Balancing a Drone Using Physics: An Overview

Overview of Control Flow

Inside the main loop, the following operations are executed in order:

1. The Arduino receives input from the gyro sensor.
2. It updates the values of key state variables, such as θ_x, ω_x, α_x, θ_y, ω_y, and α_y.
3. The function `MakeCorrection()` updates the forces each motor applies to the drone, denoted by $T_{FR}, T_{FL}, T_{BR}, T_{BL}$.

How `MakeCorrection()` Works

Before executing `MakeCorrection()`, the following state variables are available: θ_x, ω_x, α_x, θ_y, ω_y, α_y, and the current motor forces $T_{FR}, T_{FL}, T_{BR}, T_{BL}$.
To update these forces, we set our target values:

- Future motor forces: $T'_{FR}, T'_{FL}, T'_{BR}, T'_{BL}$
- Required angular acceleration: α'_x

Derivation of Correction Equations

The following equation needs to be satisfied:

$$I_x\alpha_x + b_x\omega_x + D_x\theta_x = 0 \tag{1}$$

Rewriting the expression, we have:

$$\alpha'_x = -\frac{b_x}{I_x}\omega_x - \frac{D_x}{I_x}\theta_x \tag{1}$$

Here, α'_x represents the angular acceleration we want to achieve.
Definitions Define the external angular acceleration:

$$\alpha_{ext} := \alpha_x - \alpha_M \tag{2}$$

where:

- α_x: measured angular acceleration
- α_M: angular acceleration due to motor forces

Then, we want:

$$\alpha_{ext} + \alpha'_M = \alpha'_x \tag{3}$$

where α'_M is the angular acceleration caused by the updated motor forces.

Combining Equations

Substitute into equation (3) using (1) and solve:

$$\alpha_x - \alpha_M + \alpha'_M = -\frac{b_x}{I_x}\omega_x - \frac{D_x}{I_x}\theta_x \tag{2}$$

$$I_x\alpha'_M = -I_x\alpha_x - b_x\omega_x - D_x\theta_x + I_x\alpha_M \tag{3}$$

$$T'_{FR} + T'_{FL} - T'_{BR} - T'_{BL} = c_1 \tag{A}$$

For the y-axis:

$$-T'_{FR} + T'_{FL} - T'_{BR} + T'_{BL} = c_2 \tag{B}$$

Thrust Requirement

$$T'_{FR} + T'_{FL} + T'_{BR} + T'_{BL} = c_3 \tag{C}$$

Angular Momentum Conservation For rotation around the z-axis:

$$\sqrt{T'_{FR}} - \sqrt{T'_{FL}} - \sqrt{T'_{BR}} + \sqrt{T'_{BL}} = c_4 \tag{D}$$

Solving the System of Equations

We solve the following system:

$$\begin{aligned} T'_{FR} + T'_{FL} - T'_{BR} - T'_{BL} &= c_1 \\ -T'_{FR} + T'_{FL} - T'_{BR} + T'_{BL} &= c_2 \\ T'_{FR} + T'_{FL} + T'_{BR} + T'_{BL} &= c_3 \\ \sqrt{T'_{FR}} - \sqrt{T'_{FL}} - \sqrt{T'_{BR}} + \sqrt{T'_{BL}} &= c_4 \end{aligned}$$

By substitution and algebraic simplification, we reduce this system to a single-variable function:

$$f(x) = \sqrt{\frac{c_1 + c_3}{2} - x} - \sqrt{x} - \sqrt{x - \frac{c_1 + c_2}{2}} + \sqrt{\frac{c_2 + c_3}{2} - x} - c_4 \tag{4}$$

This function can be solved using binary search within the domain:

$$D_f = \left[\max\left(0, \frac{c_1 + c_2}{2}\right), \min\left(\frac{c_1 + c_3}{2}, \frac{c_2 + c_3}{2}\right)\right] \tag{5}$$

Conclusion

By solving this system of equations, we determine the updated forces $T'_{FR}, T'_{FL}, T'_{BR}$, and T'_{BL} that must be applied to each motor. This ensures that the drone maintains stability by approximating the damped oscillation behavior we require. Specifically, by controlling the forces in this manner, we aim to satisfy the relationship

$$I_x \alpha_x + b_x \omega_x + D_x \theta_x = 0$$

along the x- and y-axes as closely as possible, thereby maintaining stability and reducing oscillatory motion.
This control approach is conceptually similar to the Proportional-Integral-Derivative (PID) controllers commonly used in modern drones. In fact, the constants I, D, and b in our system directly correlate to the parameters in PID control:

- I corresponds to the integral term (I) in PID, accounting for accumulated angular displacement,
- b represents a damping term, similar to the derivative (D) in PID, counteracting angular velocity,
- D functions analogously to the proportional (P) term in PID, balancing angular position.

Thus, this method provides an approximate, physics-based approach that mirrors the dynamics achieved by PID controllers in drone stabilization. By applying these calculated motor forces, we achieve similar effects as modern PID controllers, allowing the drone to respond to external disturbances smoothly and maintain balanced flight.

THE END