Assignment 4

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1 Question 1

Firstly we know the approximated eigenvalue $\tilde{\lambda} = 0.9$. The eigenvector $\tilde{x}$ where $\|x\|_2 = 1$ and $\|(A - \tilde{\lambda}I)\tilde{x}\|_2 \leq 10^{-2}$. Then by the theorem about eigenvalue perturbation. $\|(A - \tilde{\lambda}I)\tilde{x}\|_2 \leq 10^{-2}$ and $\|x\|_2 = 1 \implies \|\Delta A\|_2 \leq 10^{-2}$ where ΔA is the **absolute backward error**.
We also know

$$|\delta\lambda| \leq \frac{\|x\|\|y\|}{|x^H y|}\|\Delta A\|$$

. Therefore, for this question, we know the **absolute forward error** is bounded above by

$$|\delta\lambda| \leq \frac{\|y\|}{|x^H y|}10^{-2}$$

where y is the left eigenvector.
Remark: We haven't been given enough information to determine the value of the conditional number, therefore, that is the best result we can get.
If we have a real symmetric matrix A, then the hermitian form of A is the same as A. Moreover, the left eigenvectors are identical to the right eigenvectors. Since x=y, then we have

$$|\delta\lambda| \leq 10^{-2}$$

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2 Question 2

The SVD of A is given by $A = U\Sigma V^T$. Then H can be expressed as: $\begin{pmatrix} 0 & V\Sigma U^T \\ U\Sigma V^T & 0 \end{pmatrix}$
Using the hint in the question we know if σ is a singular value of A then $\pm\sigma$ are eigenvalues of H.
If Σ is a n by n matrix containing eigenvalues of A. Then

$$\begin{pmatrix} \Sigma & \\ & -\Sigma \end{pmatrix}$$

is the eigenvalues of H. Then if we want to find the eigenvalue of H. We can simply solve the following equation: (let $P = \begin{pmatrix} P_1 & P_2 \\ P_3 & P_4 \end{pmatrix}$)

$$\begin{pmatrix} 0 & V\Sigma U^T \\ U\Sigma V^T & 0 \end{pmatrix}\begin{pmatrix} P_1 & P_2 \\ P_3 & P_4 \end{pmatrix} = \begin{pmatrix} P_1 & P_2 \\ P_3 & P_4 \end{pmatrix}\begin{pmatrix} \Sigma & \\ & -\Sigma \end{pmatrix}$$

where $P_1,\ P_2,\ P_3,\ P_4$ are the eigenvectors of A.
By comparing the LHS and RHS of the equation. We obtain the following simultaneous equation:

$$\begin{cases} VU^TP_3 = P_1 \\ UV^TP_1 = P_3 \\ VU^TP_4 = -P_2 \\ UV^TP_2 = -P_4 \end{cases}$$

Therefore we get

$$\begin{pmatrix} P_1 & P_2 \\ P_3 & P_4 \end{pmatrix} = \begin{pmatrix} V & -V \\ U & U \end{pmatrix}$$