Tests of the Matching Algorithm

Nathan Carter
May 2019

These tests are taken from the test suite for an old algorithm I wrote. In that repository's source files, they are much more difficult to read, so they've been typed up here to make them easier to read and understand.

The notation $P(\!|x|\!)$ is used to indicate the application of a function P (mapping metavariables to expressions) to a metavariable x. It is not the same as $P(x)$, which is the notation we use for the ordinary function application operation. Thus for example if P and x are both variables, we would write $P(x)$, but if P is not an element of syntax, but a function from metavariables to expressions, we would write $P(\!|x|\!)$.

We will write infix operators and understand the regular conversion to prefix form is assumed internally. That is, $2{+}3 = 5$ means $equal(plus(2,3),5)$.

Unless otherwise stated, every variable in the pattern is a metavariable. Symbols like $=$, $+$, and so on are not variables. Note that in the next version of matching algorithm, we do not expect to mark bound variables as metavariables, so if using this document to test that algorithm, you can safely ignore the instantiations of any metavariables.

Small tests

Test 1.

Pattern:	**Instance:**
$P(\!\|x\|\!)$	$b(2)$
$P(\!\|y\|\!)$	$b(3)$
Results (2 solutions):	
$x : 2$	$x : b(2)$
$y : 3$	$y : b(3)$
$P : \lambda v.b(v)$	$P : \lambda v.v$

Test 2.

Pattern:	**Instance:**
$P(\!\|x\|\!)$	$2 + 3 = 5$
$P(\!\|y\|\!)$	$5 = 5$

Results (2 solutions):

$x : 2 + 3$	$x : 2 + 3 = 5$
$y : 5$	$y : 5 = 5$
$P : \lambda v.v = 5$	$P : \lambda v.v$

Test 3.

Pattern:	**Instance:**	**Result:**
$P(\!\lvert x \rvert\!)$	$A(1, 2, 3)$	$x : A(1, 2, 3)$
$P(\!\lvert y \rvert\!)$	$A(2, 1, 3)$	$y : A(2, 1, 3)$
		$P : \lambda v.v$

Test 4.

Pattern:	**Instance:**	**Result:**
$P(\!\lvert x \rvert\!)$	$A(1, 2, 3)$	
$P(\!\lvert x \rvert\!)$	$A(2, 1, 3)$	no solutions

Test 5.

Pattern:	**Instance:**
$P(\!\lvert x \rvert\!)$	$f(1, 2)$
$P(\!\lvert y \rvert\!)$	$f(1, 2)$

Results (5 solutions):

$x : f$	$x : 1$	$x : 2$
$y : f$	$y : 1$	$y : 2$
$P : \lambda v.v(1, 2)$	$P : \lambda v.f(v, 2)$	$P : \lambda v.f(1, v)$
$x : f(1, 2)$	$x :$ any	
$y : f(1, 2)$	$y :$ any	
$P : \lambda v.v$	$P : \lambda v.f(1, 2)$	

Test 6.

Pattern:	**Instance:**
$P(\!\lvert x \rvert\!)$	$g(k, e(2))$

Results (7 solutions):

$x : g(k, e(2))$ $P : \lambda v.v$	$x : g$ $P : \lambda v.v(k, e(2))$	$x : k$ $P : \lambda v.g(v, e(2))$	$x : e(2)$ $P : \lambda v.g(k, v)$
$x : e$ $P : \lambda v.g(k, v(2))$	$x : 2$ $P : \lambda v.g(k, e(v))$	$x :$ any $P : \lambda v.g(k, e(2))$	

Test 7.

Pattern:	**Instance:**
$P(\!\lvert x \rvert\!)$	$f(a, a)$

Results (6 solutions):

$x : f(a, a)$ $P : \lambda v.v$	$x : f$ $P : \lambda v.v(a, a)$	$x : a$ $P : \lambda v.f(a, v)$
$x : a$ $P : \lambda v.f(v, a)$	$x : a$ $P : \lambda v.f(v, v)$	$x :$ any $P : \lambda v.f(a, a)$

Test 8.

Pattern:	**Instance:**	**Result:**
$P(\!\lvert x \rvert\!)$	$f(a, a)$	$x : b$
x	b	$P : \lambda v.f(a, a)$

Test 9.

Pattern:	**Instance:**
$P(\!\lvert x \rvert\!)$	$f(a, a)$
x	f

Results (2 solutions):

$x : f$ $P : \lambda v.v(a, a)$	$x : f$ $P : \lambda v.f(a, a)$

Test 10.

Pattern:	**Instance:**
$P(\!\lvert a \rvert\!)$	$3 = 3$
$Q(\!\lvert b \rvert\!)$	$5 > 4$

Results (30 solutions):
We organize them in a 5-row, 6-column grid, some of which appears below. The reader will need to figure out what the other rows/columns contain.

$a : 3 = 3$ $P : \lambda v.v$ $b : 5 > 4$ $Q : \lambda v.v$	a : equals $P : \lambda v.v(3,3)$ $b : 5 > 4$ $Q : \lambda v.v$	$\cdots$	a : any $P : \lambda v.3 = 3$ $b : 5 > 4$ $Q : \lambda v.v$
$a : 3 = 3$ $P : \lambda v.v$ $b :>$ $Q : \lambda v.v(5,4)$	a : equals $P : \lambda v.v(3,3)$ $b :>$ $Q : \lambda v.v(5,4)$	$\cdots$	a : any $P : \lambda v.3 = 3$ $b :>$ $Q : \lambda v.v(5,4)$
$\vdots$	$\vdots$		$\vdots$
$a : 3 = 3$ $P : \lambda v.v$ b : any $Q : \lambda v.5 > 4$	a : equals $P : \lambda v.v(3,3)$ b : any $Q : \lambda v.5 > 4$	$\cdots$	a : any $P : \lambda v.3 = 3$ b : any $Q : \lambda v.5 > 4$

Equality elimination rule

Test 11.

Pattern:	**Instance:**	**Result:**
$a = b$	$t = 1$	$a : t$
$P(\!\lvert a \rvert\!)$	$t > 0$	$b : 1$
$P(\!\lvert b \rvert\!)$	$1 > 0$	$P : \lambda v.v > 0$

Test 12.

Pattern:	**Instance:**	
$a = b$	$t = 1$	**Result:**
$P(\!\lvert a \rvert\!)$	$1 > 0$	no solutions
$P(\!\lvert b \rvert\!)$	$t > 0$	

Test 13.

Pattern:	**Instance:**	**Result:**
$a = b$	$t = 1$	$a : t$
$P(\!\lvert a \rvert\!)$	$t + 1 = 2$	$b : 1$
$P(\!\lvert b \rvert\!)$	$1 + 1 = 2$	$P : \lambda v.v + 1 = 2$

Test 14.

Pattern:	**Instance:**	
$a = b$	$t = 1$	**Result:**
$P(\!\lvert a \rvert\!)$	$1 + 1 = 2$	no solutions
$P(\!\lvert b \rvert\!)$	$t + 1 = 2$	

Test 15.

Pattern:	**Instance:**	**Result:**
$a = b$	$1 = 2$	$a : 1$
$P(\!\lvert a \rvert\!)$	$1 + 1 = 2$	$b : 2$
$P(\!\lvert b \rvert\!)$	$2 + 2 = 2$	$P : \lambda v.v + v = 2$

Test 16.

Pattern:	**Instance:**	**Result:**
$a = b$	$1 = 2$	$a : 1$
$P(\!\lvert a \rvert\!)$	$1 + 1 = 2$	$b : 2$
$P(\!\lvert b \rvert\!)$	$2 + 1 = 2$	$P : \lambda v.v + 1 = 2$

Test 17.

Pattern:	**Instance:**	**Result:**
$a = b$	$1 = 2$	$a : 1$
$P(\!\lvert a \rvert\!)$	$1 + 1 = 2$	$b : 2$
$P(\!\lvert b \rvert\!)$	$1 + 2 = 2$	$P : \lambda v.1 + v = 2$

Test 18.

Pattern:	**Instance:**	**Result:**
$a = b$	$1 = 2$	$a : 1$
$P(\!\lvert a \rvert\!)$	$1 + 1 = 2$	$b : 2$
$P(\!\lvert b \rvert\!)$	$1 + 1 = 2$	$P : \lambda v.1 + 1 = 2$

Test 19.

Pattern:	**Instance:**	
$a = b$	$1 = 2$	**Result:**
$P(\!\lvert a \rvert\!)$	$1 + 1 = 2$	no solutions
$P(\!\lvert b \rvert\!)$	$2 + 2 = 1$	

Test 20.

Pattern:	**Instance:**	
$a = b$	$1 = 2$	**Result:**
$P(\!\lvert a \rvert\!)$	$1 + 1 = 2$	no solutions
$P(\!\lvert b \rvert\!)$	$1 + 1 = 1$	

Test 21.

Pattern:	**Instance:**	
$a = b$	$x = y$	**Result:**
$P(\!\lvert a \rvert\!)$	$\exists y.y \neq x$	no solutions
$P(\!\lvert b \rvert\!)$	$\exists y.y \neq y$	

(Although it may seem as though $a : x, b : y, P : \lambda v..\exists y.y \neq v$ could be a solution, computing $P(\!\lvert b \rvert\!)$ would not produce $\exists y.y \neq y$ because that would cause variable capture. Rather, it would produce something like $\exists z.z \neq y$ instead.)

Universal elimination rule

Test 22.

Pattern:	**Instance:**	**Result:**
$\forall x.P(\!\|x\|\!)$	$\forall x.x \geq 0$	$x : x$
$P(\!\|t\|\!)$	$7 \geq 0$	$P : \lambda v.v \geq 0$
		$t : 7$

Test 23.

Pattern:	**Instance:**	**Result:**
$\forall x.P(\!\|x\|\!)$	$\forall x.x \geq 0$	no solutions
$P(\!\|t\|\!)$	$7 \geq 7$	

Test 24.

Pattern:	**Instance:**	**Result:**
$\forall x.P(\!\|x\|\!)$	$\forall x.Q$	$x : x$
$P(\!\|t\|\!)$	Q	$P : \lambda v.Q$
		$t :$ any

Test 25.

Pattern:	**Instance:**	**Result:**
$\forall x.P(\!\|x\|\!)$	$\forall s.s^2 = s$	$x : s$
$P(\!\|t\|\!)$	$1^2 = 1$	$P : \lambda v.v^2 = v$
		$t : 1$

Test 26.

Pattern:	**Instance:**	**Result:**
$\forall x.P(\!\|x\|\!)$	$\forall x.R(x, y)$	no solutions
$P(\!\|t\|\!)$	$R(x, 3)$	

Test 27.

Pattern:	**Instance:**	**Result:**
$\forall x.P(\!\|x\|\!)$	$\forall x.R(x, y)$	$x : x$
$P(\!\|t\|\!)$	$R(3, y)$	$P : \lambda v.R(v, y)$
		$t : 3$

Test 28.

Pattern:	**Instance:**	**Result:**
$\forall x.P(\!\lvert x \rvert\!)$	$\forall x.R(x,x)$	$x : x$
$P(\!\lvert t \rvert\!)$	$R(3,3)$	$P : \lambda v.R(v,v)$
		$t : 3$

Test 29.

Pattern:	**Instance:**	**Result:**
$\forall x.P(\!\lvert x \rvert\!)$	$\forall x.R(x,x)$	no solutions
$P(\!\lvert t \rvert\!)$	$R(3,x)$	

Test 30.

Pattern:	**Instance:**	**Result:**
$\forall x.P(\!\lvert x \rvert\!)$	$\forall x.R(x,x)$	no solutions
$P(\!\lvert t \rvert\!)$	$R(x,3)$	

Test 31.

Pattern:	**Instance:**	**Result:**
$\forall x.P(\!\lvert x \rvert\!)$	$\forall x.R(x,x)$	$x : x$
$P(\!\lvert t \rvert\!)$	$R(x,x)$	$P : \lambda v.R(v,v)$
		$t : x$

Test 32.

Pattern:	**Instance:**	**Result:**
$\forall x.P(\!\lvert x \rvert\!)$	$\forall s.s + s = r$	no solutions
$P(\!\lvert t \rvert\!)$	$t + s = r$	

Test 33.

Pattern:	**Instance:**	**Result:**
$\forall x.P(\!\lvert x \rvert\!)$	$\forall x.x = x$	$x : x$
$P(\!\lvert t \rvert\!)$	$(P \Leftrightarrow Q) = (P \Leftrightarrow Q)$	$P : \lambda v.v = v$
		$t : P \Leftrightarrow Q$

Test 34.

Pattern:	**Instance:**	**Result:**
$\forall x.P(\!\|x\|\!)$	$\forall x.\exists y.x < y$	no solutions
$P(\!\|t\|\!)$	$\exists y.y < y$	

(See notes above about variable capture.)

Universal introduction rule

Test 35.

Pattern:	**Instance:**	**Result:**
Subproof[x, $P(\!\|x\|\!)$]	Subproof[a, $r(a,a)$]	no solutions
$\forall x.P(\!\|x\|\!)$	$\forall b.r(b,b)$	

NOTE: In the next version of the matching package, we will want to treat α-equivalent expressions as equal, so this test should begin to yield one solution, $x : a$, $P : \lambda v.r(v,v)$.

Test 36.

Pattern:	**Instance:**	**Result:**
Subproof[x, $P(\!\|x\|\!)$]	Subproof[a, $r(a,a)$]	$x : a$
$\forall y.P(\!\|y\|\!)$	$\forall b.r(b,b)$	$P : \lambda v.r(v,v)$
		$y : b$

Test 37.

Pattern:	**Instance:**	**Result:**
Subproof[x, $P(\!\|x\|\!)$]	Subproof[a, $a > 3$]	$x : a$
$\forall y.P(\!\|y\|\!)$	$\forall a.a > 3$	$P : \lambda v.v > 3$
		$y : a$

Test 38.

Pattern:	**Instance:**	**Result:**
$\text{Subproof}[\, x,\, P(\!\|x\|\!)\,]$ $\forall y.P(\!\|y\|\!)$	$\text{Subproof}[\, a,\, a > 3\,]$ $\forall x.x > 3$	$x : a$ $P : \lambda v.v > 3$ $y : x$

Test 39.

Pattern:	**Instance:**	**Result:**
$\text{Subproof}[\, x,\, P(\!\|x\|\!)\,]$ $\forall y.P(\!\|y\|\!)$	$\text{Subproof}[\, T,\, R(T,T)\,]$ $\forall T.R(T,T)$	$x : T$ $P : \lambda v.R(v,v)$ $y : T$

Test 40.

Pattern:	**Instance:**	**Result:**
$\text{Subproof}[\, x,\, P(\!\|x\|\!)\,]$ $\forall y.P(\!\|y\|\!)$	$\text{Subproof}[\, T,\, R(T,T)\,]$ $\forall x.R(T,x)$	no solutions

Test 41.

Pattern:	**Instance:**	**Result:**
$\text{Subproof}[\, x,\, P(\!\|x\|\!)\,]$ $\forall y.P(\!\|y\|\!)$	$\text{Subproof}[\, y,\, 0 \neq 1\,]$ $\forall z.0 \neq 1$	$x : y$ $P : \lambda v.0 \neq 1$ $y : z$

Test 42.

Pattern:	**Instance:**	**Result:**
$\text{Subproof}[\, x,\, P(\!\|x\|\!)\,]$ $\forall y.P(\!\|y\|\!)$	$\text{Subproof}[\, b,\, b - b = 0\,]$ $\forall c.b - c = 0$	no solutions

Test 43.

Pattern:	**Instance:**	**Result:**
$\text{Subproof}[\, x,\, P(\!\|x\|\!)\,]$ $\forall x.P(\!\|x\|\!)$	$\text{Subproof}[\, a,\, a > 3\,]$ $\forall a.a > 3$	$x : a$ $P : \lambda v.v > 3$

Test 44.

Pattern:	**Instance:**	**Result:**
Subproof[x, $P(\!\lvert x \rvert\!)$]	Subproof[a, $a > 3$]	no solutions
$\forall x.P(\!\lvert x \rvert\!)$	$\forall x.x > 3$	

Test 45.

Pattern:	**Instance:**	**Result:**
Subproof[x, $P(\!\lvert x \rvert\!)$]	Subproof[T, $R(T,T)$]	$x : T$
$\forall x.P(\!\lvert x \rvert\!)$	$\forall T.R(T,T)$	$P : \lambda v.R(v,v)$

Test 46.

Pattern:	**Instance:**	**Result:**
Subproof[x, $P(\!\lvert x \rvert\!)$]	Subproof[T, $R(T,T)$]	no solutions
$\forall x.P(\!\lvert x \rvert\!)$	$\forall x.R(T,x)$	

Test 47.

Pattern:	**Instance:**	**Result:**
Subproof[x, $P(\!\lvert x \rvert\!)$]	Subproof[y, $0 \neq 1$]	$x : y$
$\forall x.P(\!\lvert x \rvert\!)$	$\forall y.0 \neq 1$	$P : \lambda v.0 \neq 1$

Test 48.

Pattern:	**Instance:**	**Result:**
Subproof[x, $P(\!\lvert x \rvert\!)$]	Subproof[x, $x = x$]	$x : x$
$\forall y.P(\!\lvert y \rvert\!)$	$\forall x x = x$	$y : x$
		$P : \lambda v.v = v$

Test 49.

Pattern:	**Instance:**	**Result:**
Subproof[x, $P(\!\lvert x \rvert\!)$]	Subproof[x, $\exists y.x < y$]	no solutions
$\forall x.P(\!\lvert x \rvert\!)$	$\forall y.\exists y.y < y$	

(See notes above about variable capture.)

Existential introduction rule

Test 50.

Pattern:	**Instance:**	**Result:**
$P(\!\lvert t \rvert\!)$	$1 > 0$	$P : \lambda v.v > 0$
$\exists x.P(\!\lvert x \rvert\!)$	$\exists x.x > 0$	$t : 1$
		$x : x$

Test 51.

Pattern:	**Instance:**	**Result:**
$P(\!\lvert t \rvert\!)$	$\binom{6}{3} = 20$	$P : \lambda v.\binom{6}{v} = 20$
$\exists x.P(\!\lvert x \rvert\!)$	$\exists n.\binom{6}{n} = 20$	$t : 3$
		$x : n$

Test 52.

Pattern:	**Instance:**	**Result:**
$P(\!\lvert t \rvert\!)$	$t^x < 5$	no solutions
$\exists x.P(\!\lvert x \rvert\!)$	$\exists x.x^x < 5$	

(This fails because the solution $P : \lambda v.v^x < 5$ and $t : t$ would require $\lambda v.v^x < 5$ to be substituted into $\exists x.P(\!\lvert x \rvert\!)$ for P, which would bind the x in $\lambda v.v^x < 5$, resulting in an α conversion, yielding something like $\exists y.P(\!\lvert y \rvert\!)$, which is $\exists y.y^x < 5$, the correct conclusion, not matching $\exists x.x^x < 5$, the purported conclusion.)

Test 53.

Pattern:	**Instance:**	**Result:**
$P(\!\lvert t \rvert\!)$	$x \neq t$	$P : \lambda v.v \neq t$
$\exists x.P(\!\lvert x \rvert\!)$	$\exists y.y \neq t$	$t : x$
		$x : y$

Test 54.

Pattern:	**Instance:**	**Result:**
$P(\!\lvert t \rvert\!)$	$x \neq t$	no solutions
$\exists x.P(\!\lvert x \rvert\!)$	$\exists x.x \neq x$	

(Similar to last comment.)

Test 55.

Pattern:	**Instance:**	**Result:**
$P(\!\lvert t \rvert\!)$	$\forall t.t = t$	no solutions
$\exists x.P(\!\lvert x \rvert\!)$	$\exists x.\forall t.x = t$	

(This is actually a logical conclusion from the given first line, but it doesn't match this pattern.)

Induction on $\mathbb{N}$

Test 56.

Pattern:	**Instance:**	**Result:**
$P(\!\lvert 0 \rvert\!)$	$0 \geq 0$	$P : \lambda v.v \geq 0$
$\forall k.P(\!\lvert k \rvert\!) \Rightarrow P(\!\lvert k+1 \rvert\!)$	$\forall n.n \geq 0 \Rightarrow n+1 \geq 0$	$k : n$
$\forall n.P(\!\lvert n \rvert\!)$	$\forall n.n \geq 0$	$n : n$

Test 57.

Pattern:	**Instance:**	**Result:**
$P(\!\lvert 0 \rvert\!)$	$0 + 0 = 0$	$P : \lambda v.v + 0 = v$
$\forall k.P(\!\lvert k \rvert\!) \Rightarrow P(\!\lvert k+1 \rvert\!)$	$\forall m.m + 0 = m \Rightarrow$	$k : m$
$\forall n.P(\!\lvert n \rvert\!)$	$\quad (m+1) + 0 = m + 1$	$n : k$
	$\forall k.k + 0 = k$	

Test 58.

Pattern:	**Instance:**	**Result:**
$P(\!\lvert 0 \rvert\!)$	$P(0)$	$P : \lambda v.P(v)$
$\forall k.P(\!\lvert k \rvert\!) \Rightarrow P(\!\lvert k+1 \rvert\!)$	$\forall k.P(k) \Rightarrow P(k+1)$	$k : k$
$\forall n.P(\!\lvert n \rvert\!)$	$P(k+1)$	$n : n$

(In the preceding problem, the distinction between $P(\!|x|\!)$ and $P(x)$ is everything.)

Test 59.

Pattern:	**Instance:**	**Result:**
$P(\!|0|\!)$	$7 = 5$	$P : \lambda v.7 = 5$
$\forall k.P(\!|k|\!) \Rightarrow P(\!|k+1|\!)$	$\forall n.7 = 5 \Rightarrow 7 = 5$	$k : n$
$\forall n.P(\!|n|\!)$	$\forall n.7 = 5$	$n : n$

Test 60.

Pattern:	**Instance:**	**Result:**
$P(\!|0|\!)$	$R(n, 1)$	
$\forall k.P(\!|k|\!) \Rightarrow P(\!|k+1|\!)$	$\forall m.R(m, 1) \Rightarrow$ $R(m+1, 1)$	no solutions
$\forall n.P(\!|n|\!)$	$\forall m.R(m, 1)$	

Test 61.

Pattern:	**Instance:**	**Result:**
$P(\!|0|\!)$	$k \geq 0$	
$\forall k.P(\!|k|\!) \Rightarrow P(\!|k+1|\!)$	$\forall k.k \geq k \Rightarrow k \geq k+1$	no solutions
$\forall n.P(\!|n|\!)$	$\forall n.n \geq k$	

Test 62.

Pattern:	**Instance:**	**Result:**
$P(\!|0|\!)$	$n \geq 0$	
$\forall k.P(\!|k|\!) \Rightarrow P(\!|k+1|\!)$	$\forall k.n \geq k \Rightarrow n \geq k+1$	no solutions
$\forall n.P(\!|n|\!)$	$\forall n.n \geq n$	

Test 63.

Pattern:	**Instance:**	**Result:**
$P(\!|0|\!)$	$0 \geq 0$	
$\forall k.P(\!|k|\!) \Rightarrow P(\!|k+1|\!)$	$\forall n.n \geq 0 \Rightarrow n+1 \geq 0$	no solutions
$\forall n.P(\!|n|\!)$	$\forall n.0 \geq 0$	

Existential elimination rule

Test 64.

Pattern:	**Instance:**	**Result:**
$\exists x.P(\!\|x\|\!)$	$\exists x.x^2 = 1$	$x : x$
$\forall x.(P(\!\|x\|\!) \Rightarrow Q)$	$\forall x.x^2 = 1 \Rightarrow 1 \geq 0$	$P : \lambda v.v^2 = 1$
Q	$1 \geq 0$	$Q : 1 \geq 0$

Test 65.

Pattern:	**Instance:**	
$\exists x.P(\!\|x\|\!)$	$\exists x.x^2 = 1$	**Result:**
$\forall x.(P(\!\|x\|\!) \Rightarrow Q)$	$\forall x.x^2 = 1 \Rightarrow x \leq 1$	no solutions
Q	$x \leq 1$	

Test 66.

Pattern:	**Instance:**	
$\exists x.P(\!\|x\|\!)$	$\exists x.x > 0$	**Result:**
$\forall x.(P(\!\|x\|\!) \Rightarrow Q)$	$\forall x.x > 0 \Rightarrow -1 > 0$	no solutions
Q	$-1 > 0$	

Test 67.

Pattern:	**Instance:**	**Result:**
$\exists x.P(\!\|x\|\!)$	$\exists x.x > 0$	$x : x$
$\forall x.(P(\!\|x\|\!) \Rightarrow Q)$	$\forall x.x > 0 \Rightarrow -1 > 0$	$P : \lambda v.v > 0$
Q	$-1 > 0$	$Q : -1 > 0$

Test 68.

Pattern:	**Instance:**	
$\exists x.P(\!\|x\|\!)$	$\exists m.m > 0$	**Result:**
$\forall x.(P(\!\|x\|\!) \Rightarrow Q)$	$\forall n.n > 0 \Rightarrow -1 > 0$	no solutions
Q	$-1 > 0$	

Test 69.

Pattern:	**Instance:**	**Result:**
$\exists x.P(\!\lvert x \rvert\!)$	$\exists x.x > 0$	$x : x$
$\forall x.(P(\!\lvert x \rvert\!) \Rightarrow Q)$	$\forall x.x > 0 \Rightarrow -1 > 0$	$y : x$
Q	$-1 > 0$	$P : \lambda v.v > 0$
		$Q : -1 > 0$

Test 70.

Pattern:	**Instance:**	**Result:**
$\exists x.P(\!\lvert x \rvert\!)$	$\exists m.m > 0$	$x : m$
$\forall x.(P(\!\lvert x \rvert\!) \Rightarrow Q)$	$\forall n.n > 0 \Rightarrow -1 > 0$	$y : n$
Q	$-1 > 0$	$P : \lambda v.v > 0$
		$Q : -1 > 0$

Test 71.

Pattern:	**Instance:**	**Result:**
$\exists x.P(\!\lvert x \rvert\!)$	$\exists n.n < a$	
$\forall x.(P(\!\lvert x \rvert\!) \Rightarrow Q)$	$\forall a.a < a \Rightarrow a < a$	no solutions
Q	$a < a$	