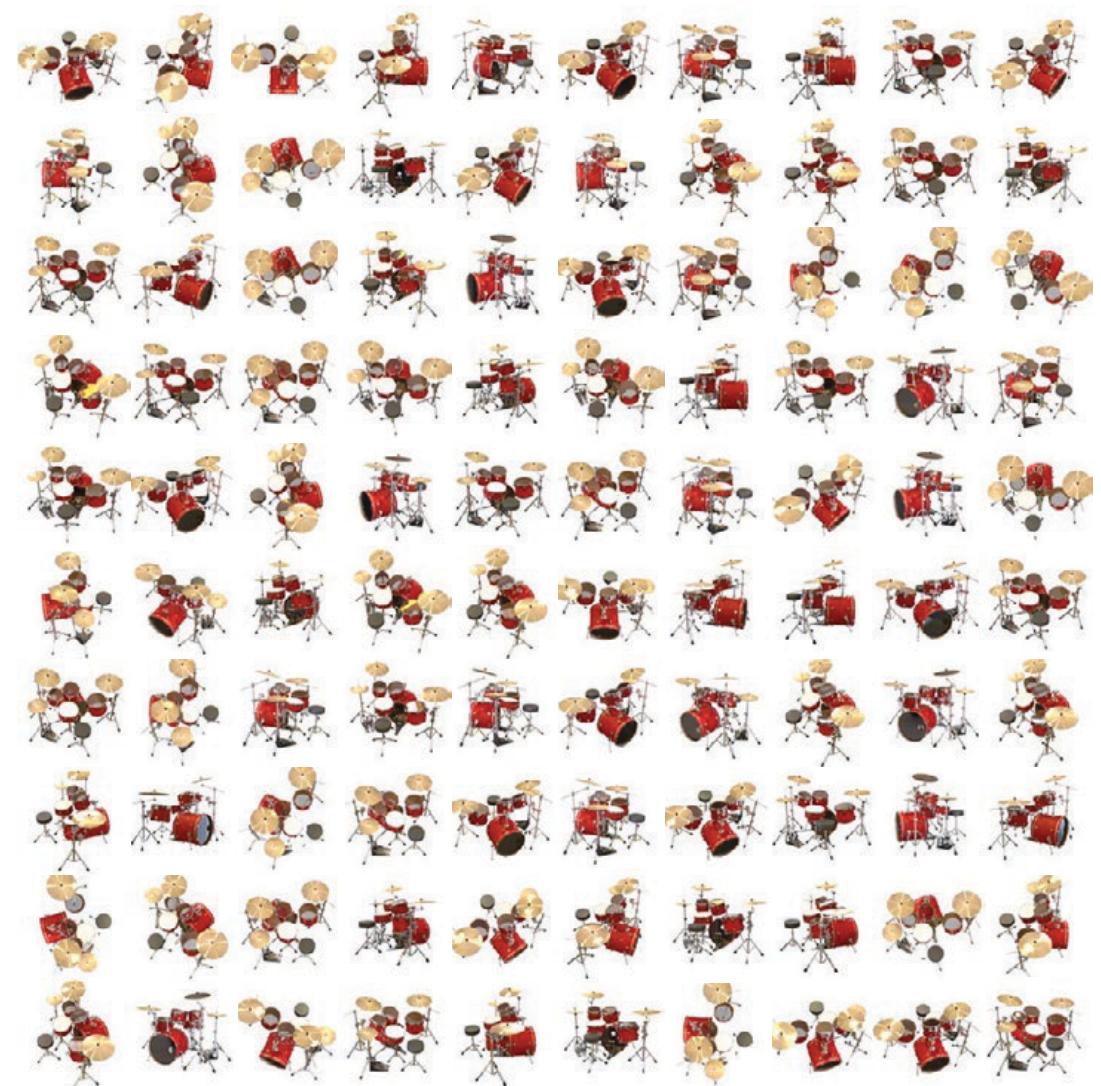


3D Scene



Camera Poses

How to get camera poses?

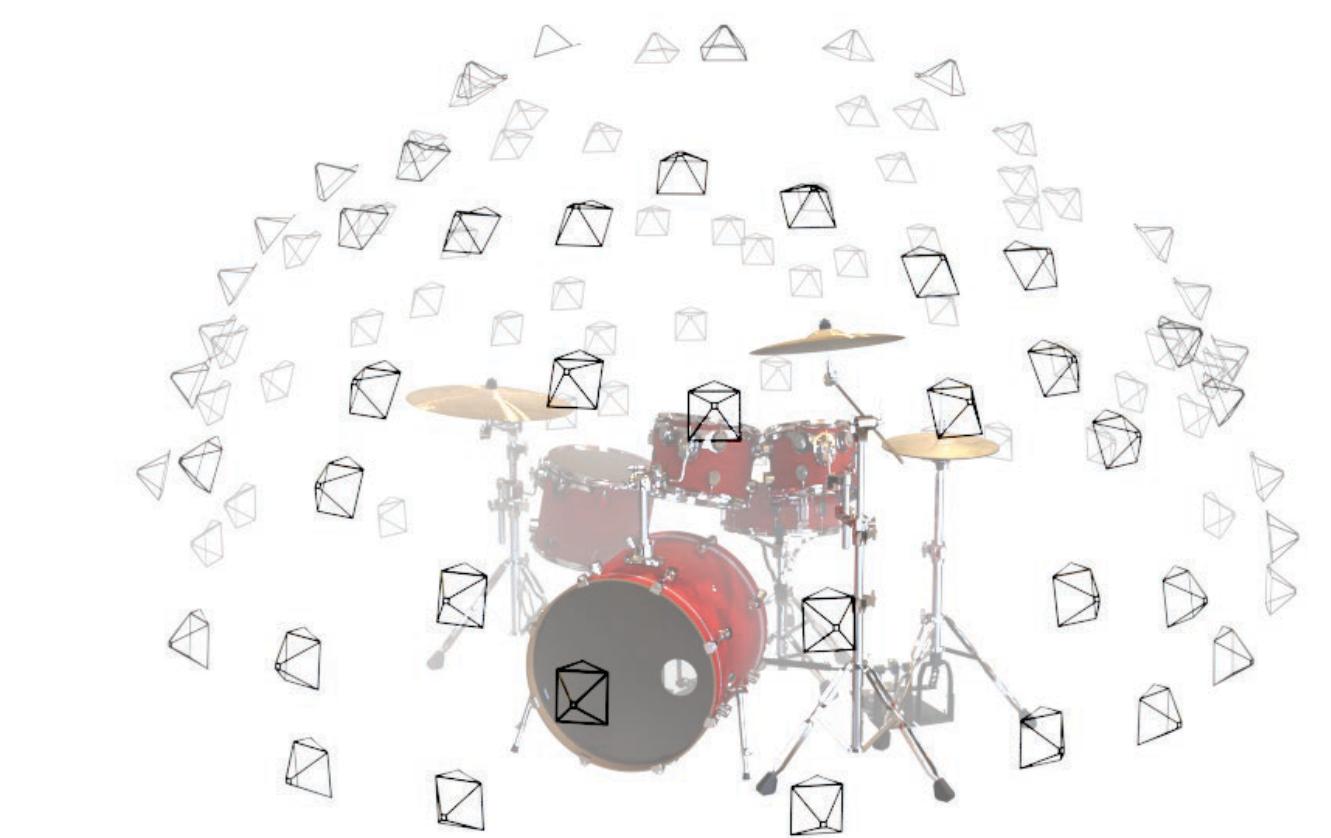


Reconstruct



Images

3D Scene



Camera Poses

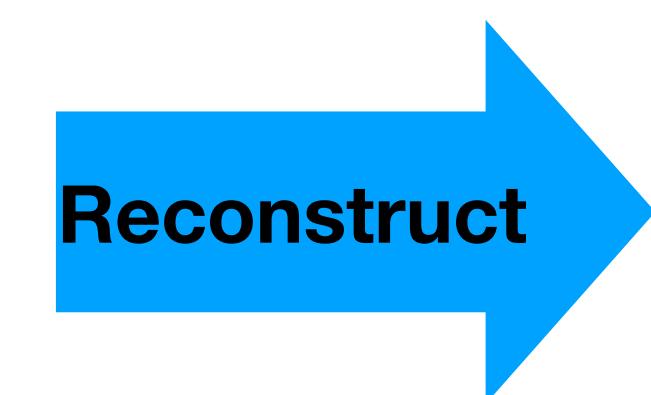
Can assume we know the camera poses.



Images

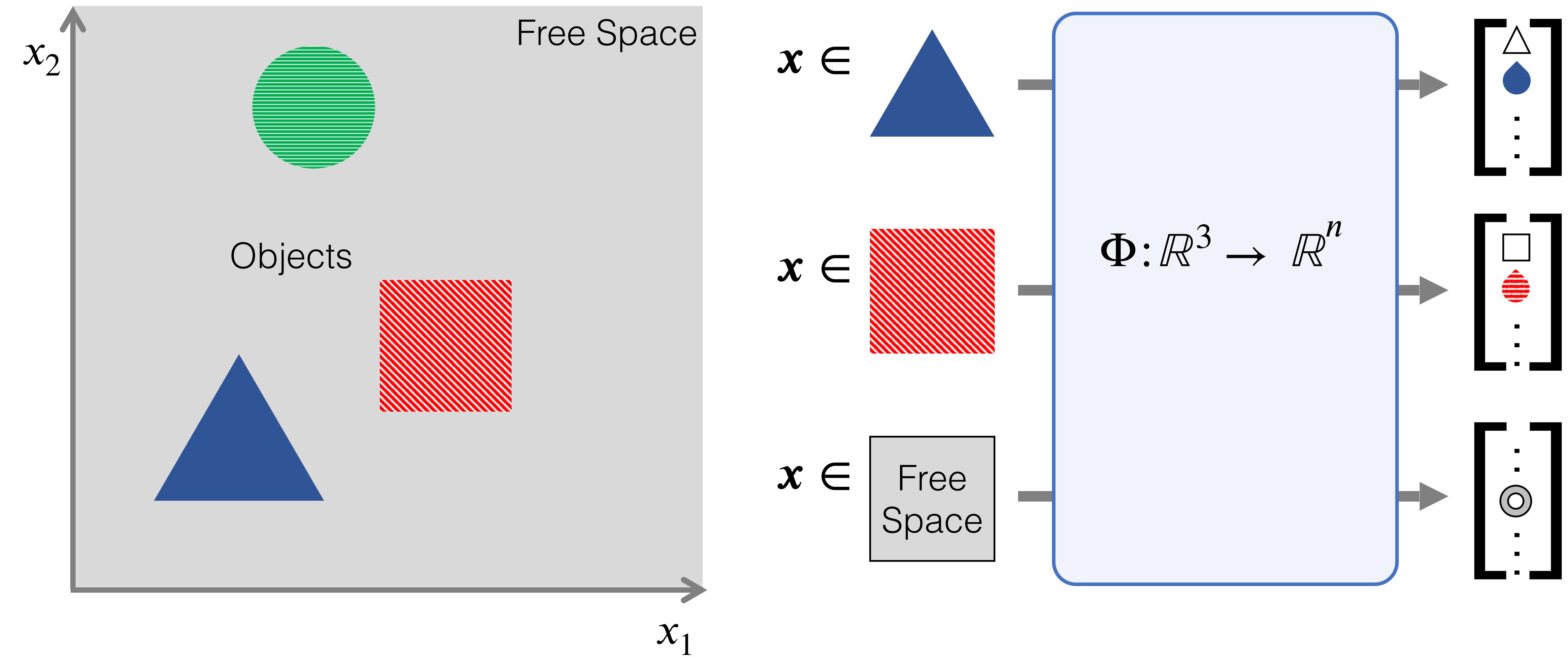


Camera Poses

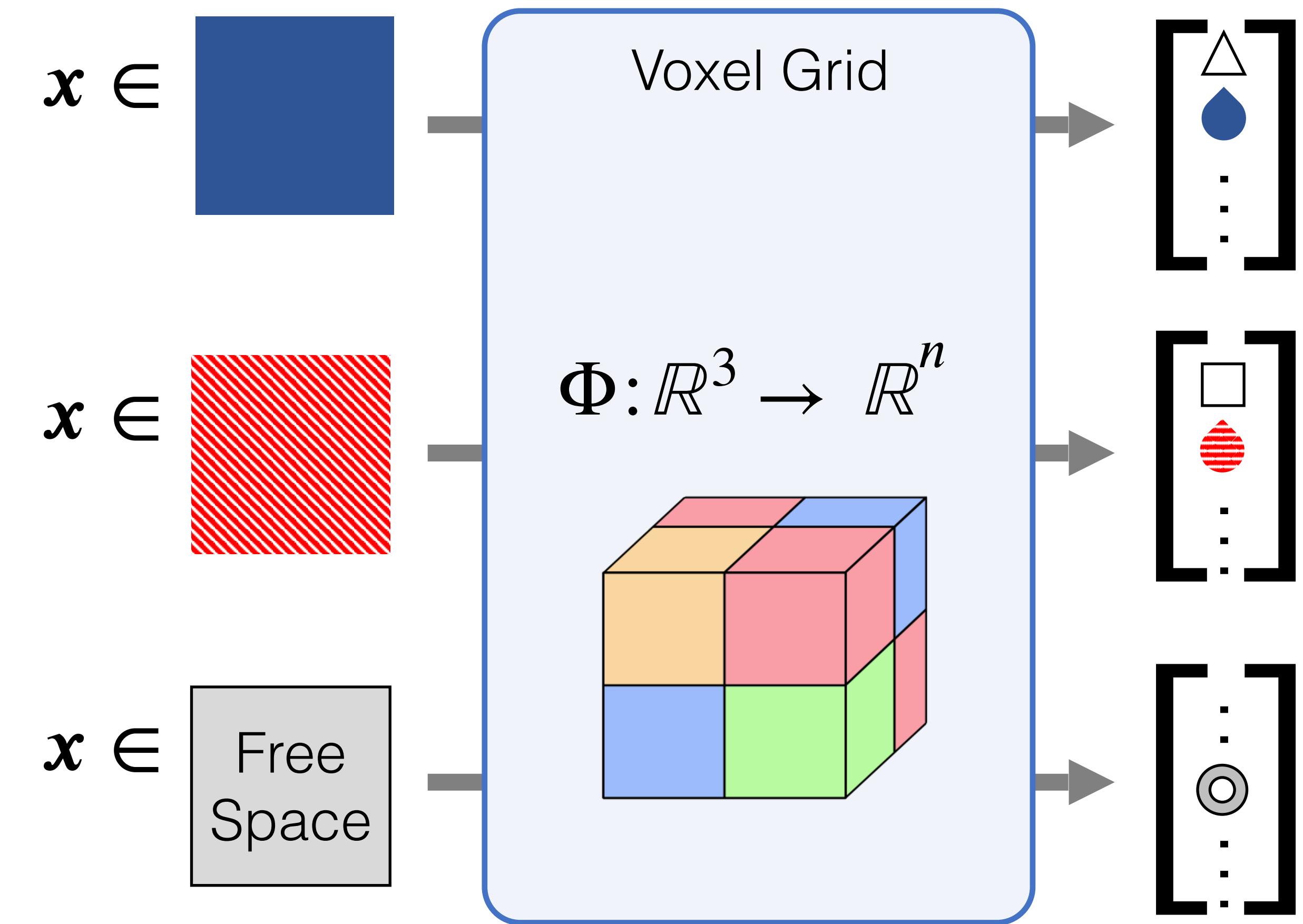
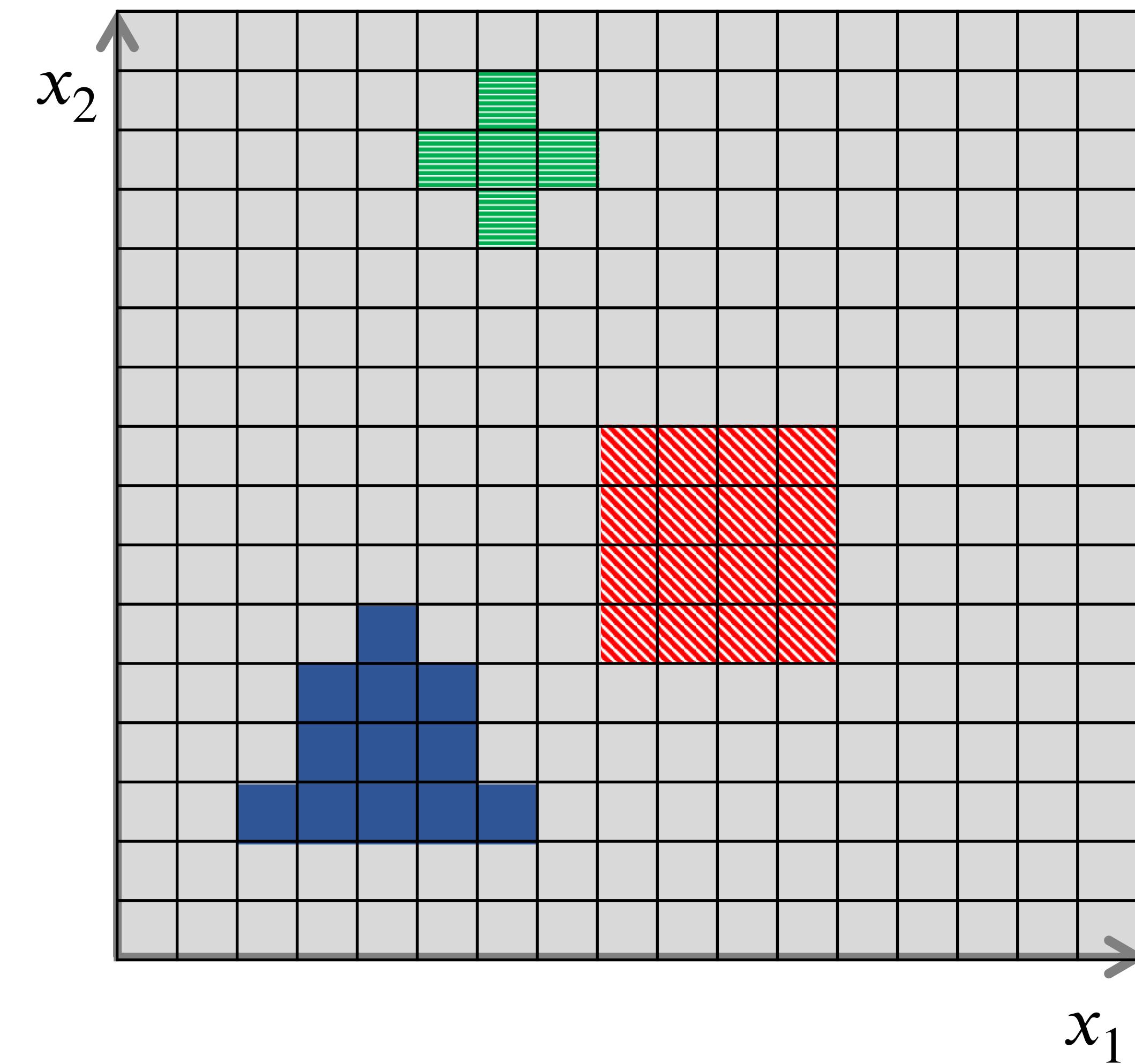


3D Scene

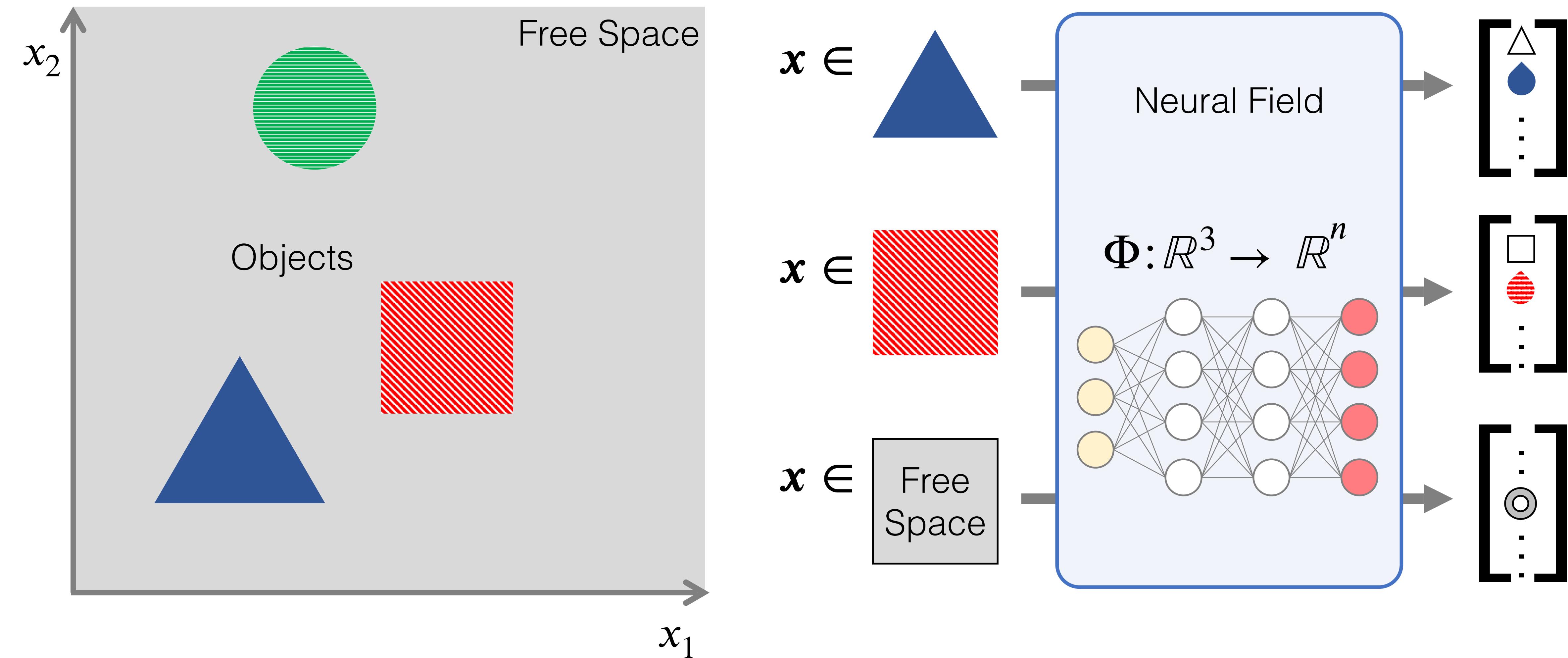
Starting Point: 3D Scene Representation



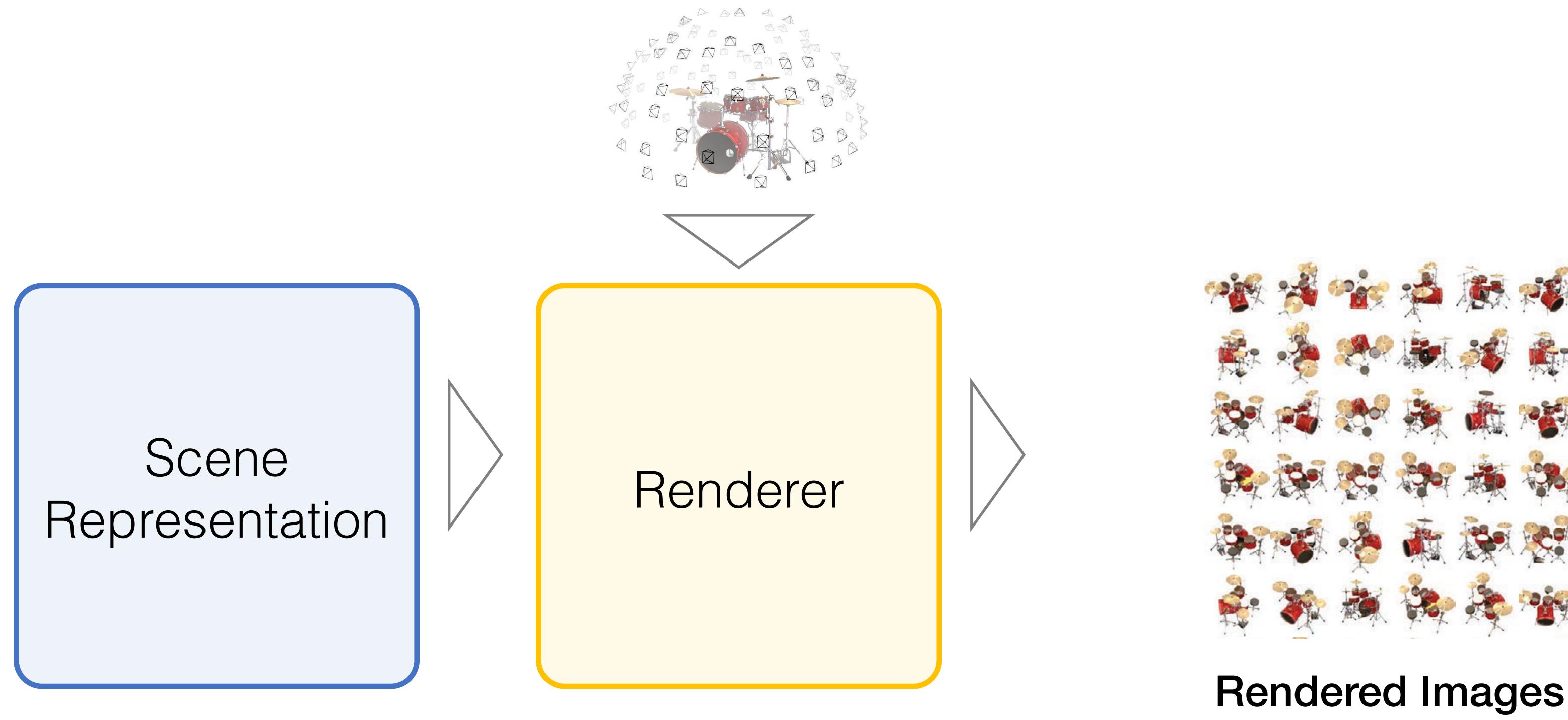
Starting Point: 3D Scene Representation



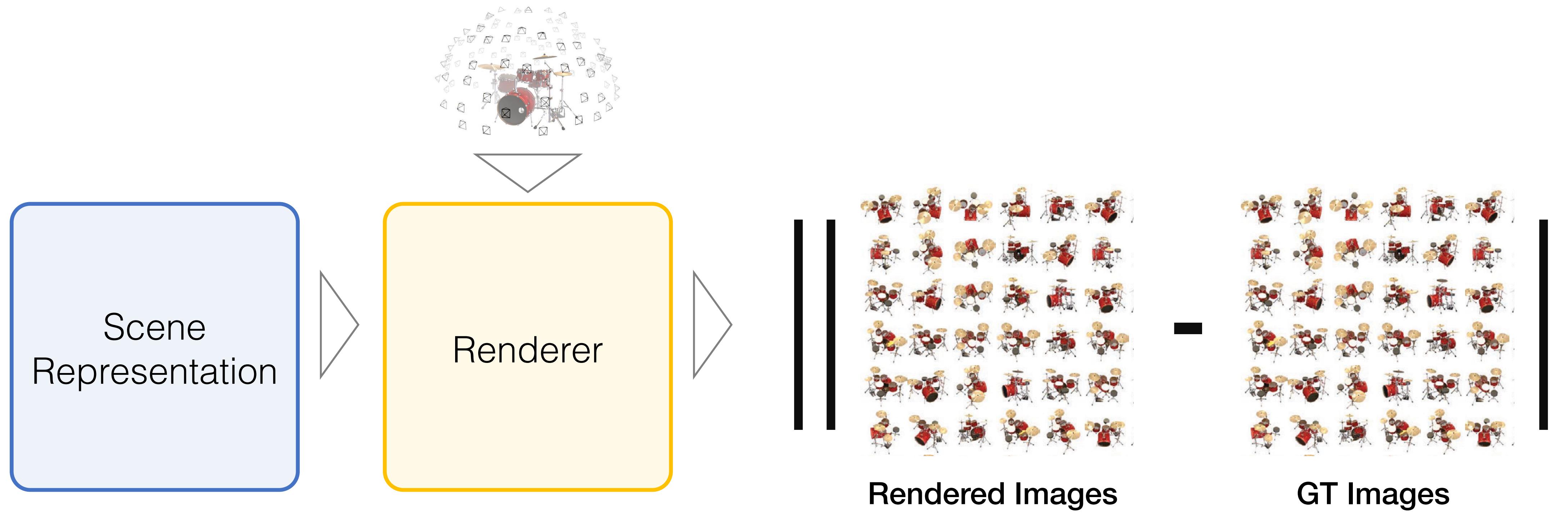
Starting Point: 3D Scene Representation



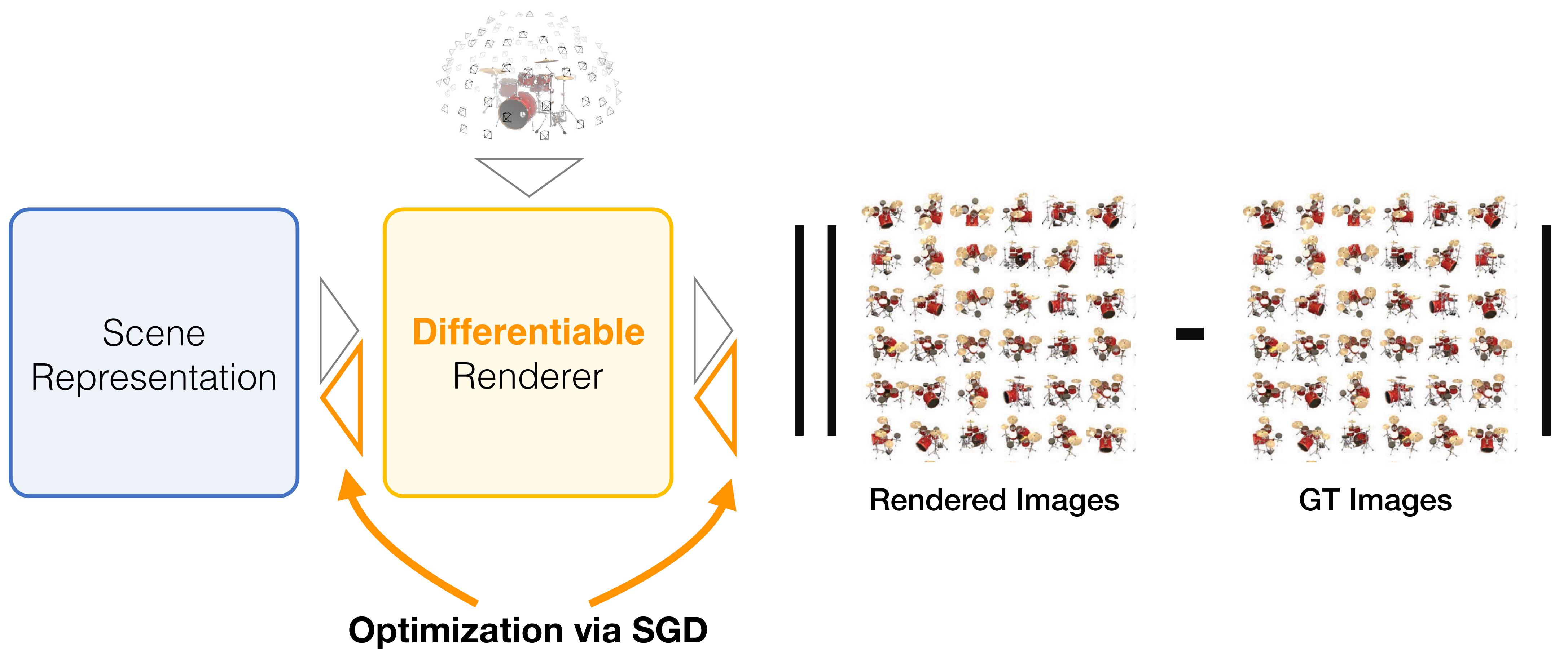
Differentiable Rendering



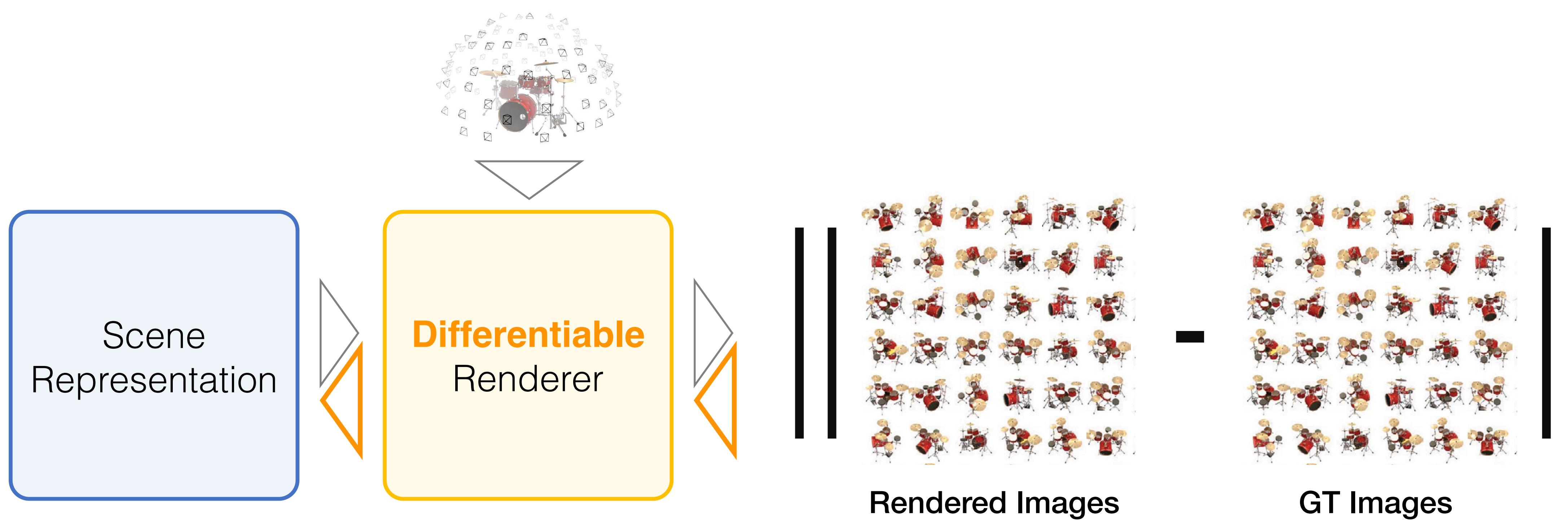
Differentiable Rendering



Differentiable Rendering



Differentiable Rendering

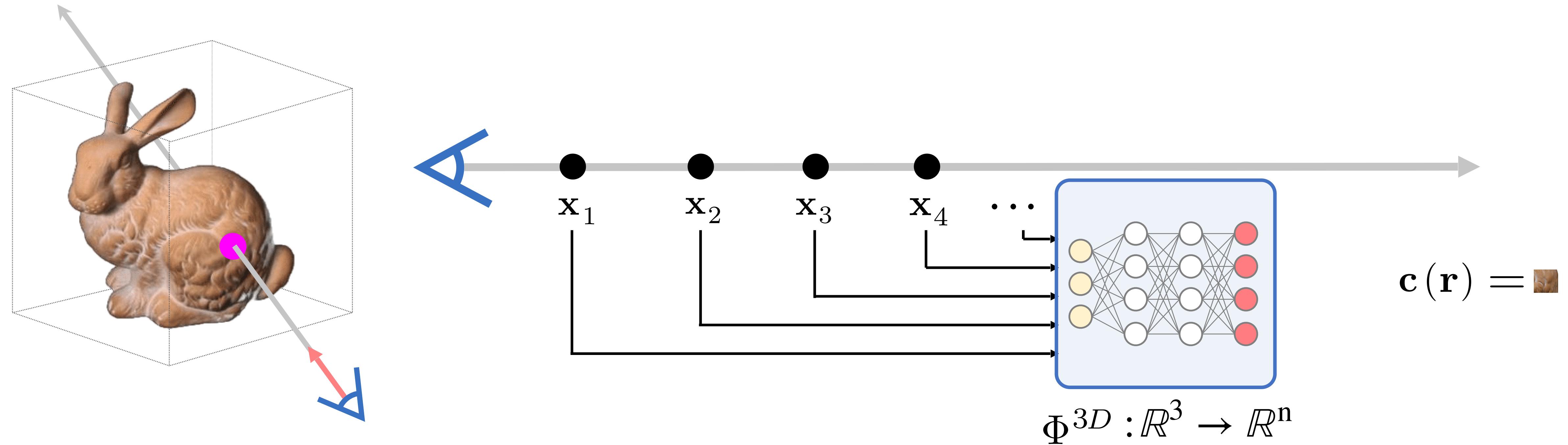


Given an *observable* variable (pixel colors), we will build a differentiable forward model that we then use to estimate *unobserved (latent) variables* (geometry, appearance)!

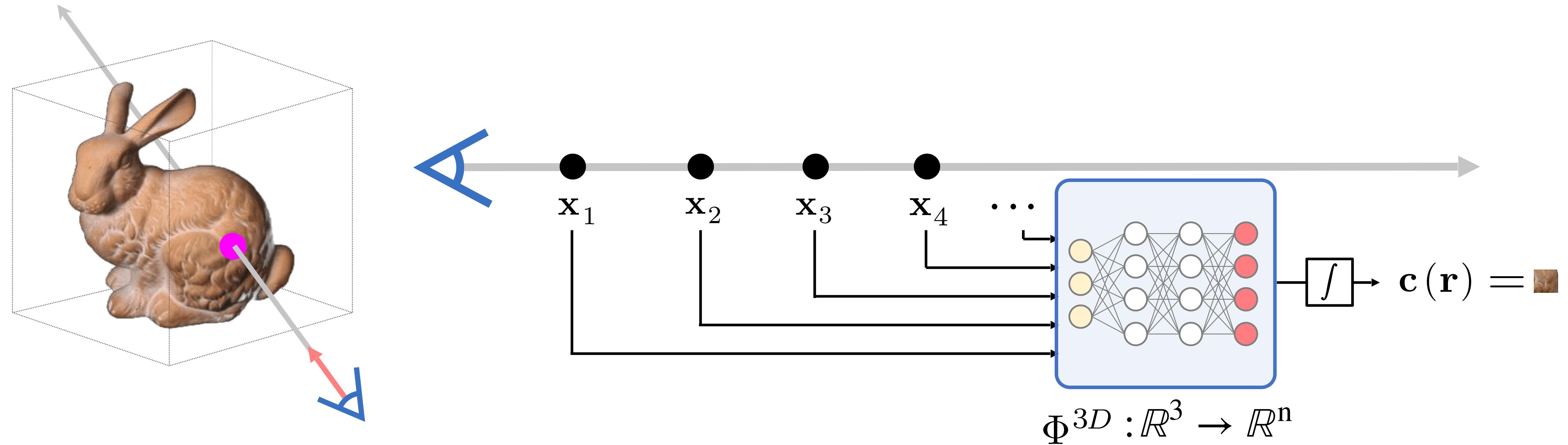
General structure of Neural Renderers for 3D Fields



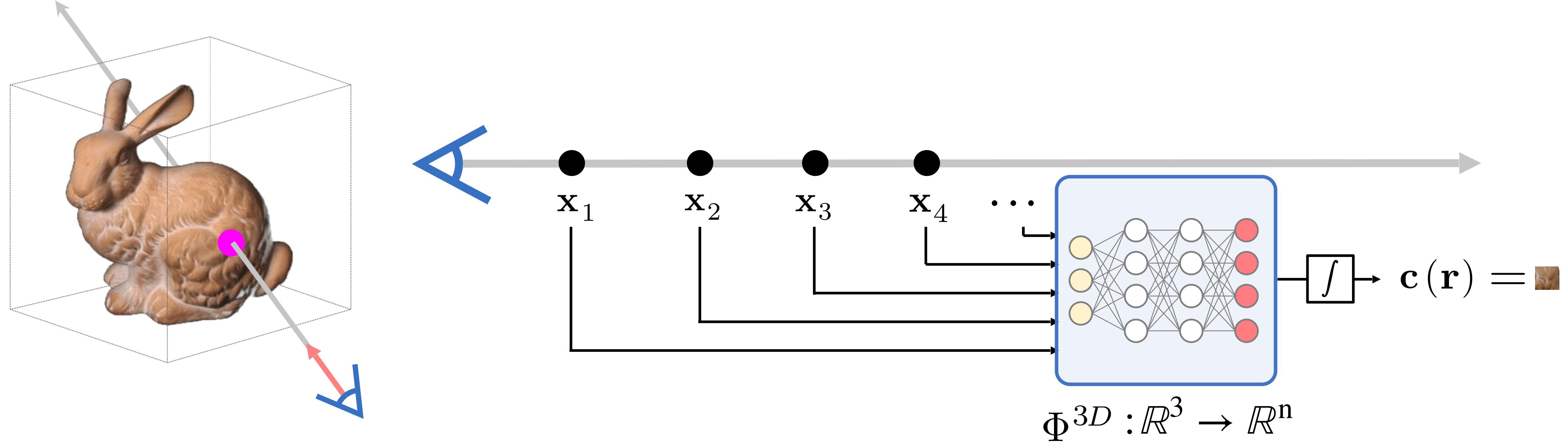
General structure of Neural Renderers for 3D Fields



General structure of Neural Renderers for 3D Fields



General structure of Neural Renderers for 3D Fields



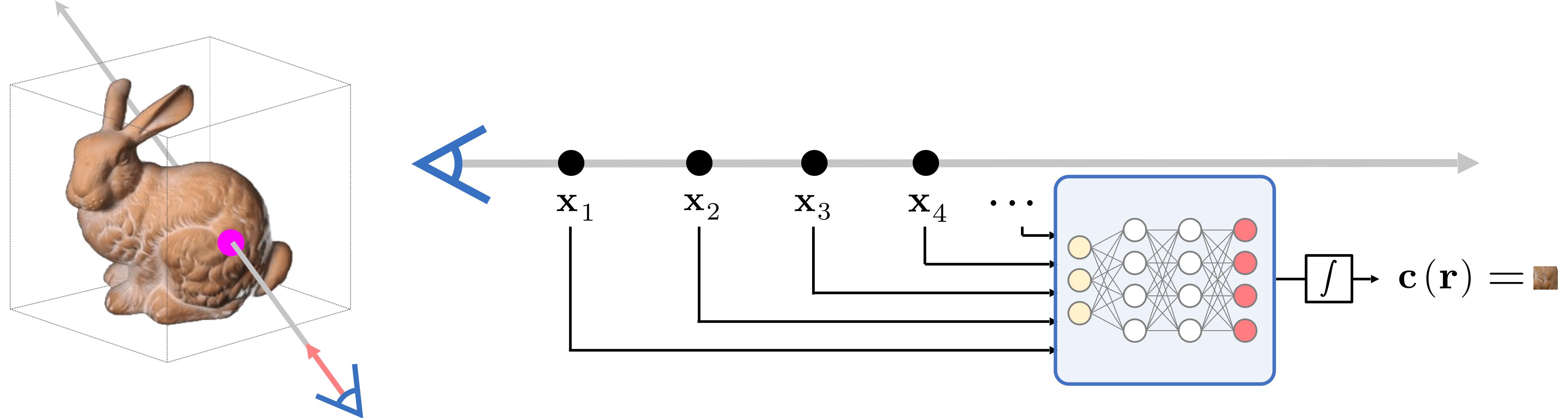
Sphere-Tracing
[JC Hart, 1996]

Volumetric

Hybrid implicit-volumetric

Learned aggregation

General structure of Neural Renderers for 3D Fields



Sphere-Tracing
[JC Hart, 1996]

Volumetric

Hybrid implicit-volumetric

Learned aggregation

Summary: volume rendering integral estimate

Rendering model for ray $\mathbf{r}(t) = \mathbf{o} + t\mathbf{d}$:

$$\mathbf{c} \approx \sum_{i=1}^n T_i \alpha_i \mathbf{c}_i$$

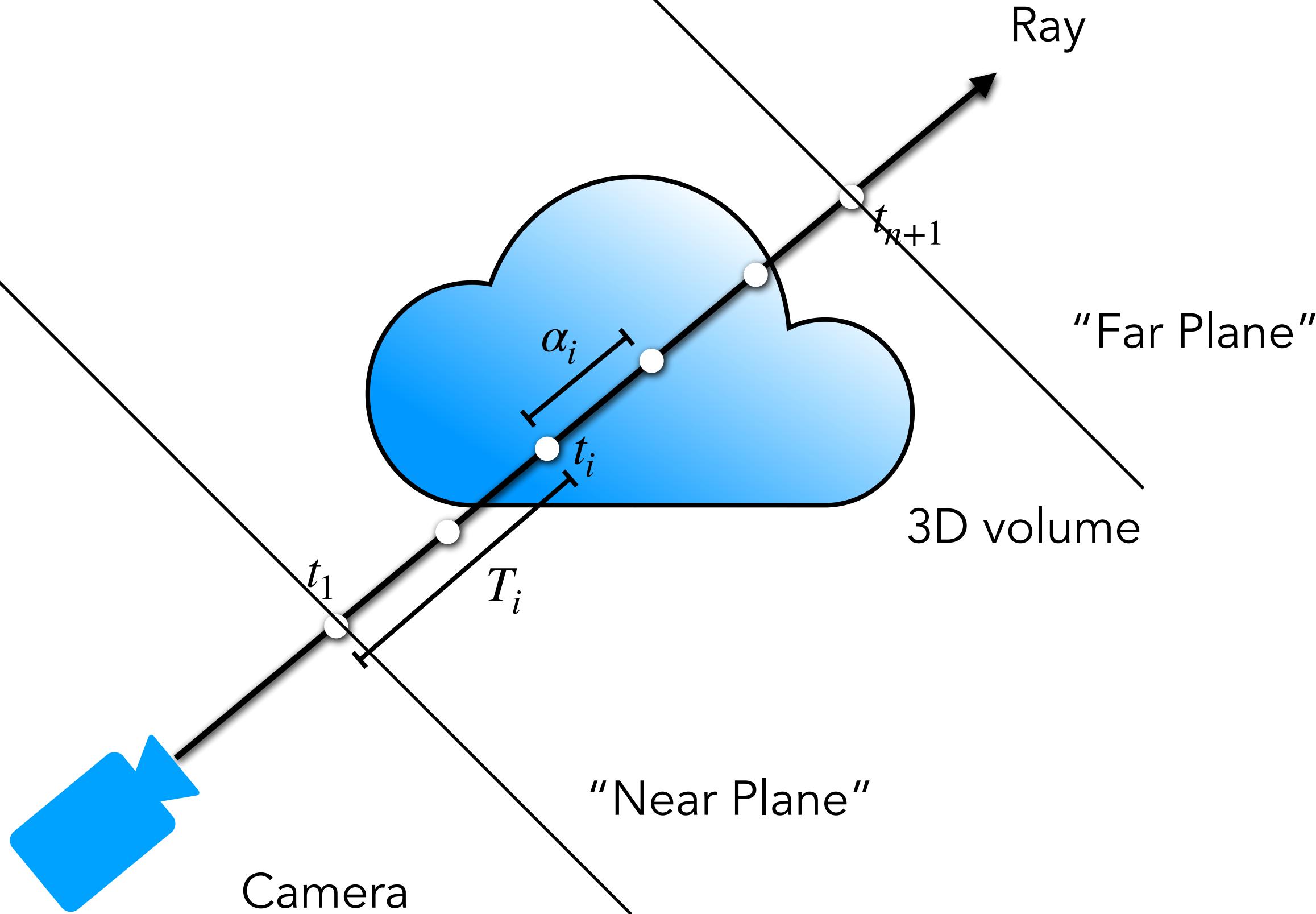
↑
weights colors

How much light is blocked earlier along ray:

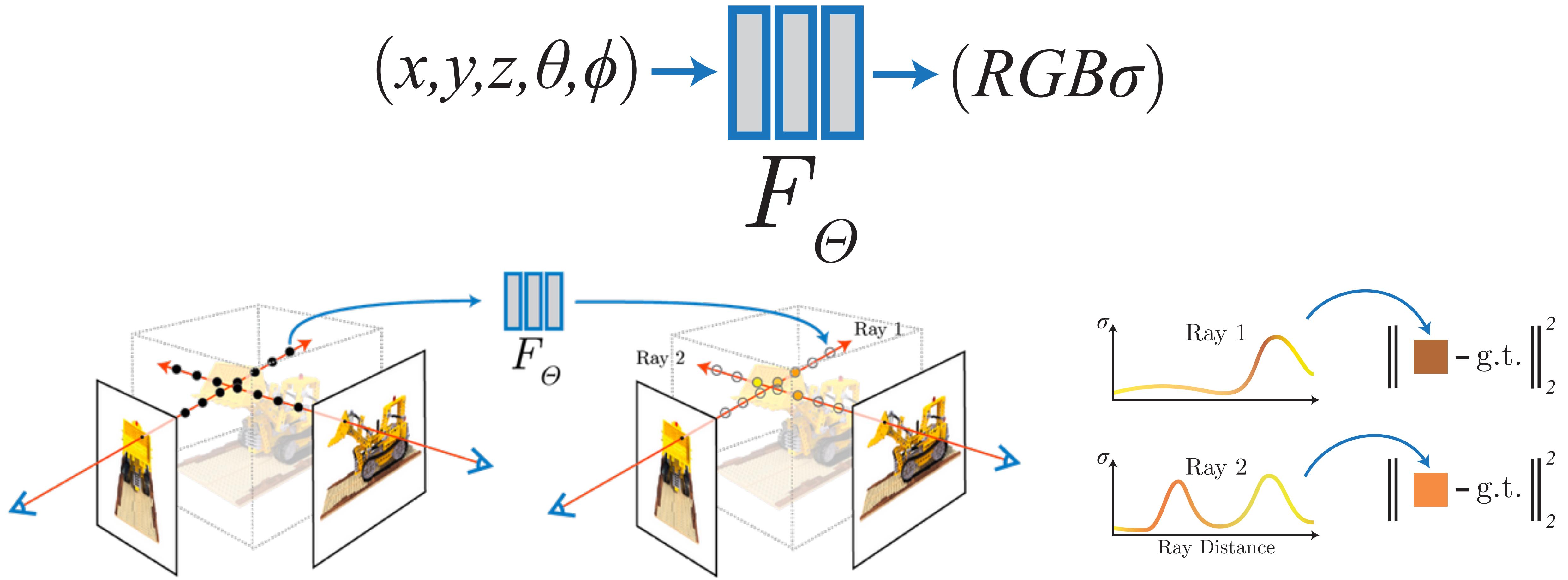
$$T_i = \prod_{j=1}^{i-1} (1 - \alpha_j)$$

How much light is contributed by ray segment i :

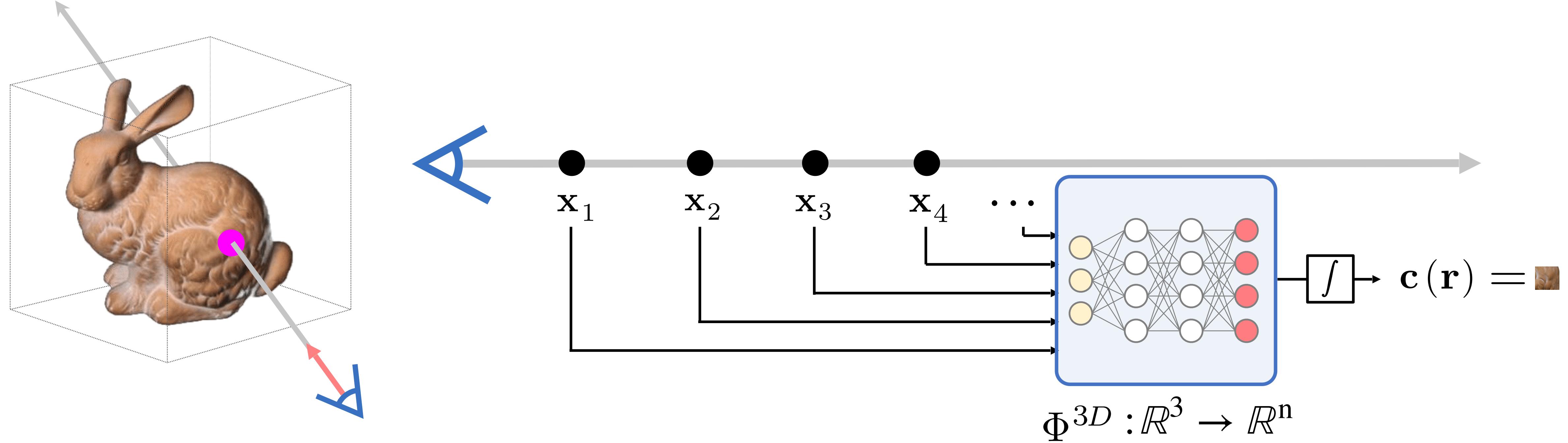
$$\alpha_i = 1 - \exp(-\sigma_i \delta_i)$$



Neural Radiance Field: Parameterize Radiance Field via neural network



General structure of Neural Renderers for 3D Fields

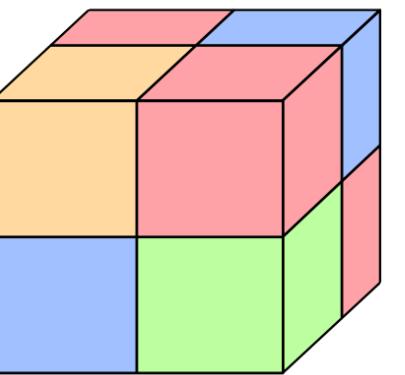


Sphere-Tracing
[JC Hart, 1996]

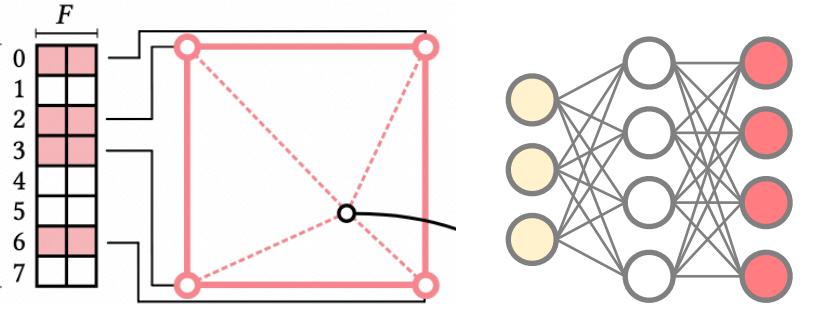
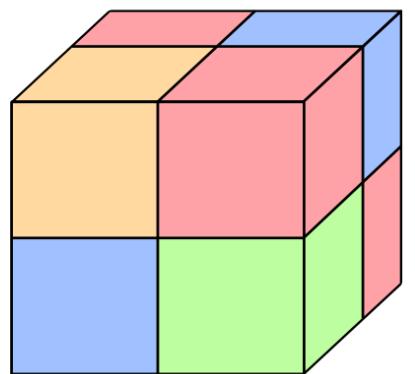
Volumetric

Hybrid implicit-volumetric

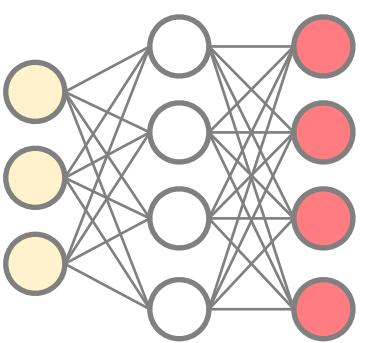
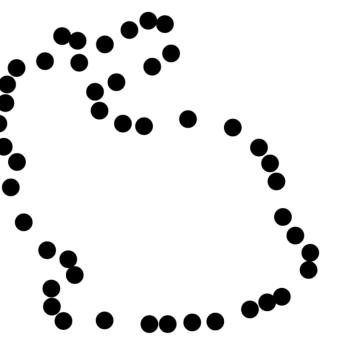
Learned aggregation



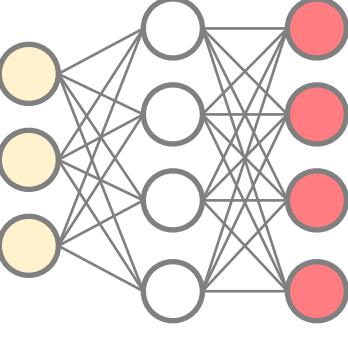
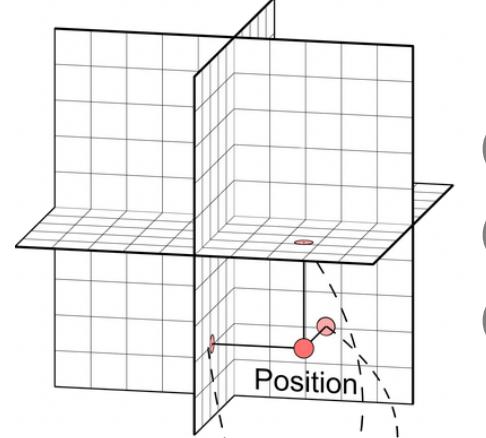
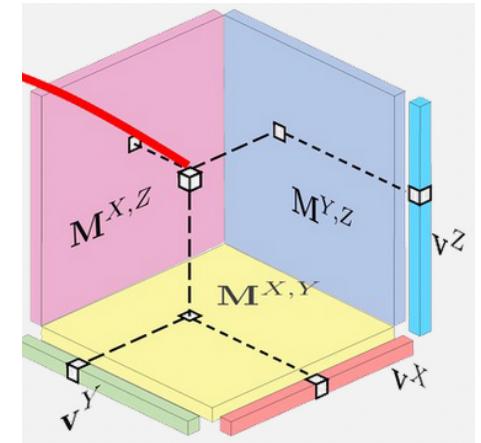
Voxel Grid



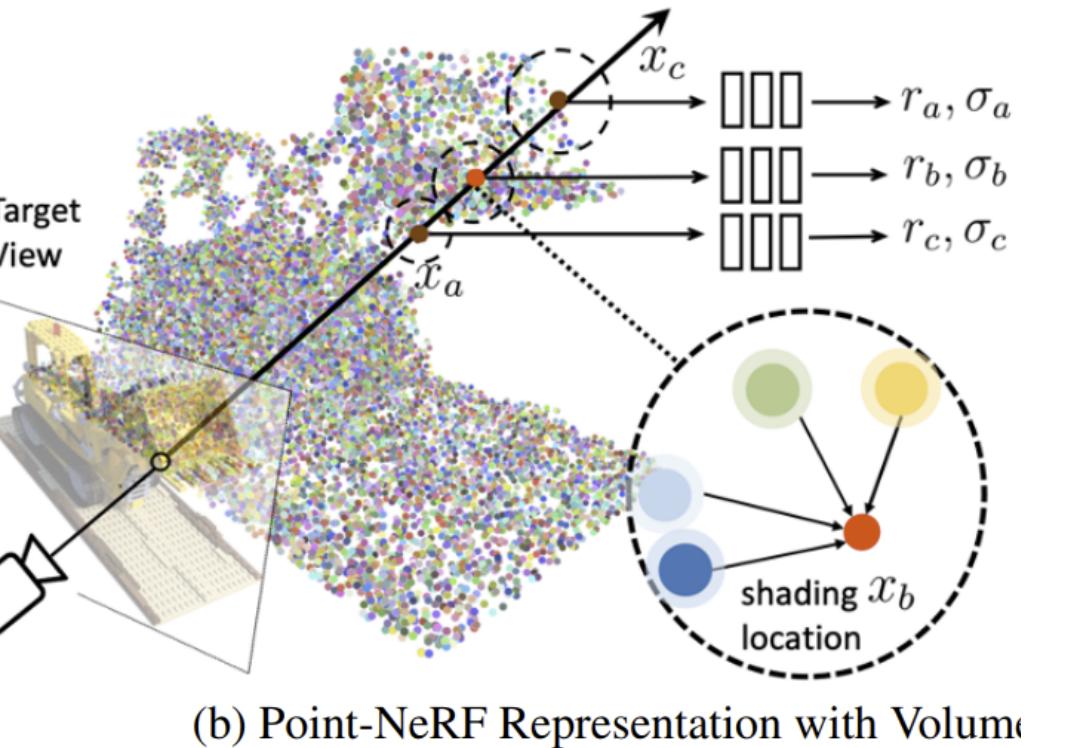
Voxel Grid + Hashmap + MLP



Point Cloud + MLP



Tensor Factorization / Triplane + MLP



(b) Point-NeRF Representation with Volume



Plenoxels: Radiance Fields ...

[Yu et al. 2022]

Direct Voxel Grid Optimization

[Sun et al. 2021]

InstantNGP: Instant Neural ...

[Müller et al. 2022]

PointNeRF: Point-based Neural ...

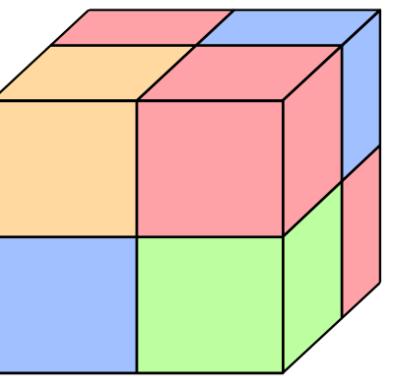
[Xu et al. 2022]

Efficient Geometry-aware 3D...

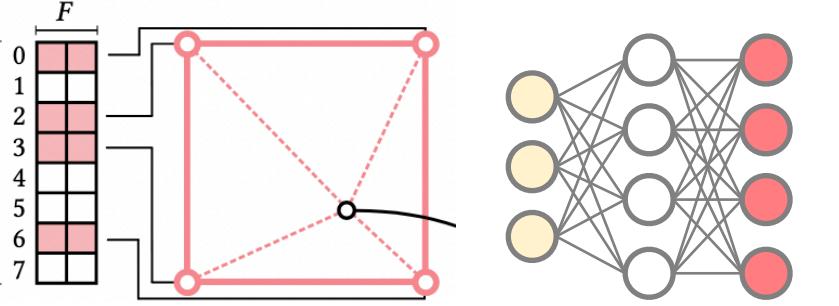
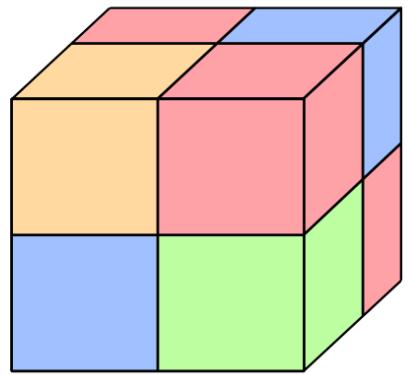
[Chan et al. 2022]

TensorRF: Tensor Radiance Fields

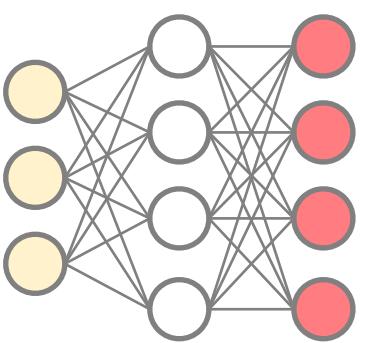
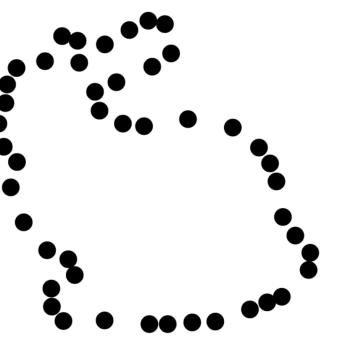
[Chen & Xu et al. 2022]



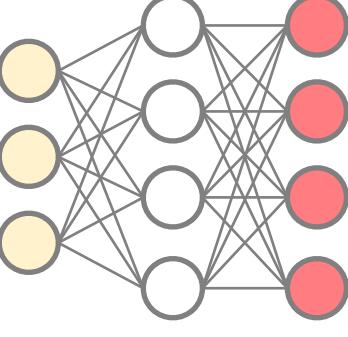
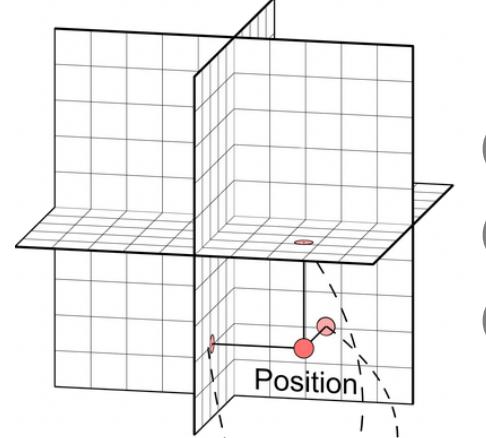
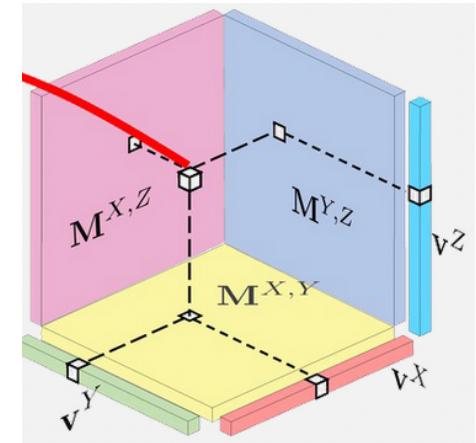
Voxel Grid



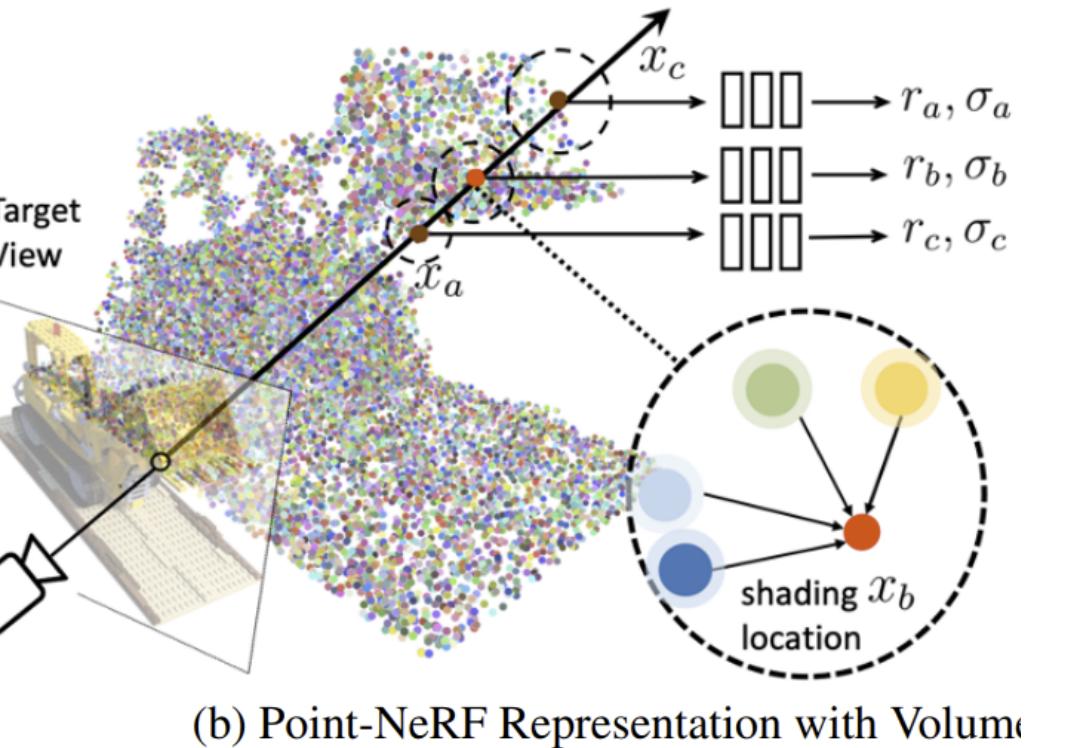
Voxel Grid + Hashmap + MLP



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[Chan et al. 2022]

TensorRF: Tensor Radiance Fields

[Chen & Xu et al. 2022]

Hybrid Multi-Scale Grid,
HashMap, Neural Field

Instant-NGP

9.0

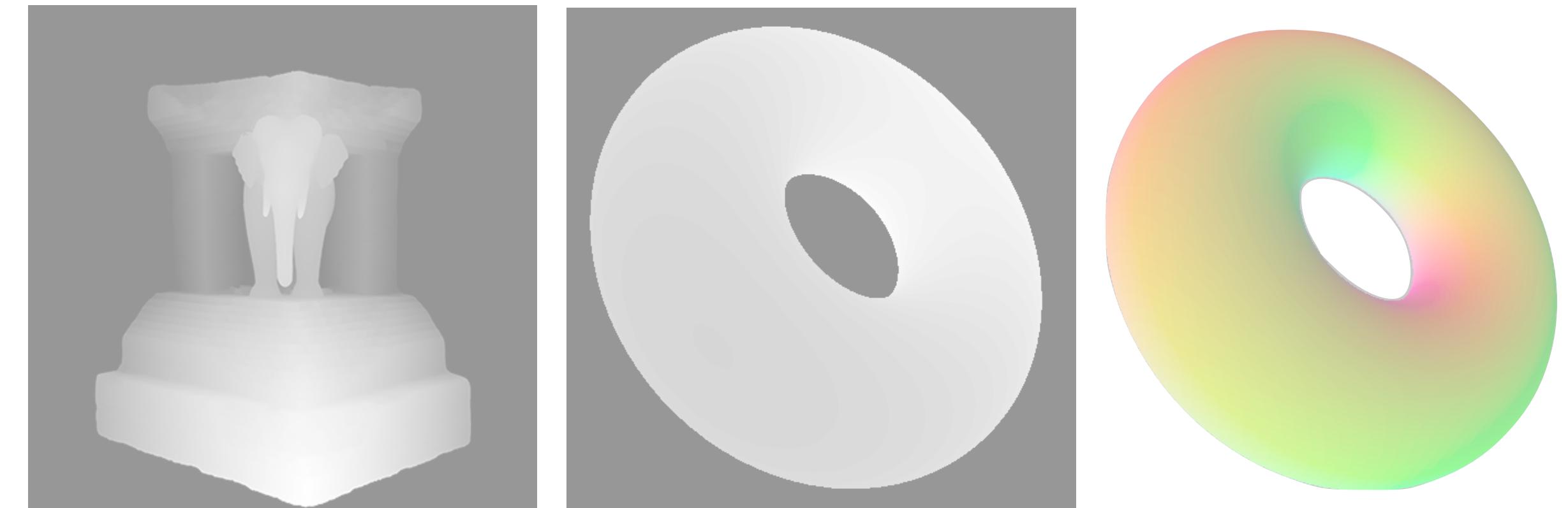
0.07

MipNeRF360

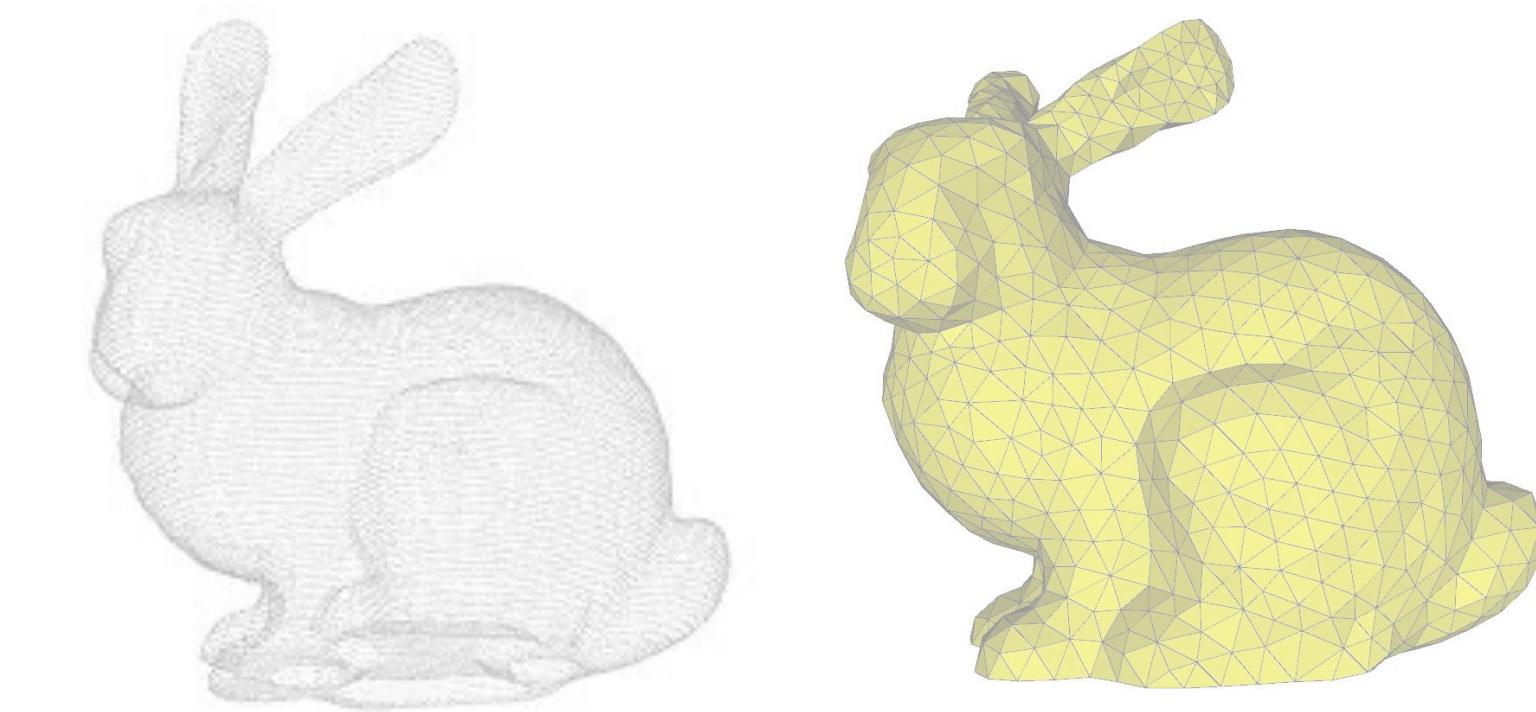
FPS

Neural Field

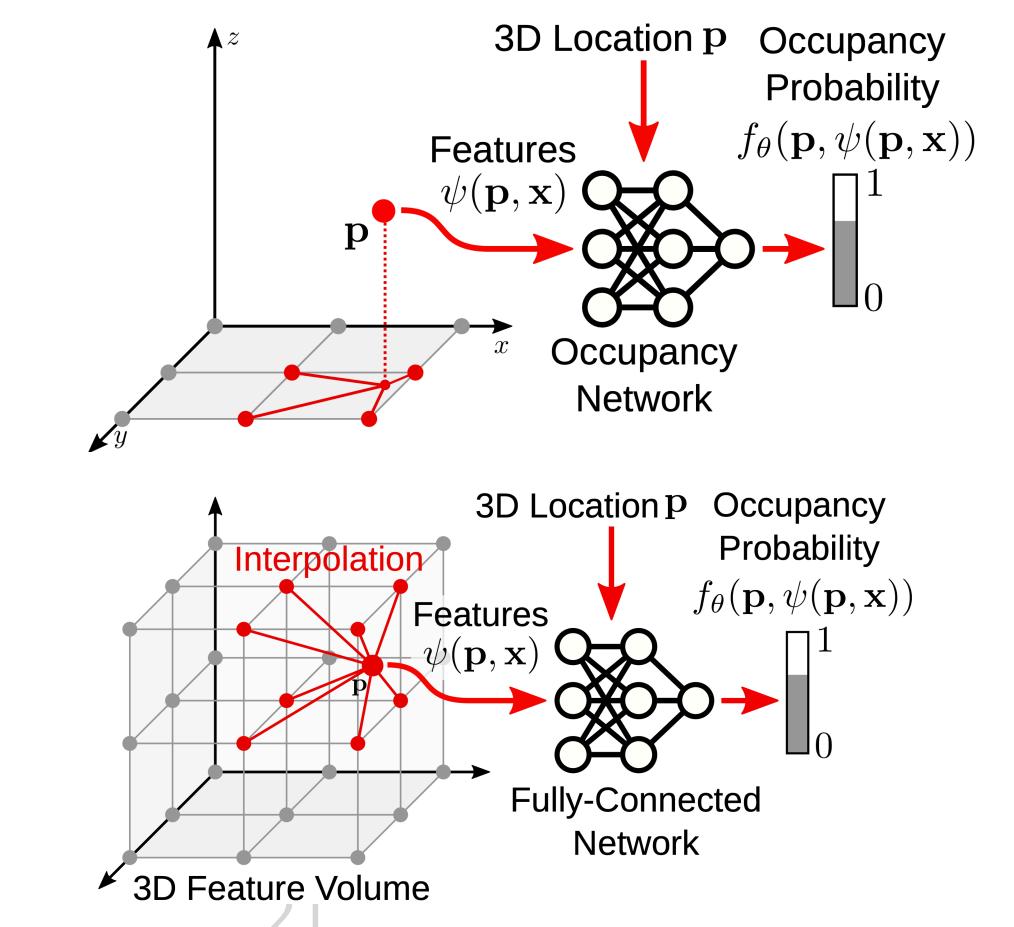
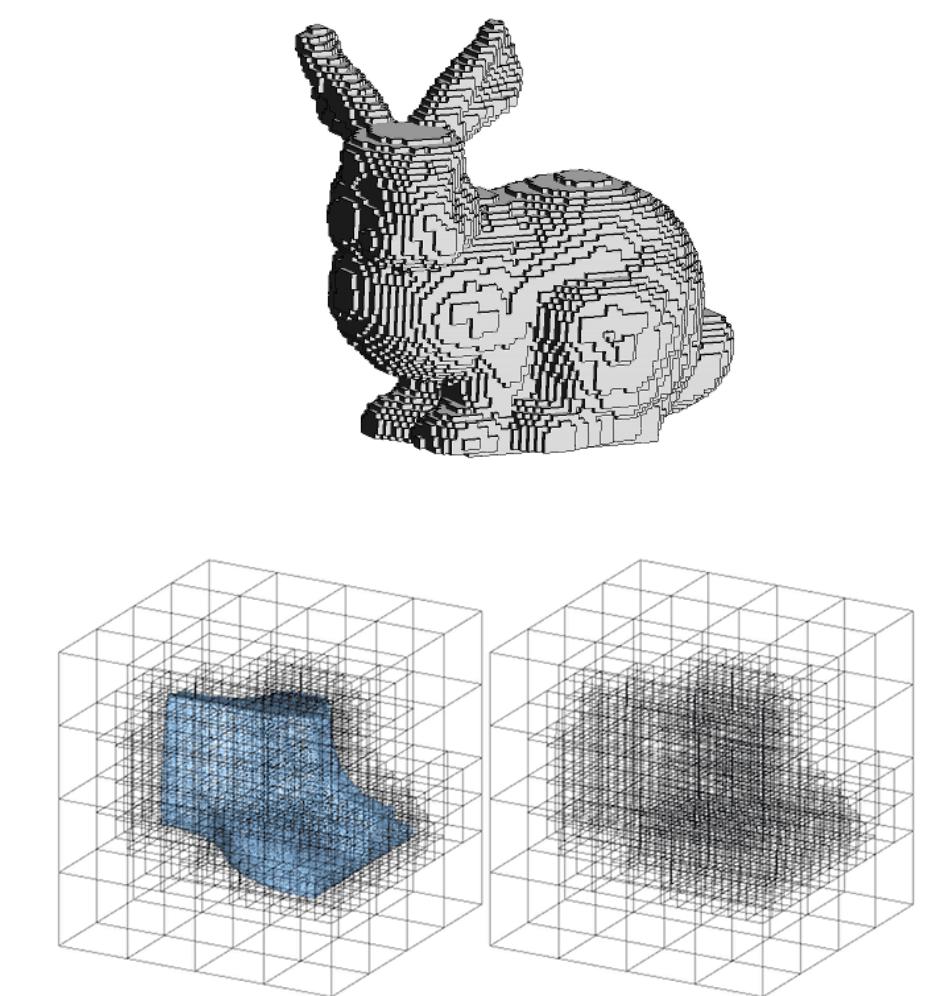
2.5D Representations



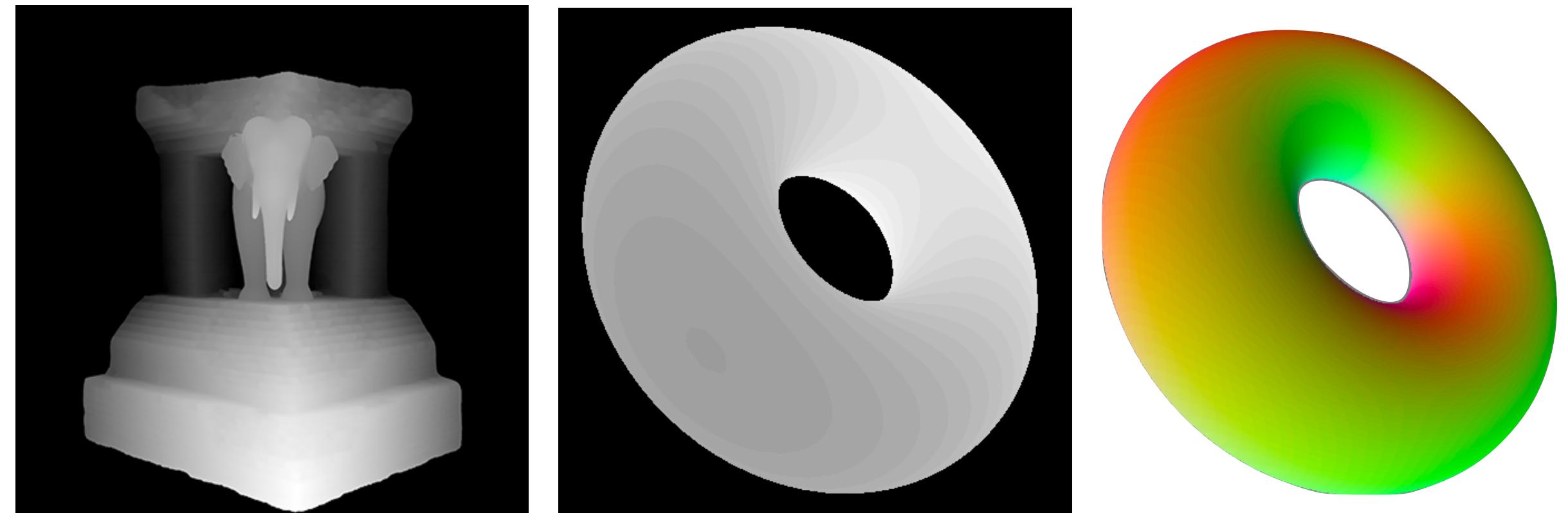
Surface Representations



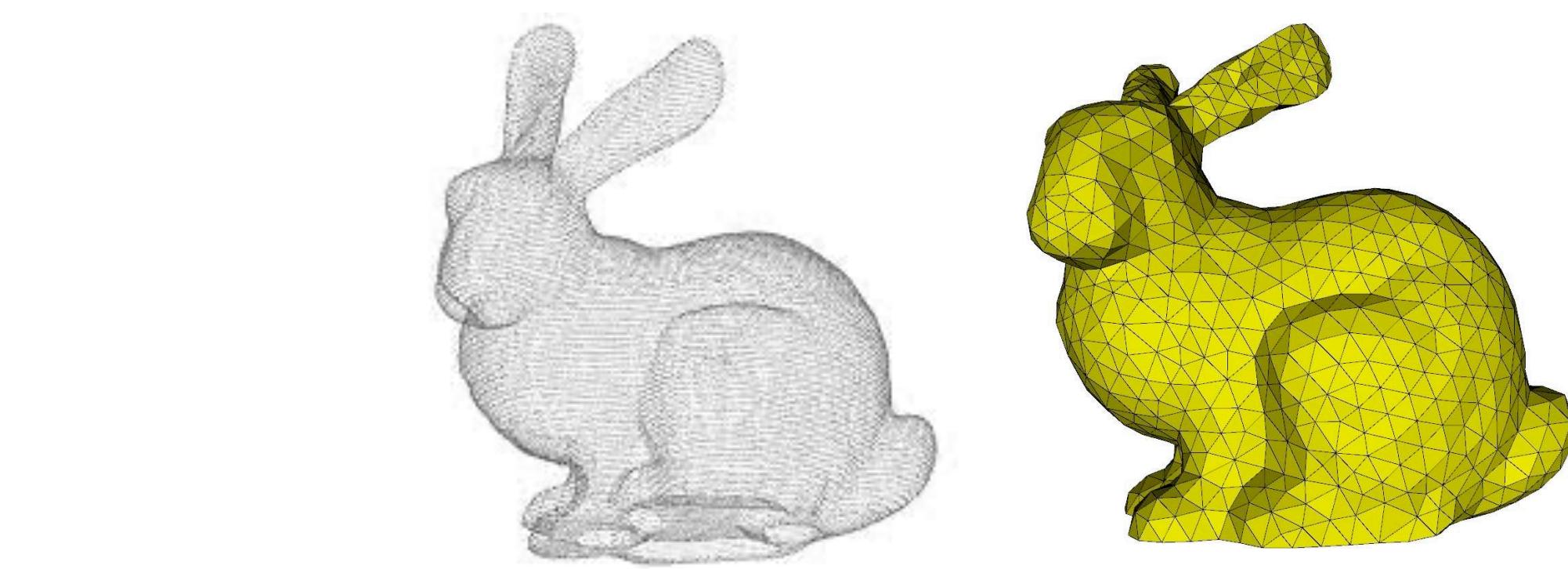
Field Parameterizations



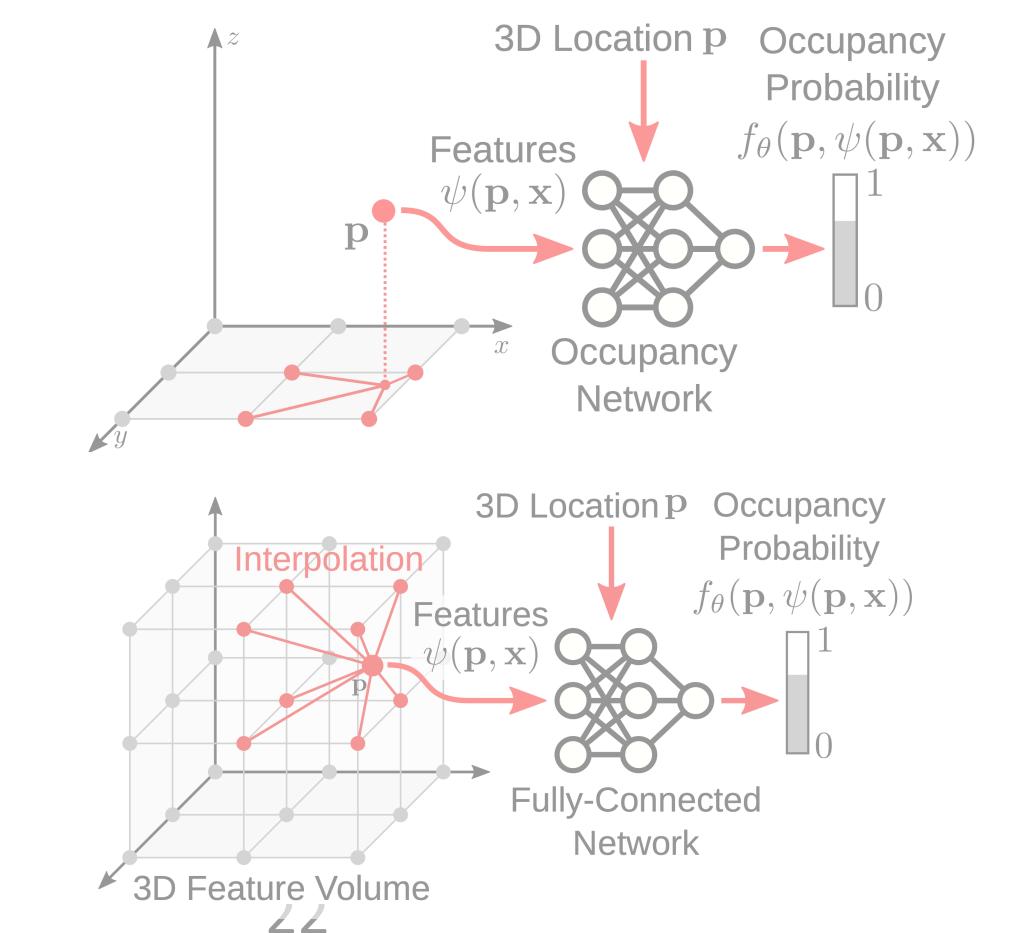
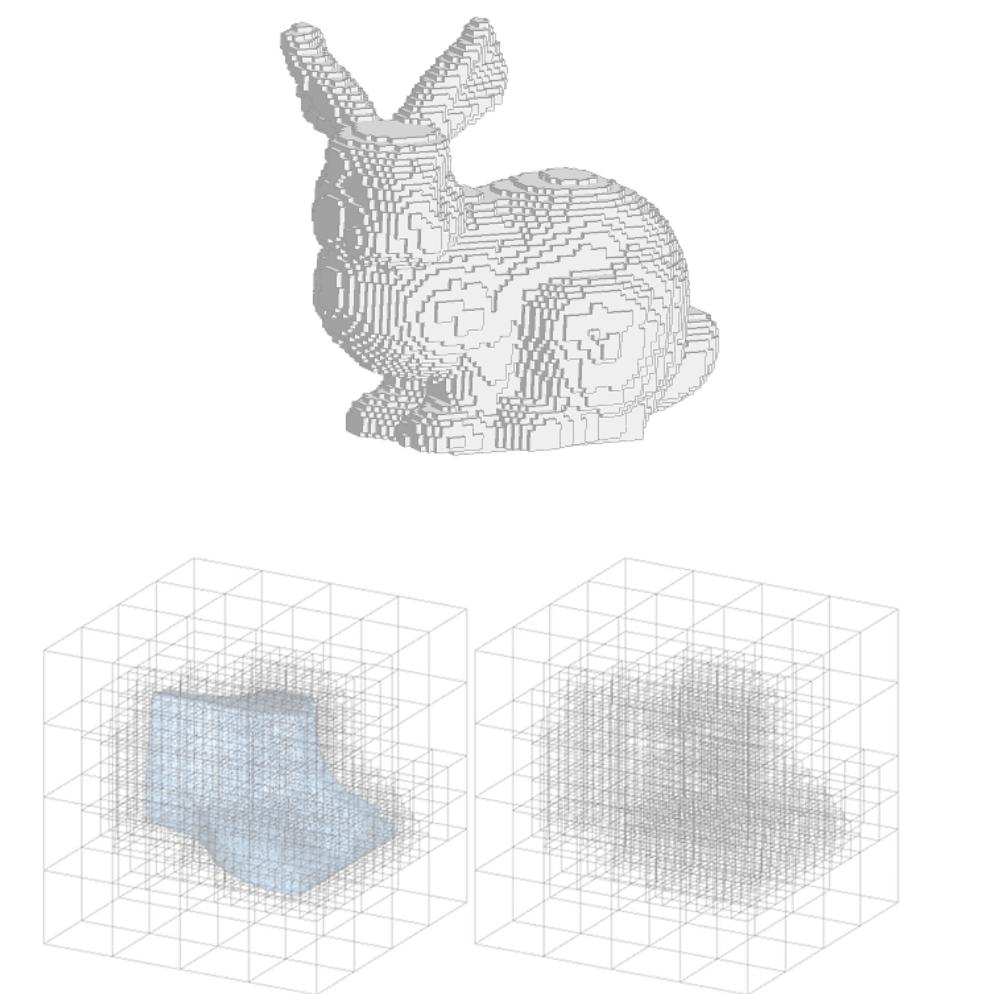
2.5D Representations



Surface Representations



Field Parameterizations



3D Gaussian Splatting for Real-Time Radiance Field Rendering

BERNHARD KERBL*, Inria, Université Côte d’Azur, France

GEORGIOS KOPANAS*, Inria, Université Côte d’Azur, France

THOMAS LEIMKÜHLER, Max-Planck-Institut für Informatik, Germany

GEORGE DRETTAKIS, Inria, Université Côte d’Azur, France

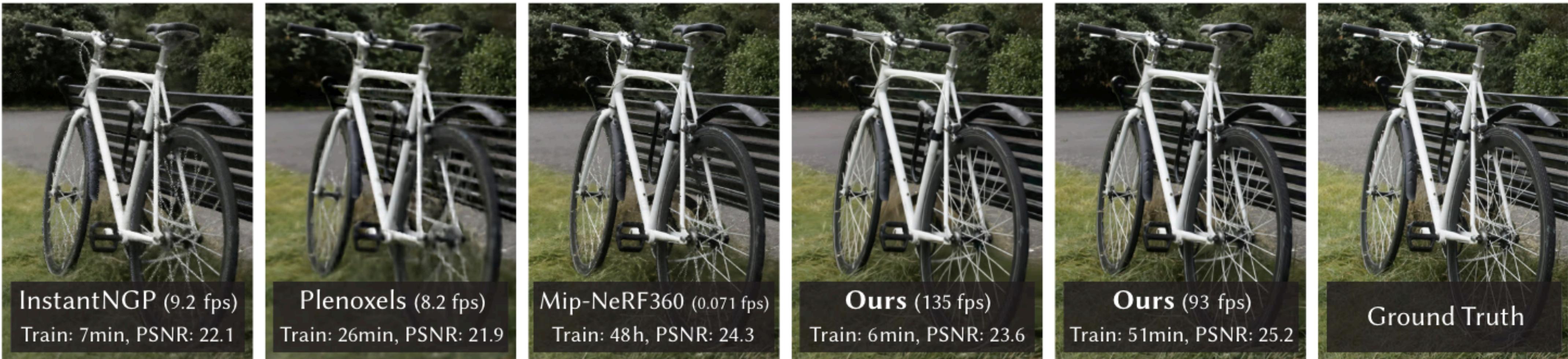
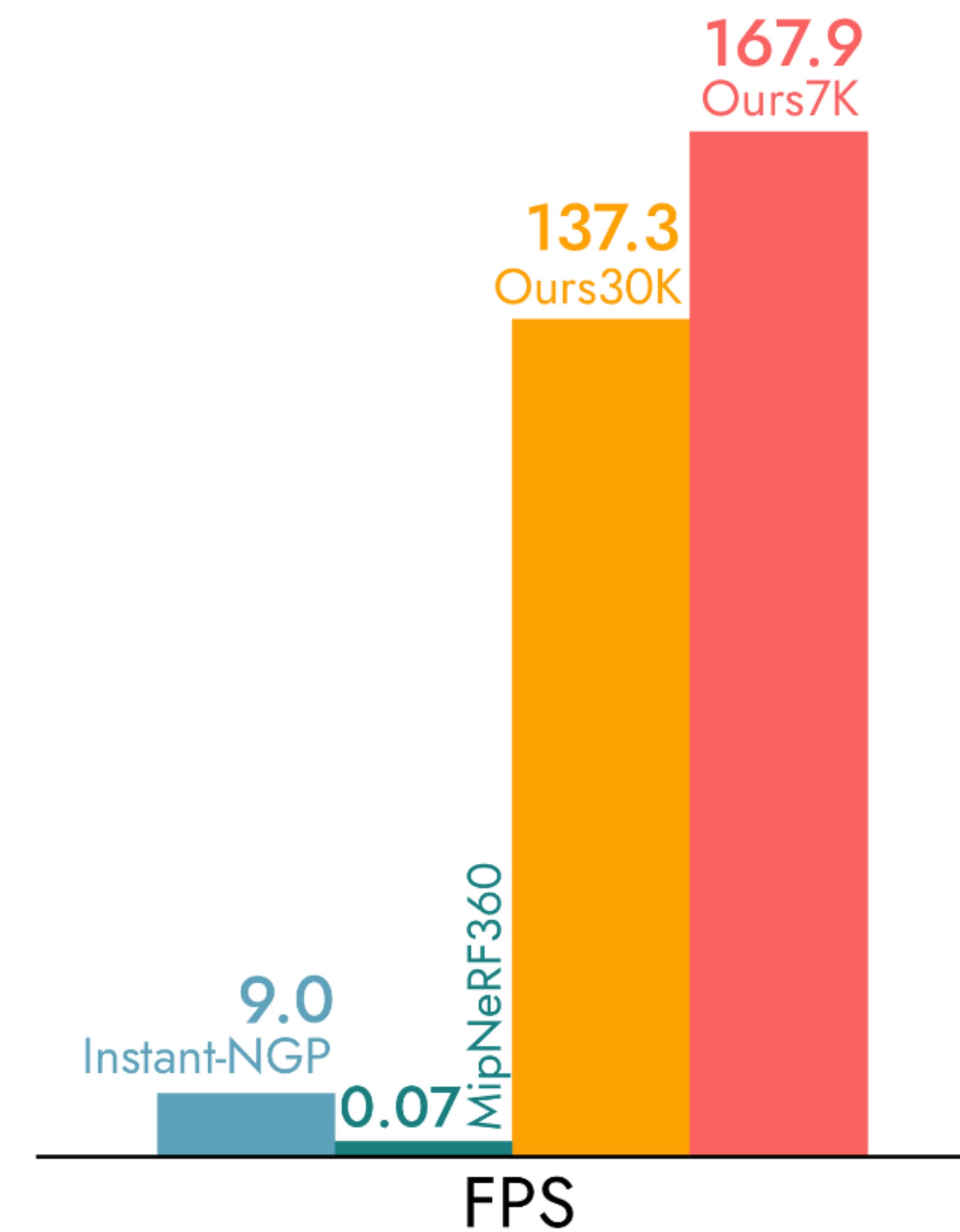
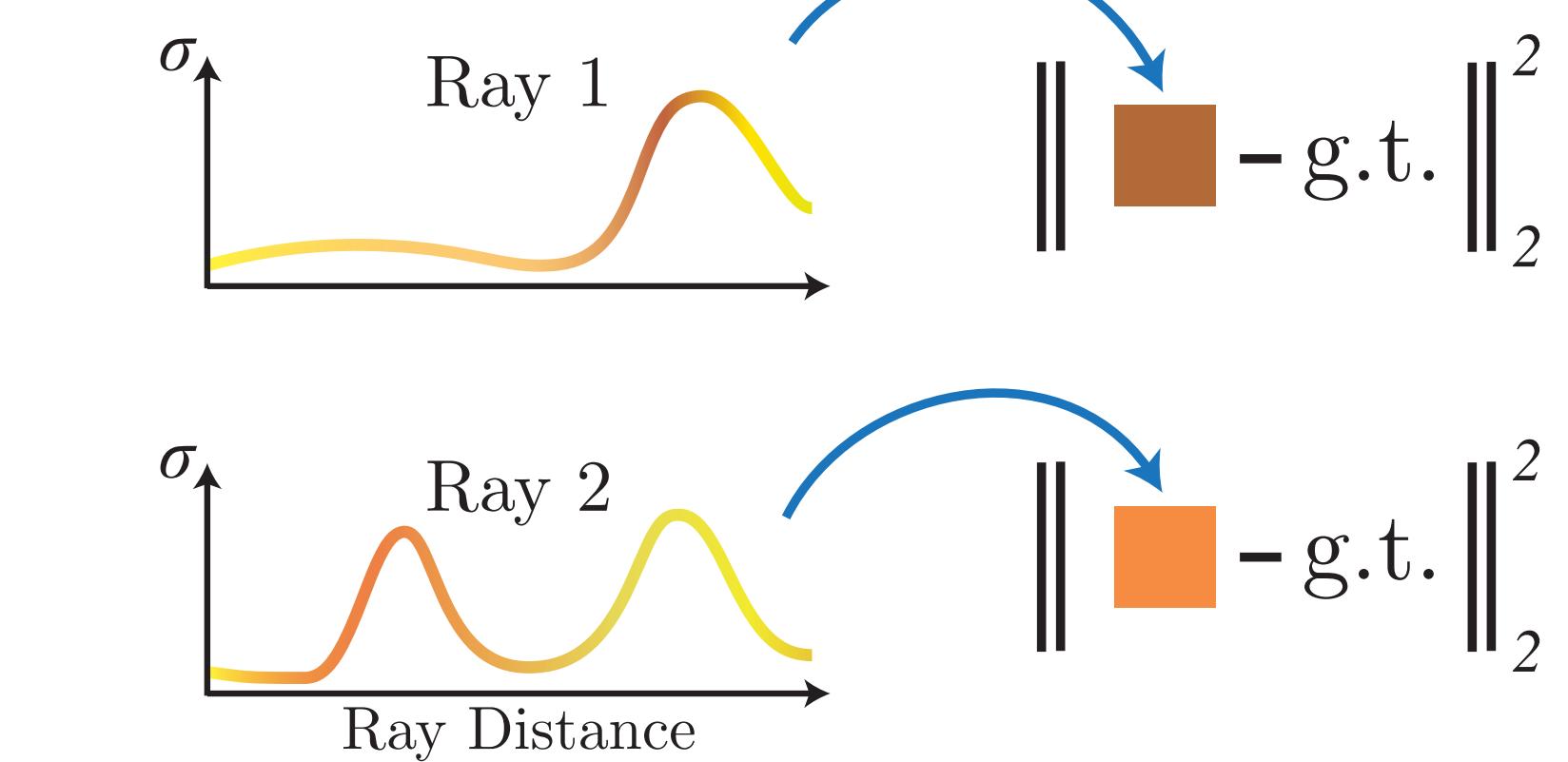
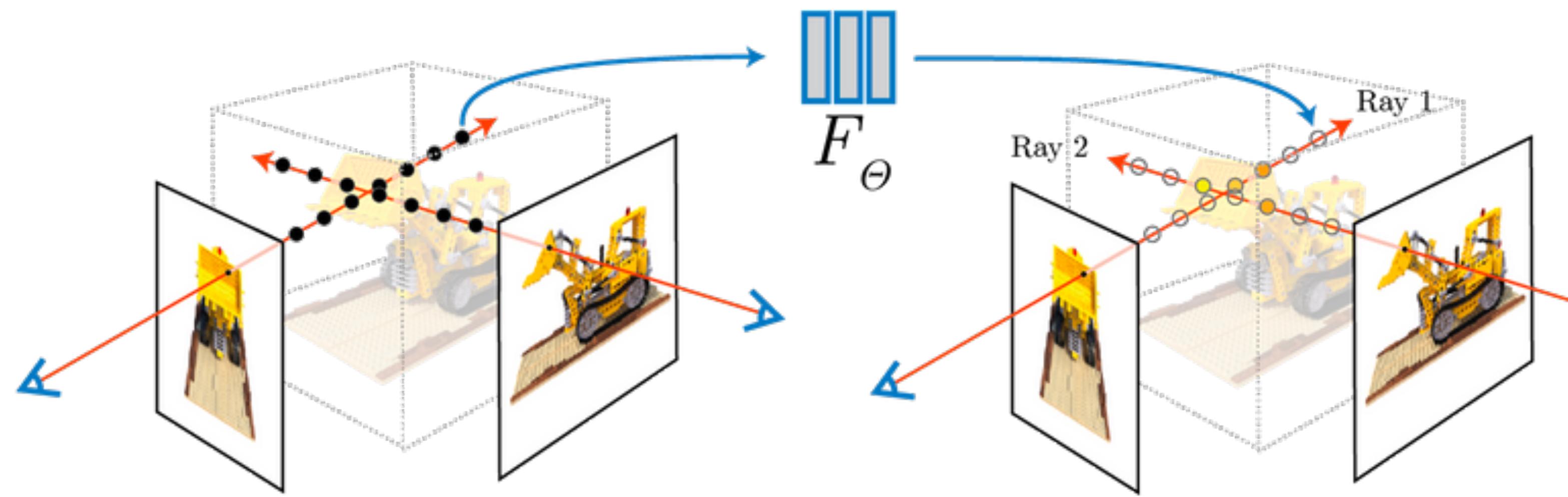
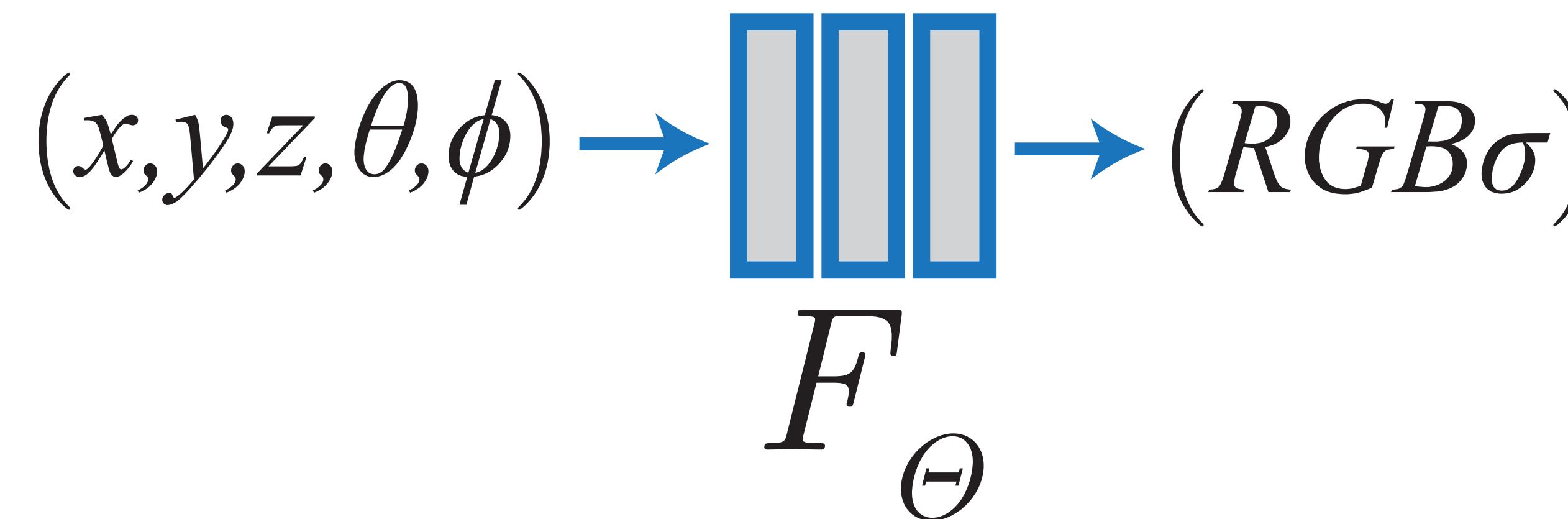


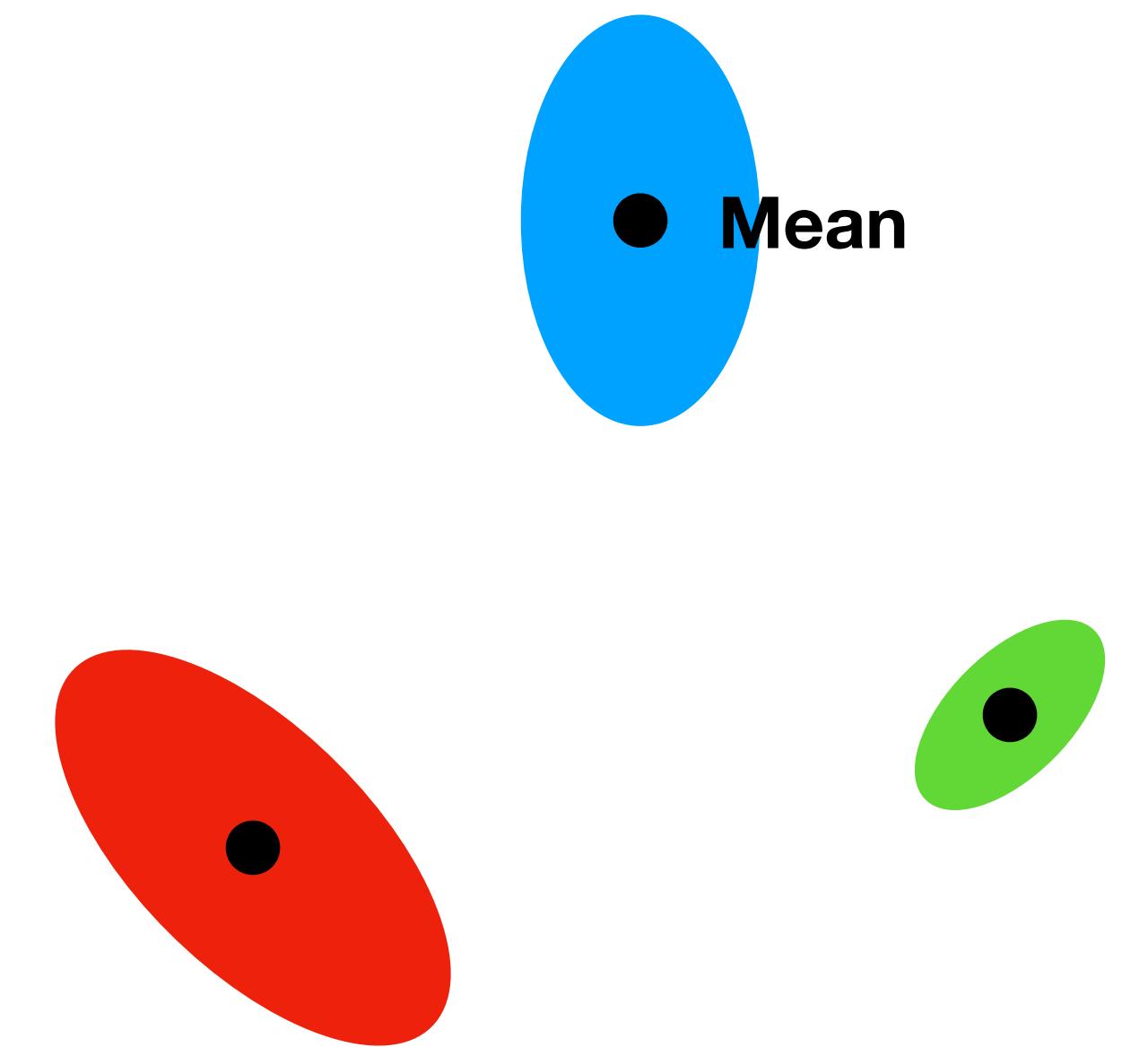
Fig. 1. Our method achieves real-time rendering of radiance fields with quality that equals the previous method with the best quality [Barron et al. 2022], while only requiring optimization times competitive with the fastest previous methods [Fridovich-Keil and Yu et al. 2022; Müller et al. 2022]. Key to this performance is a novel 3D Gaussian scene representation coupled with a real-time differentiable renderer, which offers significant speedup to both scene optimization and novel view synthesis. Note that for comparable training times to InstantNGP [Müller et al. 2022], we achieve similar quality to theirs; while this is the maximum quality they reach, by training for 51min we achieve state-of-the-art quality, even slightly better than Mip-NeRF360 [Barron et al. 2022].



Neural Radiance Field: Parameterize Radiance Field densely, at *every* point in space

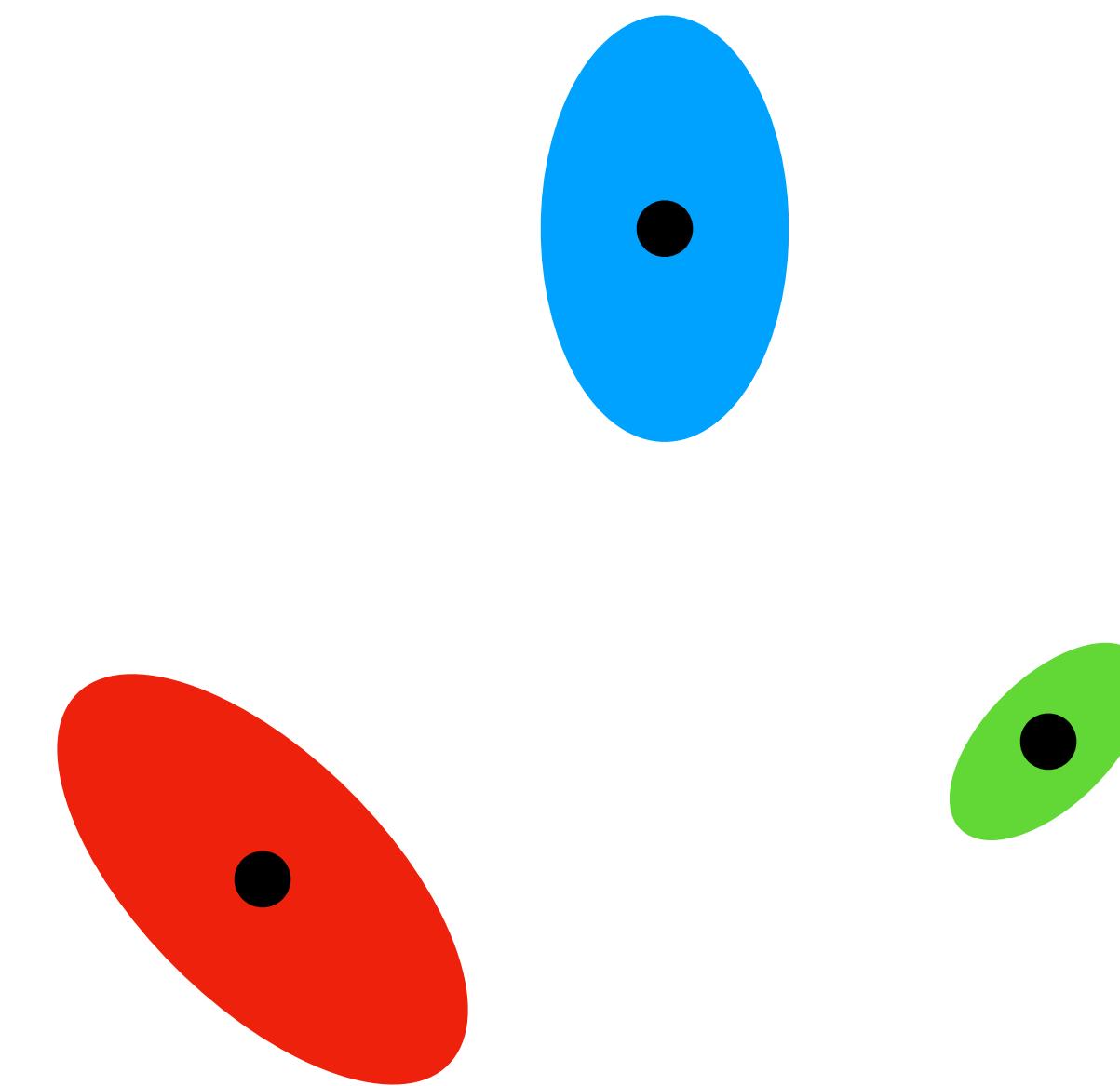


**Key Idea: Parameterize Radiance Field *sparsely*,
only where density is nonzero**



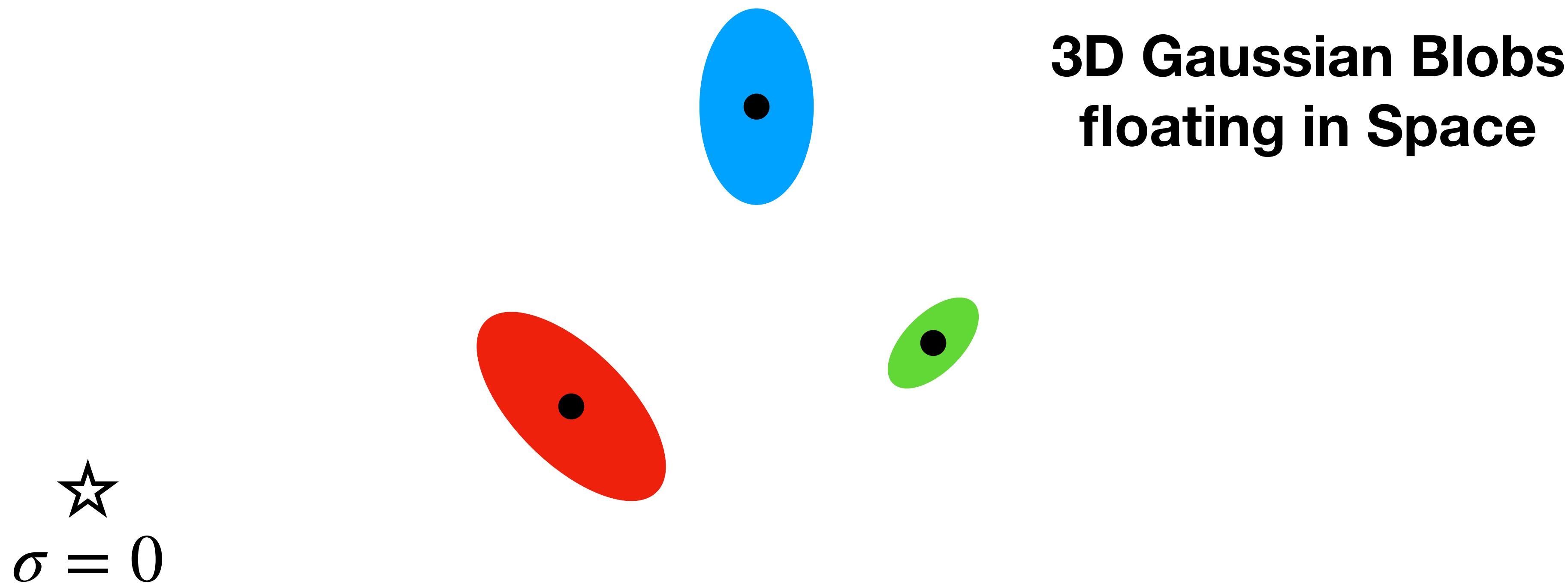
**3D Gaussian Blobs
floating in Space**

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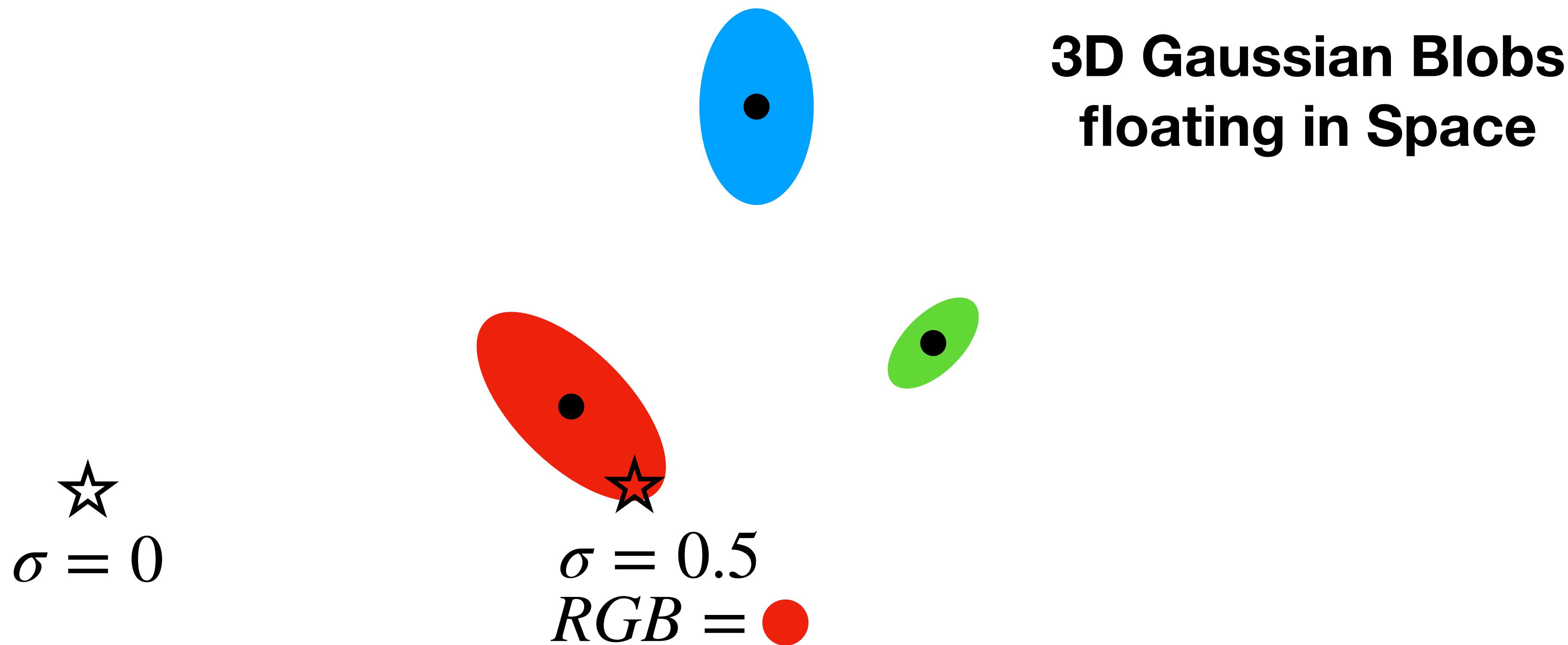


**3D Gaussian Blobs
floating in Space**

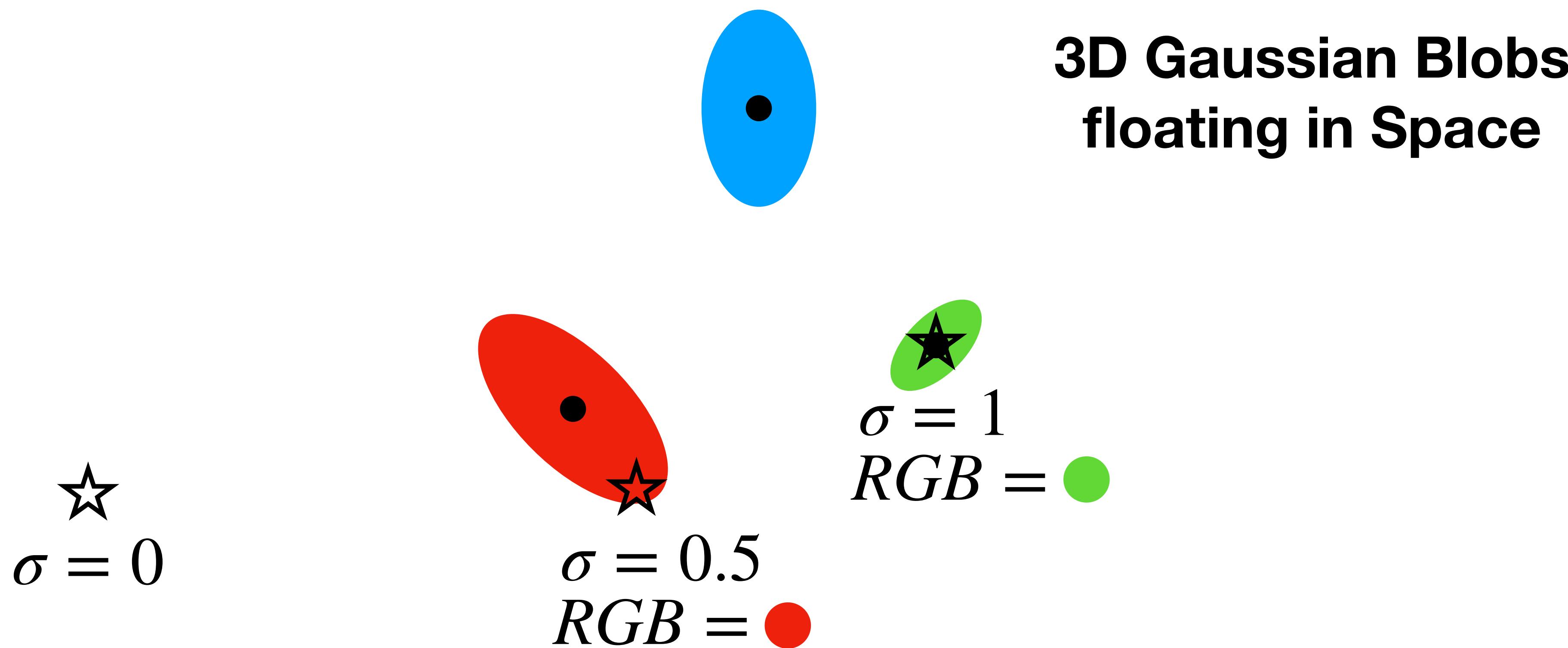
**Key Idea: Parameterize Radiance Field *sparsely*,
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Anisotropic Volumetric 3D Gaussians



Final Rendering

3D Gaussian Visualization

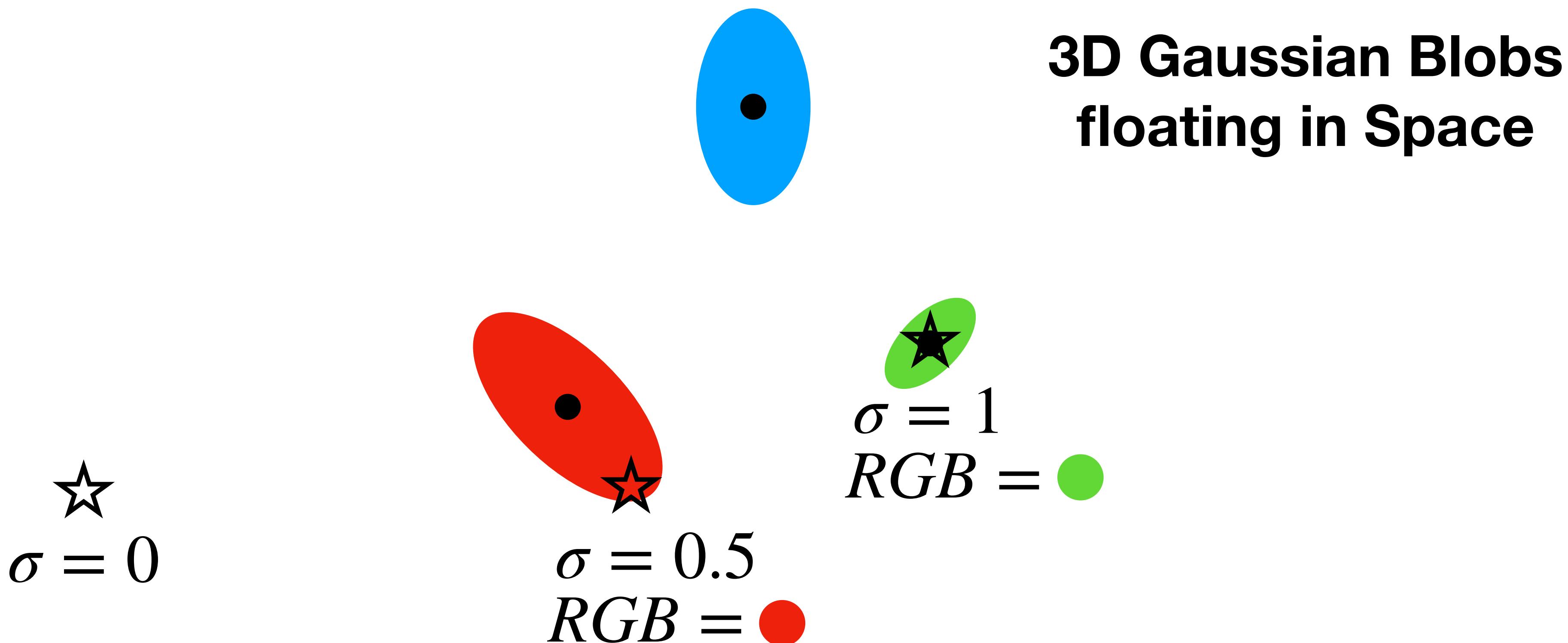
Anisotropic Volumetric 3D Gaussians



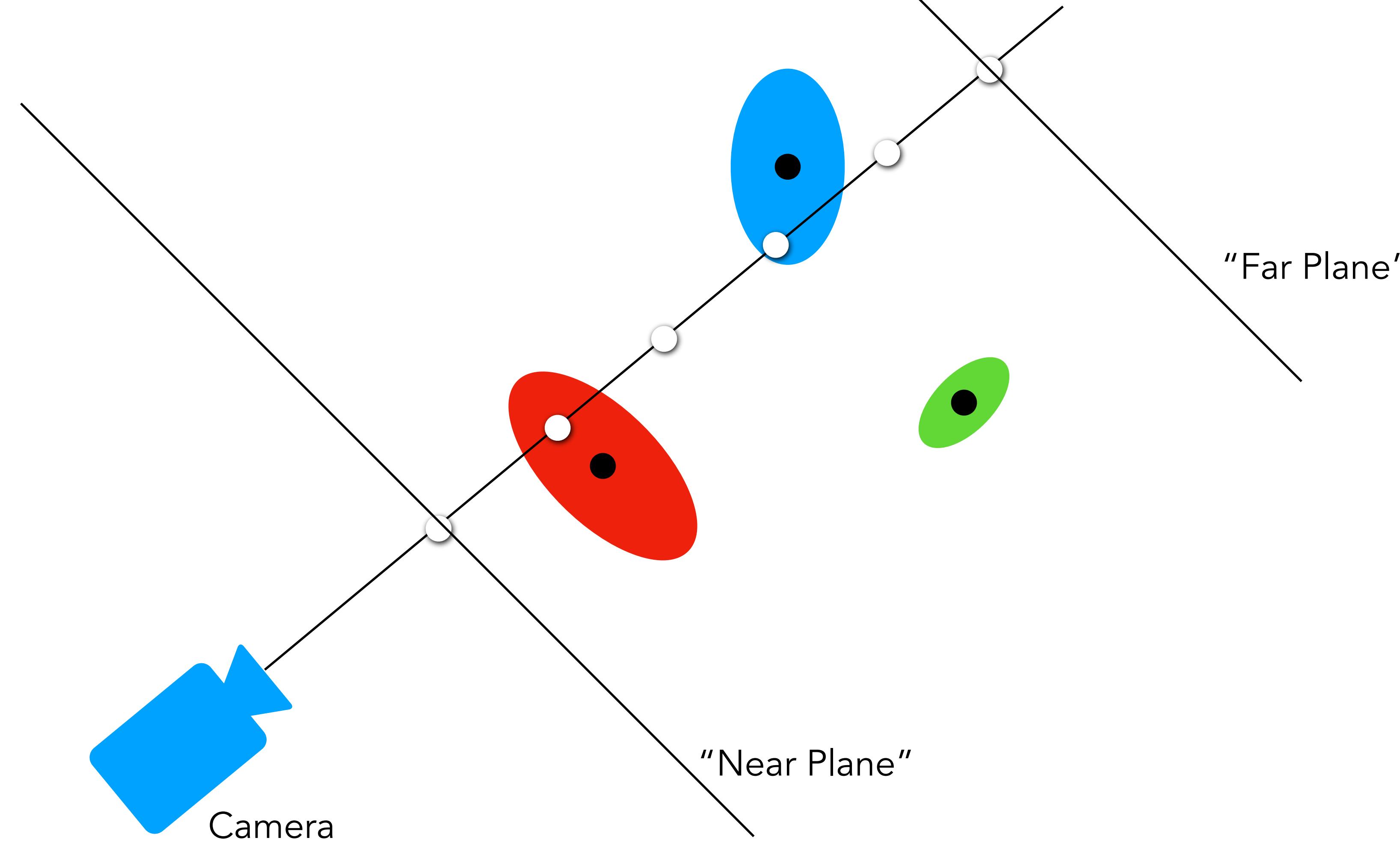
Final Rendering

3D Gaussian Visualization

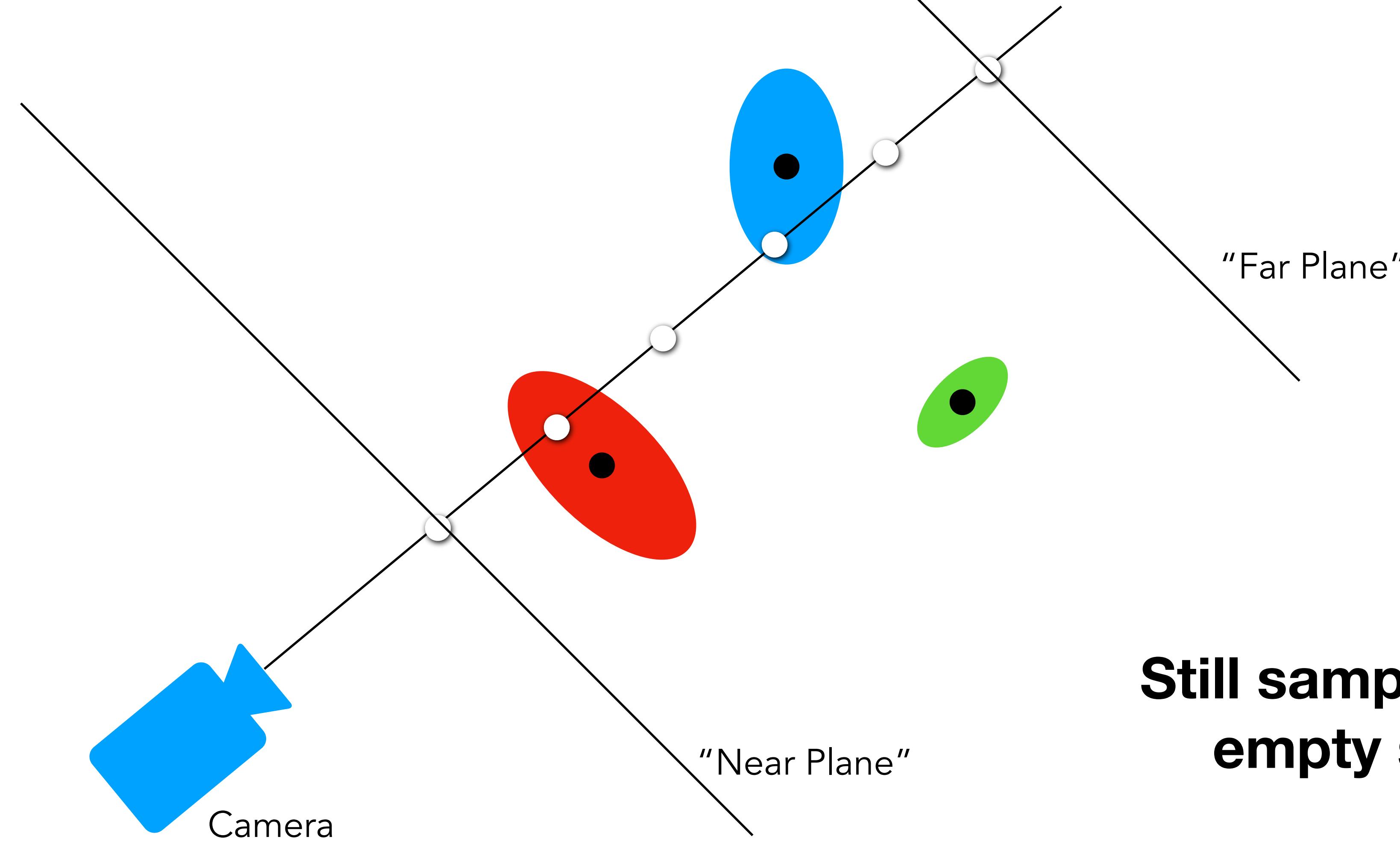
How to Render?



Same Volume Rendering Integral!

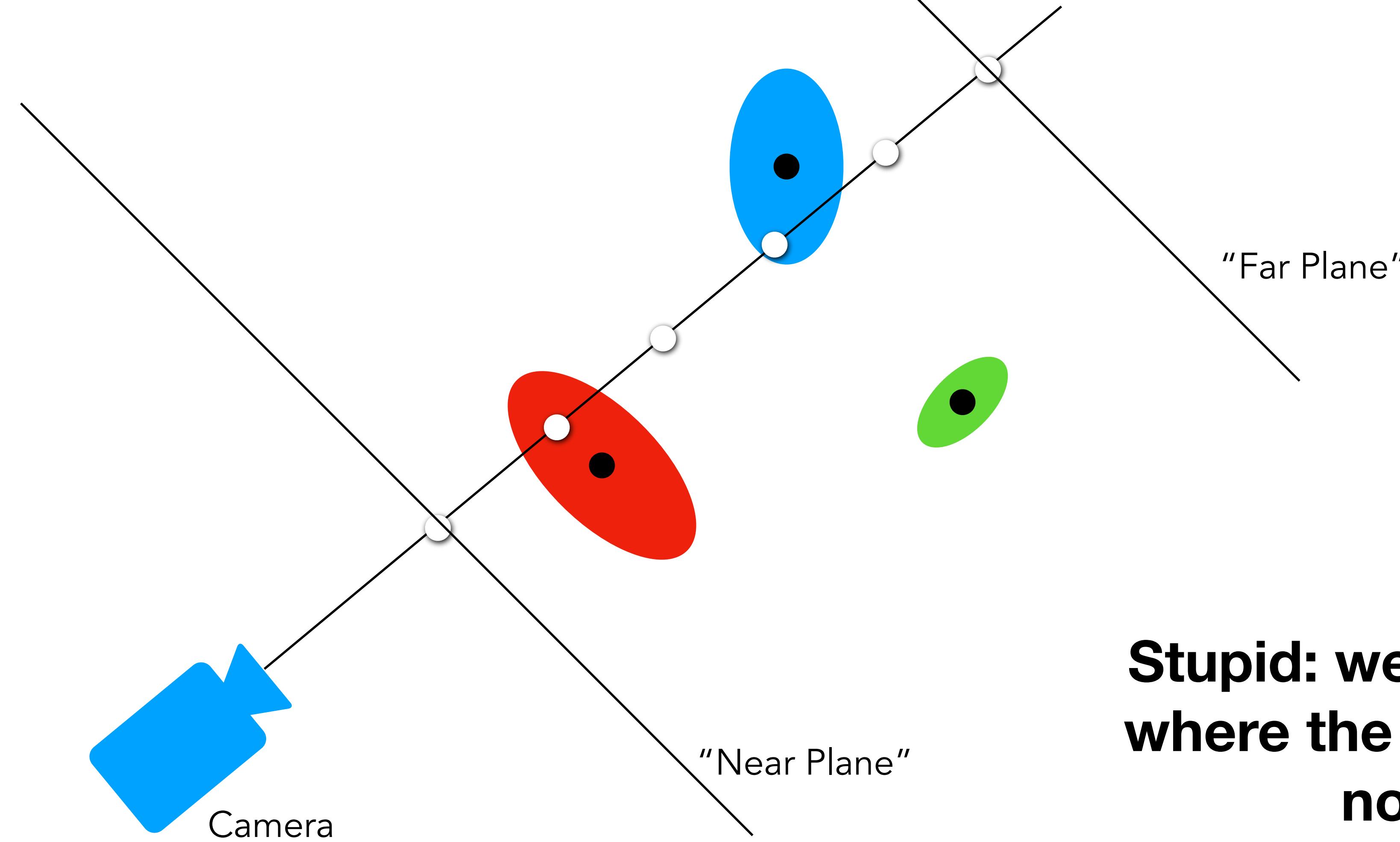


Same Volume Rendering Integral!



Still sampling lots of
empty space...

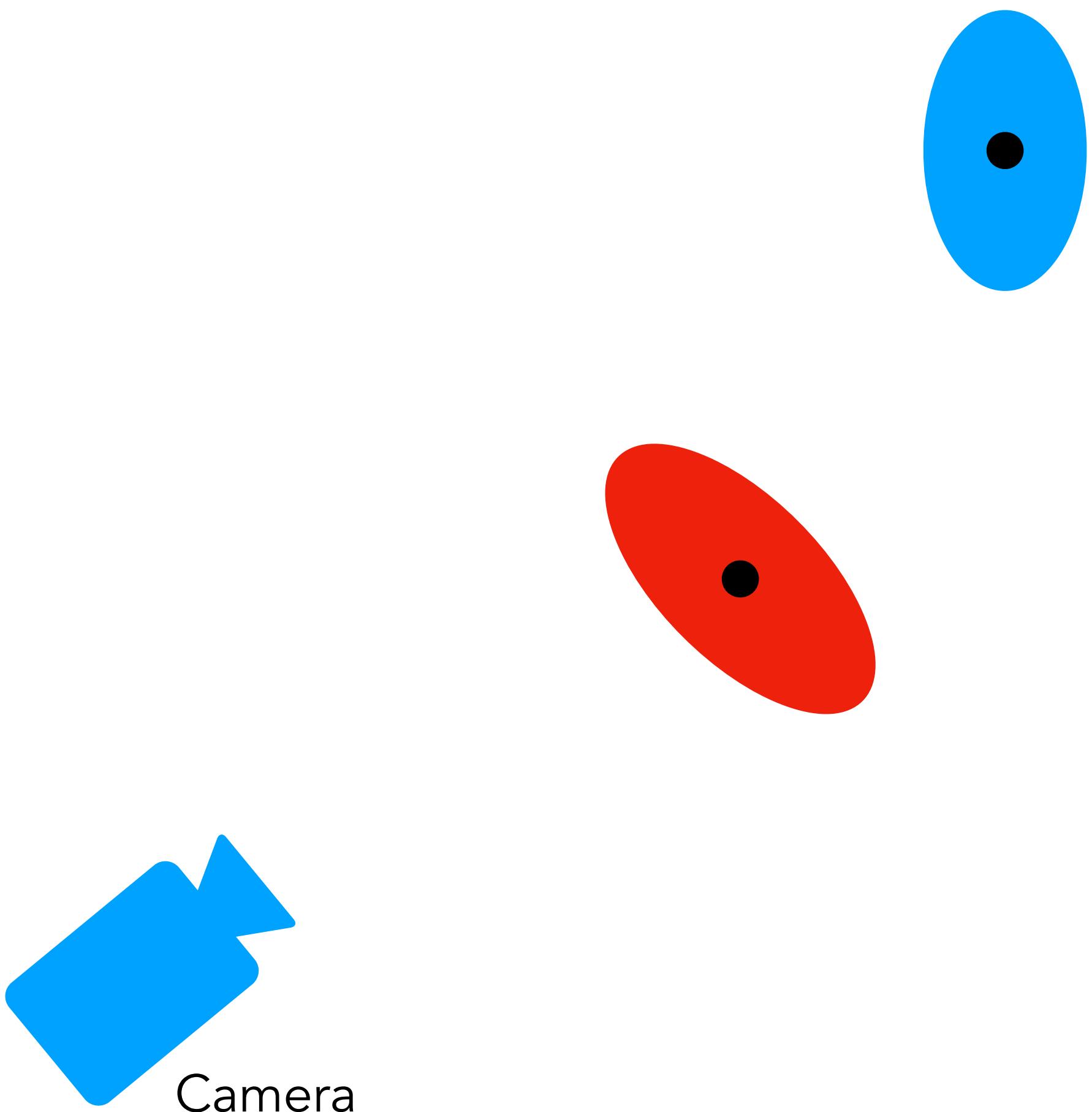
Same Volume Rendering Integral!



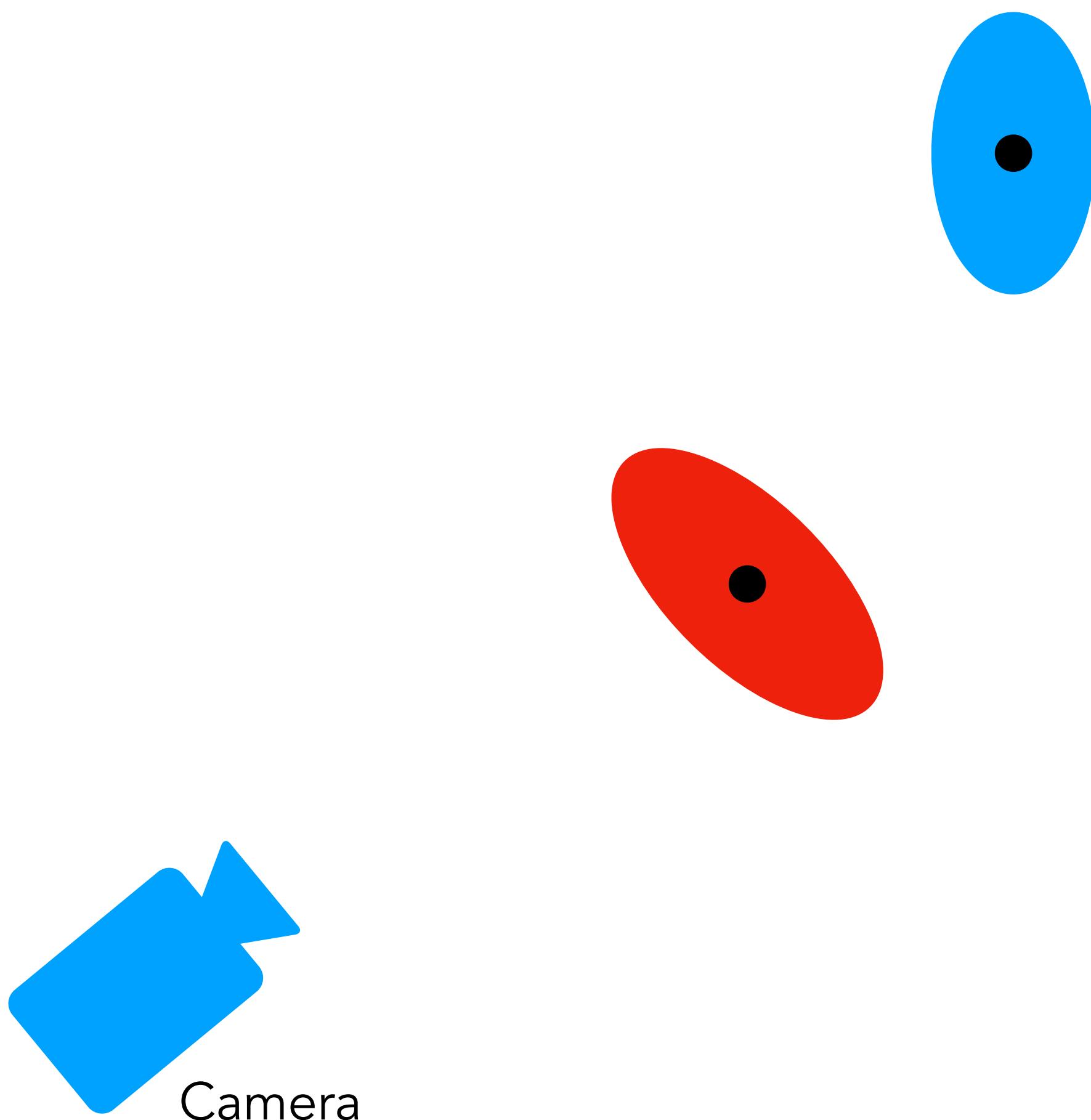
Gaussians are closed under affine transforms, integration

$$g_{\mathbf{V}}(\mathbf{x} - \mathbf{p}) = \frac{1}{2\pi|\mathbf{V}|^{\frac{1}{2}}} e^{-\frac{1}{2}(\mathbf{x}-\mathbf{p})^T \mathbf{V}^{-1} (\mathbf{x}-\mathbf{p})}$$

↑
3D Covariance!



Gaussians are closed under affine transforms, integration



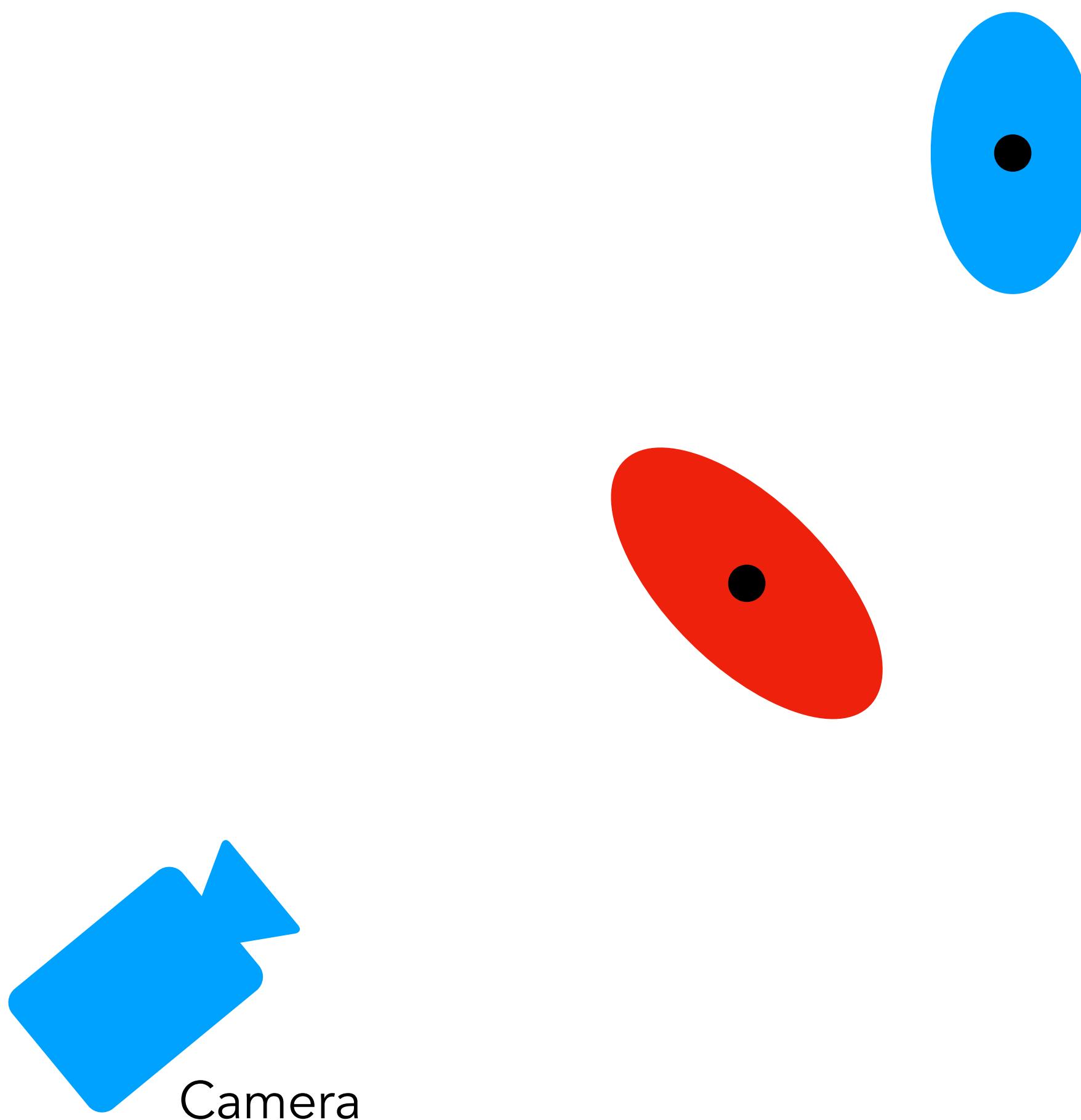
$$G_{\mathbf{V}}(\mathbf{x} - \mathbf{p}) = \frac{1}{2\pi|\mathbf{V}|^{\frac{1}{2}}} e^{-\frac{1}{2}(\mathbf{x}-\mathbf{p})^T \mathbf{V}^{-1} (\mathbf{x}-\mathbf{p})}$$

↑
3D Covariance!

Affine mapping $\Phi = \mathbf{M}\mathbf{x} + \mathbf{p}$ of coordinates (such as cam2world matrix!):

$$G_{\mathbf{V}}(\Phi^{-1}(\mathbf{u}) - \mathbf{p}) = \frac{1}{|\mathbf{M}^{-1}|} G_{\mathbf{M}\mathbf{V}\mathbf{M}^T}(\mathbf{u} - \Phi(\mathbf{p}))$$

Gaussians are closed under affine transforms, integration



$$\mathcal{G}_{\mathbf{V}}(\mathbf{x} - \mathbf{p}) = \frac{1}{2\pi|\mathbf{V}|^{\frac{1}{2}}} e^{-\frac{1}{2}(\mathbf{x}-\mathbf{p})^T \mathbf{V}^{-1} (\mathbf{x}-\mathbf{p})}$$

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Integrate along axis:

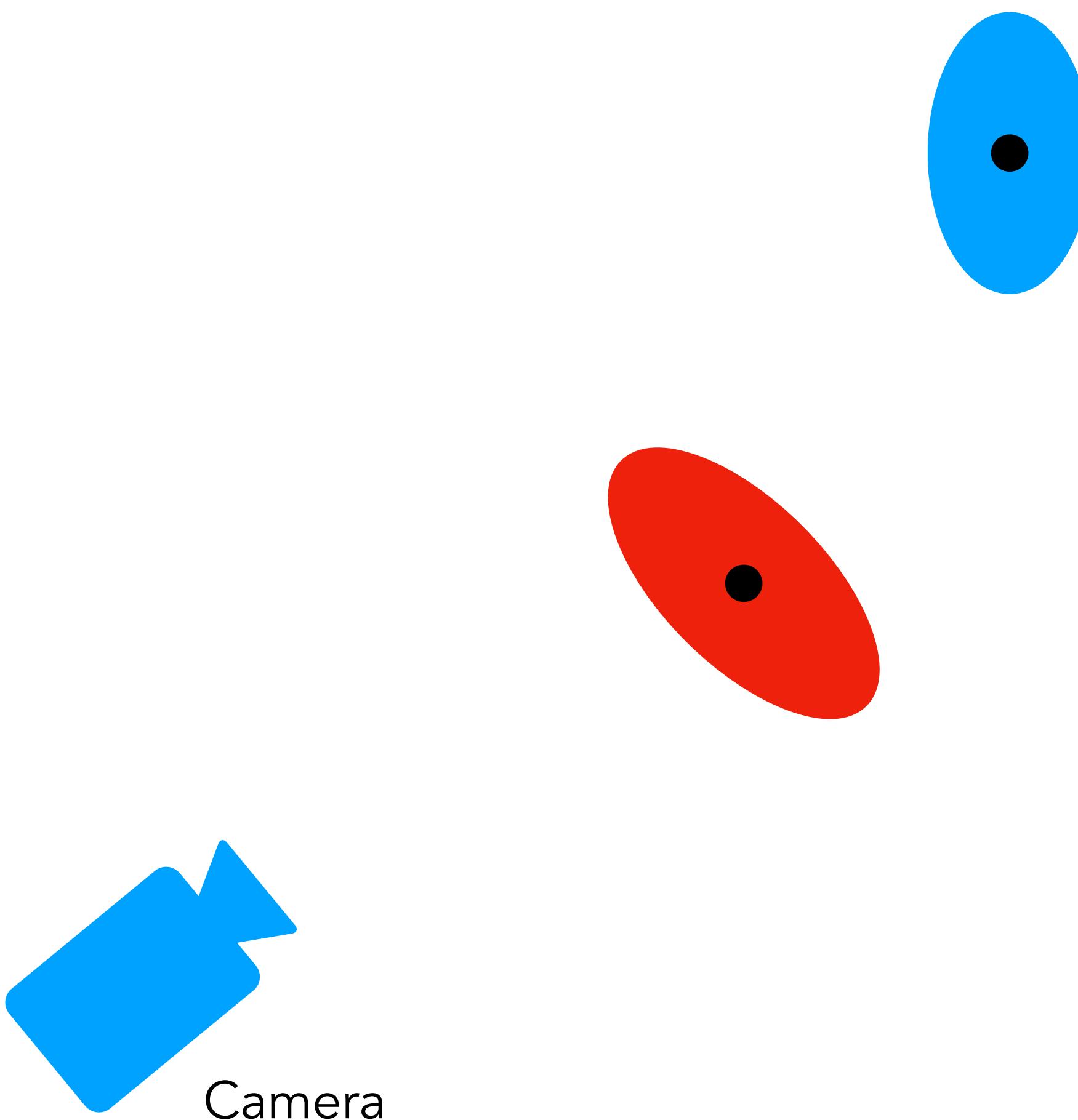
$$\int_{\mathbb{R}} \mathcal{G}_{\mathbf{V}}^3(\mathbf{x} - \mathbf{p}) dx_2 = \mathcal{G}_{\hat{\mathbf{V}}}^2(\hat{\mathbf{x}} - \hat{\mathbf{p}})$$

$$\mathbf{V} = \begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix} \Leftrightarrow \begin{pmatrix} a & b \\ b & d \end{pmatrix} = \hat{\mathbf{V}}$$

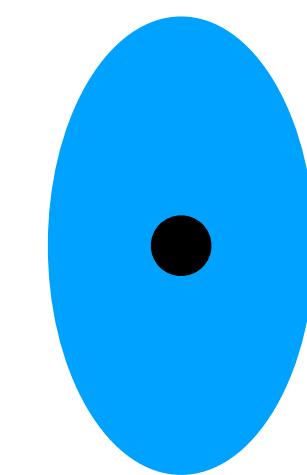
Step 1: Transform Gaussians into Camera Coordinates

Cam2world is affine mapping $\phi(x) = \mathbf{Wx} + \mathbf{p}$:

$$\mathcal{G}_{\mathbf{V}_k''}(\varphi^{-1}(\mathbf{u}) - \mathbf{t}_k) = \frac{1}{|\mathbf{W}^{-1}|} \mathcal{G}_{\mathbf{V}_k'}(\mathbf{u} - \mathbf{u}_k) = r'_k(\mathbf{u})$$



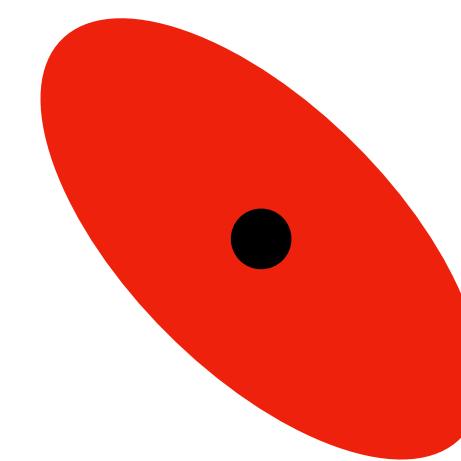
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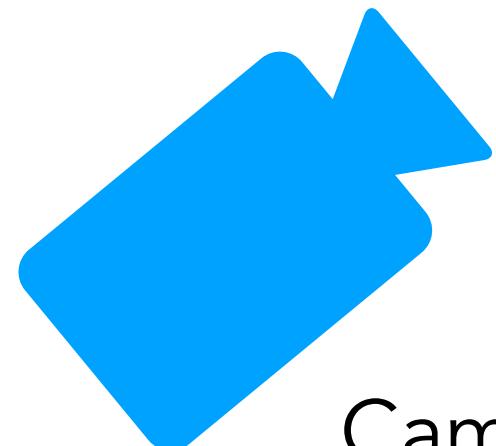
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$$\mathcal{G}_{\mathbf{V}_k''}(\varphi^{-1}(\mathbf{u}) - \mathbf{t}_k) = \frac{1}{|\mathbf{W}^{-1}|} \mathcal{G}_{\mathbf{V}_k'}(\mathbf{u} - \mathbf{u}_k) = r'_k(\mathbf{u})$$

Projection $\mathbf{m}(u)$ is *not* an affine mapping :/



$$\begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix} = \mathbf{m}(\mathbf{u}) = \begin{pmatrix} u_0/u_2 \\ u_1/u_2 \\ \|(u_0, u_1, u_2)^T\| \end{pmatrix}$$
$$\begin{pmatrix} u_0 \\ u_1 \\ u_2 \end{pmatrix} = \mathbf{m}^{-1}(\mathbf{x}) = \begin{pmatrix} x_0/l \cdot x_2 \\ x_1/l \cdot x_2 \\ 1/l \cdot x_2 \end{pmatrix},$$



Camera

Step 1: Transform Gaussians into Camera Coordinates

Cam2world is affine mapping $\phi(x) = \mathbf{Wx} + \mathbf{p}$:

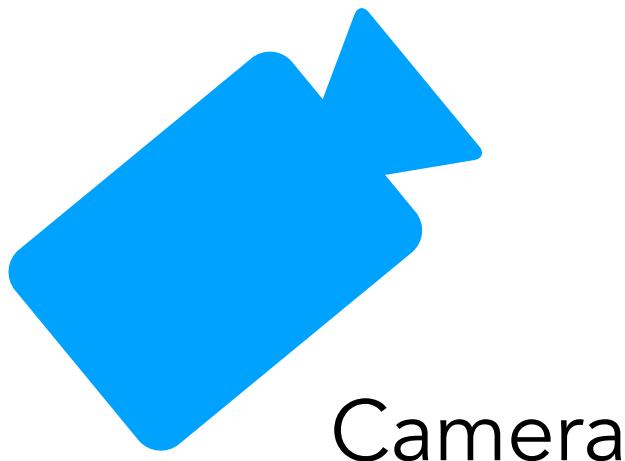
$$\mathcal{G}_{\mathbf{V}_k''}(\varphi^{-1}(\mathbf{u}) - \mathbf{t}_k) = \frac{1}{|\mathbf{W}^{-1}|} \mathcal{G}_{\mathbf{V}_k'}(\mathbf{u} - \mathbf{u}_k) = r'_k(\mathbf{u})$$

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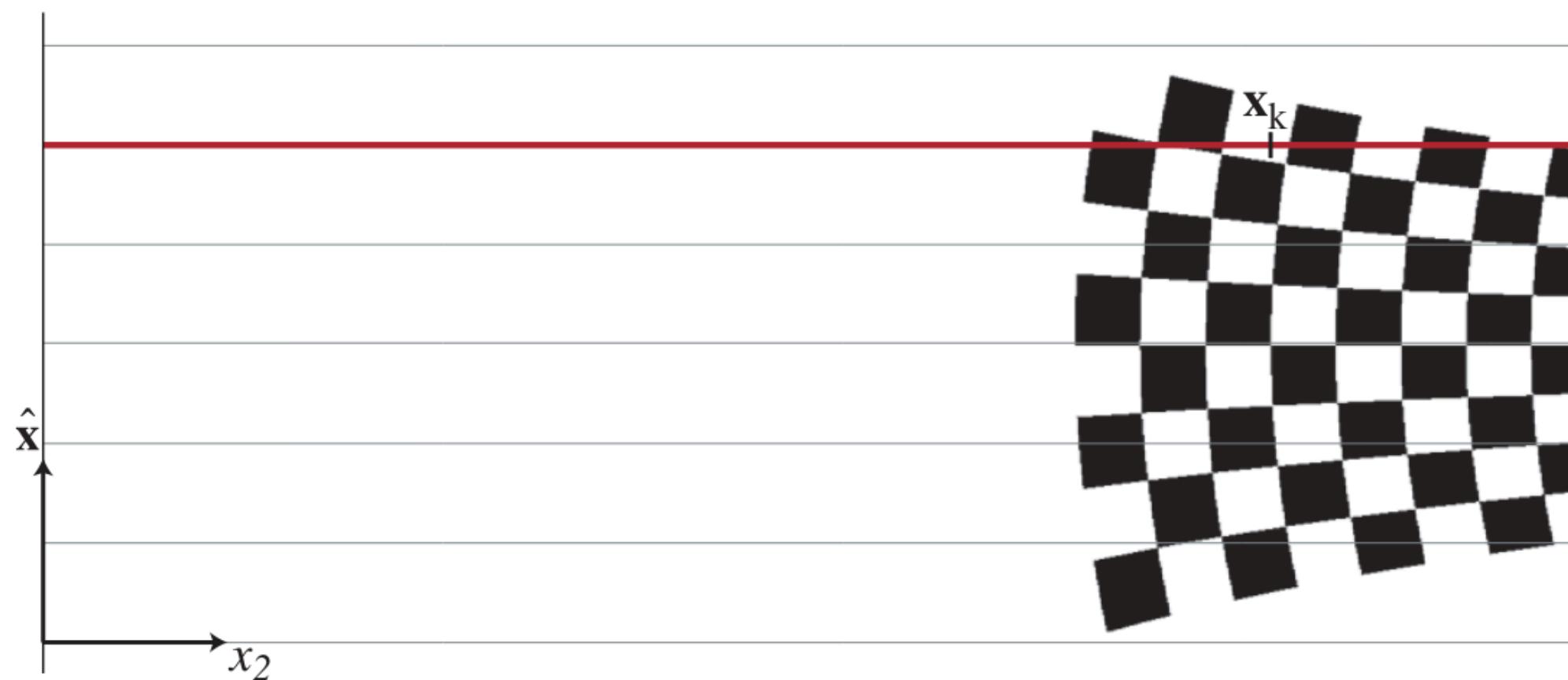
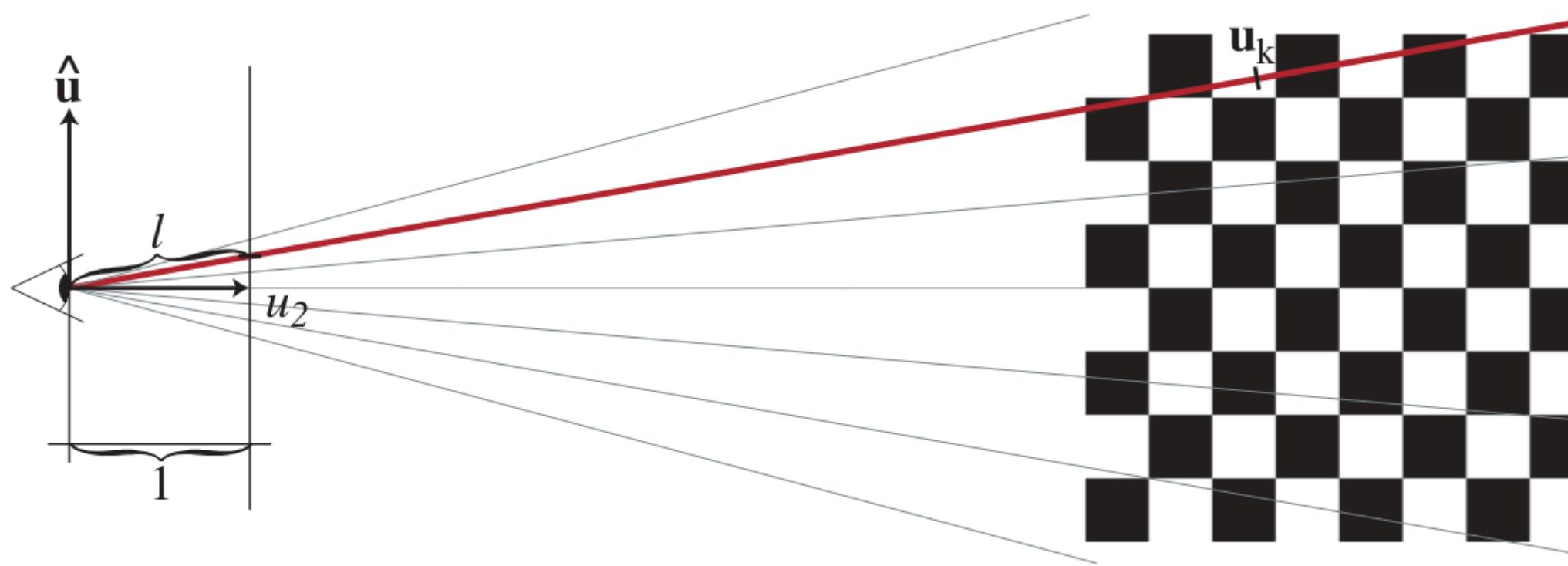
But can approximate with first-order Taylor Expansion as:

$$\mathbf{m}_{\mathbf{u}_k}(\mathbf{u}) = \mathbf{x}_k + \mathbf{J}_{\mathbf{u}_k} \cdot (\mathbf{u} - \mathbf{u}_k) \quad \mathbf{J}_{\mathbf{u}_k} = \frac{\partial \mathbf{m}}{\partial \mathbf{u}}(\mathbf{u}_k)$$



Step 1: Transform Gaussians into Camera Coordinates

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Step 1: Transform Gaussians into Camera Coordinates

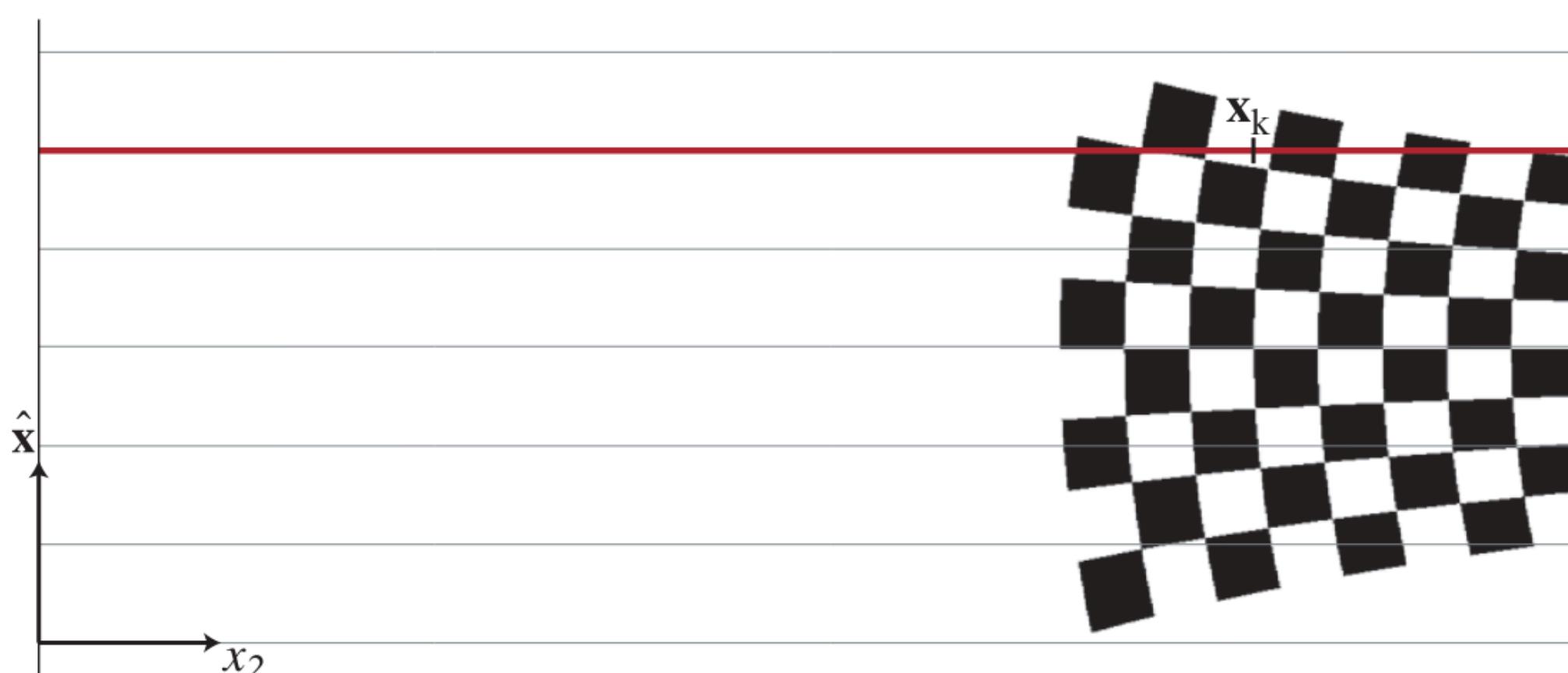
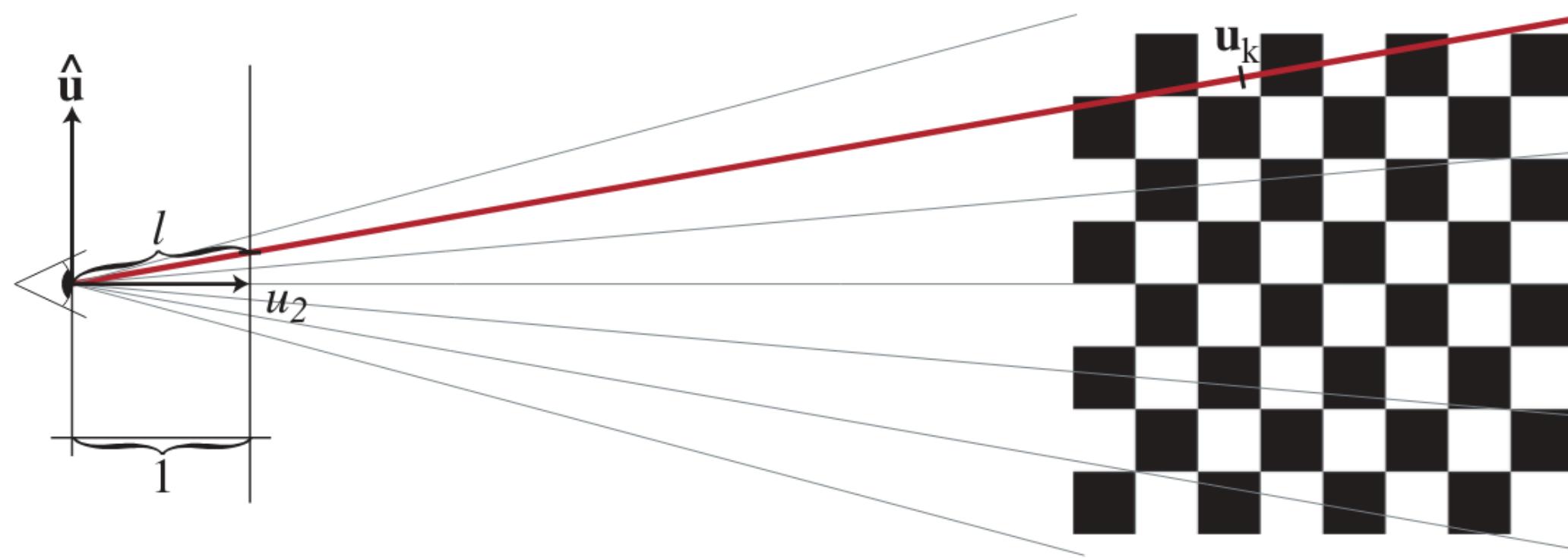
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Projected, 2D Gaussians are then:

$$\frac{1}{|\mathbf{W}^{-1}| |\mathbf{J}^{-1}|} \mathcal{G}_{\mathbf{V}_k}(\mathbf{x} - \mathbf{x}_k)$$

$$\begin{aligned}\mathbf{V}_k &= \mathbf{J} \mathbf{V}'_k \mathbf{J}^T \\ &= \mathbf{J} \mathbf{W} \mathbf{V}''_k \mathbf{W}^T \mathbf{J}^T.\end{aligned}$$



Step 1: Transform Gaussians into Camera Coordinates

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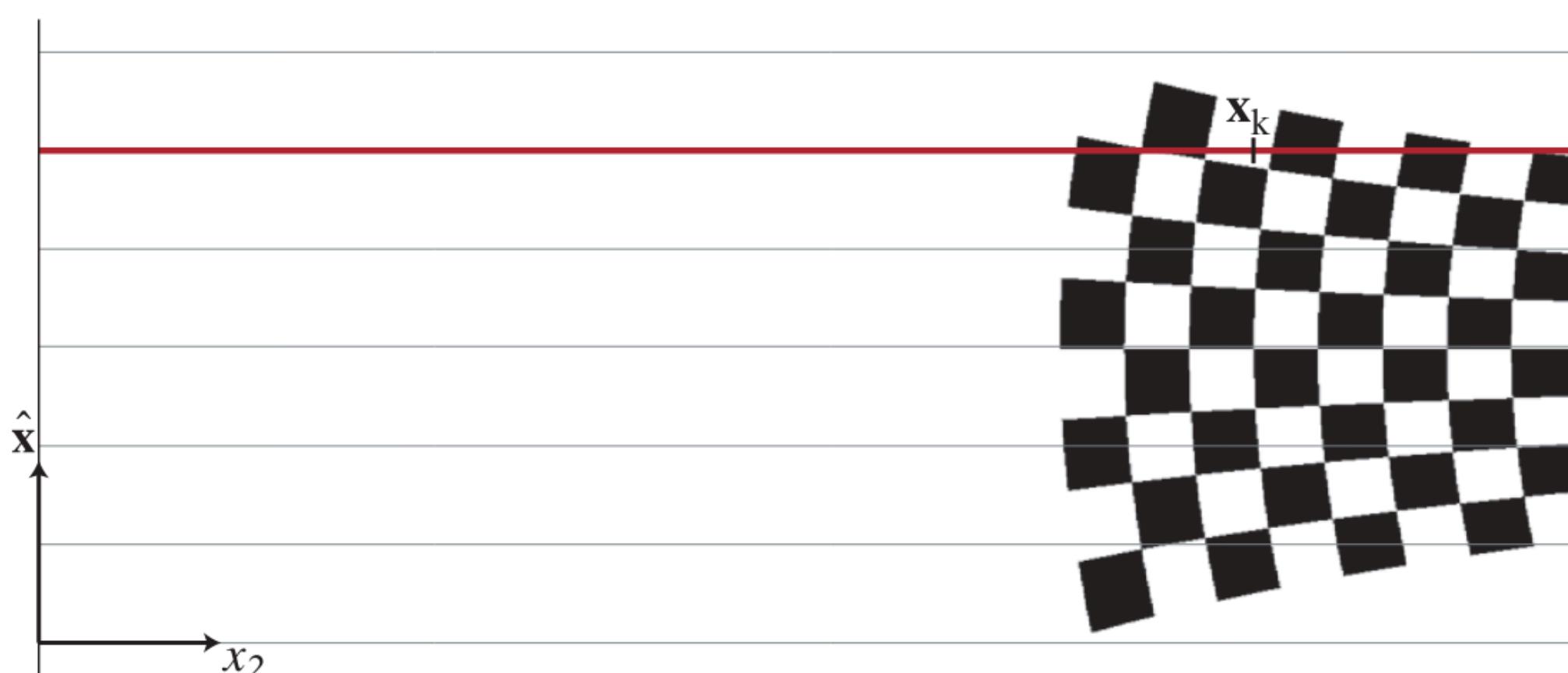
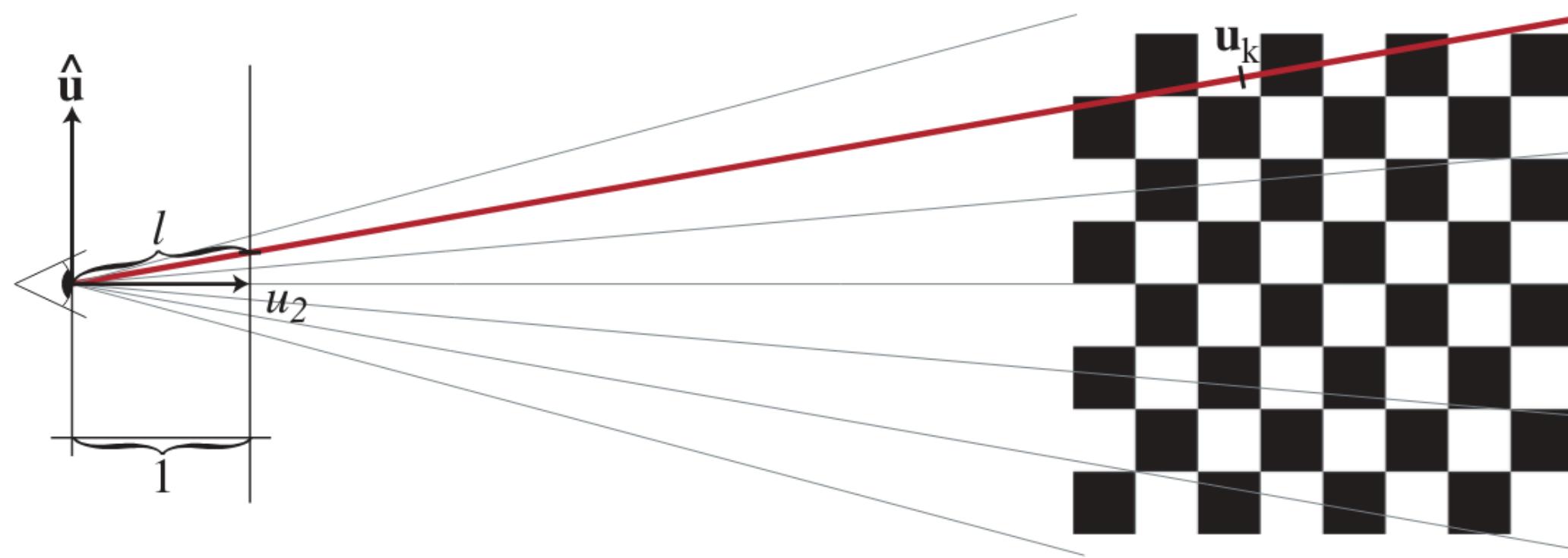
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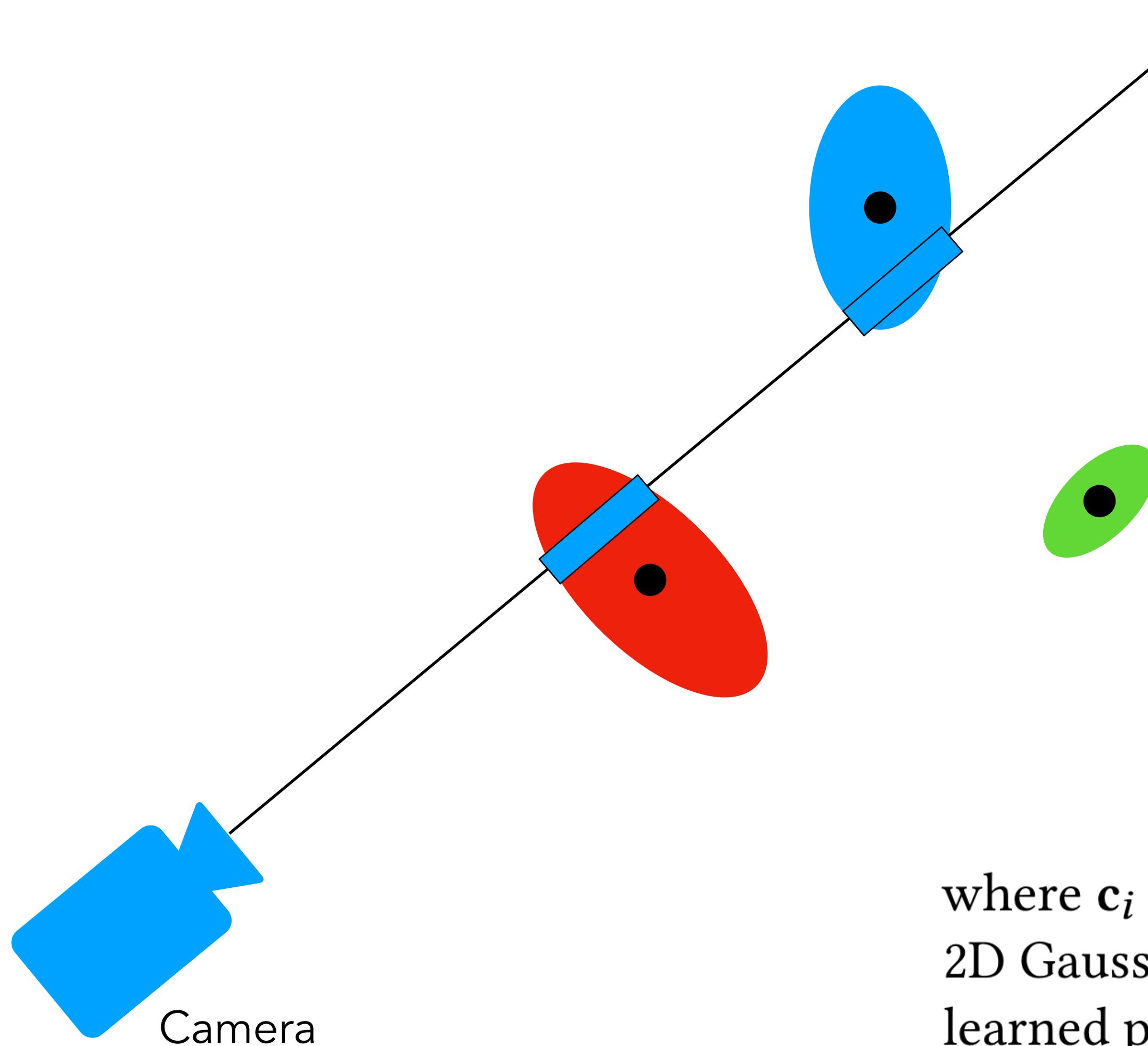
$$\begin{aligned} \mathbf{V}_k &= \mathbf{J} \mathbf{V}'_k \mathbf{J}^T \\ &= \mathbf{J} \mathbf{W} \mathbf{V}''_k \mathbf{W}^T \mathbf{J}^T. \end{aligned}$$

Finally, can integrate along rays:

$$\begin{aligned} q_k(\hat{\mathbf{x}}) &= \int_{\mathbb{R}} \frac{1}{|\mathbf{J}^{-1}| |\mathbf{W}^{-1}|} \mathcal{G}_{\mathbf{V}_k}(\hat{\mathbf{x}} - \hat{\mathbf{x}}_k, x_2 - x_{k2}) dx_2 \\ &= \frac{1}{|\mathbf{J}^{-1}| |\mathbf{W}^{-1}|} \mathcal{G}_{\hat{\mathbf{V}}_k}(\hat{\mathbf{x}} - \hat{\mathbf{x}}_k) \end{aligned}$$



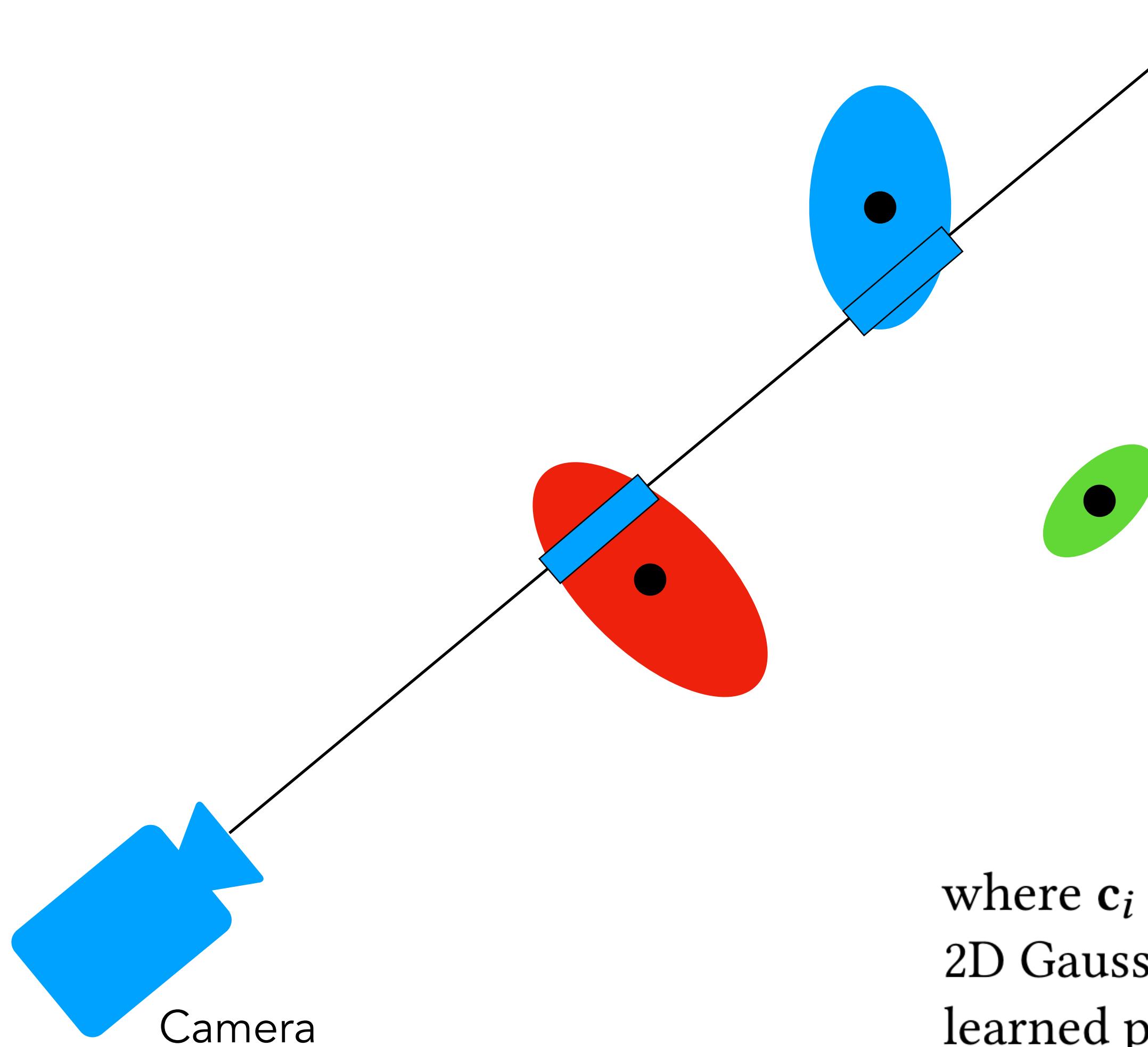
Can compute volume rendering integral without ever sampling a single 3D point in space!



$$C = \sum_{i \in \mathcal{N}} c_i \alpha_i \prod_{j=1}^{i-1} (1 - \alpha_j), \quad (3)$$

where c_i is the color of each point and α_i is given by evaluating a 2D Gaussian with covariance Σ [Yifan et al. 2019] multiplied with a learned per-point opacity.

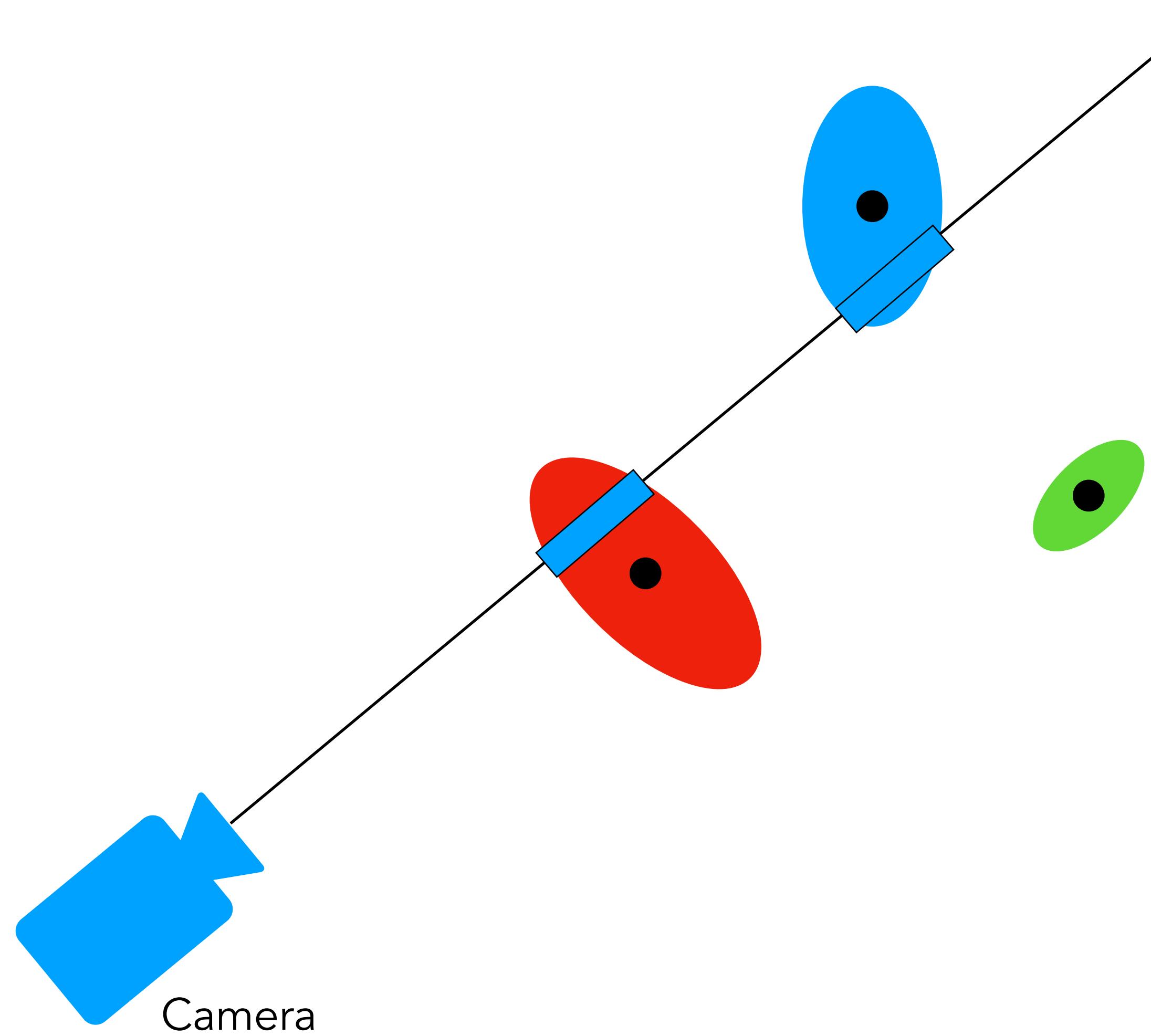
Any problems for inverse graphics, though...?



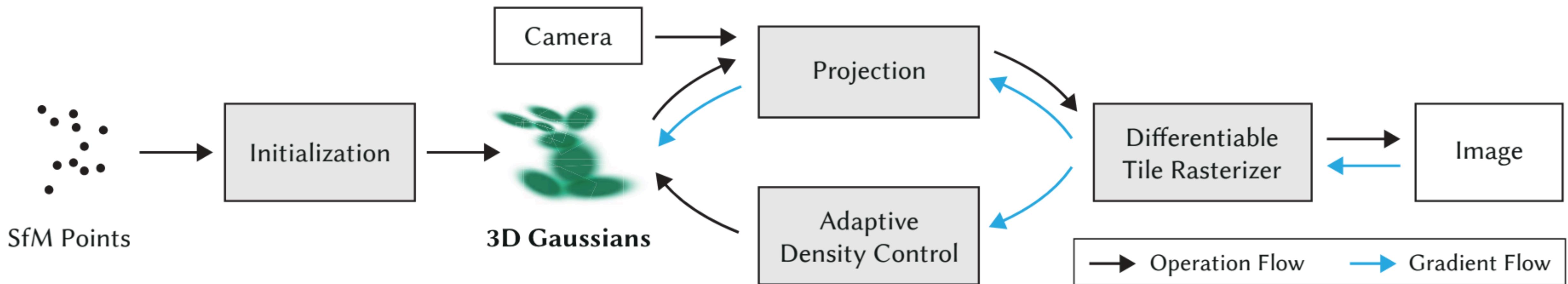
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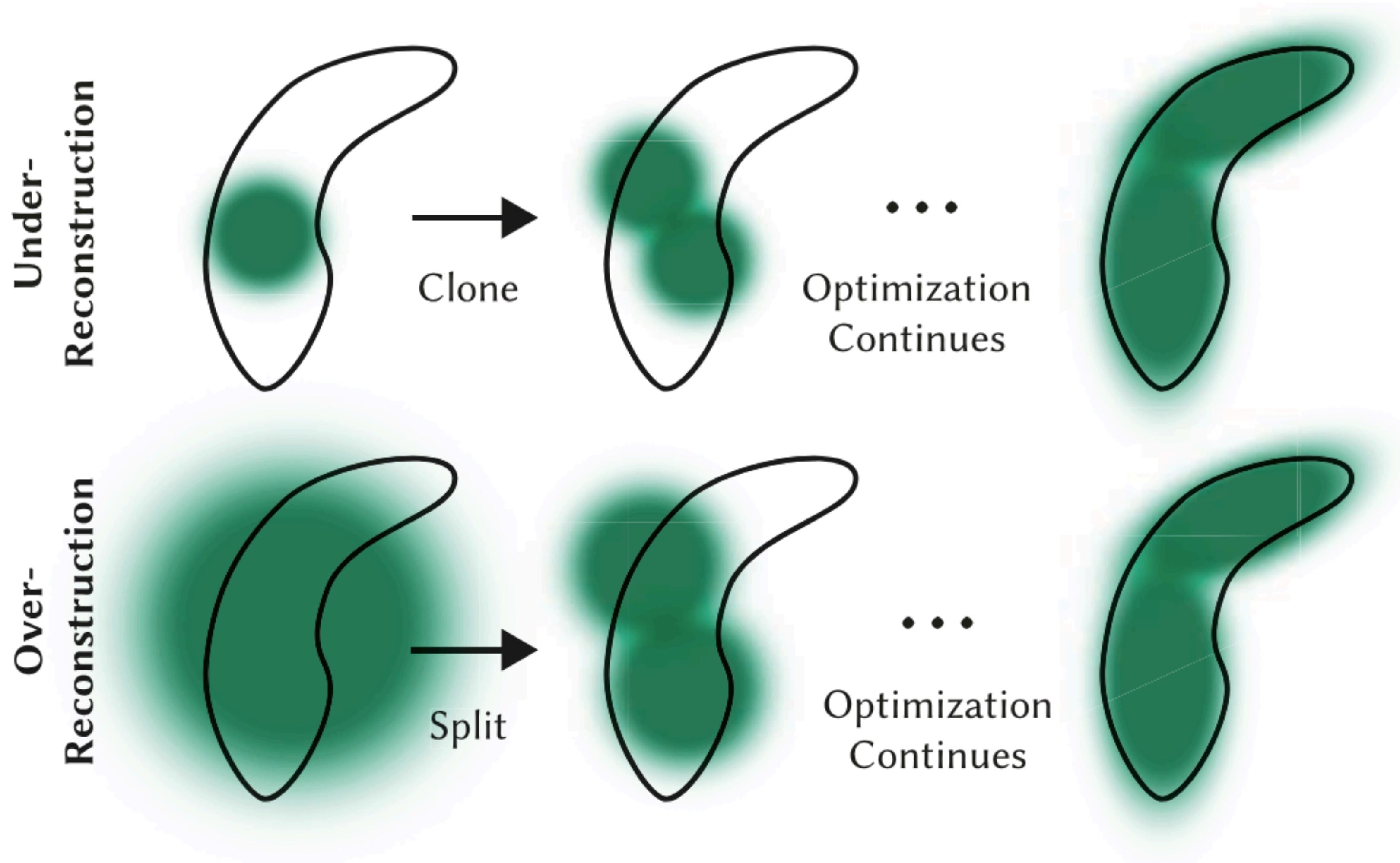
Problem: Local minima...



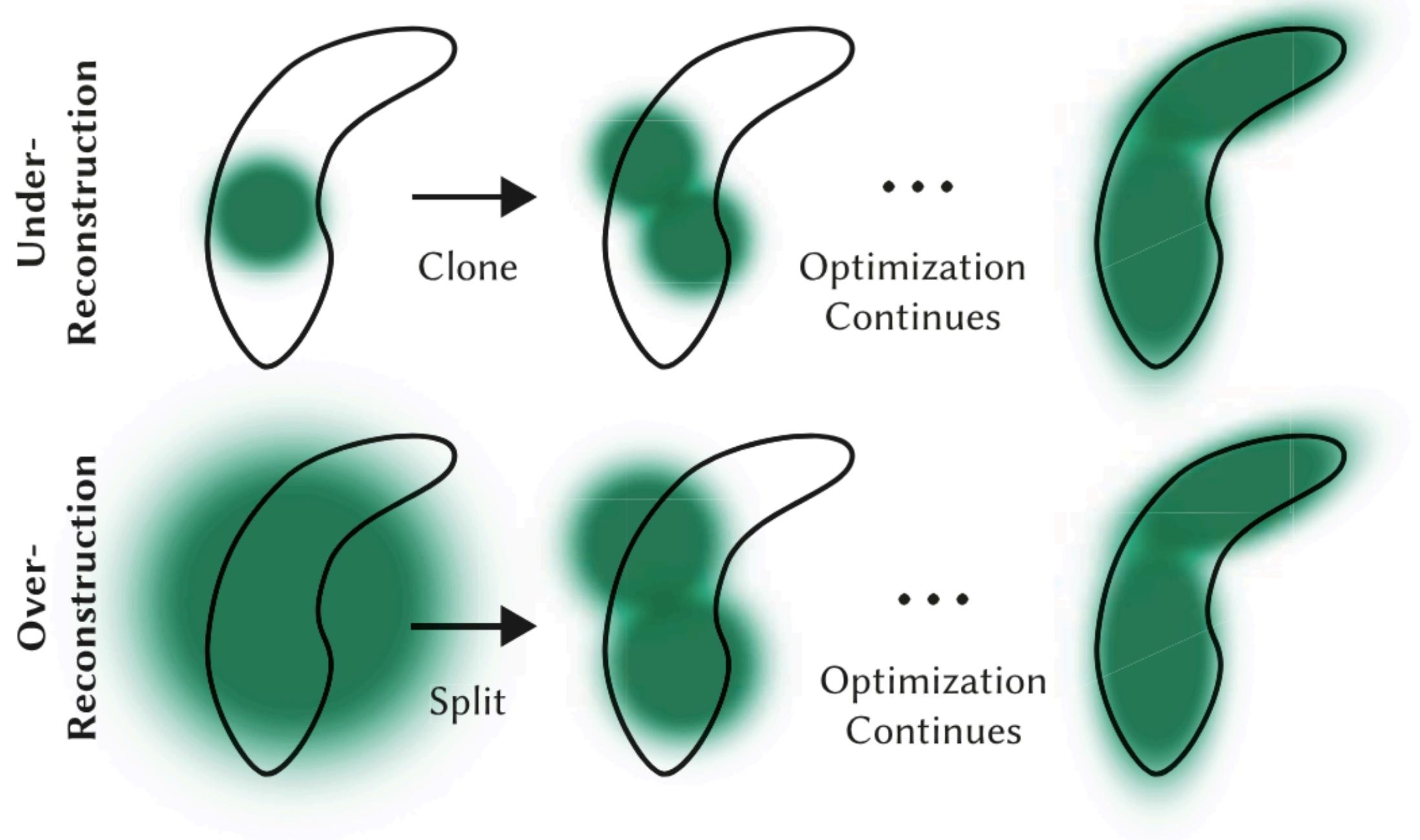
Fix 1: Start from SFM point cloud.



Fix 2: Heuristic *pruning* and *spawning* operations



Fix 2: Heuristic *pruning* and *spawning* operations



Our adaptive control of the Gaussians needs to populate empty areas. It focuses on regions with missing geometric features (“under-reconstruction”), but also in regions where Gaussians cover large areas in the scene (which often correspond to “over-reconstruction”). We observe that both have *large* view-space positional gradients. Intuitively, this is likely because they correspond to regions that are not yet well reconstructed, and the optimization tries to move the Gaussians to correct this.

Since both cases are good candidates for densification, we densify Gaussians with an average magnitude of view-space position gradients above a threshold τ_{pos} , which we set to 0.0002 in our tests.

Timelapse of the Optimization (NeRF-Synthetic Dataset)



Timelapse of the Optimization (NeRF-Synthetic Dataset)



All interactive sessions are recorded at 1080p with an A6000



▼ Metrics
92.95 (10.76 ms)

Playroom

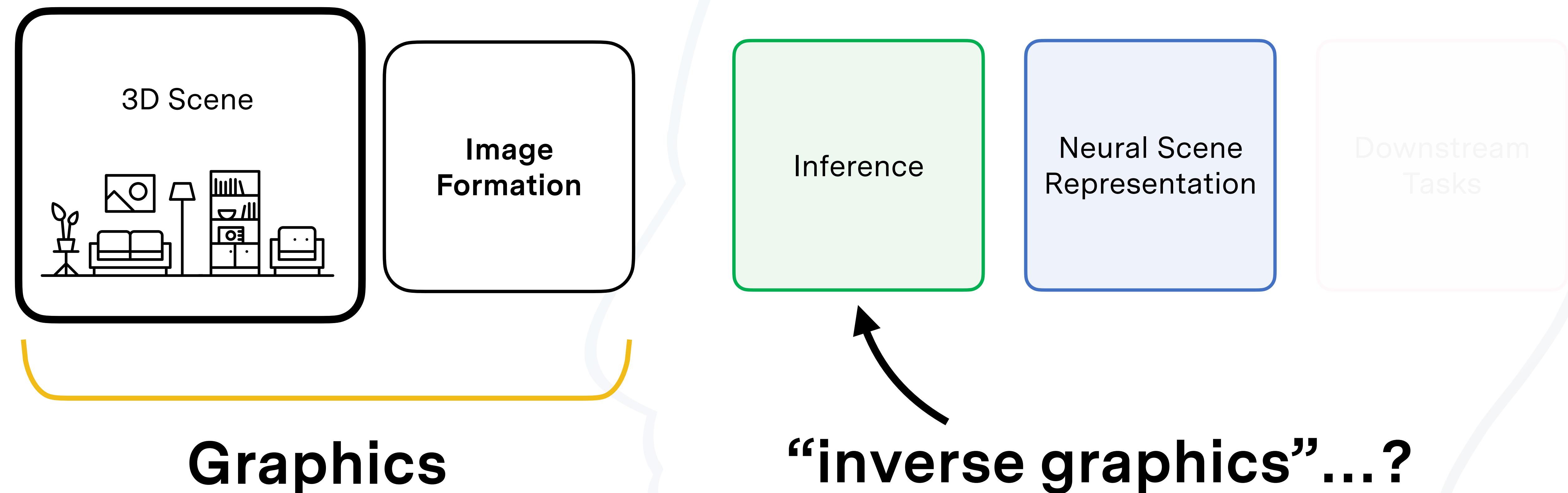
All interactive sessions are recorded at 1080p with an A6000



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92.95 (10.76 ms)

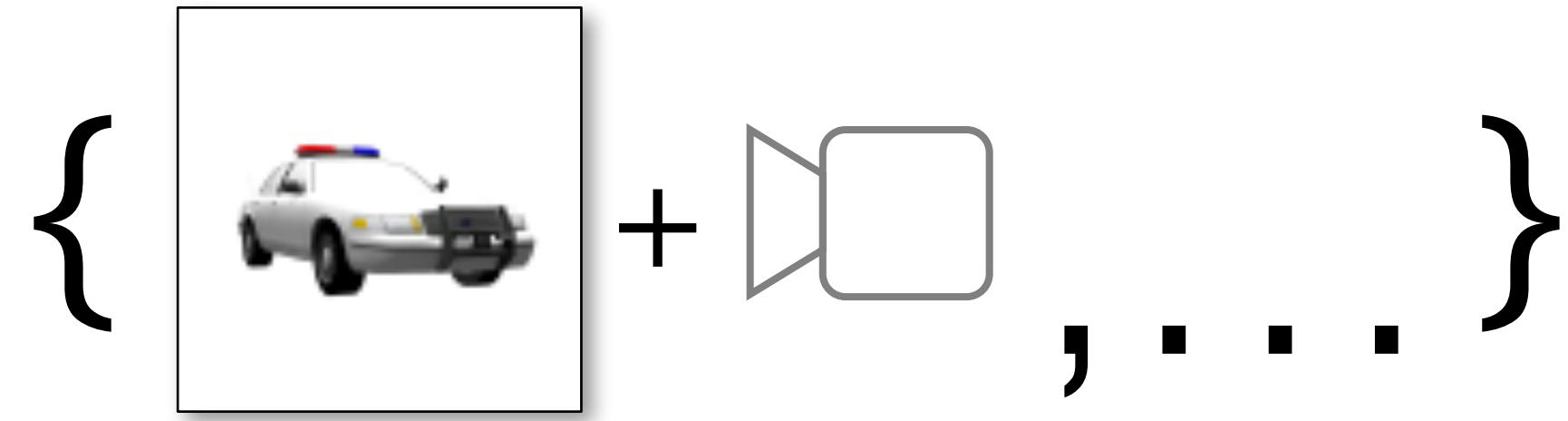
Playroom

Today: Differentiable Rendering, AKA “Inverse Graphics”

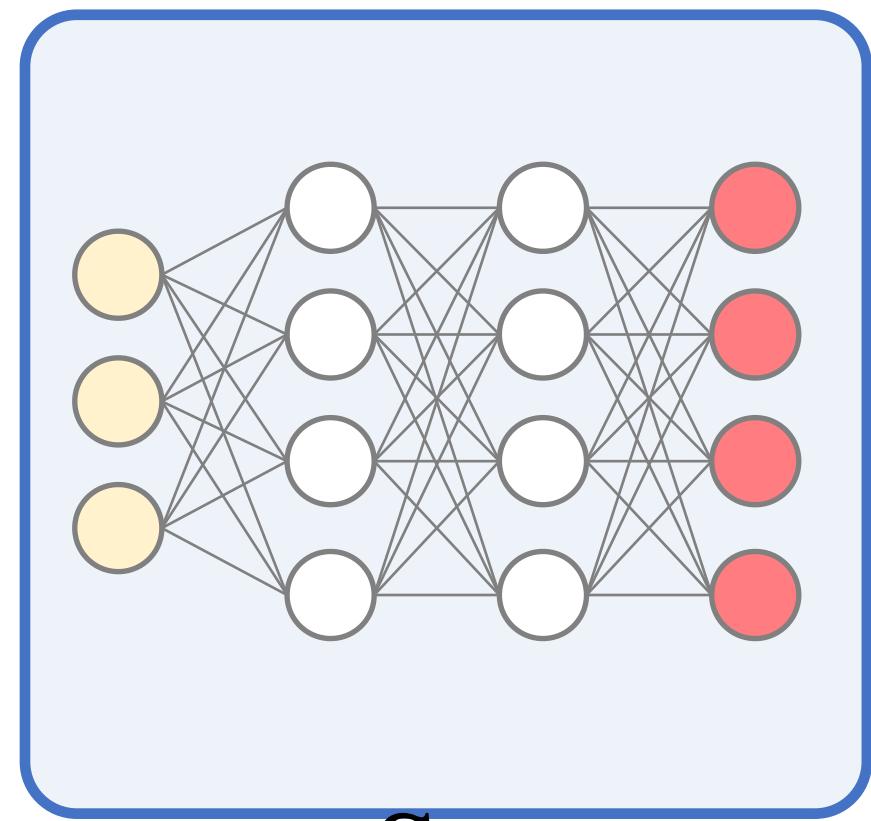
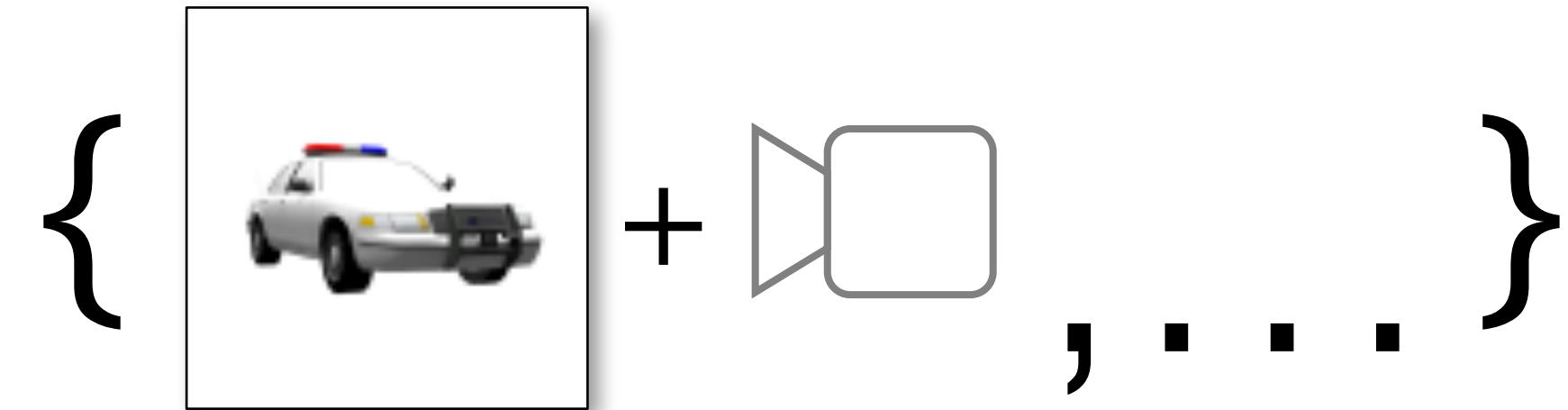


Overfitting case: Inference = Fitting via Gradient Descent

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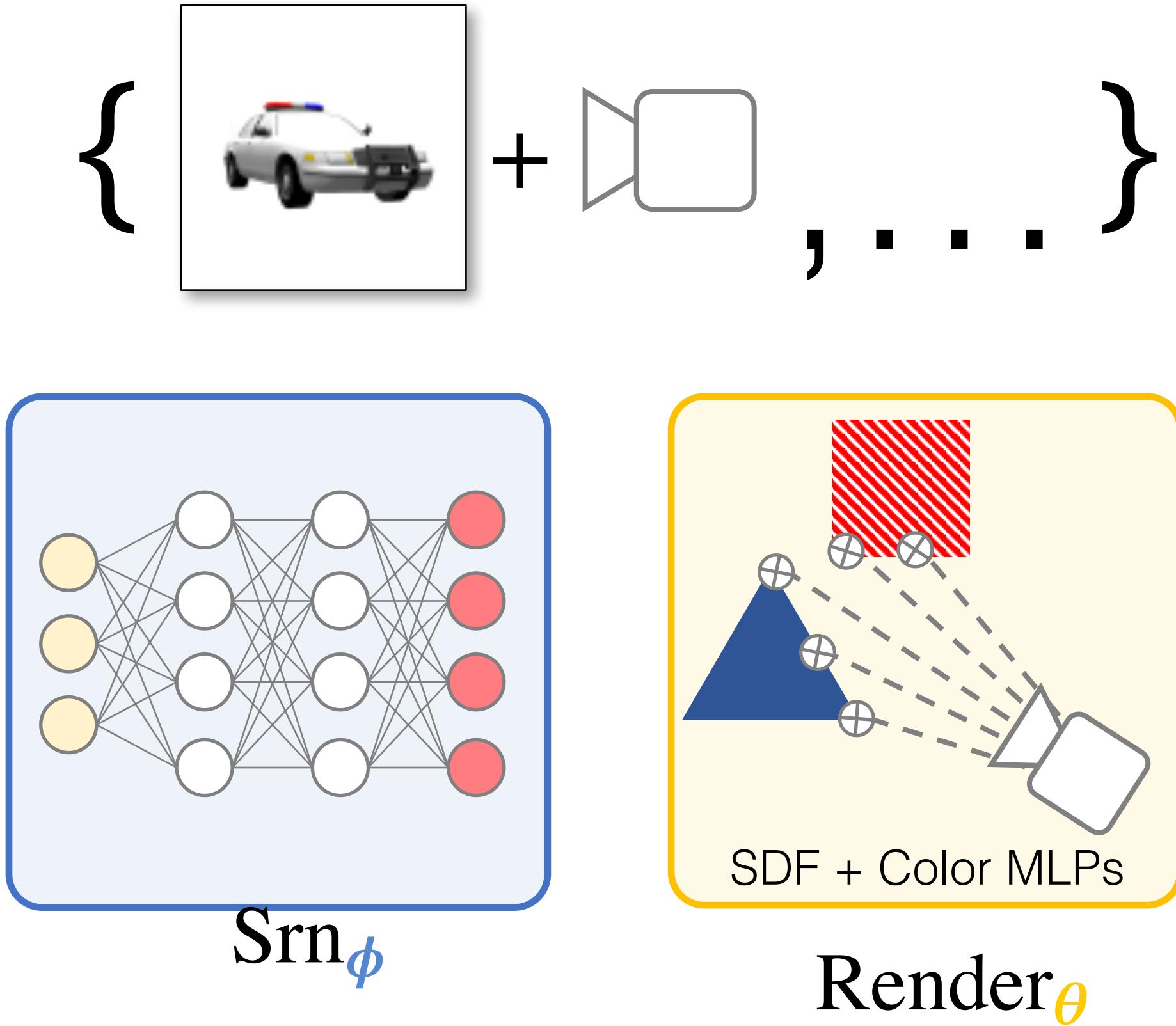


Overfitting case: Inference = Fitting via Gradient Descent

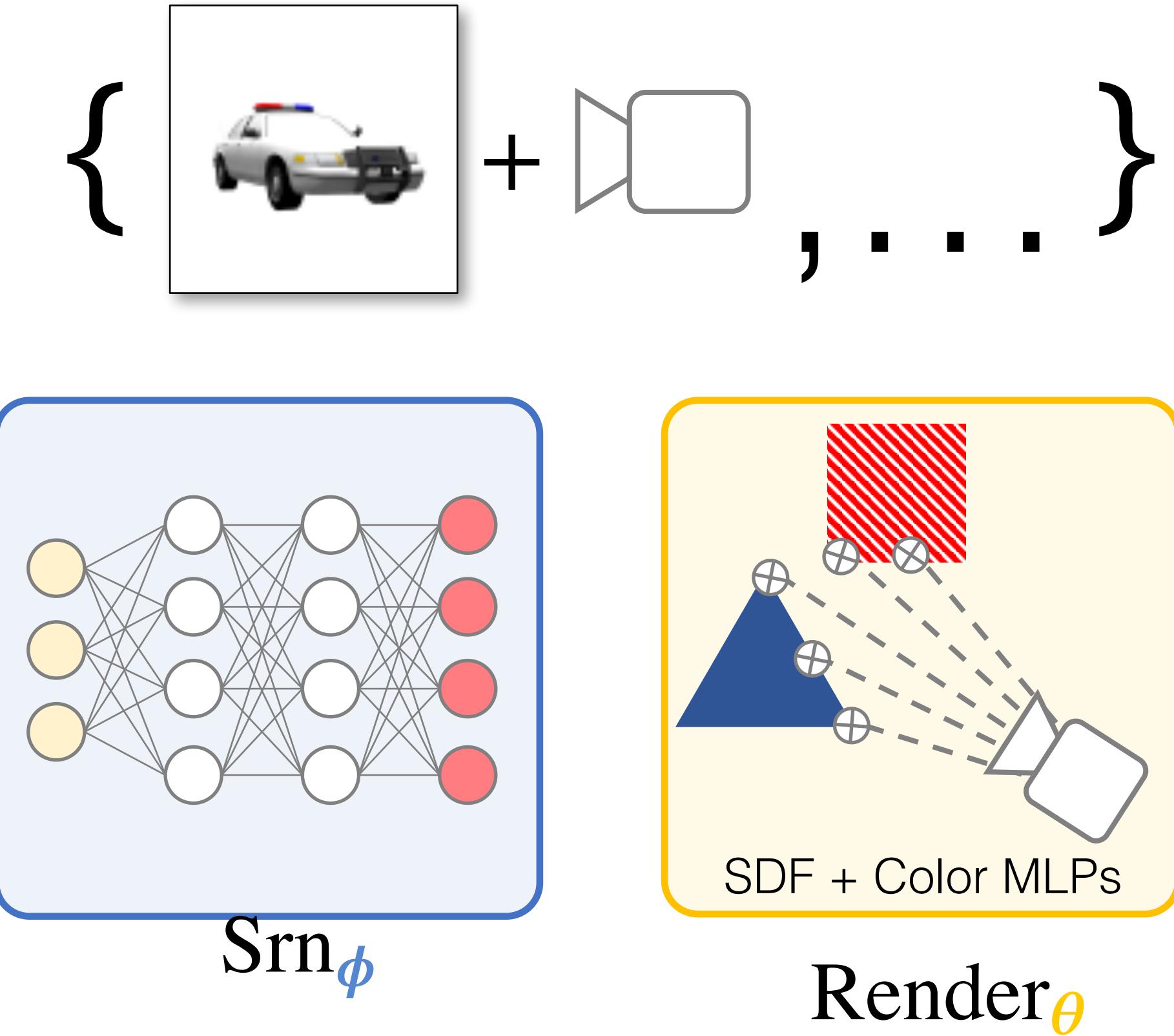


Srn_ϕ

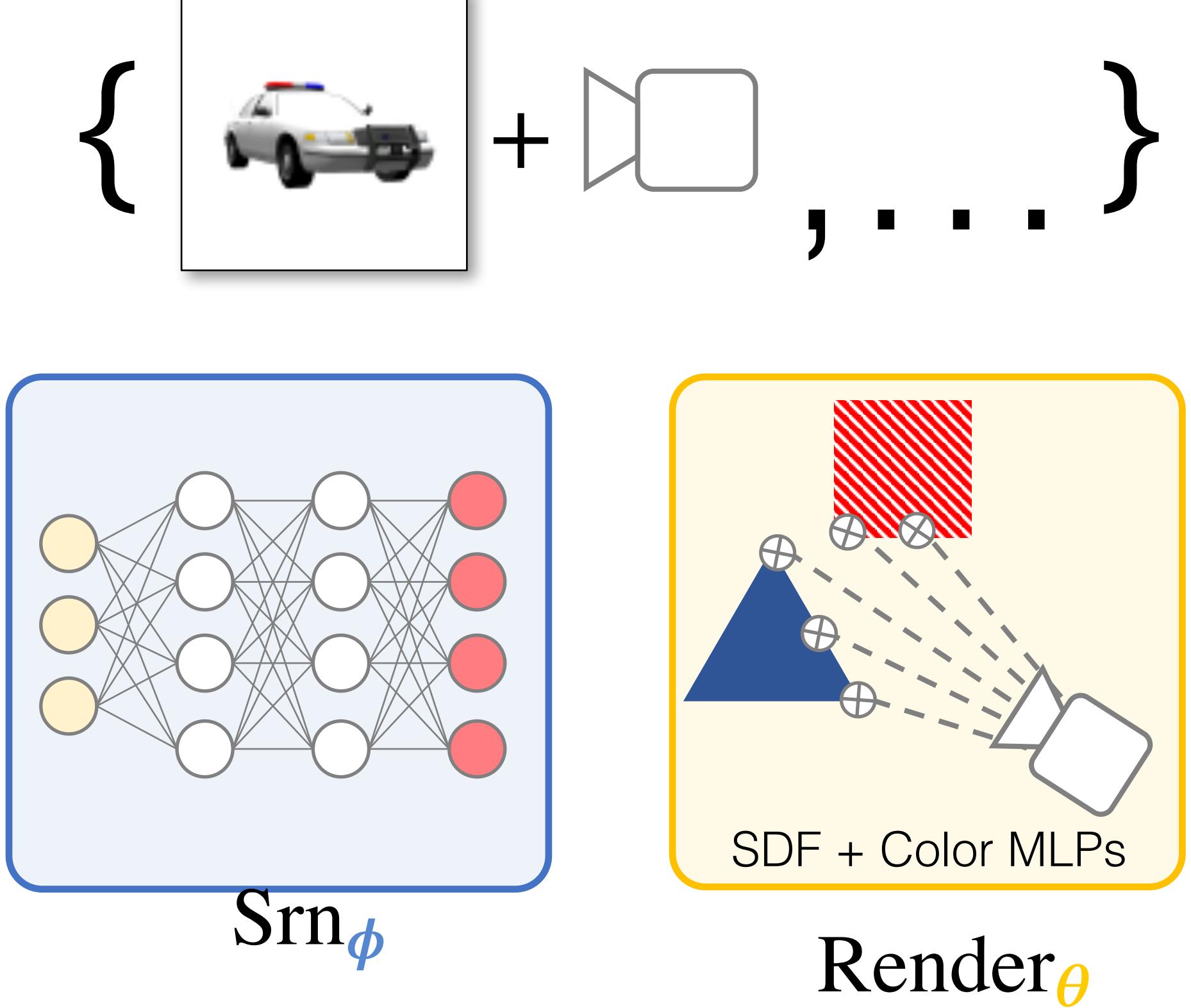
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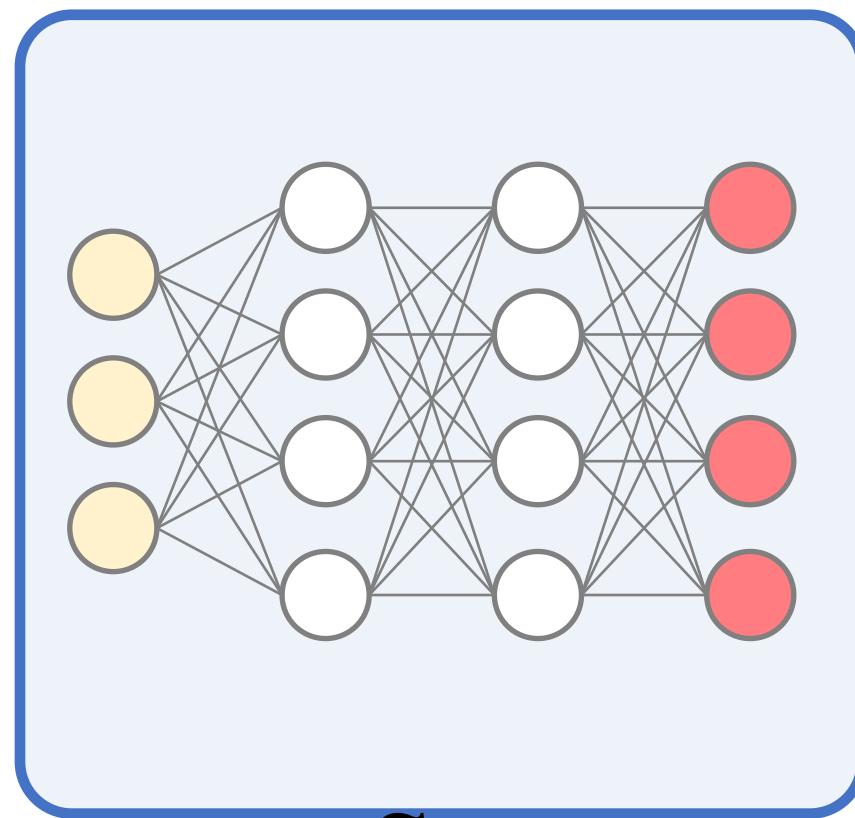
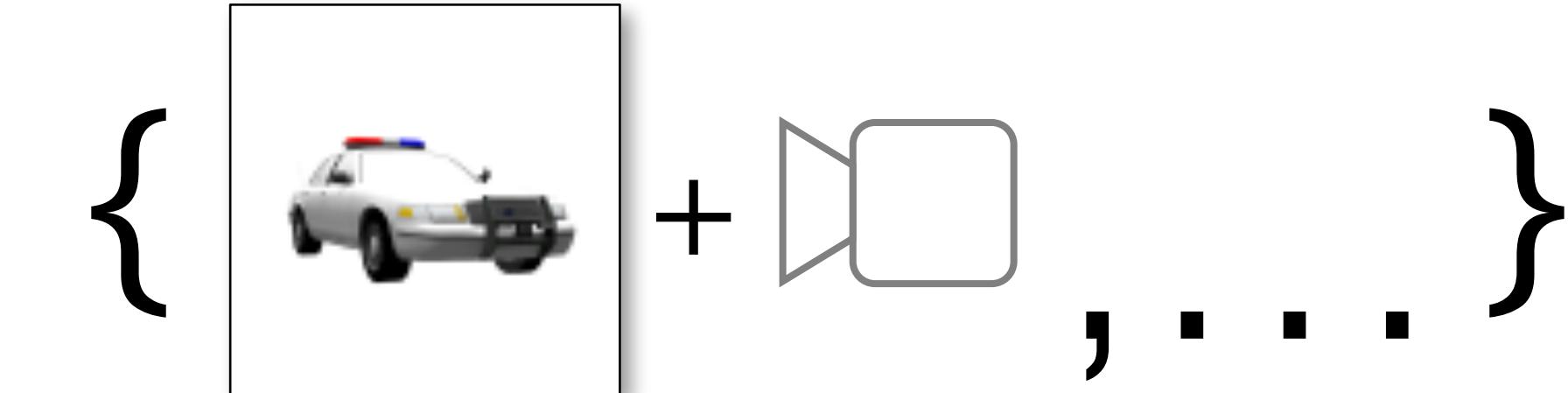


Overfitting case: Inference = Fitting via Gradient Descent Optimization

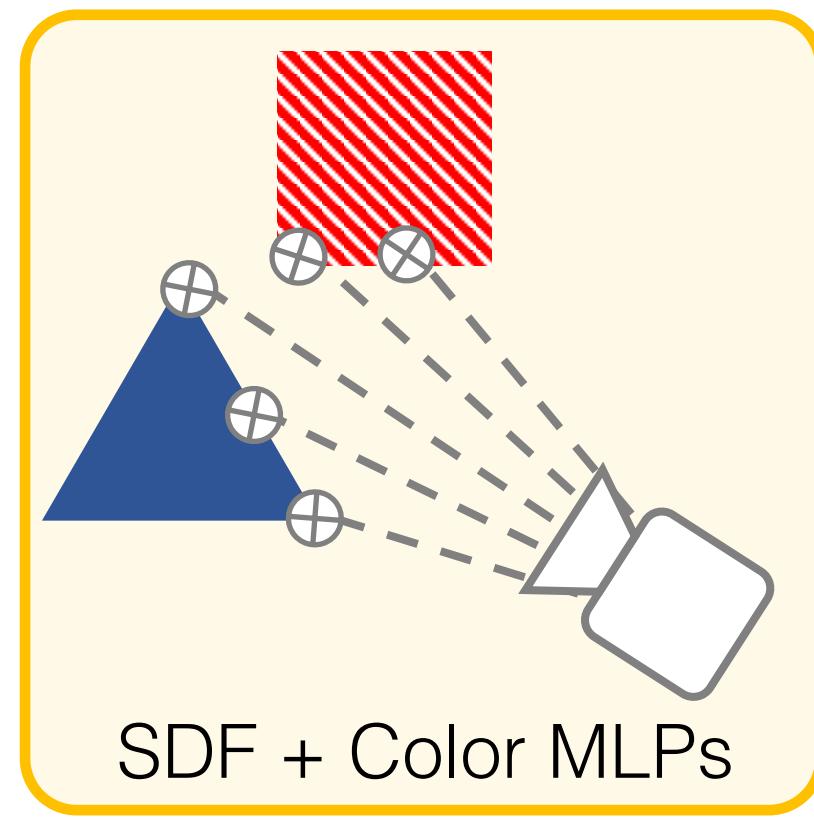


$$\min \left\| \text{Render}_\theta(\text{Srn}_\phi, \xi_i) - \mathcal{I}_i \right\|$$

Overfitting case: Inference = Fitting via Gradient Descent Optimization



Srn_ϕ

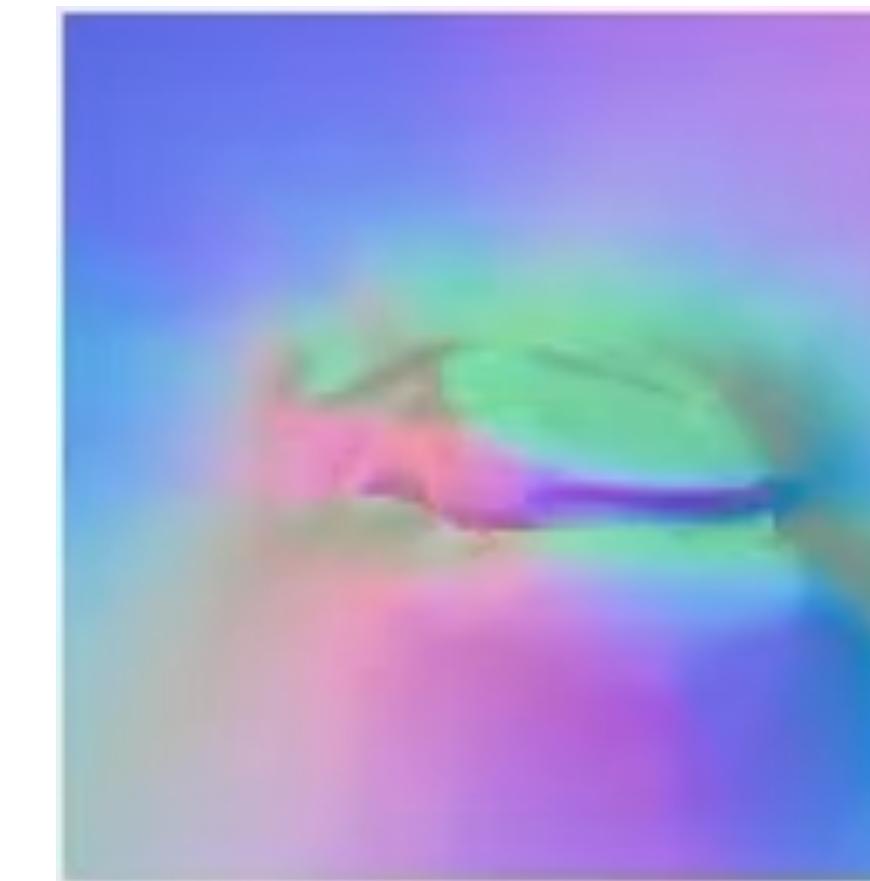


Render $_\theta$

$$\min \left\| \text{Render}_\theta(\text{Srn}_\phi, \xi_i) - \mathcal{I}_i \right\|$$



Novel View Synthesis

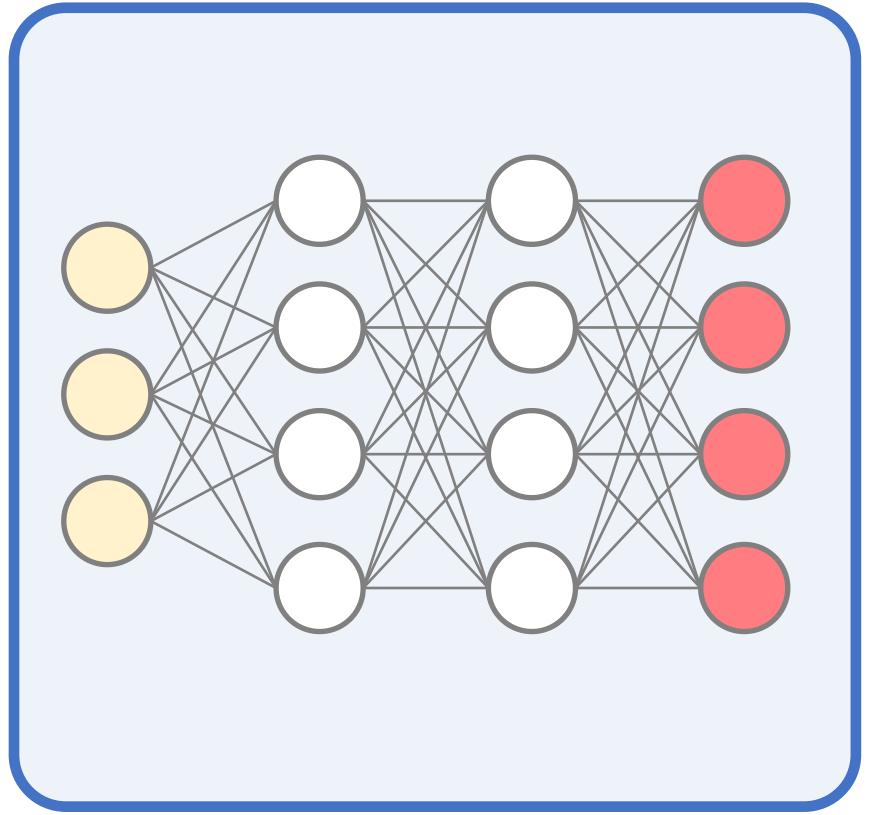


Normal map

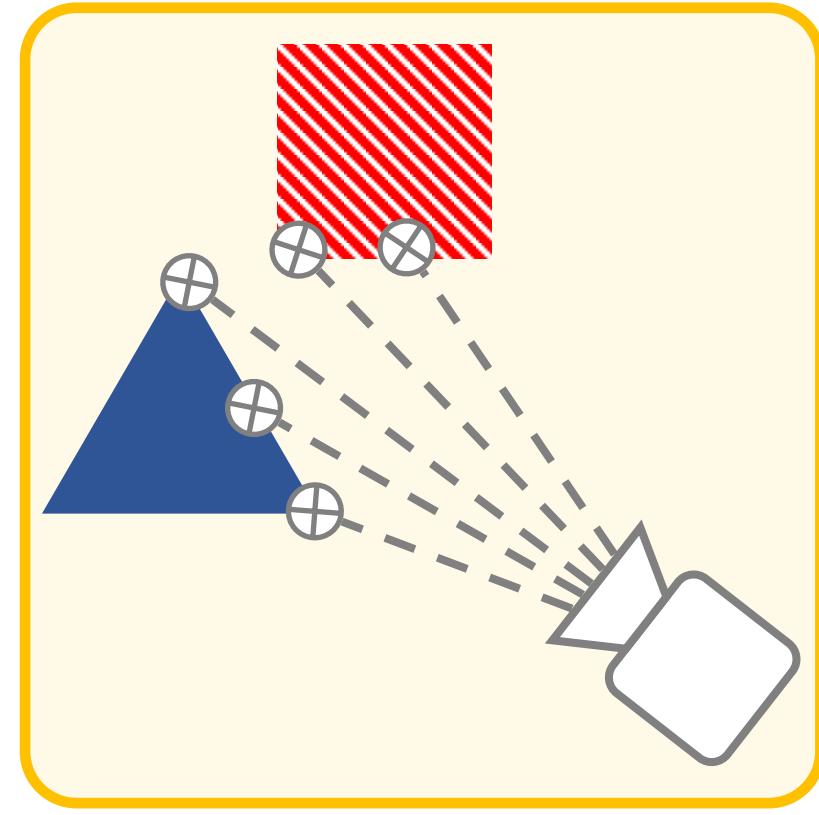


RGB

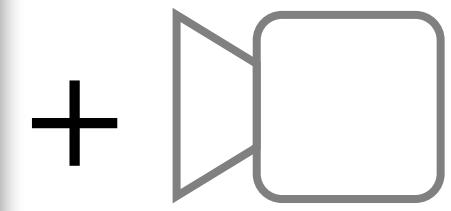
What if observations don't constrain scene representation?



Srn_ϕ



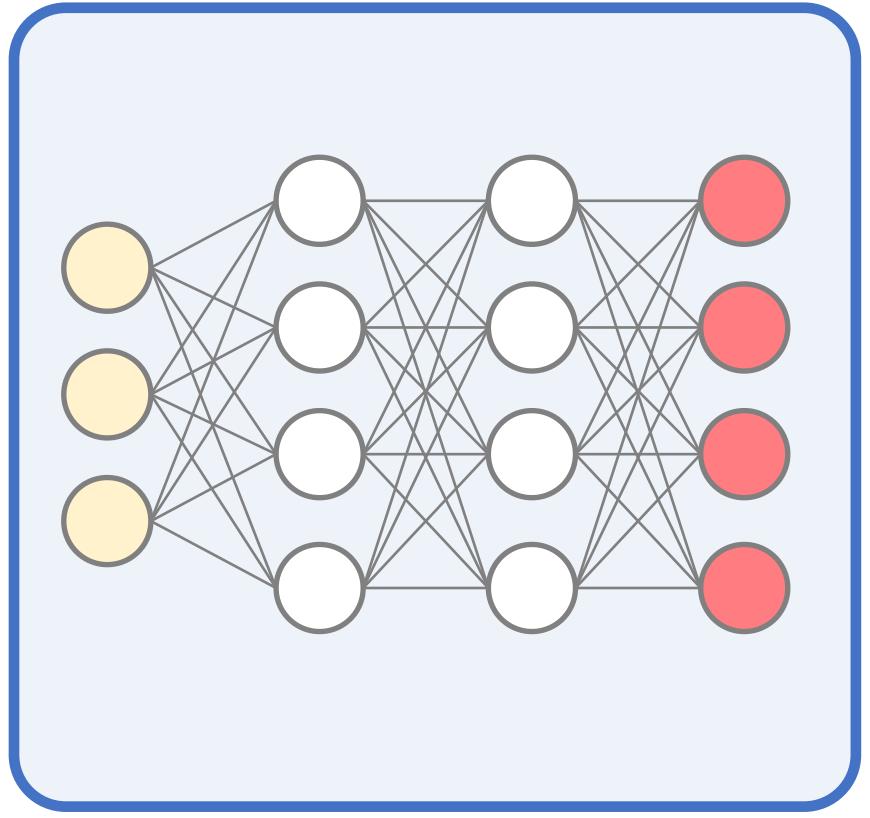
Render_θ



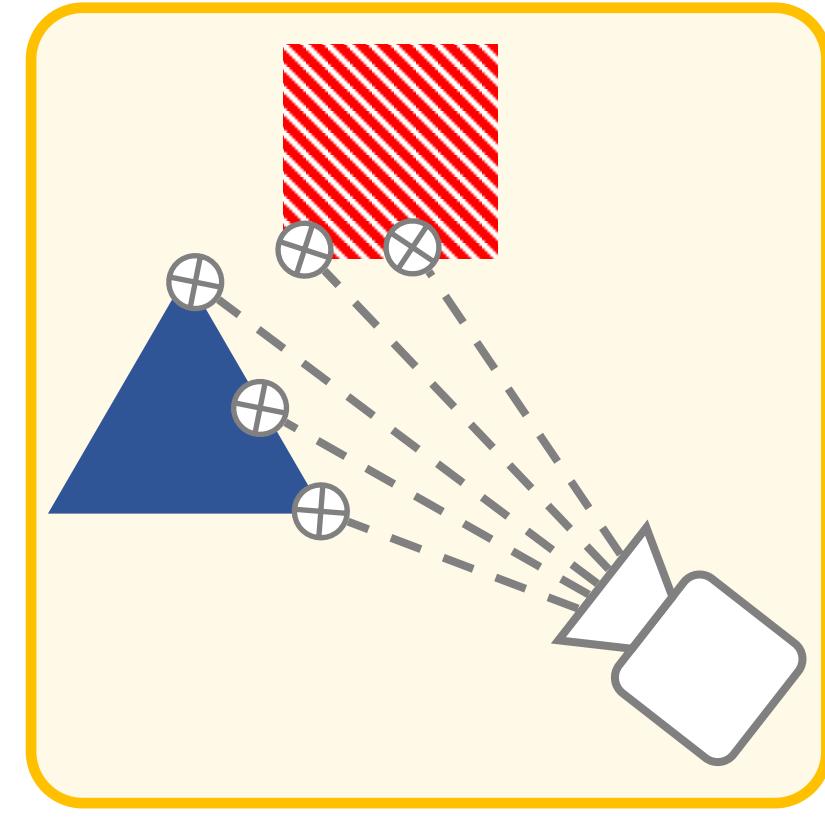
(\mathcal{I}, ξ)

$$\underset{\phi, \theta}{\operatorname{argmin}} \| \text{Render}_\theta(\text{Srn}_\phi, \xi) - \mathcal{I} \|$$

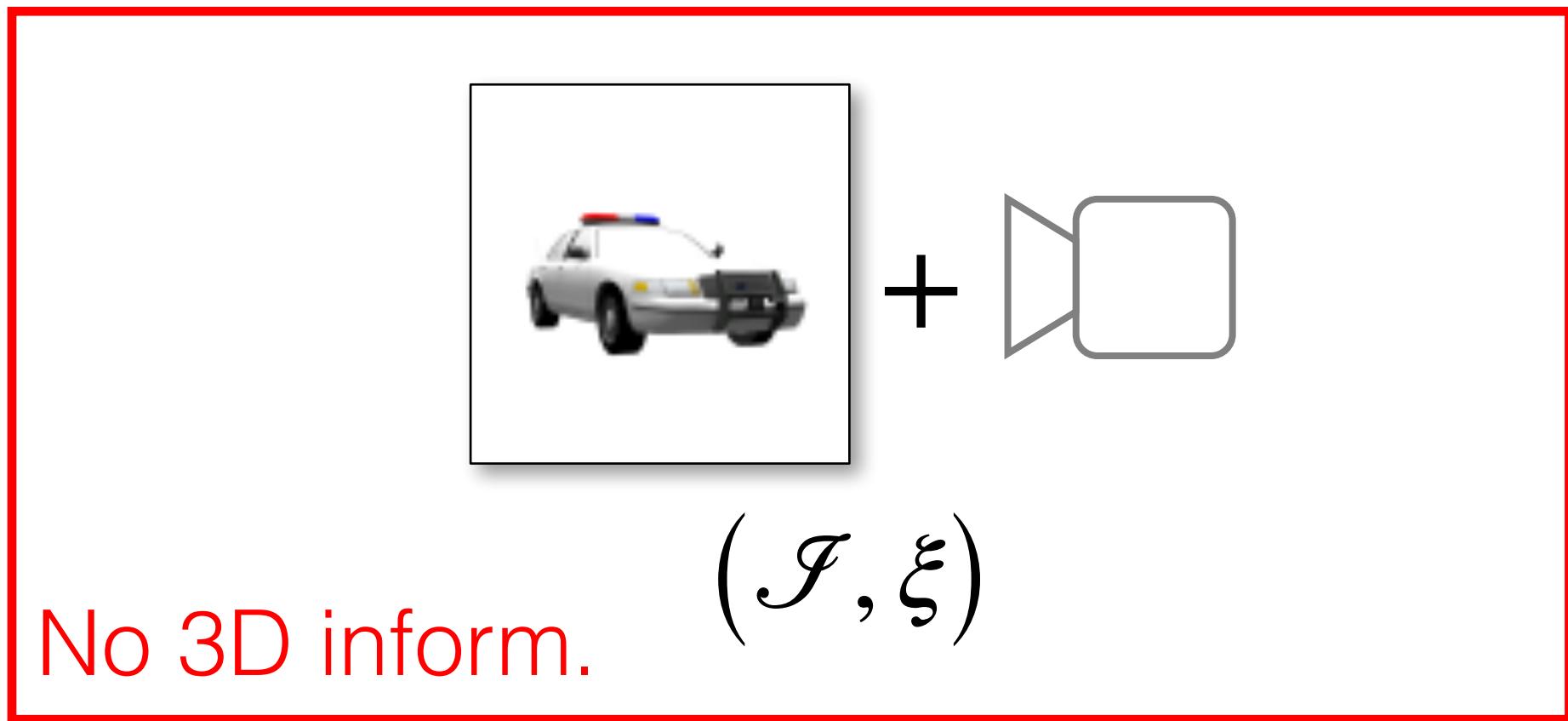
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Srn_ϕ

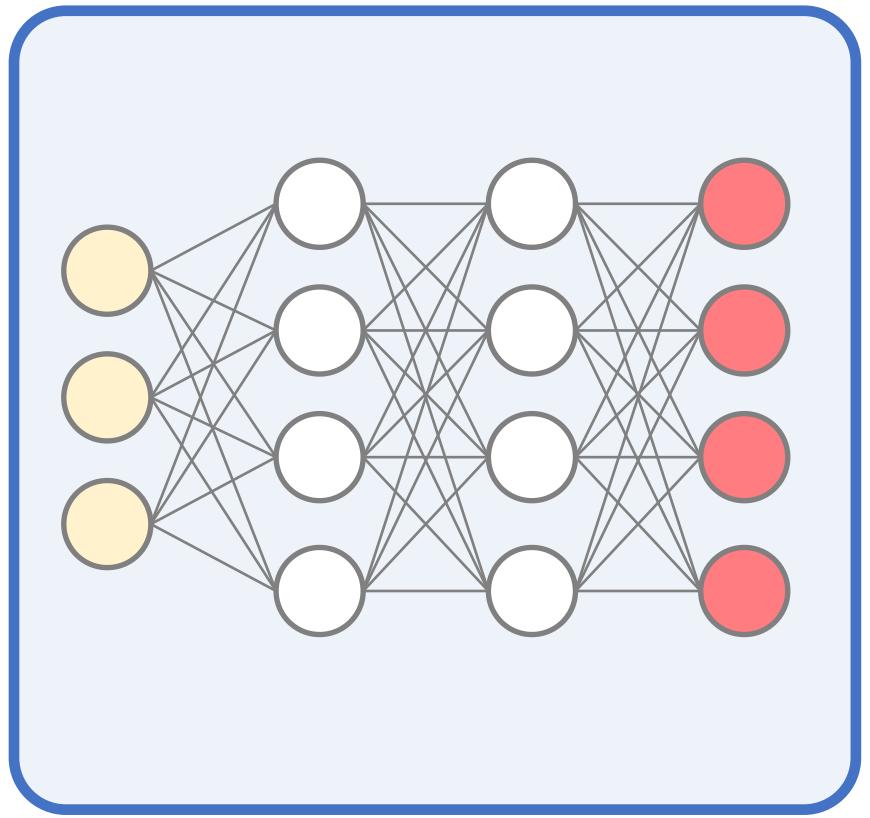


Render_θ

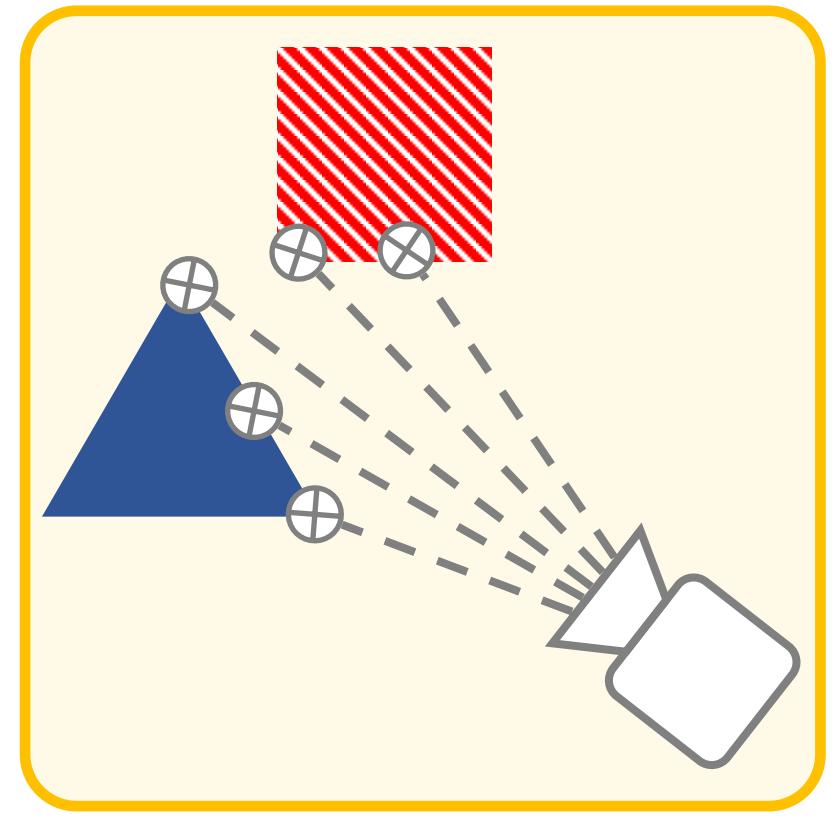


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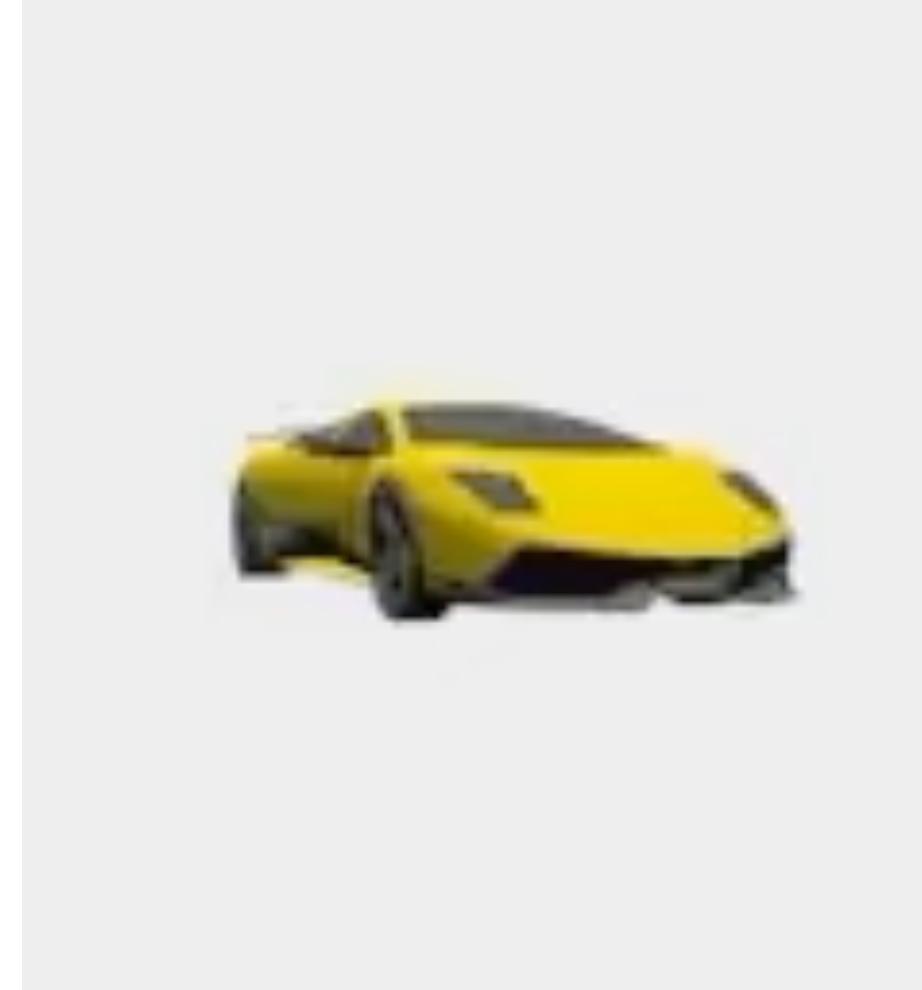
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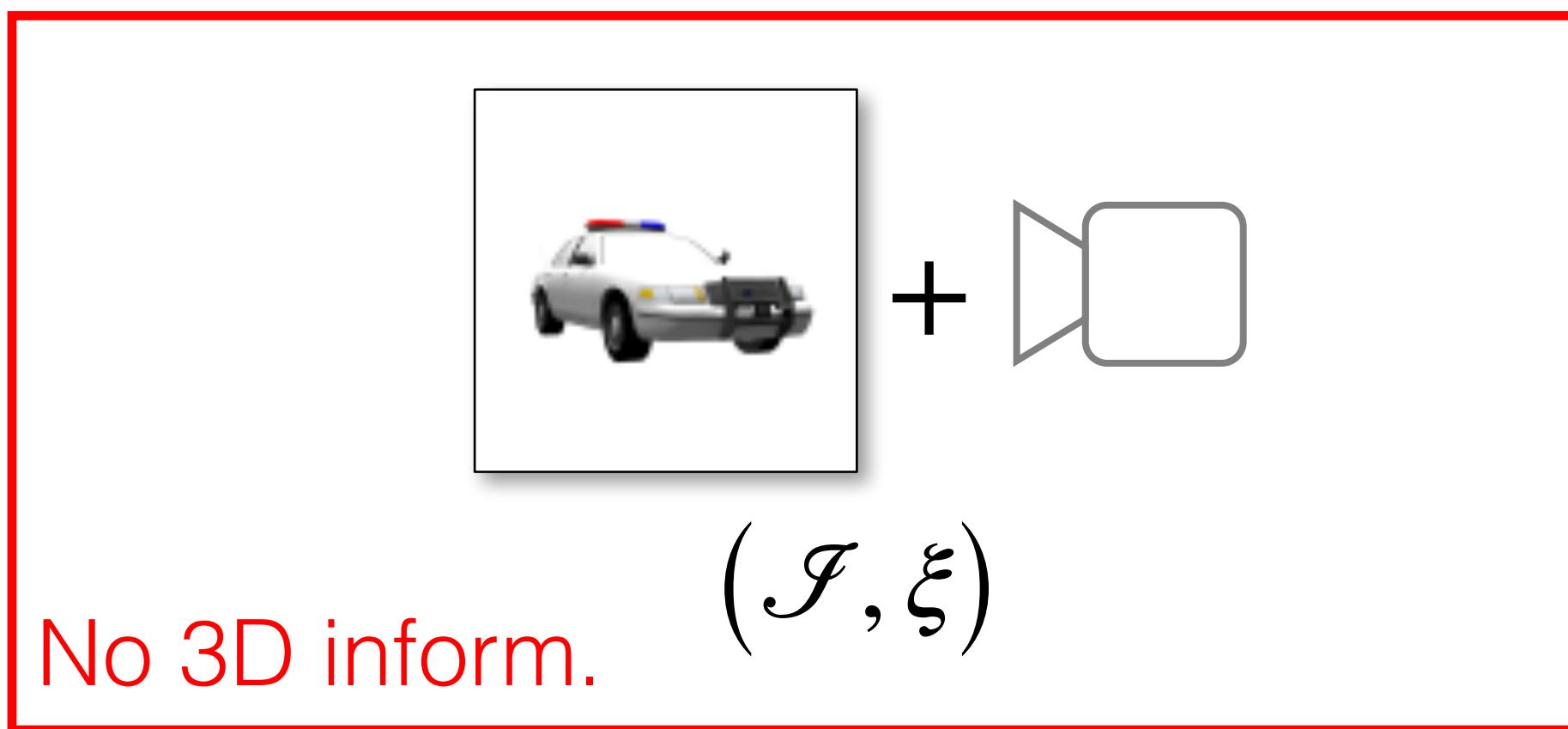
Srn_ϕ



Render_θ

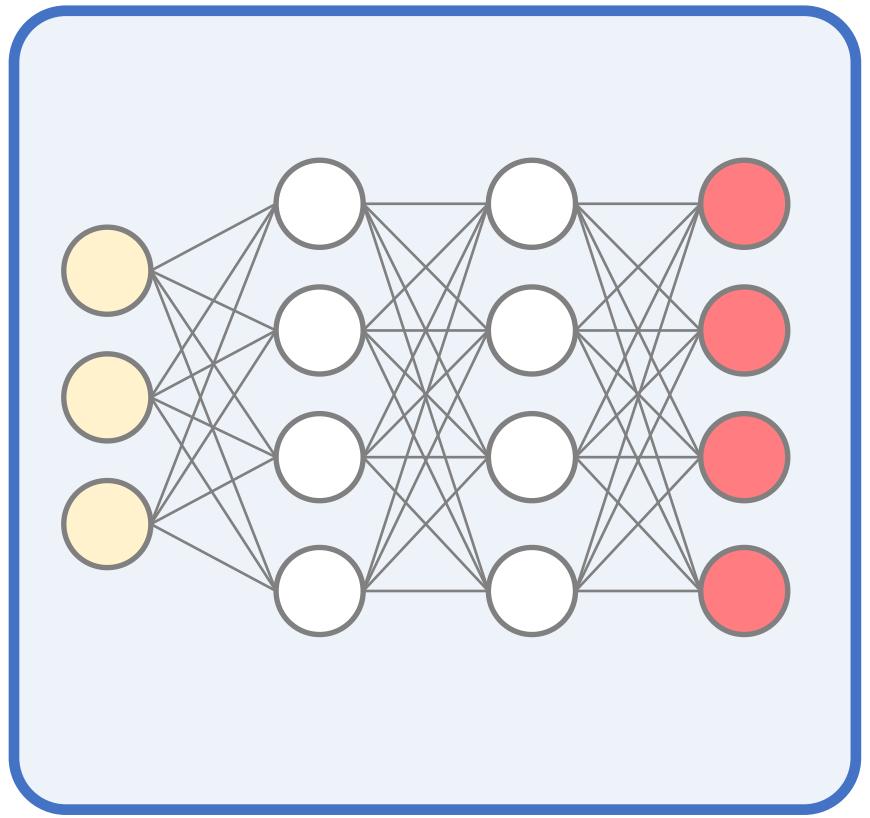


Input

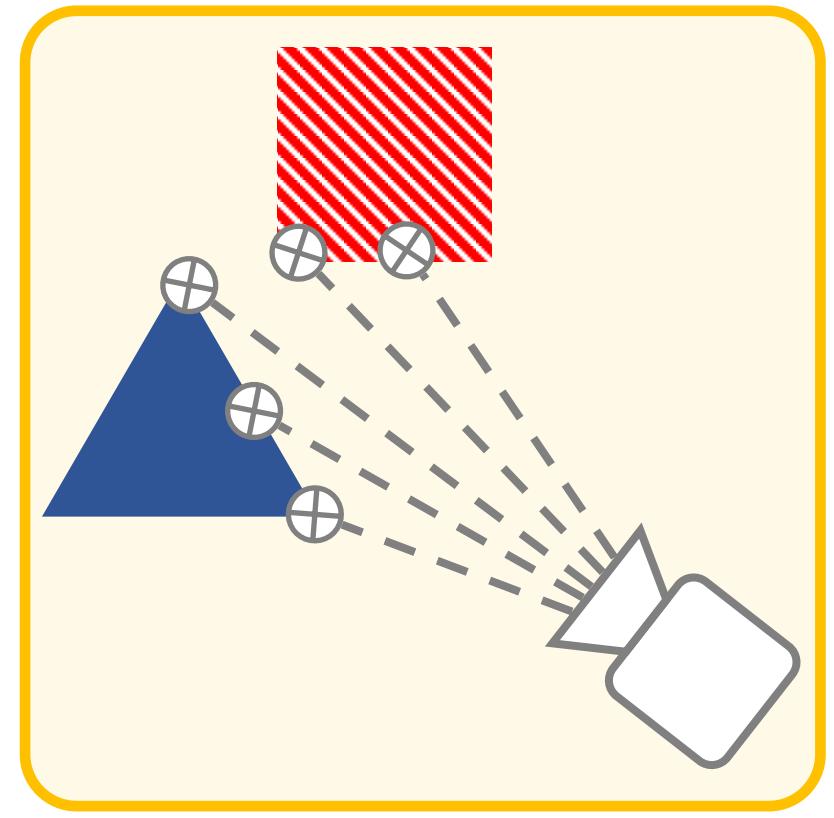


$$\operatorname{argmin}_{\phi, \theta} \| \text{Render}_\theta(\text{Srn}_\phi, \xi) - \mathcal{I} \|$$

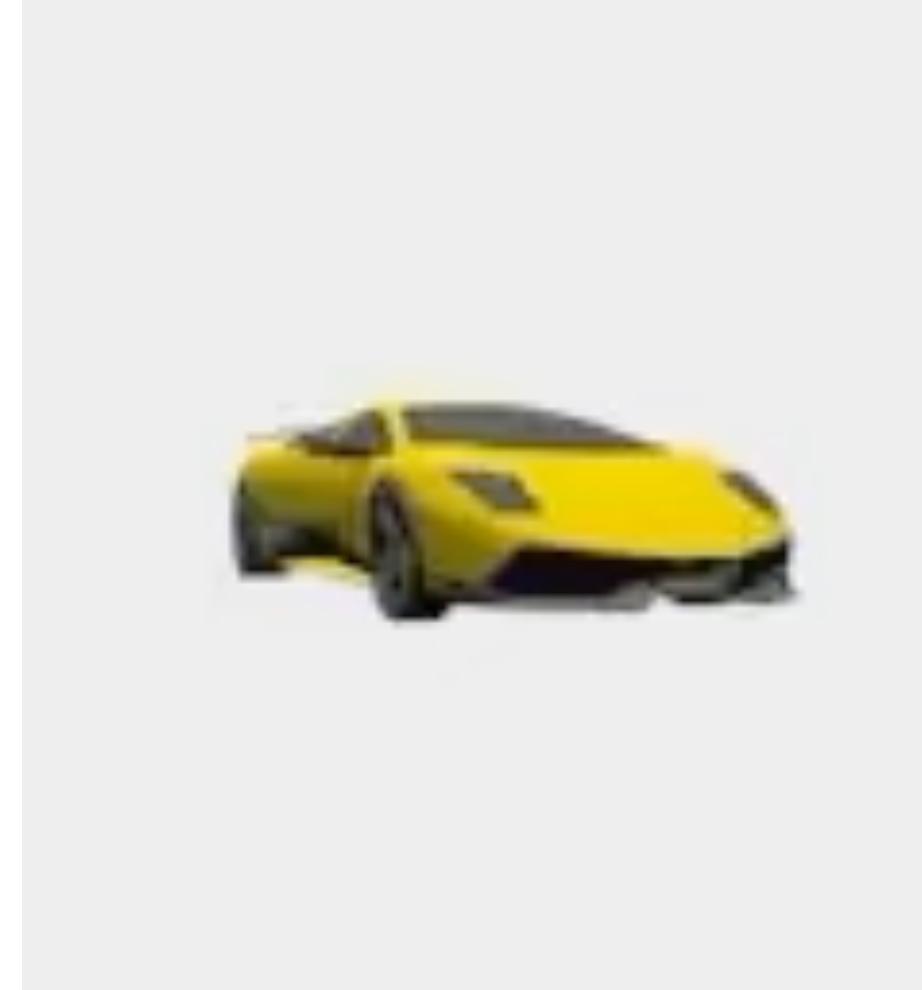
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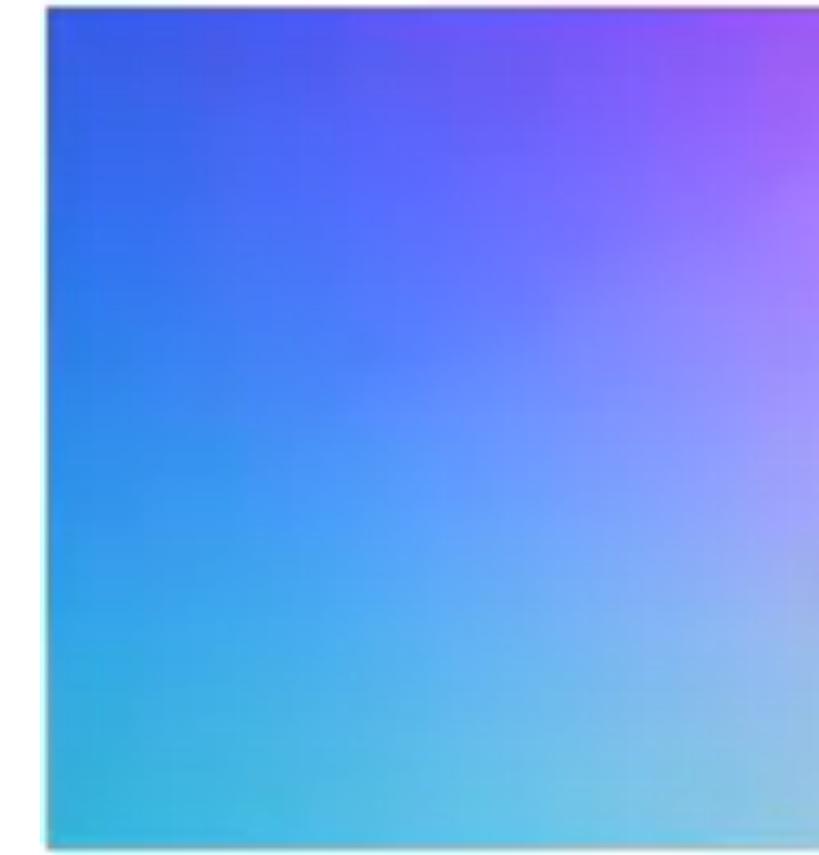
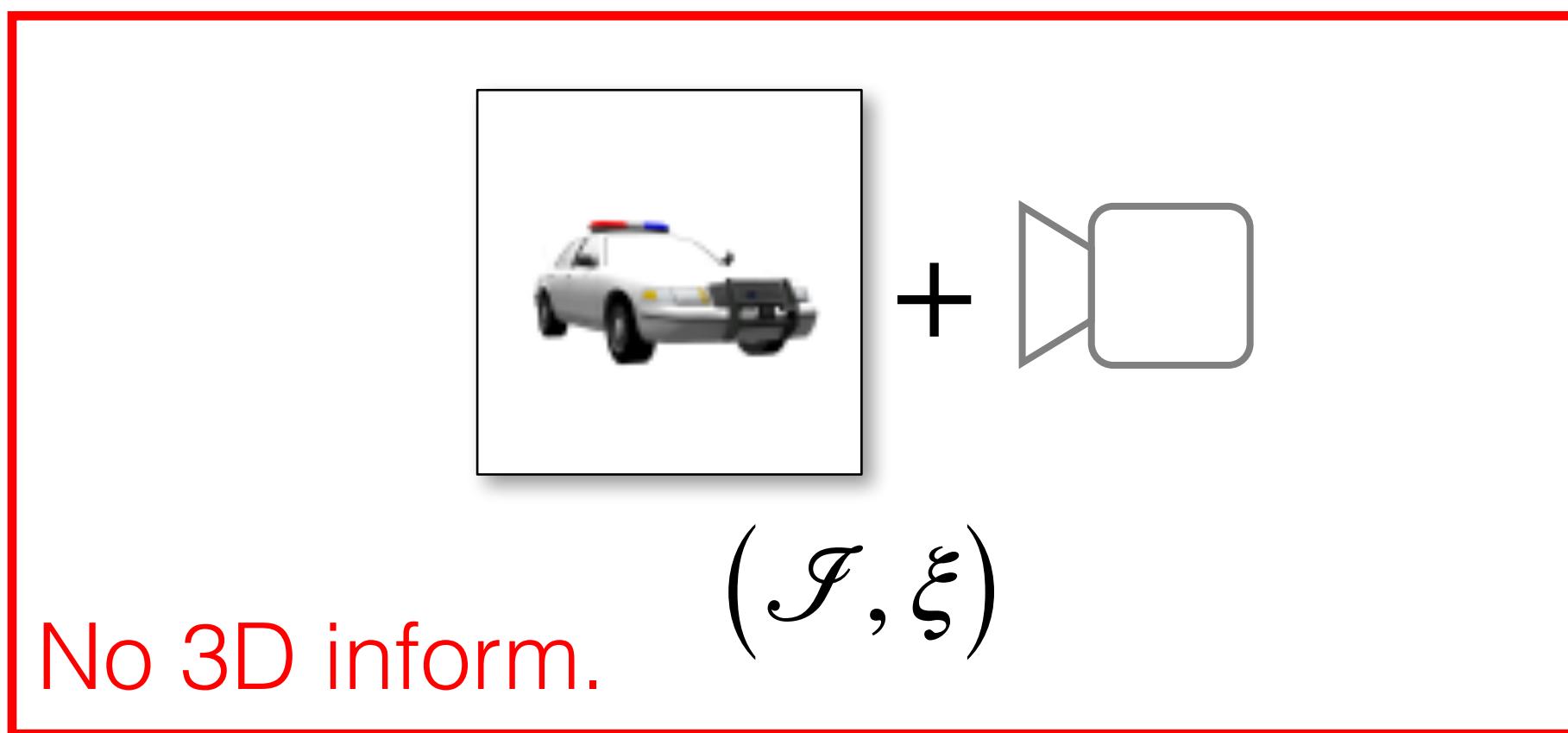
Srn_ϕ



Render_θ



Input



Normal map



GT

$$\operatorname{argmin}_{\phi, \theta} \| \text{Render}_\theta(\text{Srn}_\phi, \xi) - \mathcal{I} \|$$

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