

# CS 251, Spring 2023, Assignment 1

Due Friday, June 2, 10:00 PM

Late submission accepted until Monday, June 5, 10:00 PM with no penalty

**You are required to read, complete and sign, and submit (as well as follow) the following statement of Academic Integrity.**

Statement of Academic Integrity for CS 251 Spring 2023, **Assignment 1**

I declare the following statements to be true:

- I have not used any unauthorized aids.
- I recognize that while I can discuss the questions in this assignment on Piazza and other forums with the instructors and with other students in the class, the write up that I am submitting is my own.
- I am aware that misconduct related to course work can result in significant penalties, including failing the course and suspension (this is covered in Policy 71:)

<https://uwaterloo.ca/secretariat/policies-procedures-guidelines/policy-71>

**Student Name:**

**UW ID#:**

**Signature:**

**Date:**

The purpose of this assignment is to practice designing and implementing combinational digital logic circuits.

Coverage material for this assignment can be found on the course website as:

- Laws of Boolean Algebra Reference Sheet on the Assignment Information webpage, and
- On the Lecture Notes webpage in the
  - May 16th Lecture on Introduction to Digital Logic Design,
  - May 18th Lecture on Digital Logic Design - Combinational Logic, and
  - May 25th Lecture on Digital Logic Design - Gates, Decoders, Multiplexors and Transistors

For this assignment, make note of the following details:

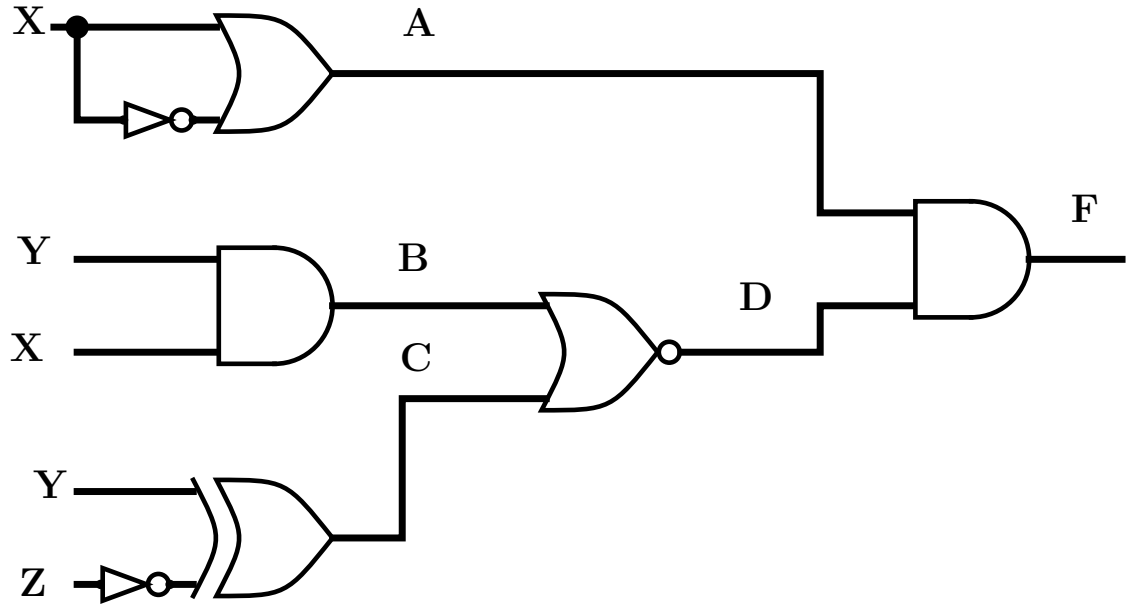
- Any diagrams that are part of your solution must be drawn neatly, using rectilinear lines and clearly labelled inputs and outputs.
- You may assume that for any valid input you also have its inverse, e.g. for input  $A$  you also have  $\overline{A}$  without using an inverter (NOT) gate explicitly.

We have an [A1 Official Post](#) on piazza. In this post, we will include all A1 assignment related information such as

- updates/corrections made to the assignment, if any
- remark request due date
- FAQs

1. (10 points)

- (a) (8 points) Complete the truth table for the combinational logic circuit given below with output  $F$  and intermediate outputs  $A$ ,  $B$ ,  $C$  and  $D$ .



$X$	$Y$	$Z$	$A$	$B$	$C$	$D$	$F$
0	0	0					
0	0	1					
0	1	0					
0	1	1					
1	0	0					
1	0	1					
1	1	0					
1	1	1					

- (b) (2 point) Give the **simplified** Boolean equation for  $F$  in terms of  $X$ ,  $Y$ , and  $Z$ .

2. (22 points) Consider the truth table given below, with inputs  $A, B$ , and  $C$  and output function  $F$ .

$A$	$B$	$C$	$F$
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

- (a) (5 points) Write the *unreduced* Boolean equation in **sum-of-products** form for the output function  $F$ .

- (b) (3 points) Write the *unreduced* Boolean equation in **product-of-sums** form for the output function  $F$ .

(c) (2 points) Write the **reduced** Boolean equation for the output function  $F$ . Show your steps for simplification for full points.

(d) (4 points) Implement the output function  $F$  using only AND and, or OR gate(s).

(e) (3 points) Implement the output function  $F$  using a minimum number of NAND gate(s).

(f) (5 points) Implement the output function  $F$  using only a 4:1 MUX.

3. (10 points) Assume that a combinational logic circuit has two inputs  $S$  and  $X$  and an output function  $F$ . The input  $X$  consists of three bits,  $X0, X1, X2$ , where  $X0$  and  $X2$  are the least and most significant bits, respectively. For example, for input  $X = 011$ ,  $X0 = 1$ ,  $X1 = 1$ , and  $X2 = 0$ . The output function  $F$  also consists of three bits  $F0, F1$ , and  $F2$ , where  $F0$  and  $F2$  are the least and most significant bits, respectively.

- (a) (6 points) Construct the truth table for the output function  $F$ , which is the result of shifting the input  $X$ , left when  $S = 0$ . When  $S = 1$ ,  $F$  is the result of shifting the input  $X$  right.

We have provided the truth table with two entries as an example.

When  $S = 0$ , the operation to perform is left shift. For input  $X = 110$ , the output  $F = 100$ . Note that the bits are shifted left by one position and the most significant bit is dropped and a zero is added in the least significant bit position.

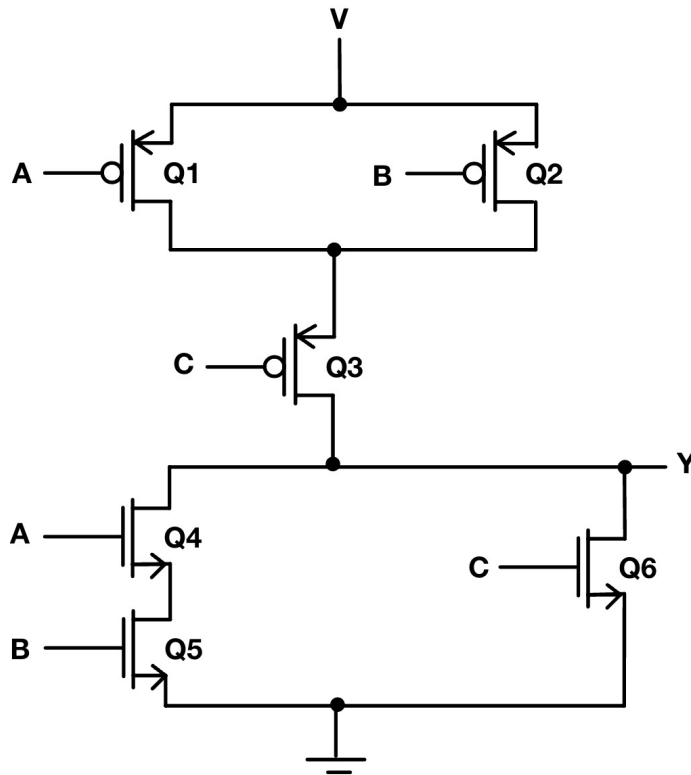
When  $S = 1$ , the operation to perform is right shift for input  $X = 110$ , the output  $F = 011$ . Note that the bits are shifted right by one position and the least significant bit is dropped and a zero is added in the most significant bit position.

$S$	$X2$	$X1$	$X0$	$F2$	$F1$	$F0$
0	0	0	0			
0	0	0	1			
0	0	1	0			
0	0	1	1			
0	1	0	0			
0	1	0	1			
0	1	1	0	1	0	0
0	1	1	1			
1	0	0	0			
1	0	0	1			
1	0	1	0			
1	0	1	1			
1	1	0	0			
1	1	0	1			
1	1	1	0	0	1	1
1	1	1	1			

- (b) (4 points) Implement the output function  $F$  with respect to the individual output bits  $F0, F1$  and  $F2$  using a Programming Logic Array (PLA).



4. (8 points) Consider the transistor circuit below, where the inputs  $A$ ,  $B$  and  $C$  are supplied as voltage at the gate of the transistors labelled  $Q1$  to  $Q6$  as illustrated in the figure. The output from the circuit is  $Y$ .



Complete the table below for the output  $Y$  and the internal resistance of each transistor  $Q1$  to  $Q6$  for the inputs  $A$ ,  $B$  and  $C$ . The resistance of the transistors should be High or Low (or 'H' or 'L'). The output  $Y$  can be 0, 1 or high-impedance  $Z$ . You may assume all resistances are accurate when determining the final output  $Y$ .

$A$	$B$	$C$	$Q1$	$Q2$	$Q3$	$Q4$	$Q5$	$Q6$	$Y$
0	0	0							
0	0	1							
0	1	0							
0	1	1							
1	0	0							
1	0	1							
1	1	0							
1	1	1							