5 Question 4b

5.1 Part 1

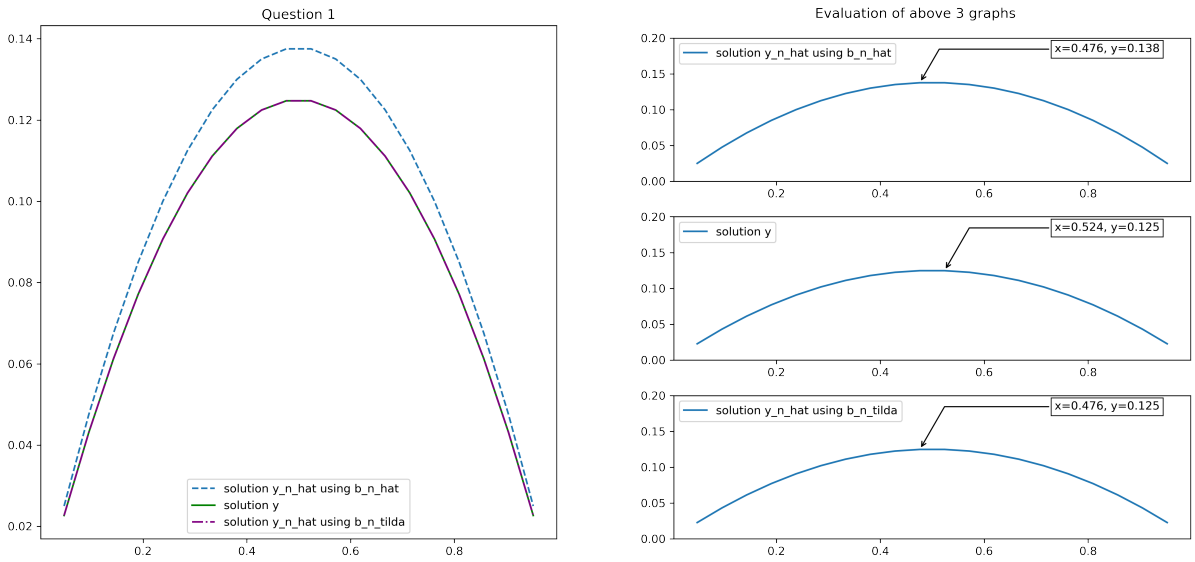

Figure 1: Part 1

5.2 Part 2

Here is the numerical result ($\hat{e}_n$) I got for $n \in \{20, 40, 80, 160\}$.

1. When $n = 20$, we get: en value when n=20 is [[-0.00232426] [-0.0044161] [-0.00627551] [-0.00790249] [-0.00929705] [-0.01045918] [-0.01138889] [-0.01208617] [-0.01255102] [-0.01278345] [-0.01278345] [-0.01255102] [-0.01208617] [-0.01138889] [-0.01045918] [-0.00929705] [-0.00790249] [-0.00627551] [-0.0044161] [-0.00232426]]

2. When $n = 40$, we get: en value when n=40 is [[-0.00060232] [-0.00117452] [-0.00171661] [-0.00222858] [-0.00271044] [-0.00316218] [-0.0035838] [-0.00397531] [-0.0043367] [-0.00466798] [-0.00496914] [-0.00524018] [-0.00548111] [-0.00569192] [-0.00587262] [-0.0060232] [-0.00614366] [-0.00623401] [-0.00629424] [-0.00632436] [-0.00632436] [-0.00629424] [-0.00623401] [-0.00614366] [-0.0060232] [-0.00587262] [-0.00569192] [-0.00548111] [-0.00524018] [-0.00496914] [-0.00466798] [-0.0043367] [-0.00397531] [-0.0035838] [-0.00316218] [-0.00271044] [-0.00222858] [-0.00171661] [-0.00117452] [-0.00060232]]

3. When $n = 80$, we get: en value when n=80 is [[-0.00015337] [-0.0003029] [-0.0004486] [-0.00059047] [-0.0007285] [-0.0008627] [-0.00099306] [-0.00111959] [-0.00124228] [-0.00136114] [-0.00147617] [-0.00158736] [-0.00169472] [-0.00179824] [-0.00189793] [-0.00199379] [-0.00208581] [-0.002174] [-0.00225835] [-0.00233887] [-0.00241555] [-0.0024884] [-0.00255742] [-0.0026226] [-0.00268395] [-0.00274146] [-0.00279514] [-0.00284498] [-0.00289099] [-0.00293317] [-0.00297151] [-0.00300602] [-0.00303669] [-0.00306353] [-0.00308654] [-0.00310571] [-0.00312105] [-0.00313255] [-0.00314022] [-0.00314405] [-0.00314405] [-0.00314022] [-0.00313255] [-0.00312105] [-0.00310571] [-0.00308654] [-0.00306353] [-0.00303669] [-0.00300602] [-0.00297151] [-0.00293317] [-0.00289099] [-0.00284498] [-0.00279514] [-0.00274146] [-0.00268395] [-0.0026226] [-0.00255742] [-0.0024884] [-0.00241555] [-0.00233887] [-0.00225835] [-0.002174] [-0.00208581] [-0.00199379] [-0.00189793] [-0.00179824] [-0.00169472] [-0.00158736] [-0.00147617] [-0.00136114] [-0.00124228] [-0.00111959] [-0.00099306] [-0.0008627] [-0.0007285] [-0.00059047] [-0.0004486] [-0.0003029] [-0.00015337]]

4. When $n = 160$, we get: en value when n=160 is [[-3.86993172e-05] [-7.69148928e-05] [-1.14646727e-04] [-1.51894820e-04] [-1.88659171e-04] [-2.24939781e-04] [-2.60736649e-04] [-2.96049776e-04] [-3.30879162e-04] [-3.65224806e-04] [-3.99086708e-04] [-4.32464869e-04] [-4.65359289e-04] [-4.97769967e-04] [-5.29696904e-04] [-5.61140099e-04] [-5.92099552e-04] [-6.22575265e-04] [-6.52567236e-04] [-6.82075465e-04] [-7.11099953e-04] [-7.39640699e-04] [-7.67697704e-04] [-7.95270968e-04] [-8.22360490e-04] [-8.48966270e-04] [-8.75088309e-04] [-9.00726607e-04] [-9.25881163e-04] [-9.50551978e-04] [-9.74739051e-04] [-9.98442383e-04] [-1.02166197e-03] [-1.04439782e-03] [-1.06664993e-03] [-1.08841830e-03] [-1.10970292e-03] [-1.13050380e-03] [-1.15082094e-03]

[-1.17065434e-03] [-1.19000400e-03] [-1.20886992e-03] [-1.22725210e-03] [-1.24515053e-03] [-1.26256522e-03]
[-1.27949617e-03] [-1.29594338e-03] [-1.31190685e-03] [-1.32738658e-03] [-1.34238256e-03] [-1.35689481e-03]
[-1.37092331e-03] [-1.38446807e-03] [-1.39752909e-03] [-1.41010637e-03] [-1.42219991e-03] [-1.43380970e-03]
[-1.44493575e-03] [-1.45557807e-03] [-1.46573664e-03] [-1.47541147e-03] [-1.48460255e-03] [-1.49330990e-03]
[-1.50153351e-03] [-1.50927337e-03] [-1.51652949e-03] [-1.52330187e-03] [-1.52959051e-03] [-1.53539541e-03]
[-1.54071656e-03] [-1.54555398e-03] [-1.54990765e-03] [-1.55377758e-03] [-1.55716377e-03] [-1.56006622e-03]
[-1.56248493e-03] [-1.56441990e-03] [-1.56587112e-03] [-1.56683860e-03] [-1.56732234e-03] [-1.56732234e-03]
[-1.56683860e-03] [-1.56587112e-03] [-1.56441990e-03] [-1.56248493e-03] [-1.56006622e-03] [-1.55716377e-03]
[-1.55377758e-03] [-1.54990765e-03] [-1.54555398e-03] [-1.54071656e-03] [-1.53539541e-03] [-1.52959051e-03]
[-1.52330187e-03] [-1.51652949e-03] [-1.50927337e-03] [-1.50153351e-03] [-1.49330990e-03] [-1.48460255e-03]
[-1.47541147e-03] [-1.46573664e-03] [-1.45557807e-03] [-1.44493575e-03] [-1.43380970e-03] [-1.42219991e-03]
[-1.41010637e-03] [-1.39752909e-03] [-1.38446807e-03] [-1.37092331e-03] [-1.35689481e-03] [-1.34238256e-03]
[-1.32738658e-03] [-1.31190685e-03] [-1.29594338e-03] [-1.27949617e-03] [-1.26256522e-03] [-1.24515053e-03]
[-1.22725210e-03] [-1.20886992e-03] [-1.19000400e-03] [-1.17065434e-03] [-1.15082094e-03] [-1.13050380e-03]
[-1.10970292e-03] [-1.08841830e-03] [-1.06664993e-03] [-1.04439782e-03] [-1.02166197e-03] [-9.98442383e-04]
[-9.74739051e-04] [-9.50551978e-04] [-9.25881163e-04] [-9.00726607e-04] [-8.75088309e-04] [-8.48966270e-04]
[-8.22360490e-04] [-7.95270968e-04] [-7.67697704e-04] [-7.39640699e-04] [-7.11099953e-04] [-6.82075465e-04]
[-6.52567236e-04] [-6.22575265e-04] [-5.92099552e-04] [-5.61140099e-04] [-5.29696904e-04] [-4.97769967e-04]
[-4.65359289e-04] [-4.32464869e-04] [-3.99086708e-04] [-3.65224806e-04] [-3.30879162e-04] [-2.96049776e-04]
[-2.60736649e-04] [-2.24939781e-04] [-1.88659171e-04] [-1.51894820e-04] [-1.14646727e-04] [-7.69148928e-05]
[-3.86993172e-05]]

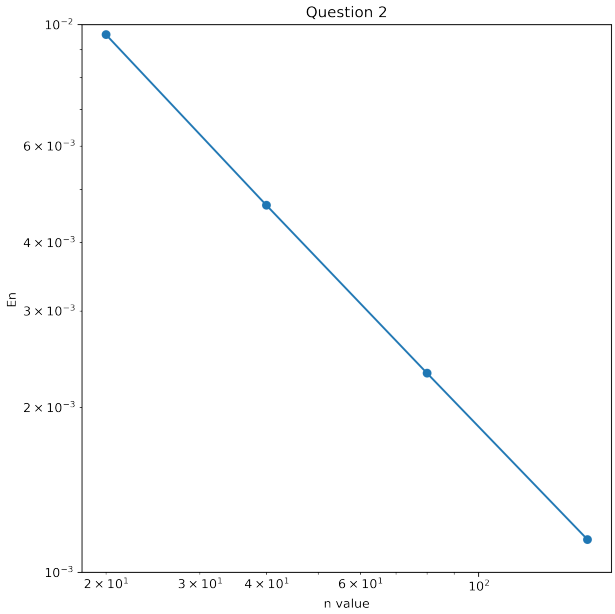

Figure 2: Log scale,Part 2.

Figure 2 shows we have a logarithm decay. We can conclude from the graph that the gradient of the graph is -1. Therefore it is $\mathcal{O}(n^{-1})$

```
Here is En for different n: [0.00958797240336126, 0.004678819260754207, 0.0023107490859802598, 0.0011482260364562053]
The gradient of the plot is: -1.020959169786405
```

Figure 3: En and gradient

5.3 Part 3

Figure 5 shows we have a logarithm growth. We can find out the gradient of the graph is approximately 2 which means $\mathcal{O}(n^2)$

```
Here is the conditional number for different n: [178.06427461086017, 680.6170700217076, 2658.4065019157188, 10504.718944451404]
The gradient of the plot is: 1.960230060929547
```

Figure 4: condition number and gradient

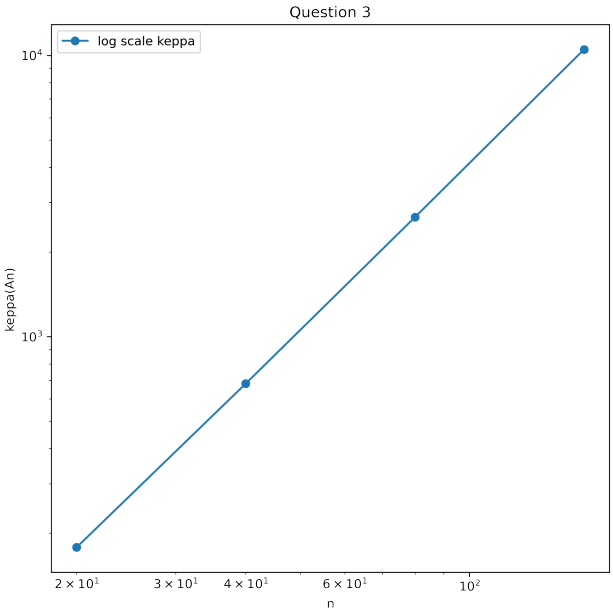

Figure 5: Conditional number.